O QUE É A ENTROPIA?

Constantino Tsallis

Centro Brasileiro de Pesquisas Físicas e Instituto Nacional de Ciência e Tecnologia de Sistemas Complexos, Rio de Janeiro, **Brasil**

Santa Fe Institute, New Mexico, **USA**

Complexity Science Hub Vienna, Austria

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CAMELS

I know that a certain percentage of the camels has blue eyes, the rest has dark eyes, but I am not informed which is which.
Consequently I have some knowledge X.

I also know that a certain percentage (same value as before!) has dark neck, the rest has light neck, but I am not informed which is which.
Consequently I have some knowledge X.

Question: How much is my total knowledge?
Desired answer: 2X

DICES

ENTROPY = IGNORANCE OR LACK OF KNOWLEDGE

Whoever knows that it came a 6: knows everything, i.e., ignores nothing

→ **ENTROPY = 0 , say 0%**

Whoever does not know what came out: knows nothing, i.e., ignores everything

→ **ENTROPY = maximum, say 100%**

Whoever only knows that came out an even number: knows something, i.e., ignores something

→ **ENTROPY S(1) = Y = ?**

Whoever only knows that came out an **even** number in each of the 2 dices knows something, i.e., ignores something, then

TOTAL ENTROPY S(2) = Y+Y = 2Y

If we had N dices, and knew that in each of them came out an **even** number, then

TOTAL ENTROPY S(N) = NY

i.e. S(N) = N S(1) (entropic extensivity)

Is there a function of probabilities (entropic functional) which is generically ADDITIVE for independent systems?

Yes, the Boltzmann-Gibbs-von Neumann-Shannon entropy!

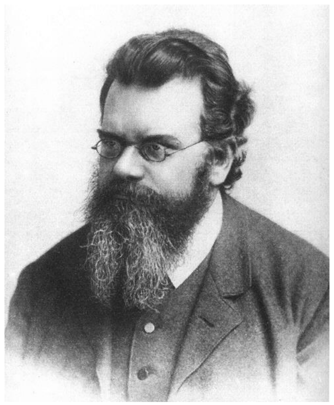
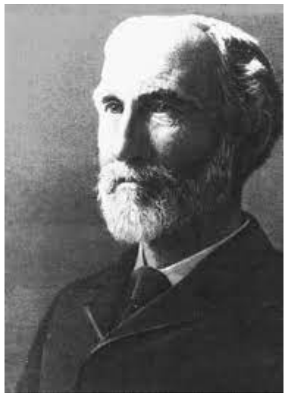
$$S_{BG} = -k \sum_{i=1}^{W} p_i \ln p_i \quad \text{with} \quad \sum_{i=1}^{W} p_i = 1$$

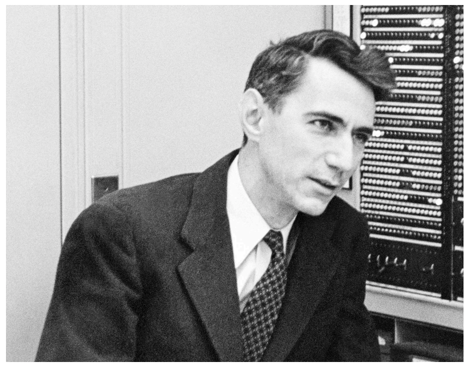
Proof:

$$S_{BG}(A) = -k \sum_{i=1}^{W_A} p_i^A \ln p_i^A \quad \text{and} \quad S_{BG}(B) = -k \sum_{j=1}^{W_B} p_j^B \ln p_j^B$$

Assuming independence, i.e., $p_{ij}^{A+B} = p_i^A p_j^B$, we verify

$$S_{BG}(A+B) = S_{BG}(A) + S_{BG}(B)$$



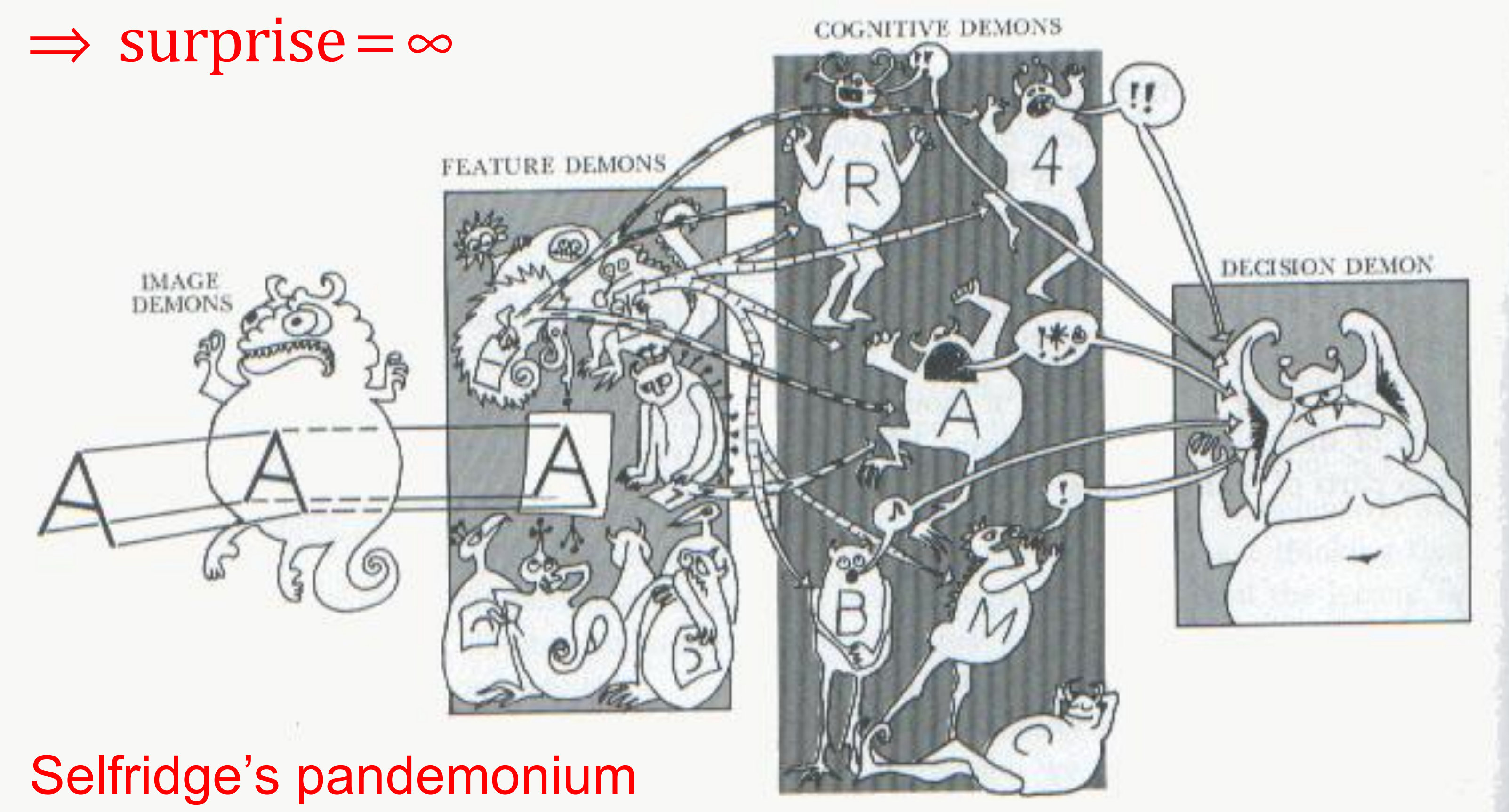
ENTROPY AND SURPRISE

$$S_{BG} = k\sum_{i=1}^{W} p_i \ln\frac{1}{p_i} = k\left\langle \ln\frac{1}{p_i} \right\rangle$$

$\ln\frac{1}{p_i} \equiv$ surprise (Watanabe 1969) or unexpectedness (Barlow 1990)

hence $\quad p_i = 1 \Rightarrow \text{surprise} = 0$

$\qquad\qquad p_i = 0 \Rightarrow \text{surprise} = \infty$



Selfridge's pandemonium

ENTROPY IN THERMODYNAMICS

$$G = U - TS + pV$$ (Legendre structure)

$$dU = TdS - pdV = \delta Q - \delta W$$

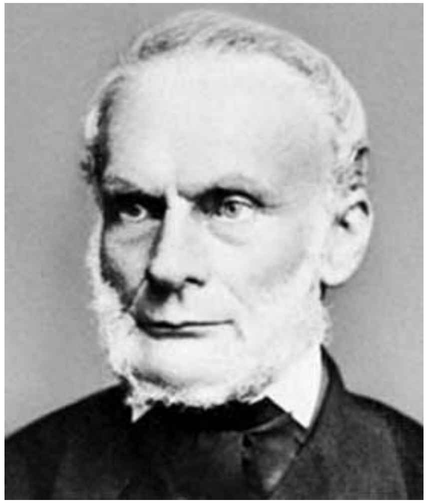
$$\Delta U = Q - W$$

$\Delta U \equiv$ variation of internal energy of the system (no mass transfer)

$Q \equiv$ heat gained by the system (disorganized energy)

$W \equiv$ work done by the system on its surroundings (organized energy)

$$dS_{syst} \geq \frac{\delta Q}{T_{surr}}$$ (= if the process is reversible)



MEPHISTOPHELES:

Denn eben wo Begriffe fehlen,

Da stellt ein Wort zur rechten Zeit sich ein.

Wolfgang von Goethe

[*Faust I, Vers 1995, Schuelerszene (1808)*]

For at the point where concepts fail,

At the right time a word is thrust in there.

Saint Augustine

What is time?

If nobody asks, I know.

If someone asks
and I want to explain,
I no longer know.

