

Effective Cosmology a la Brans-Dicke with a Non-Minimally Coupling Massive Inflaton Field Interacting with Minimally Coupling Massless Field

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We discuss an effective cosmology a la Brans-Dicke with two interacting scalar fields: a non-minimally coupling massive inflaton Higgs-like scalar field ϕ interacting with a minimally coupling massless scalar field χ . Several features are observed and discussed in some details.

Keywords: Interacting scalar fields, non-minimal and minimal couplings, quintessence

1. INTRODUCTION

The nature of the dark matter and dark energy component as one of the basic ingredients responsible for the accelerated expansion of the universe represents nowadays as one of the most profound and difficult problem in modern cosmology and theoretical physics.^{1,2,3,4,5,6} One simplest method to deal with this problem is by means of the Einstein's lambda or positive cosmological constant. This later, acts in fact, as an isotropic and homogenous de-Sitter inflationary phase-source in the sense that the cosmological equation of state is $p_\Lambda = -\rho_\Lambda = \Lambda$ and consequently, avoiding problems with fine-tuning of initial conditions in the early universe. Other options and alternatives exist in the literature: decaying cosmological constant (Λ), cold dark matter, dynamical Λ in the form of scalar field with self interacting exponential and inverse power-law potentials: quintessence and tracker field, k-essence, viscous fluid, Chaplygin gas, Brans-Dicke (BD) pressureless solutions, self-interacting BD cosmology with positive power-law potential, etc.⁷⁻¹⁹ Most of these theories are accompanied with problems and difficulties. For example, the theoretical predicted value of the cosmological constant surpasses by 120 order of magnitudes the observational value. As for the well-known quintessence theory with scalar field ϕ and equation of state $p_\phi = \omega\rho_\phi$ acting as fluctuating dark energy, fine tuning parameters and several constraints are required. A lot of works have been devoted to the investigation of a realistic cosmological model with the non-minimal coupling between gravity and inflaton scalar field and to their connection with inflationary cosmology and phase transitions (spontaneous symmetry breaking (SSB)) in the early Universe.

For minimally coupled scalar field theories, $\omega \geq -1$, but recent observations and phenomenological non-minimally coupled theories have showed that $\omega < -1$ are also welcome.^{20,21,22,23} Minimally coupled theories with $\omega < -1$ were proved to possess spatial gradient instabilities that would be ruled out by CMB observations.²⁴ It is noteworthy that if the kinetic term has the wrong term, it is possible to achieve a minimally coupled theory with $\omega < -1$. Due to such growing interests of scalar-tensor theories of gravitation, it is required to study the theory in some more detail. It is noteworthy that the inflaton scalar field ϕ satisfies in a curved spacetime the Klein-Gordon equation and one need

to set up, in general, a non-minimal coupling term that accommodates both the scalar field and the Ricci curvature of spacetime.

On the other hand, the non-minimal coupling represents an interesting deviation from the equivalence principle because it respects the geometrical nature of gravity and gauge symmetry of electromagnetism. It is in reality introduced by renormalization even if it is absent at the classical level. It is also required in classical general relativity by the Einstein equivalence principle.^{22,23,25,26,27} It is remarkable that scalar fields have been often used to explain the dark energy problem, and referred as quintessence scalar fields. The main parameter for these appealing cosmological models arises from the potential of the scalar field $V(\phi)$, and one can note for example that scaling potentials have been highly investigated. It is also recognized that scalar fields can account either for dark matter or for dark energy. In both cases, the behaviour of the scalar field is determined by its scalar potential. If one desires to use the same scalar field to unify both problems, the difficulty is to determine which scalar potential can provide matter behaviour at local cosmological scales and satisfy the observational constraints. In reality, it is possible to build up a cosmological model which could advantageously restore a model containing two dark components. This fact may have interesting consequences in different aspects of gravity theories including supergravities.^{27,28}

However, it has been recognized that reheating process and SSB may be performed through two scenarios using the Hartree mean-field approximation: the first one is that the inflaton scalar field ϕ is converted through a self-interacting quartic potential $\lambda\phi^4$ ($\lambda \approx 10^{-12}$ from COBE observations) into many particles and the second one, is the presence of a new (real) source field χ created through some coupling with the inflaton as $g^2\phi^2\chi^2$, where g is a coupling constant acting as a free-parameter.^{29,30,31,32} The non-minimally coupled of the scalar field ϕ to R for massive inflaton field were done and was proved to be successful to describe a geometric reheating model with large negative ξ . Surprisingly, the field χ was found to be amplified and that GUT scale gauge boson with mass $\approx 10^{16} GeV$ can be produced in the preheating stage, a scenario not successfully described with a massive-inflation quadratic potential alone with $|\xi| \approx 10^{-3}$.³³ Besides, it was recently argued that large $|\xi|$ ($\approx 10^4$) allows using the Standard Model Higgs field for inflation. At the same time the case $|\xi| \approx 10^{-3}$ makes inflaton with quartic potential compatible with the CMB observations. For the coupling constant $|\xi| \approx 10^{-3}$ very small amount of tensor modes is expected for the quartic inflation. Interestingly, the non-minimally cou-

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pled inflation with quadratic potential and $|\xi| \approx 10^{-3}$ is no longer compatible with observations, generating too small spectral index.³⁴

In this paper, we consider a more general dynamics of a non-minimally coupling inflaton scalar field ϕ with the spacetime curvature R interacting with a minimally coupling massless and scalar field χ . Despite the fact that static massless scalar fields play a crucial role in Schwarzschild-like geometry, it was argued recently that they are connected to many cosmological solutions based on some-kind of duality transformations.³⁵ They also play a crucial role in quantum fields in curved spacetime, in particular in the background of an Einstein-like universe³⁶ and dilatonic electrodynamics in flat spacetime.³⁷ On the other hand, we argue that massless static scalar field could contribute to the dark energy problem by supplying a substantial amount of energy density to the early universe, yet the astronomical data are not precise enough to pinpoint whether the dark energy is truly static or dynamical. Additionally, the term $g\phi^2\chi^2$ may have an electromagnetic origin³⁸ as in the early universe, we strongly be-

lieve that matter was highly ionized and coupled to scalar field. Latter during cooling as a result of expansion of ions combined to form neutral matter. Furthermore, massless chaotic potentials play a leading role in inflationary scenarios and physics of the early universe and supergravity^{39,40}.

