EuPhO 2018
Problema 1 – “Three Balls”

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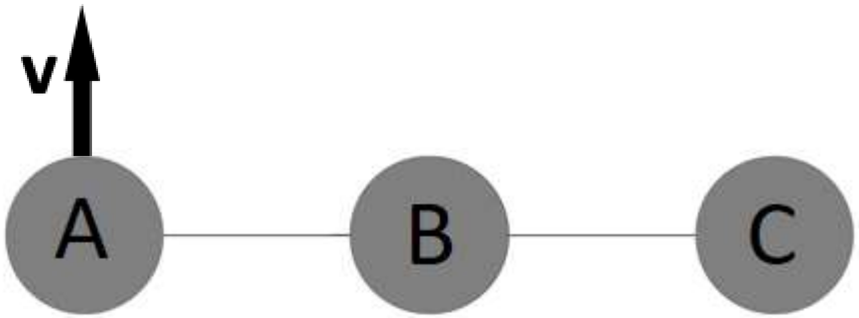
- **Enunciado**

"Three small identical balls (denoted as A, B, and C) of mass m each are connected with two massless rods of length ℓ so that one of the rods connects the balls A and B, and the other rod connects the balls B and C. The connection at the ball B is hinged, and the angle between the rods can change effortlessly. The system rests in weightlessness so that all the balls lie on one line. The ball A is given instantaneously a velocity perpendicular to the rods.

Find the minimal distance d between the balls A and C during the subsequent motion of the system. Any friction is to be neglected."



- Conservação do momento linear:

$$p = mv \qquad m_{tot} = 3m$$

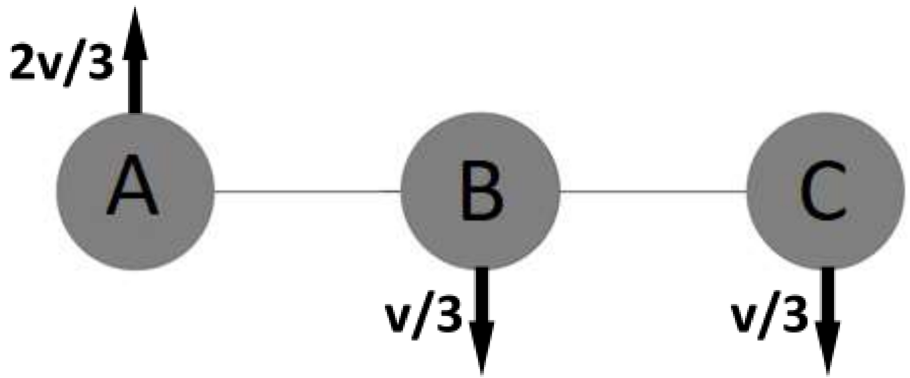
$$v_{cm} = \frac{mv}{3m} = \frac{v}{3}$$

- Energia total:

$$E = \frac{mv^2}{2} \times \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = \frac{mv^2}{3}$$

- Momento angular:

$$L = r \times p = m\frac{2v}{3}l + m\frac{v}{3}l = mvl$$



Referencial do CM

Definindo o ângulo entre as barras: $\theta = \sphericalangle ABC$
d_{AC} é mínimo quando $\dot{d}_{AC} = 0 \Rightarrow \ \theta = cte$

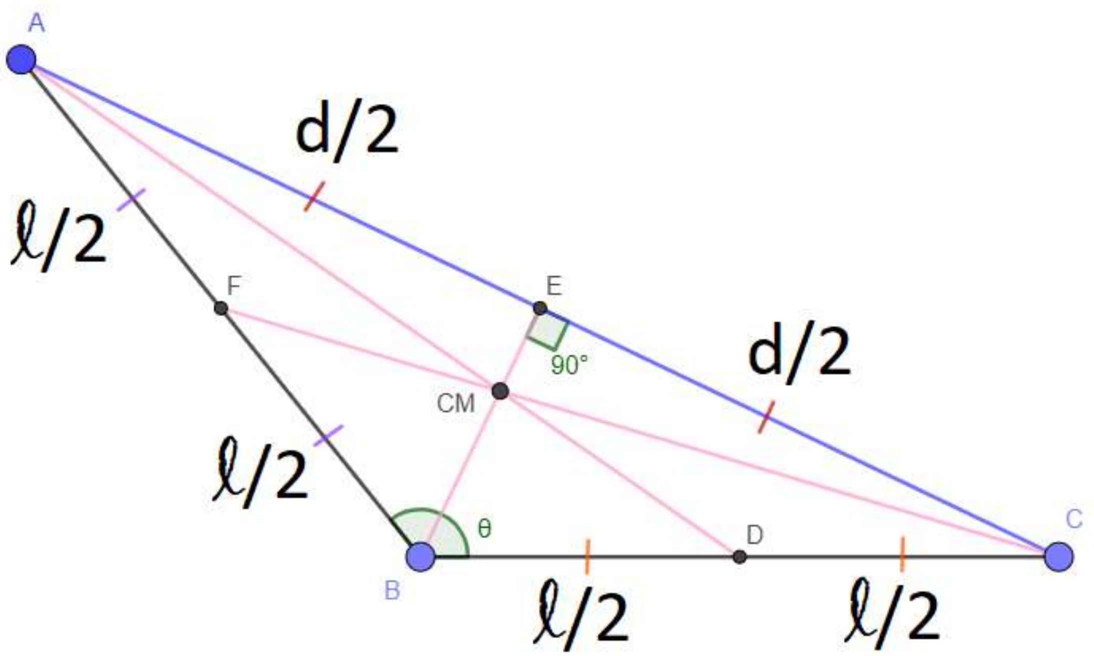
- Lei dos cossenos:

$$d_{AC} = \sqrt{l^2 + l^2 - 2l^2 \cos\theta} = l\sqrt{2(1-\cos\theta)} \Rightarrow \dot{d}_{AC} = \frac{l}{2}\sqrt{\frac{2}{1-\cos\theta}}\sin(\theta)\dot{\theta} = 0 \Rightarrow$$
$$\dot{\theta} = 0 \Rightarrow \ \theta = cte$$

$\Rightarrow$ O sistema comporta-se como um corpo rígido no instante em que a distância é mínima.

- Momento de inércia (a partir da energia e do momento angular):

$$E = \frac{I\omega^2}{2} = \frac{L^2}{2I} \Rightarrow I = \frac{L^2}{2E} = \frac{(mvl)^2}{2\frac{mv^2}{3}} = \frac{3}{2}ml^2 \ \textbf{(I)}$$



- Distâncias até o CM:

$$d_{B\rightarrow CM} = \frac{2}{3}\sqrt{l^2 - \frac{d^2}{4}} \qquad d_{A\rightarrow CM} = d_{C\rightarrow CM} = \sqrt{\frac{1}{9}l^2 + \frac{2}{9}d^2}$$

- Momento de inércia (geometricamente):

$$I = \sum mr^2 = m\left(\frac{4}{9}\left(l^2 - \frac{d^2}{4}\right) + 2\left(\frac{l^2}{9} + \frac{2d^2}{9}\right)\right) = \frac{m}{3}(2l^2 + d^2) \quad \textbf{(II)}$$

- Comparando os momentos de inércia (igualando as expressões $\mathbf{I}$ e $\mathbf{II}$):

$$\frac{3}{2}ml^2 = \frac{m}{3}(2l^2 + d^2) \Rightarrow \boxed{\boldsymbol{d = l\sqrt{\frac{5}{2}}}}$$