Enrico FERMI *Thermodynamics* (Dover, 1936)

The entropy of a system composed of several parts is very often equal to the sum of the entropies of all the parts. This is true if the energy of the system is the sum of the energies of all the parts and if the work performed by the system during a transformation is equal to the sum of the amounts of work performed by all the parts. Notice that these conditions are not quite obvious and that in some cases they may not be fulfilled. Thus, for example, in the case of a system composed of two homogeneous substances, it will be possible to express the energy as the sum of the energies of the two substances only if we can neglect the surface energy of the two substances where they are in contact. The surface energy can generally be neglected only if the two substances are not very finely subdivided; otherwise, it can play a considerable role.

ENTROPIC FUNCTIONALS

	$p_i = \frac{1}{W} \quad (\forall i)$ equiprobability	$\forall p_i \ (0 \le p_i \le 1)$ $\left(\sum_{i=1}^{W} p_i = 1\right)$
BG entropy ($q = 1$)	$k \ln W$	$-k \sum_{i=1}^{W} p_i \ln p_i$
Entropy *Sq* (q *real*)	$k \frac{W^{1-q} - 1}{1-q}$	$k \frac{1 - \sum_{i=1}^{W} p_i^q}{q-1}$

additive

Concave

Extensive

Lesche-stable

Finite entropy production per unit time

Pesin-like identity (with largest entropy production)

Composable (**unique** trace form; Enciso-Tempesta)

Topsoe-factorizable (unique)

Amari-Ohara-Matsuzoe conformally invariant geometry (unique)

Biro-Barnafoldi-Van thermostat universal independence (unique)

nonadditive (if $q \neq 1$)

Possible generalization of Boltzmann-Gibbs statistical mechanics

C.T., J. Stat. Phys. **52**, 479 (1988)

DEFINITIONS: *q-logarithm*: $\ln_q x \equiv \dfrac{x^{1-q}-1}{1-q} \quad (x>0;\ \ln_1 x = \ln x)$

q-exponential: $e_q^x \equiv [1+(1-q)\,x]^{\frac{1}{1-q}} \quad (e_1^x = e^x)$

Hence, the entropies can be rewritten:

	equal probabilities	*generic probabilities*
BG entropy $(q=1)$	$k \ln W$	$k \sum_{i=1}^{W} p_i \ln \frac{1}{p_i}$
entropy S_q $(q \in R)$	$k \ln_q W$	$k \sum_{i=1}^{W} p_i \ln_q \frac{1}{p_i}$

TYPICAL SIMPLE SYSTEMS:

$$W(N) \propto \mu^N \quad (\mu > 1)$$

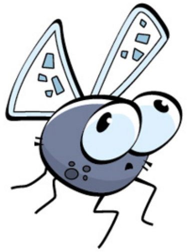
Short-range space-time correlations

Markovian processes (short memory), Additive noise

Strong chaos (positive maximal Lyapunov exponent), **Ergodic**, Riemannian geometry

Short-range many-body interactions, weakly quantum-entangled subsystems

Linear and homogeneous Fokker-Planck equations, Gaussians



→ Boltzmann-Gibbs entropy (additive)

→ Exponential dependences (Boltzmann-Gibbs weight, ...)

TYPICAL COMPLEX SYSTEMS:

$$\text{e.g.,} \quad W(N) \propto N^\rho \quad (\rho > 0)$$

Long-range space-time correlations

Non-Markovian processes (long memory), Additive and multiplicative noises

Weak chaos (zero maximal Lyapunov exponent), **Nonergodic**, Multifractal geometry

Long-range many-body interactions, strongly quantum-entangled sybsystems

Nonlinear and/or inhomogeneous Fokker-Planck equations, *q*-Gaussian

→ Entropy S_q (nonadditive)

→ *q*-exponential dependences (asymptotic power-laws)

ADDITIVITY: O. Penrose, *Foundations of Statistical Mechanics: A Deductive Treatment* (Pergamon, Oxford, 1970), page 167

An entropy is additive if, for any two probabilistically independent systems A and B,

$$S(A+B) = S(A) + S(B)$$

Therefore, since

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\,\frac{S_q(A)}{k}\,\frac{S_q(B)}{k}$$

S_{BG} and S_q^{Renyi} $(\forall q)$ are additive, and S_q $(\forall q \neq 1)$ is nonadditive .

EXTENSIVITY:

Consider a system $\Sigma \equiv A_1 + A_2 + \ldots + A_N$ made of N (not necessarily independent) identical elements or subsystems A_1 and A_2, ..., A_N.

An entropy is extensive if

$$0 < \lim_{N\to\infty} \frac{S(N)}{N} < \infty \;,\; i.e., \;\; S(N) \propto N \;\; (N \to \infty)$$

EXTENSIVITY OF THE ENTROPY $(N \to \infty)$

$W \equiv$ total number of possibilities with nonzero probability, assumed to be equally probable

If $W(N) \sim \mu^N \quad (\mu > 1)$

$$\Rightarrow \; S_{BG}(N) = k_B \ln W(N) \propto N \qquad \text{OK!}$$

If $W(N) \sim N^\rho \quad (\rho > 0)$

$$\Rightarrow \; S_q(N) = k_B \ln_q W(N) \propto [W(N)]^{1-q} \propto N^{\rho(1-q)}$$

$$\Rightarrow \; S_{q=1-1/\rho}(N) \propto N \qquad \text{OK!}$$

If $W(N) \sim \nu^{N^\gamma} \quad (\nu > 1;\; 0 < \gamma < 1)$

$$\Rightarrow \; S_\delta(N) = k_B \left[\ln W(N)\right]^\delta \propto N^{\gamma\,\delta}$$

$$\Rightarrow \; S_{\delta=1/\gamma}(N) \propto N \qquad \text{OK!}$$

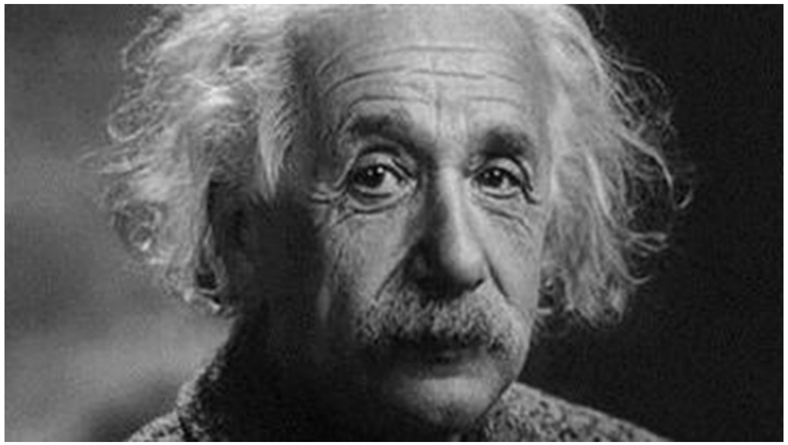
IMPORTANT: $\boxed{\mu^N >> \nu^{N^\gamma} >> N^\rho \quad \text{if} \quad N >> 1}$

All happy families are alike; each unhappy family is unhappy in its own way.

Leo Tolstoy (*Anna Karenina*, 1875-1877)

SYSTEMS $W(N)$ $(equiprobable)$	ENTROPY S_{BG} (ADDITIVE)	ENTROPY S_q $(q \neq 1)$ (NONADDITIVE)	ENTROPY S_δ $(\delta \neq 1)$ (NONADDITIVE)
$e.g.,\ \mu^N$ $(\mu > 1)$	EXTENSIVE	NONEXTENSIVE	NONEXTENSIVE
$e.g.,\ N^\rho$ $(\rho > 0)$	NONEXTENSIVE	EXTENSIVE $(q = 1 - 1/\rho)$	NONEXTENSIVE
$e.g.,\ \nu^{N^\gamma}$ $(\nu > 1;$ $0 < \gamma < 1)$	NONEXTENSIVE	NONEXTENSIVE	EXTENSIVE $(\delta = 1/\gamma)$

A theory is the more impressive the greater the simplicity of its premises is, the more different kinds of things it relates, and the more extended is its area of applicability. Therefore the deep impression that classical thermodynamics made upon me. It is the only physical theory of universal content concerning which I am convinced that, within the framework of applicability of its basic concepts, it will never be overthrown. Albert Einstein (1949)



King Thutmosis I
18th Dynasty
circa 1500 BC

COMPOSITION OF VELOCITIES OF INERTIAL SYSTEMS (d=1)

$$v_{13} = v_{12} + v_{23} \qquad \text{(Galileo)}$$

$$v_{13} = \frac{v_{12} + v_{23}}{1 + \frac{v_{12}}{c}\frac{v_{13}}{c}} \qquad \text{(Einstein)}$$

Newton mechanics:
It satisfies Galilean additivity **but** violates Lorentz invariance (hence mechanics can not be unified with Maxwell electromagnetism)

Einstein mechanics (Special relativity):
It satisfies Lorentz invariance (hence mechanics is unified with Maxwell electromagnetism) **but** violates Galilean additivity

Question: which is physically more fundamental, the additive composition of velocities **or** the unification of mechanics and electromagnetism?