The paper is organized as follows: In Sec.II, we introduce the Lagrangian of the theory and we analyze the cosmological FRW solutions around the background fields ϕ and χ . In the same section, we derive the pressure, the density and the equation of state parameter and we analyze their dynamical evolutions in time. In Sec. III, we discuss briefly the role of the fields ϕ and χ in the early evolution epoch of the universe. Sec. IV is devoted for conclusions and perspectives.

2. LAGRANGIAN AND COSMOLOGICAL EVOLUTIONS

The Lagrangian of the theory is written in one of the following forms a la Brans-Dicke:

$$\begin{aligned}
 L = & \underbrace{\frac{R}{2\kappa^2}}_{\text{purely gravitational part of the action}} - \underbrace{\frac{1}{2}\xi_{\phi}R\phi^2}_{\text{explicit interaction term between the gravitational and the field}} - \underbrace{\left(\frac{\phi_{;\mu}\phi^{;\mu}}{2} + \frac{h_{\phi}}{4}\phi^4\right)}_{\text{purely material part of the action associated with the scalar field } \phi} \\
 & - \underbrace{\left(\frac{\chi_{;\mu}\chi^{;\mu}}{2} + \frac{h_{\chi}}{4}\chi^4\right)}_{\text{purely material part of the action associated with the scalar field } \chi} - \underbrace{\frac{1}{2}g^2\phi^2\chi^2}_{\text{interaction between the massive dynamical field } \phi \text{ and the massless static field } \chi}, \\
 & = \frac{R}{2\kappa^2} - \frac{\phi_{;\mu}\phi^{;\mu}}{2} - \underbrace{\frac{1}{2}\left\{R\xi_{\phi} + \frac{1}{2}h_{\phi}\phi^2\right\}}_{\text{Higgs potential of the field } \phi}\phi^2 \\
 & - \frac{\chi_{;\mu}\chi^{;\mu}}{2} - \underbrace{\frac{1}{4}h_{\chi}\chi^4}_{\text{chaotic-like massless potential}} - \frac{1}{2}g^2\phi^2\chi^2, \\
 & \equiv \frac{R}{2\kappa^2} - \frac{\phi_{;\mu}\phi^{;\mu}}{2} - \frac{\chi_{;\mu}\chi^{;\mu}}{2} \\
 & - \underbrace{\frac{1}{4}h_{\phi}\left(\phi^2 - Rh_{\phi}\xi_{\phi}\right)^2 - \frac{1}{4}h_{\chi}\chi^4 - \frac{1}{2}g^2\phi^2\chi^2 - \frac{1}{4}h_{\phi}R^2h_{\phi}^2\xi_{\phi}^2}_{\text{term playing the role of a normalization constant}}, \\
 & \underbrace{\hspace{15em}}_{V(\phi,\chi)}
 \end{aligned}$$

$$= \underbrace{f(\phi)}_{\frac{1}{2\kappa^2} - \frac{1}{2}\xi_\phi\phi^2} R - \frac{1}{2}\phi_{;\rho}\phi^{;\rho} - \frac{1}{2}\chi_{;\rho}\chi^{;\rho} + 2 \underbrace{V(\phi, \chi)}_{\frac{h_\phi}{4}\phi^4 + \frac{1}{4}h_\chi\chi^4 + \frac{1}{2}g^2\phi^2\chi^2}, \quad (1)$$

where $\kappa^2 = 8\pi G$, G is the gravitational constant, g is the gauge coupling, ξ_ϕ is a coupling constant which corresponds for the scalar field ϕ , m_ϕ is the dynamical mass of the field ϕ , h_ϕ and h_χ are free positive parameters, Λ_ϕ is the corresponding cosmological constant. Obviously, the effective mass of the scalar field ϕ is $m_\phi^2 = \xi_\phi R + g^2\chi^2$ which in turns tends to zero for $R = 0$ (flat spacetime) as well as in the absence of a the massless field χ . Amazingly, in the majority of inflationary cosmological scenarios, the coupling constant ξ_ϕ is not a free parameter that could be tuned randomly, but its value is fixed by the gravitational theory and of the scalar field adopted. Moreover, the theoretical consistence of many inflationary scenarios is deeply affected by their proper values. Some phenomenological scenarios turn out to be theoretically contradictory, while others are viable according to the correct use of non-minimal coupling constants. Once their proper values are determined, one does not possess any longer the freedom to regulate their values and the fine-tuning problems that may outbreak the inflationary scenario re-emerge. It is noteworthy that in quantum field theory in curved spacetime, non-minimal coupling is to be expected when the spacetime curvature is large.

The field equations are obtained by varying the action (1) with respect to the metric and to the scalar fields ϕ and χ :

$$f(\phi) \left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \right) = \frac{1}{2} \left(\phi_{;\mu}\phi_{;\nu} - \frac{1}{2}g_{\mu\nu}\phi_{;\rho}\phi^{;\rho} \right) + \frac{1}{2} \left(\chi_{;\mu}\chi_{;\nu} - \frac{1}{2}g_{\mu\nu}\chi_{;\rho}\chi^{;\rho} \right) + f_{;\mu\nu} - g_{\mu\nu}f_{;\rho}^{;\rho} + g_{\mu\nu}V(\phi, \chi), \quad (2)$$

$$\phi_{;\rho}^{;\rho} + \frac{df}{d\phi}R = -2\frac{dV(\phi, \chi)}{d\phi}, \quad (3)$$

$$\chi_{;\rho}^{;\rho} + \frac{df}{d\chi}R = -2\frac{dV(\phi, \chi)}{d\chi}, \quad (4)$$

where

$$f(\phi) = \frac{1}{2\kappa^2} - \frac{1}{2}\xi_\phi\phi^2, \quad (5)$$

and

$$V(\phi, \chi) = h_\phi \frac{\phi^4}{4} + h_\chi \frac{\chi^4}{4} + \frac{1}{2}g^2\phi^2\chi^2, \quad (6)$$