Euclid set of axioms *including his celebrated 5th postulate* yields the magnificent Euclidean geometry

Violation of the 5th postulate yields Riemannian geometries

- Carl Friedrich Gauss 1813
- Ferdinand Karl Schweikart 1818
- János Bolyai 1830
- Nikolai Ivanovich Lobachevsky 1830
- Bernhard Riemann 1854

If we stubbornly insisted that the 5th postulate was not only proposed by Euclid but was mandated by *God*, then General Relativity would not exist! ☹

PHYSICAL REVIEW E **78**, 021102 (2008)

Nonadditive entropy reconciles the area law in quantum systems with classical thermodynamics

Filippo Caruso[1] and Constantino Tsallis[2,3]

[1]*NEST CNR-INFM and Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126 Pisa, Italy*

[2]*Centro Brasileiro de Pesquisas Fisicas, Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil*

[3]*Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA*

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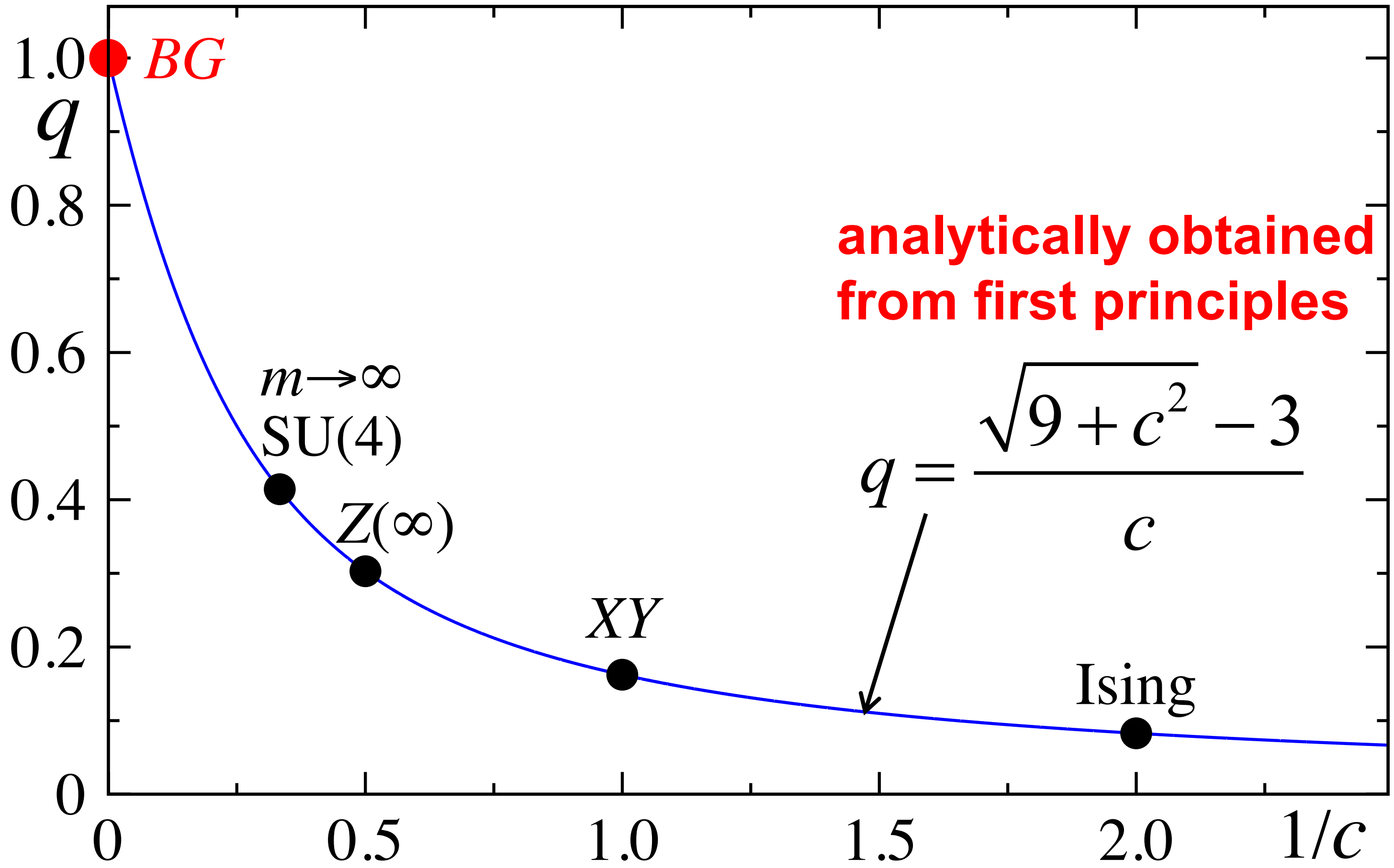
The Boltzmann–Gibbs–von Neumann entropy of a large part (of linear size L) of some (much larger) d-dimensional quantum systems follows the so-called area law (as for black holes), i.e., it is proportional to L^{d-1}. Here we show, for $d=1,2$, that the (nonadditive) entropy S_q satisfies, for a special value of $q \neq 1$, the classical thermodynamical prescription for the entropy to be extensive, i.e., $S_q \propto L^d$. Therefore, we reconcile with classical thermodynamics the area law widespread in quantum systems. Recently, a similar behavior was exhibited in mathematical models with scale-invariant correlations [C. Tsallis, M. Gell-Mann, and Y. Sato, Proc. Natl. Acad. Sci. U.S.A. **102** 15377 (2005)]. Finally, we find that the system critical features are marked by a maximum of the special entropic index q.

Block entropy for the *d=1+1* model, with central charge *c*, at its quantum phase transition at *T=0* and critical transverse "magnetic" field



Self-dual $Z(n)$ magnet $(n = 1,2,\dots)$ [FC Alcaraz, JPA 20 (1987) 2511]

$$\to c = \frac{2(n-1)}{n+2} \in [0,2]$$

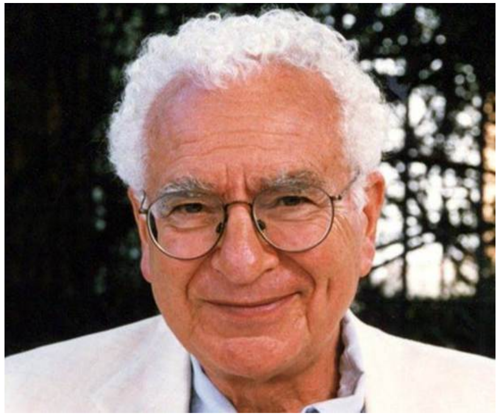
$SU(n)$ magnets $(n = 1,2,\dots;\ m = 2,3,\dots)$ [FC Alcaraz and MJ Martins, JPA 23 (1990) L1079]

$$\to c = (n-1)\left[1 - \frac{n(n+1)}{(m+n-2)(m+n-1)}\right] \in [0, n-1]$$

Milan j. math. 76 (2008), 307–328

1424-9286/010307-22, *published online* 14.3.2008
DOI 10.1007/s00032-008-0087-y

Milan Journal of Mathematics



M. Gell-Mann

On a q-Central Limit Theorem Consistent with Nonextensive Statistical Mechanics

Sabir Umarov, Constantino Tsallis and Stanly Steinberg

JOURNAL OF MATHEMATICAL PHYSICS **51**, 033502 (2010)

Generalization of symmetric α-stable Lévy distributions for $q>1$

Sabir Umarov,[1,a)] Constantino Tsallis,[2,3,b)] Murray Gell-Mann,[3,c)] and Stanly Steinberg[4,d)]
[1]*Department of Mathematics, Tufts University, Medford, Massachusetts 02155, USA*
[2]*Centro Brasileiro de Pesquisas Fisicas and National Institute of Science and Technology for Complex Systems, Rua Dr. Xavier Sigaud 150, 22290-180 Rio de Janeiro, Brazil*
[3]*Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501, USA*
[4]*Department of Mathematics and Statistics, University of New Mexico, Albuquerque, New Mexico 87131, USA*

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See also: H.J. Hilhorst, JSTAT P10023 (2010)
M. Jauregui and C. T., Phys Lett A **375**, 2085 (2011)
M. Jauregui, C. T. and E.M.F. Curado, JSTAT P10016 (2011)
A. Plastino and M.C. Rocca, Physica A and Milan J Math (2012)
A. Plastino and M.C. Rocca, Physica A **392**, 3952 (2013)
S. Umarov and C. T., J Phys A **49**, 415204 (2016)

www.nature.com/scientificreports

SCIENTIFIC REPORTS

OPEN

The standard map: From Boltzmann-Gibbs statistics to Tsallis statistics

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Ugur Tirnakli[1,*] & Ernesto P. Borges[2,3,*]

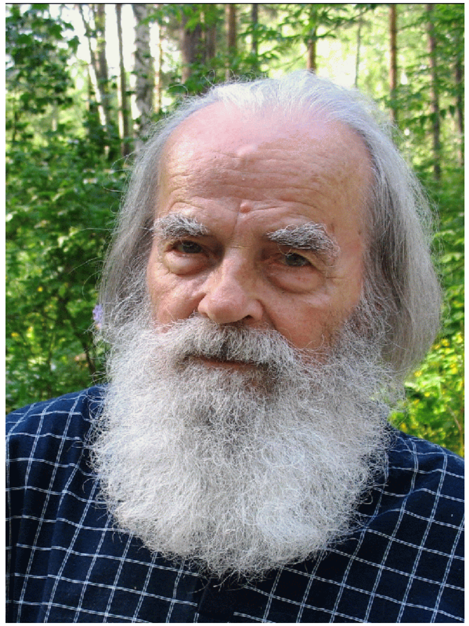
As well known, Boltzmann-Gibbs statistics is the correct way of thermostatistically approaching ergodic systems. On the other hand, nontrivial ergodicity breakdown and strong correlations typically drag the system into out-of-equilibrium states where Boltzmann-Gibbs statistics fails. For a wide class of such systems, it has been shown in recent years that the correct approach is to use Tsallis statistics instead. Here we show how the dynamics of the paradigmatic conservative (area-preserving) standard map exhibits, in an exceptionally clear manner, the crossing from one statistics to the other. Our results unambiguously illustrate the domains of validity of both Boltzmann-Gibbs and Tsallis statistical distributions. Since various important physical systems from particle confinement in magnetic traps to autoionization of molecular Rydberg states, through particle dynamics in accelerators and comet dynamics, can be reduced to the standard map, our results are expected to enlighten and enable an improved interpretation of diverse experimental and observational results.

STANDARD MAP (Chirikov 1969)

$$p_{i+1} = p_i - K \sin x_i \quad (\mathrm{mod}\ 2\pi)$$

$$x_{i+1} = x_i + p_{i+1} \quad (\mathrm{mod}\ 2\pi)$$

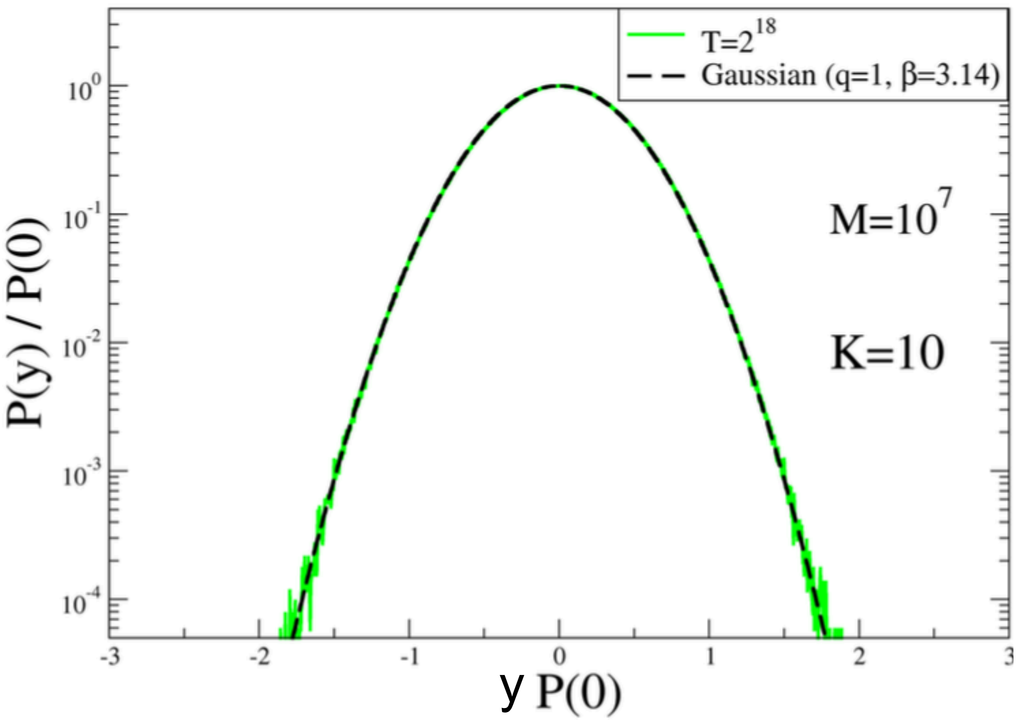
$$(i = 0,1,2,\ldots)$$

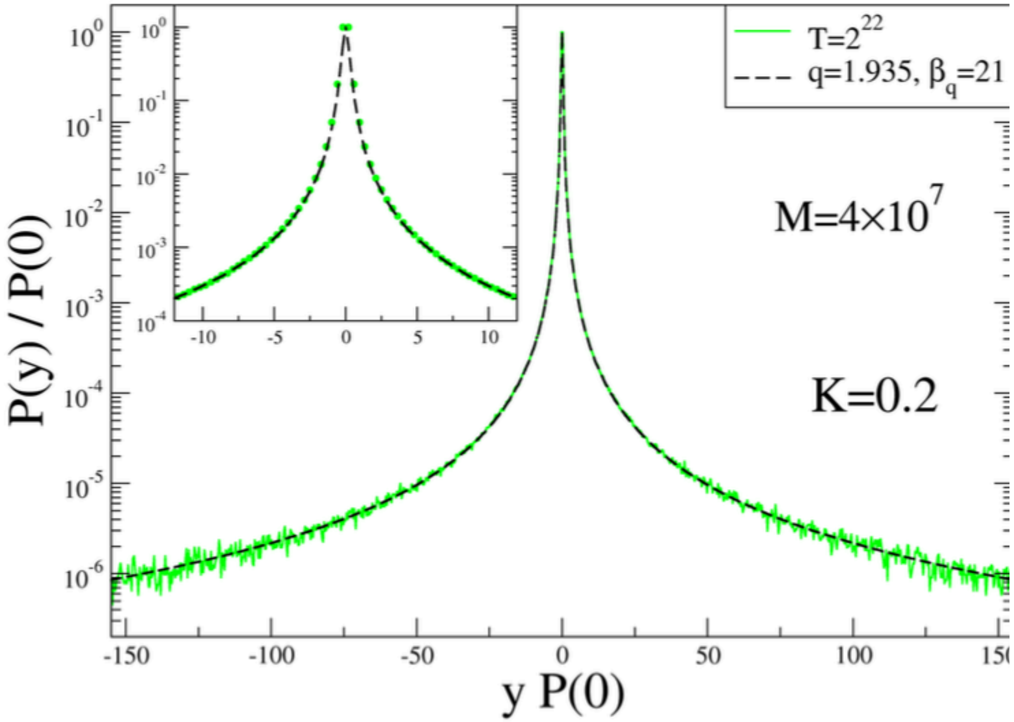


(1928-2008)

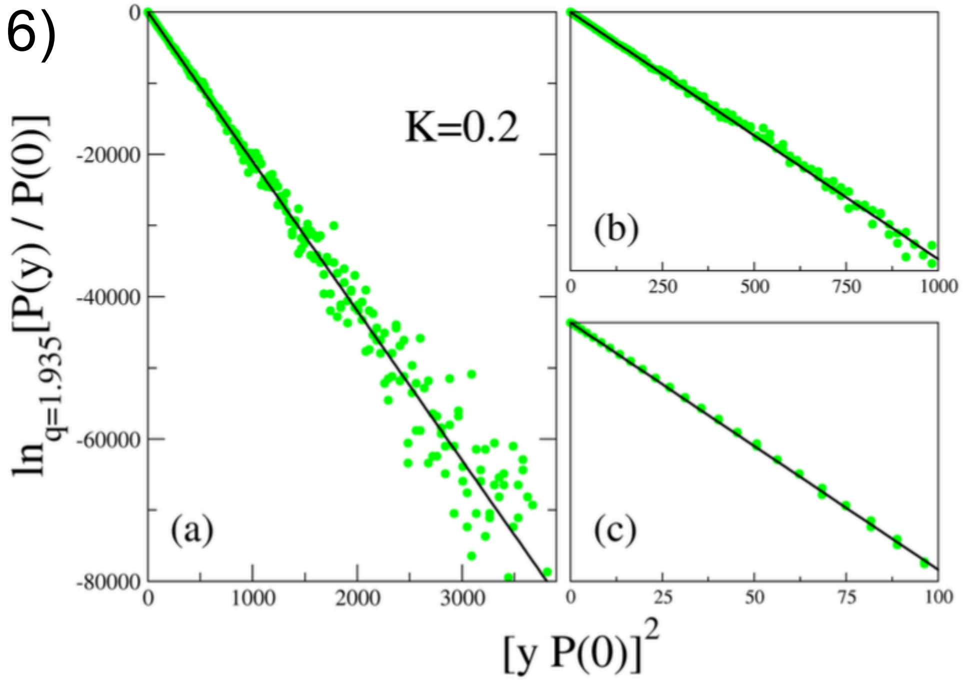
(area-preserving)

Particle confinement in magnetic traps,
particle dynamics in accelerators,
comet dynamics,
ionization of Rydberg atoms,
electron magneto-transport





Tirnakli and Borges
Nature / Scientific Reports **6**, 23644 (2016)



Recent analytic result:

$q = 2$

Bountis, Veerman and Vivaldi
Phys. Lett. A **384**, 126659 (2020)

J.W. GIBBS

Elementary Principles in Statistical Mechanics - Developed with Especial Reference to the Rational Foundation of Thermodynamics

C. Scribner's Sons, New York, 1902; Yale University Press, New Haven, (1981), page 35

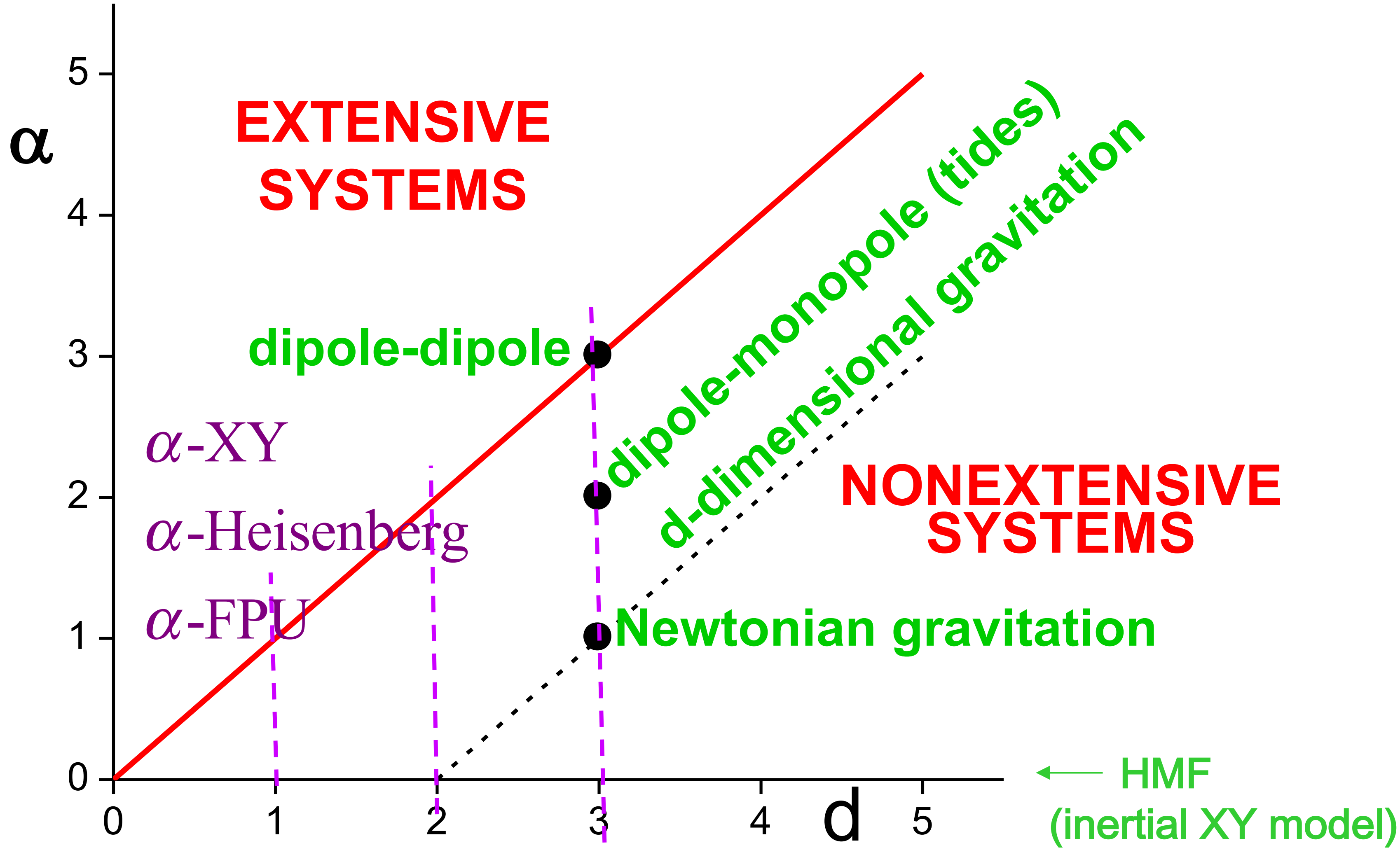
In treating of the canonical distribution, we shall always suppose the multiple integral in equation (92) [the partition function, as we call it nowadays] *to have a finite valued, as otherwise the coefficient of probability vanishes, and the law of distribution becomes illusory. This will exclude certain cases, but not such apparently, as will affect the value of our results with respect to their bearing on thermodynamics. It will exclude, for instance, cases in which the system or parts of it can be distributed in unlimited space […]. It also excludes many cases in which the energy can decrease without limit, as when the system contains material points which attract one another inversely as the squares of their distances. […]. For the purposes of a general discussion, it is sufficient to call attention to the assumption implicitly involved in the formula (92).*