To analyze the cosmological solutions around the background fields ϕ and χ , we adopt the Friedmann-Robertson-Walker (FRW) flat spacetime with metric $d\hat{s}^2 = -dt^2 + a^2(t)d\bar{x}^2$ strongly favored by cosmological observations and

where from theoretical point of views, the inflaton is distributed homogeneously. Here $a(t)$ is the scale factor of the universe. In order to explore the time-variation of the equation of state parameter, we write the Higgs interacting-fields potential of the field ϕ as $V(\phi) = (1/2)\mu^2\phi^2 + (h/4)\phi^4$ where $\mu^2 = R\xi + g^2\chi^2$ with $h \equiv h_\phi \equiv h_\chi > 0$ for simplicity. In fact making use of the Bianchi identity, the total energy-momentum tensor is not a conserved quantity. However, since we have assumed that baryon-matter and radiation are considered as non-interacting fields from the decoupling age up to the present time, we presume through this work that the covariant divergence of the energy-momentum tensors of the baryonic matter and radiation vanishes. The resulting Klein-Gordon equation for the scalar fields ϕ and χ , and the field equations are:

$$\ddot{\phi} + 3H\dot{\phi} + [\xi R + g^2\chi^2 + h\phi^2]\phi = 0, \quad (7)$$

$$\ddot{\chi} + 3H\dot{\chi} + [g^2\phi^2 + h\chi^2]\chi = 0, \quad (8)$$

$$6[1 - \xi(1 - 6\xi)\kappa^2\phi^2](\dot{H} + 2H^2) - \kappa^2(6\xi - 1)\dot{\phi}^2 + \kappa^2\dot{\chi}^2 - 4\kappa^2 \left(h\frac{\phi^4}{8} + h\frac{\chi^4}{8} + \frac{1}{2}g^2\phi^2\chi^2 \right) + 6\kappa^2\xi\phi(h\phi^3 + g^2\chi^2\phi) = 0, \quad (9)$$

$$\kappa^2(\dot{\phi}^2 + \dot{\chi}^2) + 12\xi\kappa^2 H\phi\dot{\phi} - 6H^2(1 - \kappa^2\xi\phi^2) + \kappa^2 \left(\frac{1}{2}h\phi^4 + \frac{1}{2}h\chi^4 + g^2\phi^2\chi^2 \right) = 0. \quad (10)$$

Equations (7)-(10) gives:

$$p_{eff} \equiv p = \frac{1}{2}(\dot{\phi}^2 + \dot{\chi}^2) - \left(\frac{h}{4}\phi^4 + \frac{1}{4}h\chi^4 + \frac{1}{2}g^2\phi^2\chi^2 \right) - \xi[4H\phi\dot{\phi} + 2\dot{\phi}^2 + 2\phi\ddot{\phi} + (2\dot{H} + 3H^2)\phi^2], \quad (11)$$

$$\rho_{eff} \equiv \rho = \frac{1}{2}(\dot{\phi}^2 + \dot{\chi}^2) + 3\xi H\phi(H\phi + 2\dot{\phi}) + \frac{h}{4}\phi^4 + \frac{1}{4}h\chi^4 + \frac{1}{2}g^2\phi^2\chi^2, \quad (12)$$

where $H = \dot{a}/a$ is the Hubble parameter. The effective gravitational constant is $\kappa_{eff}^2 = \kappa^2(1 - \kappa^2\xi\phi^2)^{-1}$ and the critical values of the scalar field are $\phi_c = \pm(\kappa^2\xi)^{-1/2}$ for $\xi > 0$. Hence, the effective Einstein equations are:

$$H^2 = \frac{\kappa_{eff}^2}{3}\rho_{eff}, \quad (13)$$

$$\dot{H} = -\frac{\kappa_{eff}^2}{2}(\rho_{eff} + p_{eff}). \tag{14}$$

One interesting class of solution is obtained by setting $\phi = \phi_c = \pm(\kappa^2\xi)^{-1/2}$ and absence of the gauge coupling and the scalar field χ . This gives $R = 6(\dot{H} + 2H^2) = \kappa^2(\rho_{eff} - 3p_{eff})$ and therefore $\dot{H} + 2H^2 = C$ where C is a constant. The solution is given by²²:

$$a(t) = a_0 \cosh^{1/2}(\sqrt{2C}(t - t_0)), \tag{15}$$

with

$$H = \sqrt{\frac{C}{2}} \tanh(\sqrt{2C}(t - t_0)), \tag{16}$$

for $H < \sqrt{C/2}$. Equation (15) describes an asymptotic contracting de Sitter spacetime as $t \rightarrow -\infty$, reaching a maximum at $t = t_0$ and then superaccelerating with $\dot{H} < 0$ tending when $t \rightarrow \infty$ to a de-Sitter expanding spacetime.

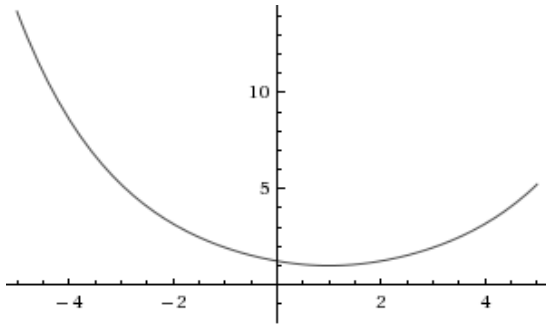


FIG. 1: Plot of $a(t) = a_0 \cosh^{1/2}(\sqrt{2C}(t - t_0))$, $a_0 = t_0 = 1, 2C = 1$.

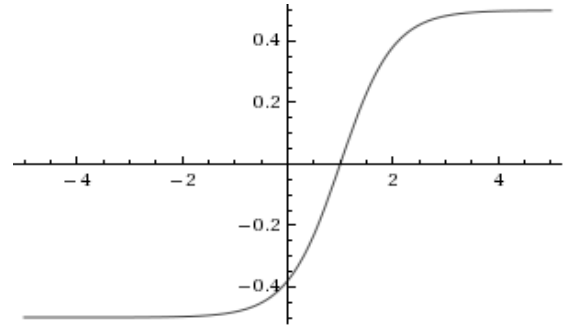


FIG. 2: Plot of $H = \sqrt{\frac{C}{2}} \tanh(\sqrt{2C}(t - t_0))$, $a_0 = 1, t_0 = 1, 2C = 1$.

In fact, by setting $\phi = \pm\phi_c$, one finds the following effective time dependent density and pressure:

$$\rho_{eff} = \frac{1}{2}\dot{\chi}^2 + 3\xi H^2 \phi_c^2 + \frac{h}{4}\phi_c^4 + \frac{1}{4}h\chi^4 + \frac{1}{2}g^2\phi_c^2\chi^2, \tag{17}$$

$$p_{eff} = \frac{1}{2}\dot{\chi}^2 - \left(\frac{h}{4}\phi_c^4 + \frac{1}{4}h\chi^4 + \frac{1}{2}g^2\phi_c^2\chi^2\right) - \xi(2\dot{H} + 3H^2)\phi_c^2, \tag{18}$$

and accordingly the corresponding state equation is

$$w = \frac{p_{eff}}{\rho_{eff}} = \frac{\frac{1}{2}\dot{\chi}^2 - \left(\frac{h}{4}\phi_c^4 + \frac{1}{4}h\chi^4 + \frac{1}{2}g^2\phi_c^2\chi^2\right) - \xi(2\dot{H} + 3H^2)\phi_c^2}{\frac{1}{2}\dot{\chi}^2 + 3\xi H^2 \phi_c^2 + \frac{h}{4}\phi_c^4 + \frac{1}{4}h\chi^4 + \frac{1}{2}g^2\phi_c^2\chi^2}. \tag{19}$$

One amazing interesting class of solution is obtained if we conjecture that $\chi = \exp(-\phi)$. As we expect that $|\phi_c| = (\kappa^2\xi)^{-1/2} \gg 1$, then we find:

$$w = -\frac{\frac{h}{4}\phi_c^4 + \frac{h}{4}e^{-4\phi_c} + \frac{1}{2}g^2\phi_c^2e^{-2\phi_c} + \xi(2\dot{H} + 3H^2)\phi_c^2}{3\xi H^2 \phi_c^2 + \frac{h}{4}\phi_c^4 + \frac{h}{4}he^{-4\phi_c} + \frac{1}{2}g^2\phi_c^2e^{-2\phi_c}} \approx -\frac{\frac{h}{4}\phi_c^2 + \xi(2\dot{H} + 3H^2)}{\frac{h}{4}\phi_c^2 + 3\xi H^2}. \tag{20}$$

Consequently for $\dot{H} \ll H^2$, $w \rightarrow -1$ which corresponds for quintessence whereas for $h \ll 1$, we obtain $w < -1$ which corresponds to phantom energy. The equation of state which

may be written like:

$$w \approx -1 - \frac{2}{3} \frac{\dot{H}}{H^2}, \tag{21}$$

suggests that the equation of state parameter may varies in time and this is quite appealing. Amazingly, for $H = \alpha/t, \alpha > 0$ (t is here the cosmological time), we find $w \approx -1 + 2/3\alpha$ and accordingly for $\alpha = 3/2$, we get $w \approx 0$. This special case corresponds for an accelerated universe dominated by pressureless matter and not dark energy. The case $\chi = \exp(-\phi)$ results into the special exponential potential:

$$V(\phi) = \frac{h}{4}(\phi^4 + \exp(-4\phi)) + \frac{1}{2}g^2\phi^2 \exp(-2\phi). \tag{22}$$

We plot in what follows the potential $V(\phi)$ for $h = 1/2, g = 1$ for illustration purpose.

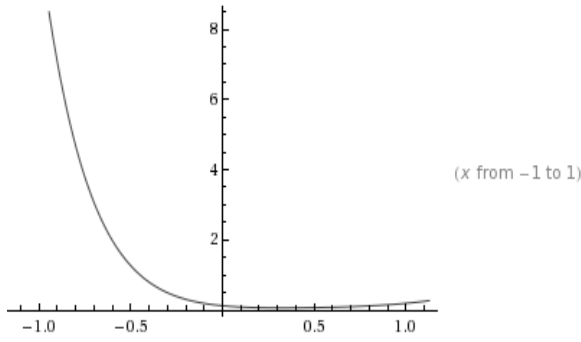


FIG. 3: Plot of $V(\phi)$, $\phi \equiv x$

It is note worthy that exponential potential plays a crucial role in inflationary cosmology^{31,32}. The case $\chi = \exp(-\phi)$ is not the only possible choice of solution. We may discuss the following independent interesting cases:

II-1: We discuss first the case of the scalar field $\chi = \phi^m$, m is a real parameter. Accordingly, the state equation takes the special form:

$$w = -\frac{\frac{h}{4}\phi_c^4 + \frac{h}{4}\phi_c^{4m} + \frac{1}{2}g^2\phi_c^{2+2m} + \xi(2\dot{H} + 3H^2)\phi_c^2}{3\xi H^2\phi_c^2 + \frac{h}{4}\phi_c^4 + \frac{h}{4}\phi_c^{4m} + \frac{1}{2}g^2\phi_c^{2+2m}}. \quad (23)$$