CLASSICAL LONG-RANGE-INTERACTING MANY-BODY HAMILTONIAN SYSTEMS

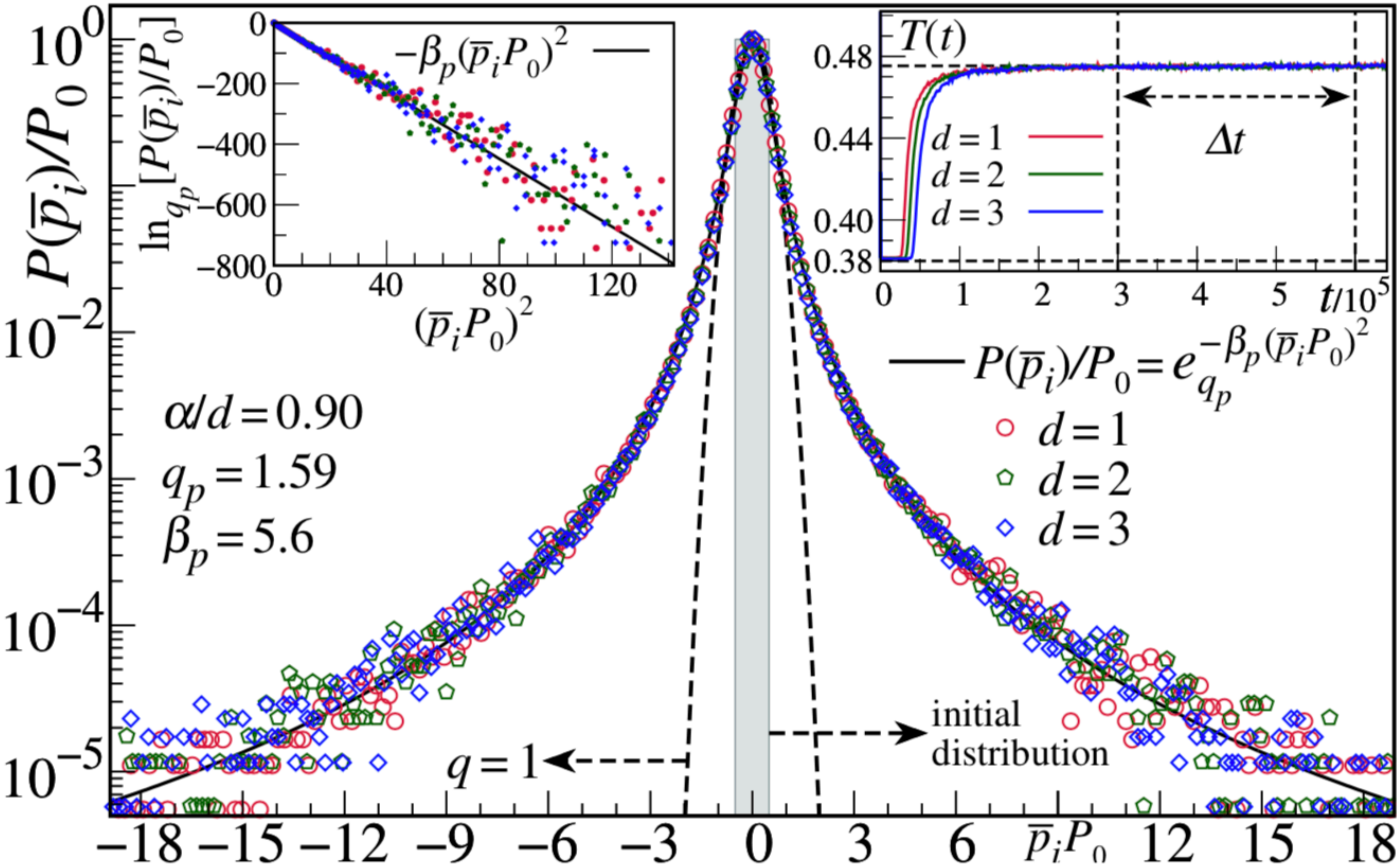
$$V(r) \sim -\frac{A}{r^{\alpha}} \qquad (r \to \infty) \qquad (A > 0, \;\; \alpha \geq 0)$$

$$\textit{integrable if} \quad \alpha / d > 1 \qquad (\textit{short-ranged})$$

$$\textit{non-integrable if} \;\; 0 \leq \alpha / d \leq 1 \qquad (\textit{long-ranged})$$

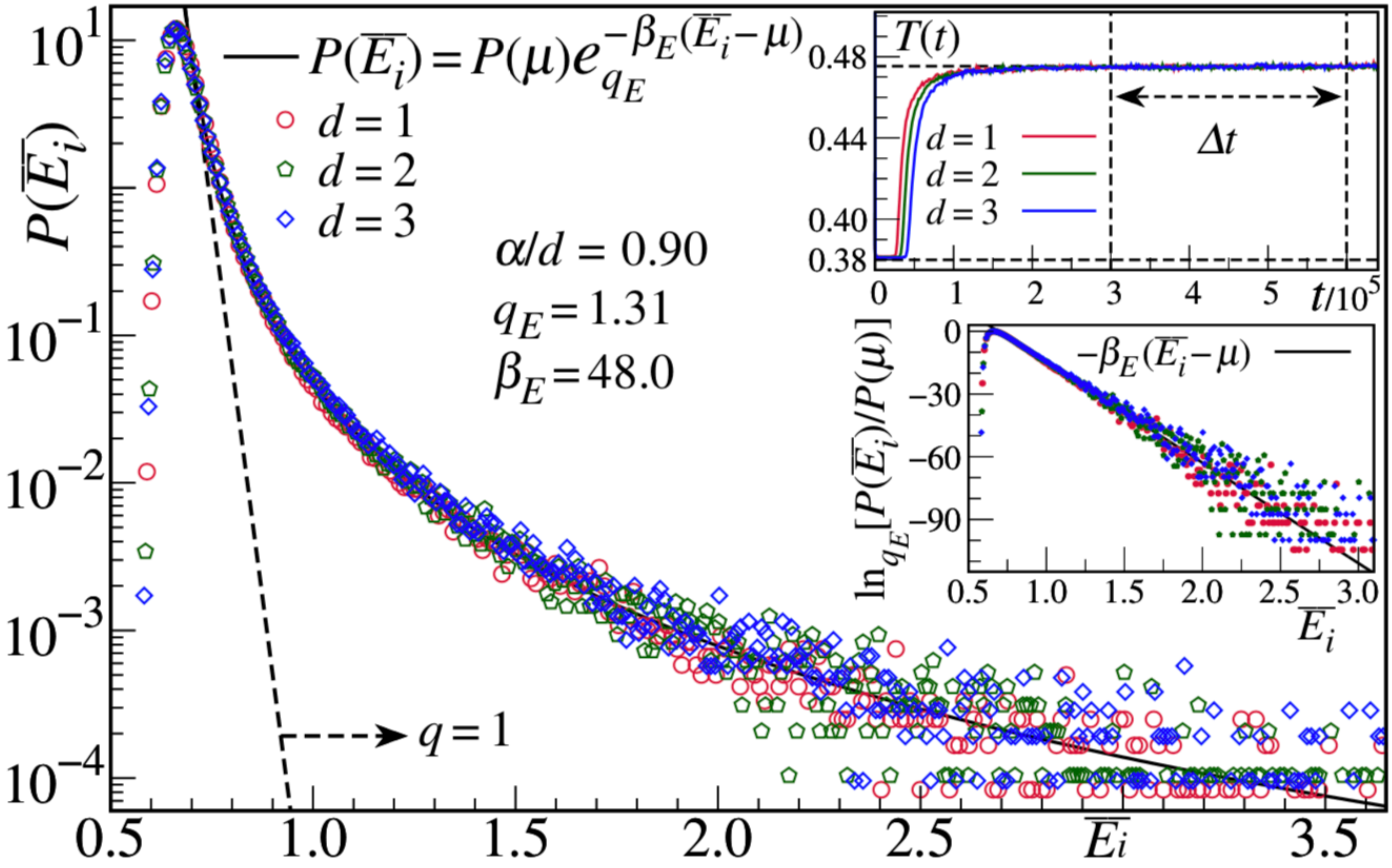


d - DIMENSIONAL XY MODEL



L.J.L Cirto, A. Rodriguez, F.D. Nobre and C.T., EPL **123**, 30003 (2018)

d - DIMENSIONAL XY MODEL



L.J.L Cirto, A. Rodriguez, F.D. Nobre and C.T., EPL **123**, 30003 (2018)