Notice that for $m < 0$ and for $|\phi_c| = (\kappa^2\xi)^{-1/2} \gg 1$, equation (23) is approximated again by:

$$w \approx -\frac{\xi(2\dot{H} + 3H^2) + \frac{h}{4}\phi_c^2}{3\xi H^2 + \frac{h}{4}\phi_c^2}, \quad (24)$$

and the same previous arguments may hold. However, for $m \gg 1$, we may approximate equation (24) by:

$$w = -\frac{\frac{h}{4}\phi_c^{4m} + \xi(2\dot{H} + 3H^2)\phi_c^2}{\frac{h}{4}\phi_c^{4m} + 3\xi H^2\phi_c^2} \rightarrow -1. \quad (25)$$

This case corresponds also for a quintessence dominated universe. The case where $\chi = \phi^m$ results on the following potential

$$V(\phi) = \frac{h}{4}(\phi^4 + \phi^{4m}) + \frac{1}{2}g^2\phi^{2+2m}. \quad (26)$$

We plot in what follows $V(\phi)$ for $h = 4, g = 1$ and four different values for m :

II-2: Another interesting cases corresponds for $H = \beta\dot{\chi}/\chi, \beta$ is a real parameter augmented by $\chi = \phi^m$, that is $H = \beta\dot{\chi}/\chi = \beta m\dot{\phi}/\phi$. Accordingly, equation (19) is reduced to:

$$w = \frac{\frac{1}{2}\dot{\chi}^2 - (\frac{h}{4}\phi_c^4 + \frac{1}{4}h\chi^4 + \frac{1}{2}g^2\phi_c^2\chi^2) - \xi(2\beta\frac{\dot{\chi}}{\chi} + \beta(3\beta - 2)\frac{\dot{\chi}^2}{\chi^2})\phi_c^2}{\frac{1}{2}\dot{\chi}^2 + 3\xi\beta^2\frac{\dot{\chi}^2}{\chi^2}\phi_c^2 + \frac{h}{4}\phi_c^4 + \frac{1}{4}h\chi^4 + \frac{1}{2}g^2\phi_c^2\chi^2}. \quad (27)$$

As $\chi = \phi^m$, then at $|\phi_c| = (\kappa^2\xi)^{-1/2} \gg 1$, equation (26) is reduced straightforwardly to $w = -1$.

II-3: At the end, we discuss the case $H = e^\phi$ and $\chi = \phi^m$. Accordingly, equation (19) is reduced again to:

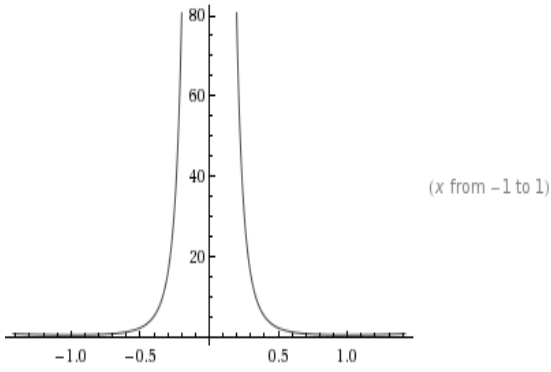
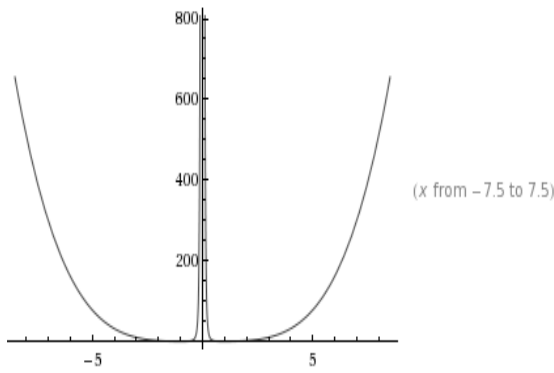
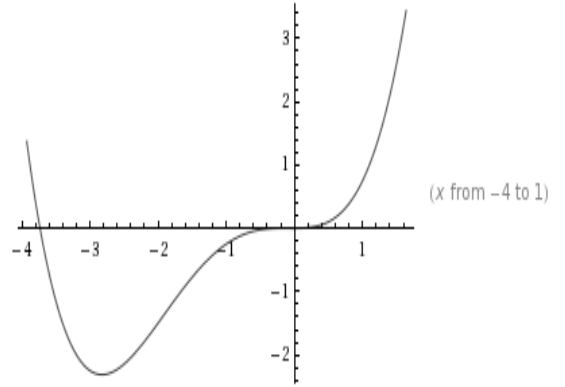
$$w = -\frac{3\xi e^{2\phi_c}\phi_c^2 + (\frac{h}{4}\phi_c^4 + \frac{1}{4}h\phi_c^{4m} + \frac{1}{2}g^2\phi_c^{2+2m})}{3\xi e^{2\phi_c}\phi_c^2 + \frac{h}{4}\phi_c^4 + \frac{1}{4}h\phi_c^{4m} + \frac{1}{2}g^2\phi_c^{2+2m}} = -1. \quad (28)$$

The previous case corresponds for certain modified gravity types having the following corresponding Lagrangians:

$$\chi = \phi^m : L = \left(\frac{1}{2\kappa^2} - \frac{1}{2}\xi\phi^2\right)R - (1 + m^2\phi^{2m-2})\frac{\phi_{;\mu}\phi^{;\mu}}{2} - \left(\frac{h}{8}(\phi^4 + \phi^{4m}) + \frac{1}{2}g^2\phi^{2+2m}\right). \quad (29)$$

More generally, we will discuss the particular case $\xi \ll 1$. For this, we assume the following ansatz $\phi = \phi_0 H^q, \chi = \chi_0 H^p$ where $(p, q) \in \mathbb{R}$ and ϕ_0, χ_0 are constants parameters

chosen such that at $t = 1, \phi = \phi_0 = 1, \chi = \chi_0 = 1$ for mathematical simplicity. These ansatzs could be important building blocks in constructing the models of dark energy. For these particular solutions, as the universe expands in time, one naturally expects that the cosmological constant and accordingly the scalar curvature should decay, thereby leading to the tiny value observed presently. In fact, cold dark matter particles with time decreasing masses were proved to have an important measurable effect in the dynamical motion of the halo of spiral galaxies. At clusters scale this could have important consequences on dark matter halos (axions and mass

FIG. 4: Plot of $V(\phi)$, $m = -1$, $\phi \equiv x$ FIG. 5: Plot of $V(\phi)$, $m = -1$, $\phi \equiv x$ FIG. 6: Plot of $V(\phi)$, $m = 1/2$, $\phi \equiv x$

$$w = \frac{q^2 \frac{\dot{H}^2}{H^2} - \frac{1}{2} (h + g^2) H^{2q} - \xi \left[2(2q + 1)\dot{H} + 2q^2 \frac{\dot{H}^2}{H^2} + 2q(q - 1) \frac{\dot{H}}{H^2} + 2q \frac{\dot{H}}{H} + 3H^2 \right]}{q^2 \frac{\dot{H}^2}{H^2} + 3\xi H^2 + 6\xi q \dot{H} + \frac{1}{2} (h + g^2) H^{2q}}. \quad (30)$$

The author may check that the quintessence solution is obtained one more if, for instance, $h = -g^2$, $\dot{H} \ll H$ and $\ddot{H} \ll H$. However, for $H = \alpha/t$, $\alpha > 0$, equation (28) is reduced to:

$$w = \frac{\left[q^2 - \frac{1}{2} (h + g^2) \alpha^{2q} + 6\alpha\xi - 2\xi - 3\alpha^2\xi \right] \frac{1}{t^2} - 3\xi q \frac{1}{t^4}}{\left[q^2 - \frac{1}{2} (h + g^2) \alpha^{2q} - 6\alpha\xi + 3\alpha^2\xi \right] \frac{1}{t^2}}. \quad (31)$$

$$\approx \frac{q^2 - \frac{1}{2} (h + g^2) \alpha^{2q} + 6\alpha\xi - 2\xi - 3\alpha^2\xi}{q^2 - \frac{1}{2} (h + g^2) \alpha^{2q} - 6\alpha\xi + 3\alpha^2\xi}. \quad (32)$$

For $h = -g^2$ and $\xi = 1/6$ which corresponds for the conformal coupling case, we obtain:

$$w \approx \frac{q^2 + \alpha - \frac{1}{3} - \frac{\alpha^2}{2}}{q^2 - \alpha + \frac{\alpha^2}{2}}. \quad (33)$$

varying neutrinos are good candidate for the cold dark matter of our universe).^{41,42,43} Accordingly, can easily check that for $p = q$:

Notice that $w > -1$ (dark energy dominance) for $q^2 > 1/6$ whereas $w < -1$ (phantom energy dominance) for $q^2 < 1/6$.

In order to analyze the dynamical equations in some details, we choose the scaling solutions which were proved to be useful to describe many interesting features of early and late-time dynamics of the cosmological evolutions. For this, we choose: $\phi = t^x$, $\chi = t^y$ and $a = t^z$, (x, y, z) are real parameters. Equations (7)-(10) are now reduced for the case of conformal coupling $\xi = 1/6$ to:

$$x(x-1)t^{x-2} + 3xzt^{x-2} + \left[\frac{1}{6}R + g^2t^{2y} + ht^{2x} \right] t^x = 0, \quad (34)$$

$$y(y-1)t^{y-2} + 3yzt^{y-2} + \left[g^2t^{2x} + ht^{2y} \right] t^y = 0, \quad (35)$$

$$6(2z-1)zt^{-2} + \kappa^2 y^2 t^{2y-2}$$

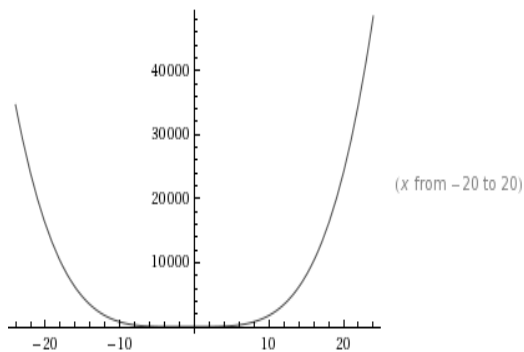


FIG. 7: Plot of $V(\phi)$, $m = 1/2$, $\phi \equiv x$

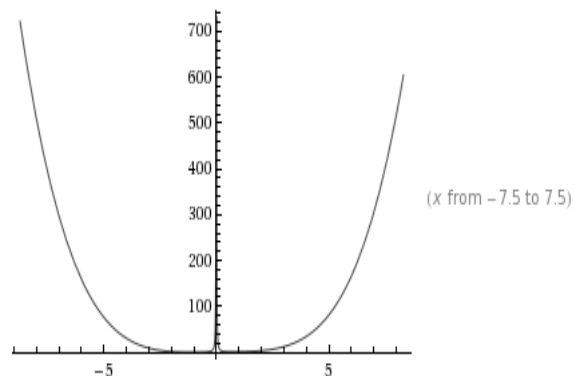


FIG. 10: Plot of $V(\phi)$, $m = -1$, $\phi \equiv x$

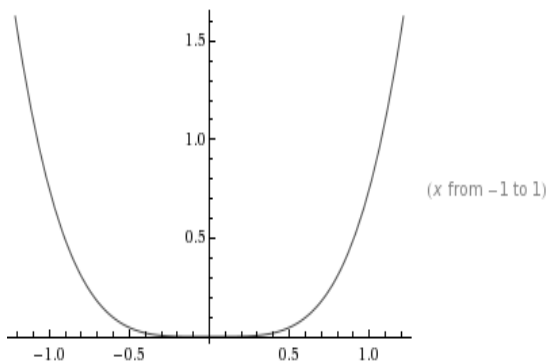


FIG. 8: Plot of $V(\phi)$, $m = 1$, $\phi \equiv x$

$$+\kappa^2 \left(\frac{h}{2} t^{4x} + \frac{1}{2} h t^{4y} + g^2 t^{2x+2y} \right) = 0. \quad (37)$$