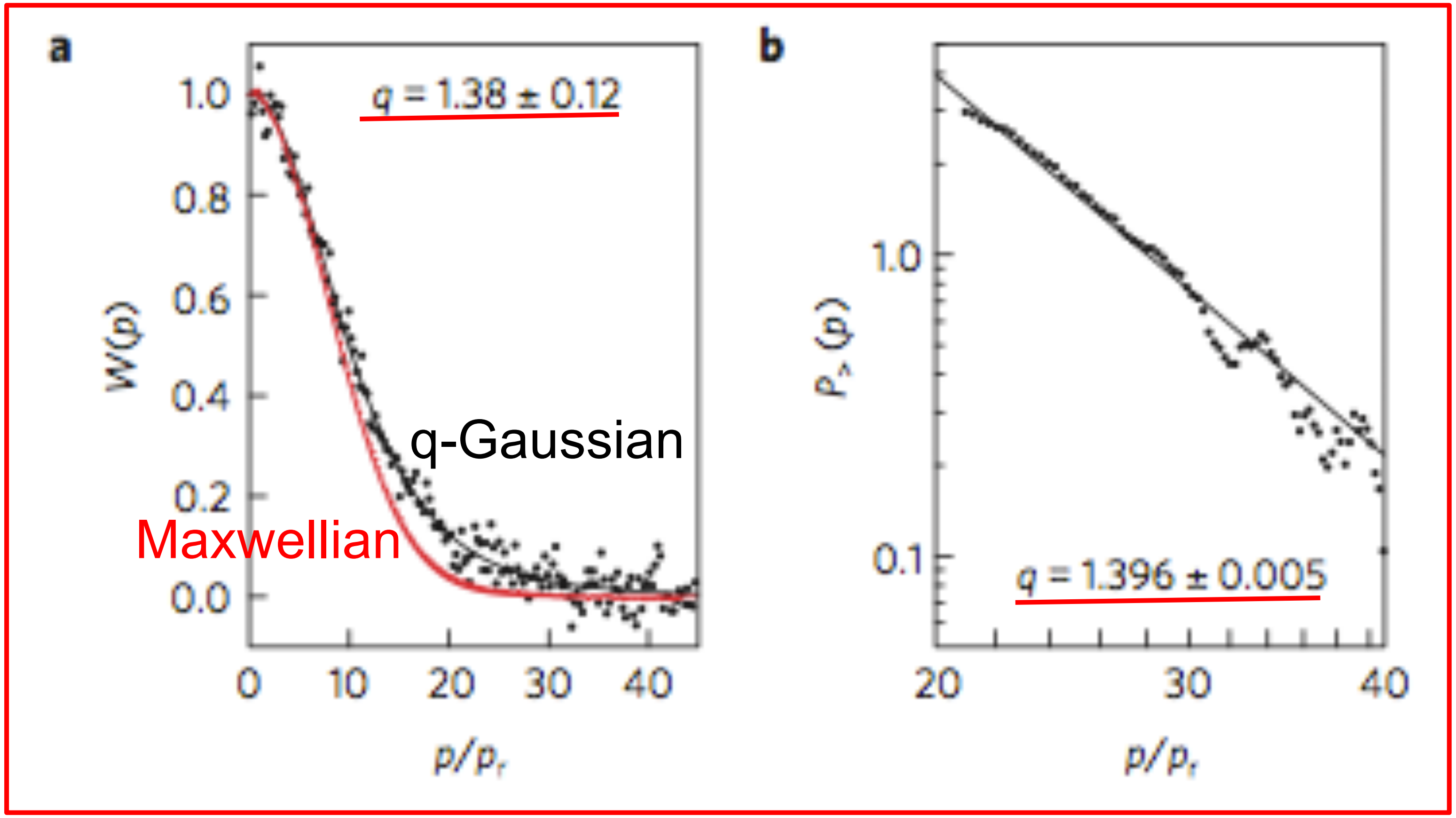
nature physics
PROGRESS ARTICLE
PUBLISHED ONLINE: 1 OCTOBER 2013 | DOI: 10.1038/NPHYS2751

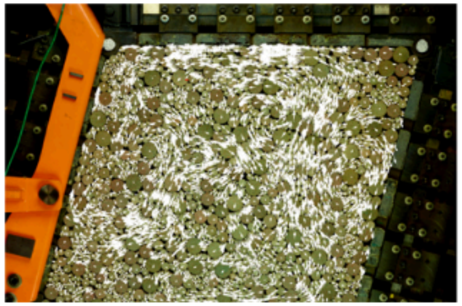
Beyond Boltzmann–Gibbs statistical mechanics in optical lattices

Eric Lutz[1,2] and Ferruccio Renzoni[3]*

Cs COLD ATOMS

$q = 1 + 44\, E_R / U_0$





EDITORS' SUGGESTION

Experimental Validation of a Nonextensive Scaling Law in Confined Granular Media

The velocity distribution of sheared granular media shows unexpected similarities with turbulent fluid flows.

Gaël Combe, Vincent Richefeu, Marta Stasiak, and Allbens P.F. Atman

Phys. Rev. Lett. **115**, 238301 (2015)

Experimental Validation of a Nonextensive Scaling Law in Confined Granular Media

Gaël Combe,* Vincent Richefeu, and Marta Stasiak
Université Grenoble Alpes, 3SR, F-38000 Grenoble, France and CNRS, 3SR, F-38000 Grenoble, France

Allbens P. F. Atman†
Departamento de Física e Matemática, National Institute of Science and Technology for Complex Systems, Centro Federal de Educação Tecnológica de Minas Gerais – CEFET-MG, Avenida Amazonas 7675, 30510-000 Belo Horizonte-MG, Brazil

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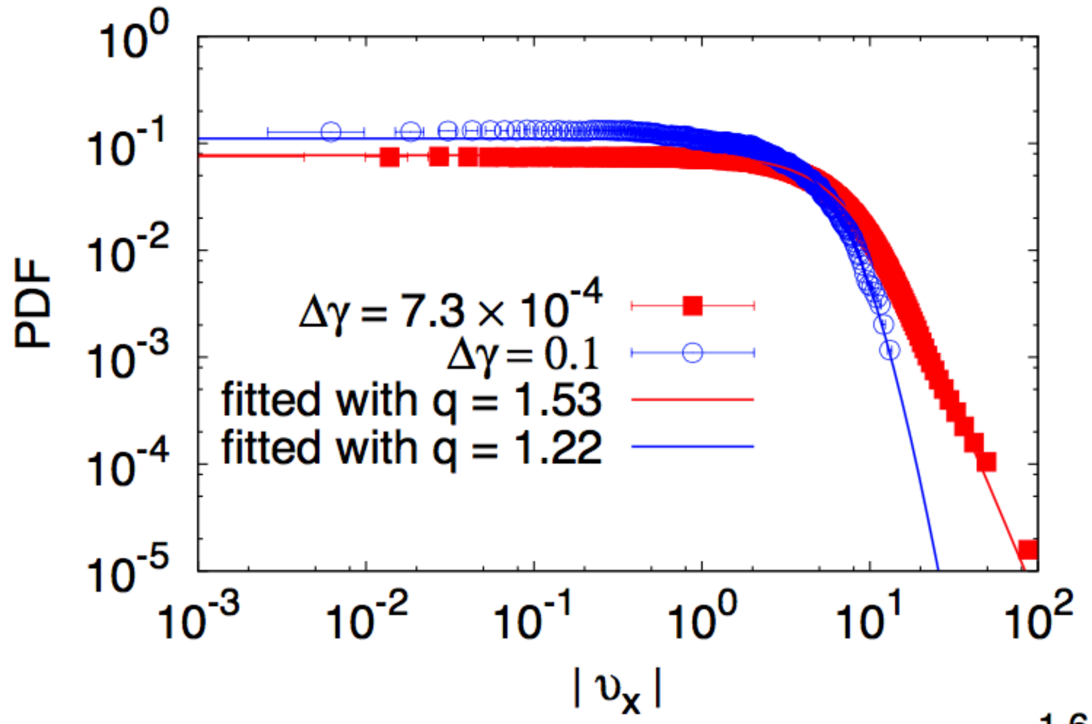
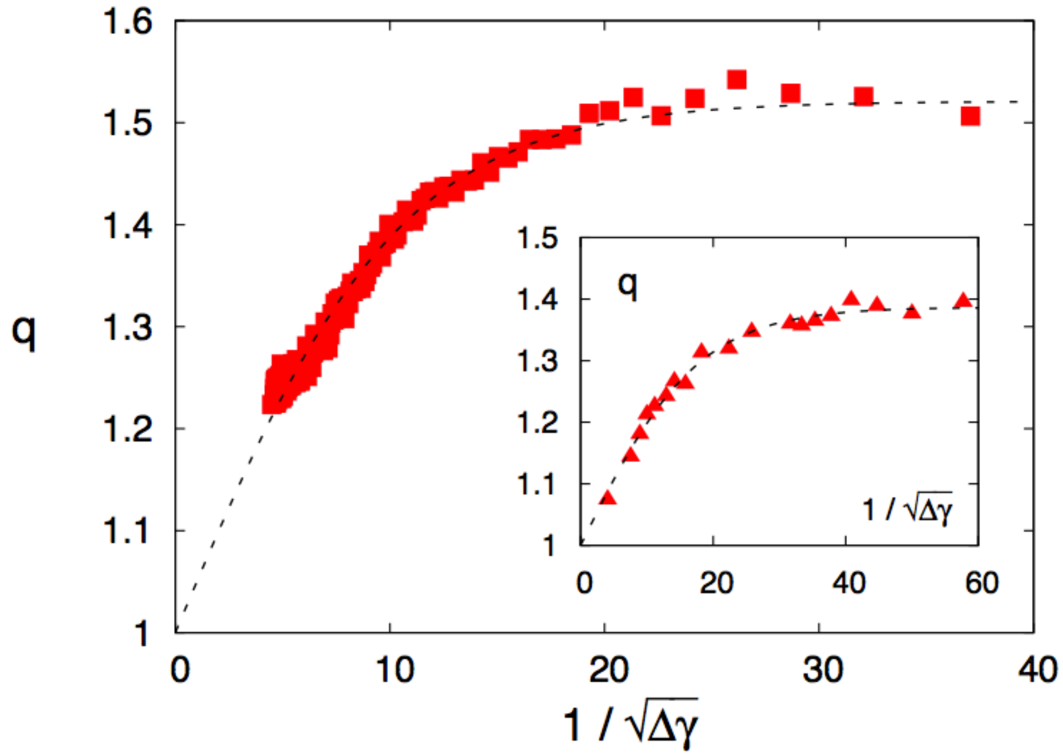




Combe, Richefeu, Stasiak and Atman
PRL **115**, 238301 (2015)

$$\langle x^2 \rangle \propto t^\alpha$$

Combe, Richefeu, Stasiak and Atman
PRL **115**, 238301 (2015)

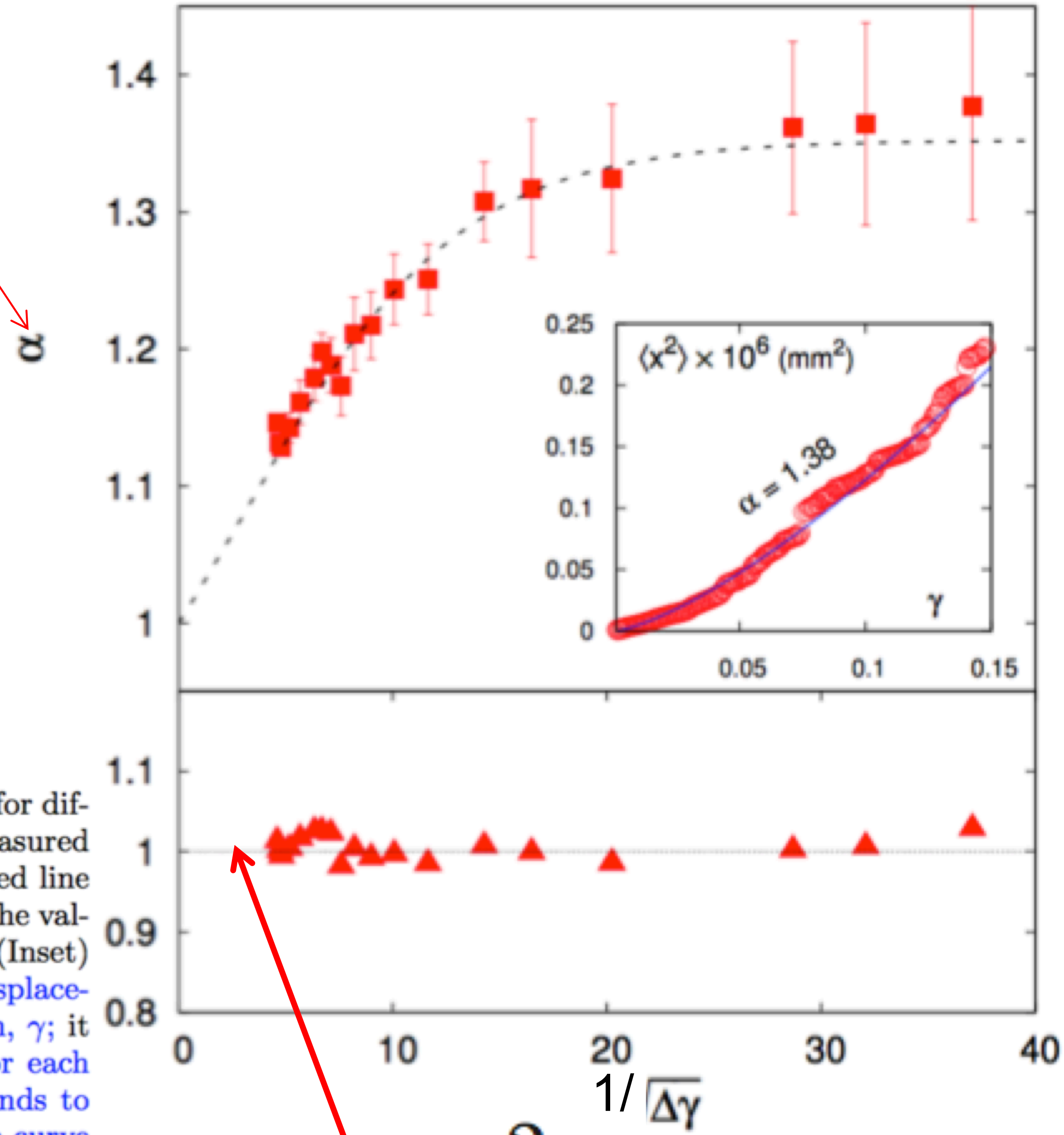


FIG. 4. Verification of the Tsallis-Bukman scaling law for different regimes of diffusion. (top) Evolution of the measured diffusion exponent α as a function of $1/\sqrt{\Delta\gamma}$ the dashed line is a direct application of the scaling law from the fit of the values shown in Fig. 3, $\alpha(1/\sqrt{\Delta\gamma}) = 2/[3 - q(1/\sqrt{\Delta\gamma})]$. (Inset) a typical diffusion curve showing the mean square displacement fluctuations, $\langle x^2 \rangle$, in function of the shear strain, γ; it allows the assessment of the diffusion exponent, α, for each strain window tested. In the case shown, it corresponds to the smallest strain window, the rightmost point in the curve at the main panel. Note that for a constant strain rate, γ is proportional to time. (Bottom) Measure of the deviation of the data relative to the scaling law prediction, as a function of $1/\sqrt{\Delta\gamma}$, showing an agreement on the order of $\pm 2\%$.

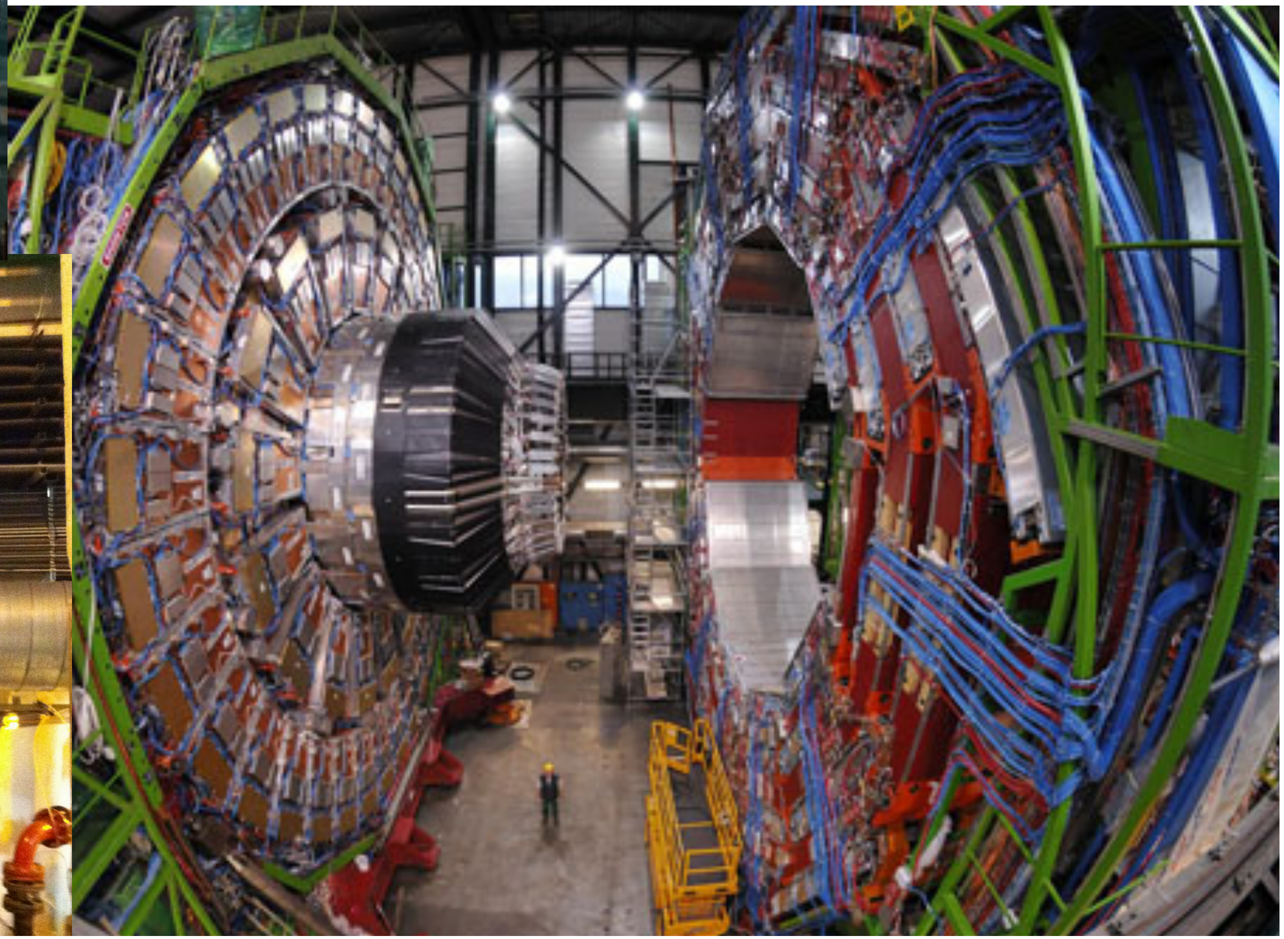
$$\alpha = \frac{2}{3 - q}$$

CT and DJ Bukman, PRE **54** (1996) R2197

LHC (Large Hadron Collider)