It is an easy task to check that consistency is obtained if, for instance, $R = 0$, $y = x = -1$ and $\kappa^2 = t^2$ (increasing gravitational coupling constant). The case $y = x = -1$ indicates that $\chi \propto \phi$. Then:

$$x(x - 1) + 3xz + [g^2 + h] = 0, \quad (38)$$

$$y(y - 1) + 3yz + [g^2 + h] = 0, \quad (39)$$

$$(1 - g^2) + 6(2z - 1)z = 0, \quad (40)$$

$$(2 - 2z + (h + g^2) + z^2) - 6z^2 = 0. \quad (41)$$

After adding equation (38) to (39) we find:

$$7z^2 - 8z + h - 3 = 0, \quad (42)$$

which gives

$$z = \frac{4 + \sqrt{100 - 28h}}{7}, \quad (43)$$

which holds for $h < 3.57$. For $h = -1$, $z \approx 2.18$ and for $h \ll -1, z \gg 1$ which corresponds for an accelerated expansion of the universe. Besides, equation (38) or (39) gives $g^2 + h = 3z - 2$. This fact corresponds for a chaotic quartic inflationary potential $V(\phi) = (h + g^2)\phi^4/2$. The effective pressure and density vary like:

$$p_{eff} \equiv p = -\frac{1}{2} (z(z - 2) + h + g^2) t^{-4}, \quad (44)$$

$$-4\kappa^2 \left(h \frac{t^{4x}}{8} + h \frac{t^{4y}}{8} + \frac{1}{2} g^2 t^{2x+2y} \right) + \kappa^2 t^x (h t^{3x} + g^2 t^{x+2y}) = 0, \quad (36)$$

$$\kappa^2 (x^2 t^{2x-2} + y^2 t^{2y-2}) + 2\kappa^2 z x t^{2x-2} - 6z^2 t^{-2} \left(1 - \frac{1}{6} \kappa^2 t^{2x} \right)$$

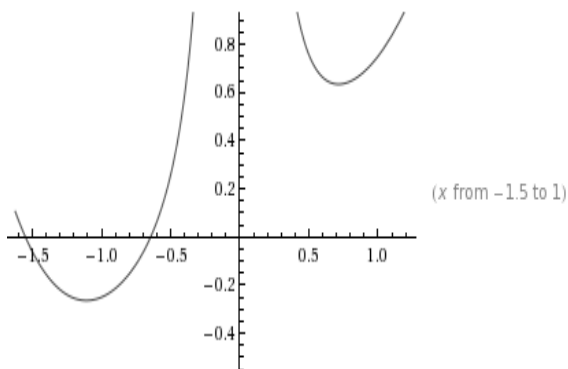


FIG. 9: Plot of $V(\phi)$, $m = -1/2$, $\phi \equiv x$

$$\rho_{eff} \equiv \rho = \left(1 + \frac{1}{2}z(z-2) + \frac{1}{2}(h+g^2)\right) t^{-4}, \quad (45)$$

and accordingly the equation of state parameter for $z \approx 2.18$ and $g^2 + h = 3z - 2$ is:

$$w = \frac{2}{z^2+z} - 1 = -0.7, \quad (46)$$

whereas $w \rightarrow -1$ for $z \gg 1$ as it is expected.

Finally, it is interesting to see that

$$\kappa_{eff}^2 = \frac{\kappa^2}{1 - \kappa^2 \xi_f^{2x}} \propto t^2, \quad (47)$$

and that the present day variation of it $\kappa_{eff}^2 / \kappa_{eff}^2 = 2H/z$ is too small for $h \ll -1, z \gg 1$ in agreement with observational limit.⁴⁴ In fact, time variation of the effective gravitational constant is tightly constrained by astronomical stellar observations with the Solar system and terrestrial laboratories. The time-variation of the effective gravitational coupling constant is not new as a variation of the gravitational constant is predicted in numerous scalar field tensor theories. Further, that there exit a lot of experimental evidence on the time variation of the 4D gravitational constant: radar ranging data to the Viking landers on Mars, lunar laser ranging experiments, measurements of the masses of young and

old neutron stars in binary pulsars. It was recently observed that for late times, a modified cosmology with varying κ_{eff} is in accordance with the observed values of the cosmological parameters.^{44,45}

3. ROLE OF THE INTERACTING DYNAMICAL-STATIC SCALAR FIELDS DURING THE QUANTUM GRAVITATIONAL EPOCH OF THE UNIVERSE

In this section, we give a brief thought on the role of the inflaton dynamical massive field ϕ interacting with the massless static scalar field χ in the early evolution epoch of the universe where the scalar field potential is:

$$V(\phi, \chi) = \frac{1}{4}h\phi^4 + \frac{1}{4}h\chi^4 + \frac{1}{2}g^2\phi^2\chi^2. \quad (48)$$

The case where $\chi = \phi^m$ results on the following potential

$$V(\phi) = \frac{h}{4}(\phi^4 + \phi^{4m}) + \frac{1}{2}g^2\phi^{2+2m}. \quad (49)$$

We choose $m = 1$ for which

$$V(\phi) = \frac{1}{2}(h+g^2)\phi^4. \quad (50)$$

The partition function is written as:²⁶

$$Z = \int D\phi e^{-\int_0^\beta d\tau \int d^3x L_H(\phi, \partial_\mu \phi)}, \quad (51)$$

$$= \int d\langle \phi \rangle e^{-(h+g^2) \int_0^\beta d\tau \int d^3x (\frac{1}{2}\langle \phi^4 \rangle)}$$

$$\times \int D\chi e^{-\int_0^\beta d\tau \int d^3x \left[\underbrace{\frac{1}{2}(\partial_\mu \Psi)^2 + \frac{1}{2}(6\langle \phi^2 \rangle)\Psi^2 + 4\Psi^3 \langle \phi \rangle + 4\Psi \langle \phi^3 \rangle + \frac{h}{4}\Psi^4}_{\text{free part}} + \underbrace{\dots}_{\text{interaction part}} \right]}, \quad (52)$$

$$\equiv \int d\langle \phi \rangle e^{-\int_0^\beta d\tau \int d^3x V_{effective}(\langle \phi \rangle)}, \quad (53)$$

where $\partial_\mu = (\partial_\tau, \nabla)$, $D\phi = \prod_i d\phi_i$ and:

$$\phi(x) = \sqrt{\frac{1}{\beta V}} \sum_k \phi_{\vec{k}n} e^{i(\vec{k}\cdot\vec{x} + \omega_n \tau)} = \psi(x) + \langle \phi \rangle. \quad (54)$$

where β is a constant having the dimension of time, \vec{k} is the wave number, $\omega_n = (2\pi/\beta)n$, $n = 0, \pm 1, \pm 2, \dots$ is the Matsubara frequency, V is the volume in which we are consider-

ing the Fourier modes of the fields, ψ here is assumed here to represent the $\vec{k} = n = 0$ -mode of the scalar field assumed to be periodic to the imaginary Euclidean time τ and where the

mass of the free part is $M^2 = 3(h + g^2) \langle \phi^2 \rangle$. All the $\vec{k} = 0$ modes are in the "v-part" with $\int d^3x \psi = 0$. The form of the effective Mexican potential will be responsible of the kind of symmetries we have in the theory. We can now expand the potential like:

$$V_{effective}(\langle \phi \rangle) = V_{effective}^{(0)} + V_{effective}^{(1)} + V_{effective}^{(2)} + \dots, \quad (55)$$

$$= (h + g^2) \left(\frac{1}{2} \langle \phi^4 \rangle + 4\psi \langle \phi^3 \rangle + 3\psi^2 \langle \phi^2 \rangle + \dots \right) \dots \quad (56)$$

The first term is the *classical term*, the second one is a *1-loop correction*, the third one is the *2-loop-correction* and so on.

4. CONCLUSIONS AND PERSPECTIVES

In this work, we have discussed numerous new features of a cosmology a la Brans-Dicke with inflaton scalar field ϕ with Higgs-like potential non-minimally coupled to the spacetime curvature R interacting with a minimally scalar field χ with a scalar fields potential of the form $V(\phi, \chi) = h\phi^4/8 + h\chi^4/8 + g^2\phi^2\chi^2/2$. The cosmological model described in this work corresponds for a homogeneous, isotropic and flat Friedmann-Robertson-Walker spacetime. It has interesting and promising features concerning the description of quintessence/ vacuum energy as non-minimally coupled scalar field as well as the important role of the interacting massive dynamics and massless static fields in the theory described. We have discussed many interesting ansatzs relating the inflaton field to a new (real) source field χ created through some coupling with the inflaton as $g^2\phi^2\chi^2$, where g is a coupling constant acting as a free-parameter. The first case corresponds to $\chi = \exp(-\phi)$ and the second case is $\chi = \phi^m$ and both cases were discussed at the critical value of the scalar field $|\phi_c| = (\kappa^2\xi)^{-1/2} \gg 1$. For the first case, it was found that for $\dot{H} \ll H^2$, the equation of state parameter $w \rightarrow -1$ which corresponds for quintessence whereas for $h \ll 1$, we obtain $w < -1$ which corresponds to phantom energy. Furthermore, it was observed that the equation of state parameter may varies in time and this is quite appealing. One additional result obtained for this special case is obtained if the Hubble parameter decays like $H = \alpha/t$, $\alpha > 0$. We found that the equation of state parameter varies like $w \approx -1 + 2/3\alpha$ and accordingly for $\alpha = 3/2$, we get $w \approx 0$. This particular case is amazing as it corresponds for an accelerated universe dominated by pressureless mat-

ter and not dark energy. The case $\chi = \exp(-\phi)$ results into the special exponential potential described by equation (22). It will be of interest to explore in the future a scalar tensor theory dominated by this type of potential. Whereas for the second case $\chi = \phi^m$, it was as well observed that similar results occur just like the previous exponential behavior with the major difference of the form of the potential obtained. As it was observed in equation (26), the potential depends on m . For $m = 1$, the potential is of the form $V(\phi) \propto \phi^4$ which is chaotic-like.

Furthermore, we have made a detailed analysis of the dynamical equations obtained in some details where we have choose the scaling solutions. Other types of solutions may exist but we left this part for a future work, nevertheless, scaling laws were proved to give a good picture of the evolution of the universe. It was found that $\chi = \phi = t^{-1}$ and that the universe is in the stage of accelerated universe dominated by dark energy for $z \approx 2.18$ and by quintessence for $z \gg 1$. In addition, it was observed that the gravitational coupling constant increases in time whereas its present time-variation is too small for $z \gg 1$, that is, in agreements with terrestrial and astrophysical observations.

We have also analyzed briefly the role of the inflaton dynamical massive field ϕ interacting with the massless static scalar field χ in the early evolution epoch of the universe. One may extends all the previous arguments to include a dynamical gauge coupling parameter, e.g. a decrease of the gauge coupling scale due to a dilution of the scalar field. Work in this direction is under progress. It is interesting to have a decreasing gauge coupling constant during the matter evolution epoch of the universe. In fact, the physics responsible for a cosmic time variation of the coupling constants takes place at energies above the unification scale.⁴⁶⁻⁵¹ In all cases, if g varies with time, this means that there exists some new physics beyond our ordinary physical theory and consequently the description of gravity from Einstein's General Relativity is incomplete. If it is the case, we need to introduce something to complete the theory and makes the scalar field vary.^{52,53,54,55} Further details and analysis are in progress.

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