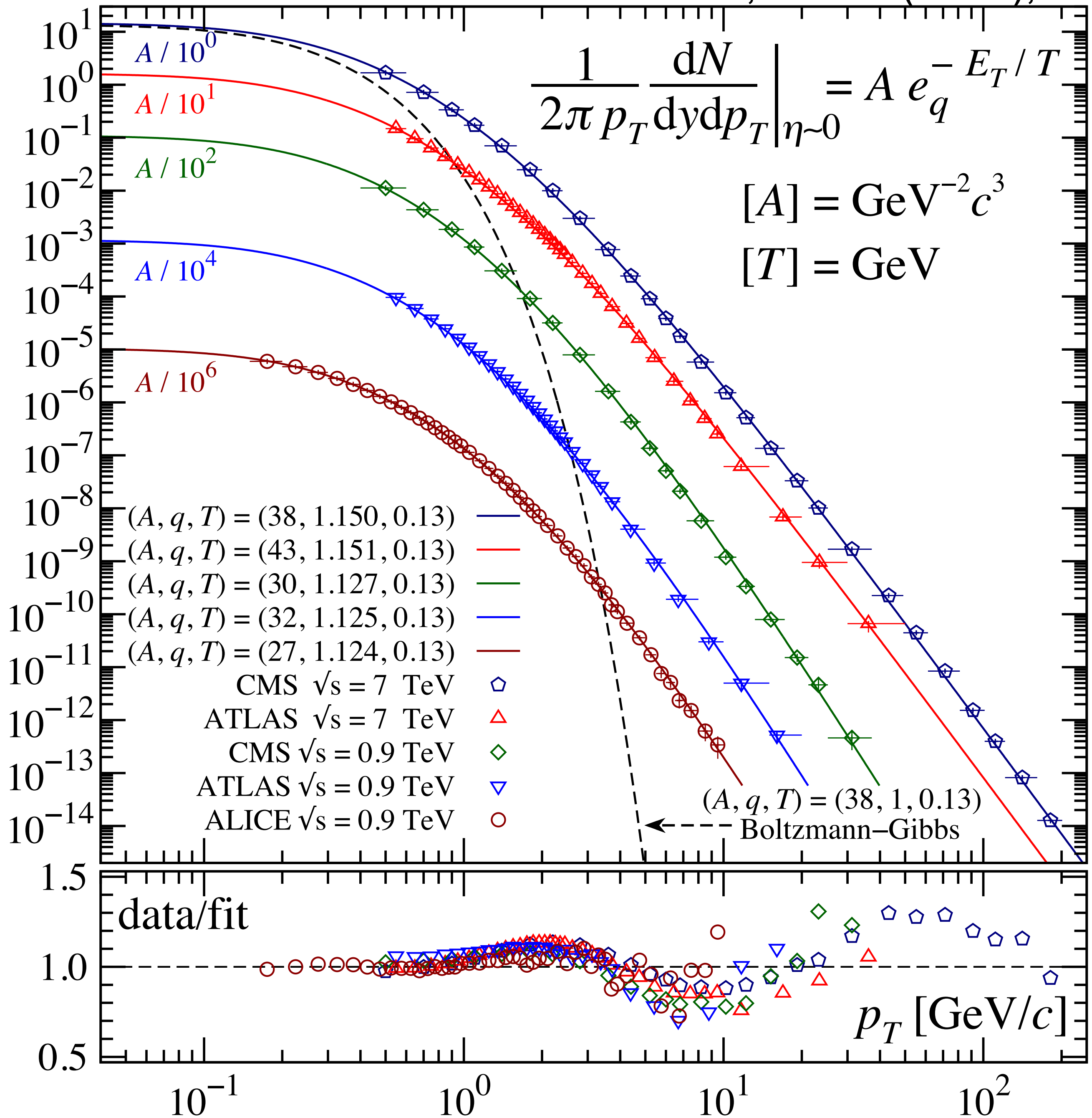
CMS, ALICE, ATLAS and LHCb detectors

~ 4000 scientists/engineers from ~ 200 institutions of ~ 50 countries



SIMPLE APROACH: TWO-DIMENSIONAL SINGLE RELATIVISTIC FREE PARTICLE

C.Y. Wong, G. Wilk, L.J.L. Cirto and C. T.,
EPJ Web of Conferences **90**, 04002 (2015), and PRD **91**, 114027 (2015)



$$E_T = \sqrt{m^2c^4 + p_T^2c^2}$$

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A novel automatic microcalcification detection technique using Tsallis entropy & a type II fuzzy index

Mohanalin *, Beenamol, Prem Kumar Kalra, Nirmal Kumar

Department of Electrical Engineering, IIT Kanpur, UP-208016, India

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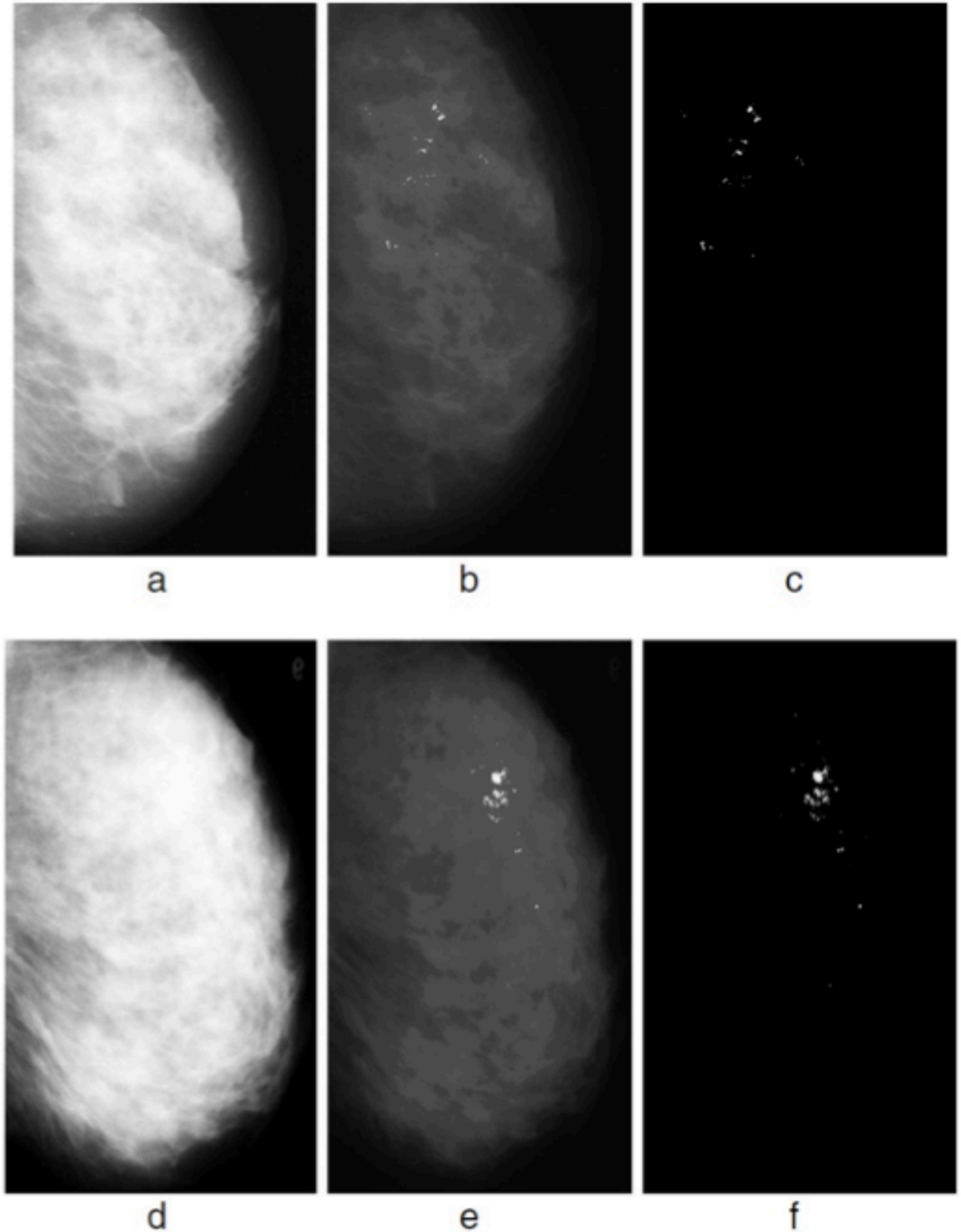
ABSTRACT

This article investigates a novel automatic microcalcification detection method using a type II fuzzy index. The thresholding is performed using the Tsallis entropy characterized by another parameter '*q*', which depends on the non-extensiveness of a mammogram. In previous studies, '*q*' was calculated using the histogram distribution, which can lead to erroneous results when pectoral muscles are included. In this study, we have used a type II fuzzy index to find the optimal value of '*q*'. The proposed approach has been tested on several mammograms. The results suggest that the proposed Tsallis entropy approach outperforms the two-dimensional non-fuzzy approach and the conventional Shannon entropy partition approach. Moreover, our thresholding technique is completely automatic, unlike the methods of previous related works. Without Tsallis entropy enhancement, detection of microcalcifications is meager: 80.21% Tps (true positives) with 8.1 Fps (false positives), whereas upon introduction of the Tsallis entropy, the results surge to 96.55% Tps with 0.4 Fps.


a
b
c
d
e
f

Tout le monde savait que c’ était impossible.

Il y avait un qui ne le savait pas.

Alors il est allé et il l’a fait.

Mark Twain, Jean Cocteau, Winston Churchill, Marcel Pagnol ...

Si l’action n’a quelque splendeur de liberté, elle n’a point de grâce ni d’honneur.

Montaigne

OBRIGADO!