Thermal Properties of Heavy-Light Quark Pseudoscalar and Vector Mesons

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The thermal behaviour of the mass, leptonic decay constant, and width of heavy-light quark peseudoscalar and vector mesons is analized in the framework of thermal Hilbert moment QCD sum rules. In all the cases, the meson leptonic decay constants decrease with increasing T, and vanish at a critical temperature T_c, while the mesons develop a width which increases dramatically, diverging when $T \to T_c$, where T_c is the temperature for chiral-symmetry restoration. The spectral function becomes a smooth function of the energy. This is interpreted as a signal for deconfinement at $T = T_c$. In contrast, the thermal masses are stable, except when $T \to T_c$, where the pseudoscalar meson mass increases slightly by 10-20 %, and the vector meson mass decreases by some 20-30 %.

Keywords: Finite temperature field theory; QCD Sum Rules

1. INTRODUCTION

The discussion of hadronic Green's functions at finite temperature, in a variety of frameworks, is crucial in understanding the dynamics of the quark-gluon plasma. One such framework is that of QCD sum rules [1], based on the Operator Product Expansion (OPE) of current correlators beyond perturbation theory, and on the notion of quark-hadron duality. This program was extended to the finite temperature scenario in [2]. It is based on two assumptions, (a) that the OPE remains valid, with temperature dependent vacuum condensates, and (b) that the quark-hadron duality also remains valid. Additional evidence supporting these assumptions was provided in [3]. This analysis suggests strongly that at finite temperature stable particles at $T = 0$ develop a non-zero width and resonances become broader, diverging at a critical deconfining temperature (T_c). This width is a result of particle absorption in the thermal bath. The analysis shows that the onset of the continuum decreases and approaches threshold near T_c. This technique provides also evidence for the equality of the critical temperatures for deconfinement and chiral-symmetry restoration [6]. When $T \to T_c$ hadrons seem to melt and disappear from the hadronic spectral functions. This scenario is further supported by the thermal behaviour of electromagnetic mean-squared radii, which also diverge at T_c [4]. On the contrary, the thermal mass evolution is not a relevant signal for deconfinement. Conceptually, given either the emergence or the broadening of an existing width, together with its divergence at T_c, the concept of mass looses its meaning. In practice, in some cases the mass increases slightly with increasing T, and in others it decreases.

In this paper we use Hilbert moment QCD sum rules for heavy-light quark pseudoscalar and vector meson correlators to determine the temperature behaviour of the hadronic masses, couplings, and widths. For more details, see [7].At $T = 0$ this problem was discussed in [8]-[9]. While there are only four ground-state pseudoscalar heavy-light quark mesons in the spectrum (D, D_s, B, and B_s), and similarly for vector mesons, it is possible to determine the decay constants for arbitrary meson masses in a self-consistent way [9]. It turns out that the meson masses are basically independent of T, except very close to T_c where they increase slightly (pseudoscalars) by 10 - 20 %, or decrease (vector mesons) by 20-30 %. Here T_c is the critical temperature for chiral-symmetry restoration. The leptonic decay constants decrease with increasing T, and vanish at the critical temperature. Pseudoscalar and vector mesons develop a non-zero hadronic width that increases with T diverging at T_c. These results provide evidence for quark deconfinement at $T = T_c$.

2. PSEUDOSCALAR MESONS

Let us consider the correlator of axial-vector divergences at finite temperature, i.e. the retarded Green's function

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$$\psi_5(q^2,T) = i\int d^4x\, e^{iqx}\ \theta(x_0) \tag{1}$$
$$<< |\, [\partial^\mu A_\mu(x)\ ,\ \partial^\nu A_\nu^\dagger(0)]\, | >>,$$

where $\partial^\mu A_\mu(x) = m_Q : \bar{q}(x)\, i\gamma_5\, \bar{Q}(x) :$, q (Q) refers to the light (heavy) quark, and $m_Q >> m_q$ is assumed. The matrix element above is the Gibbs average

$$<< A\cdot B >>= \frac{1}{Z}\sum_n exp(-E_n/T) < n|A\cdot B|n > , \tag{2}$$

where $|n>$ is any complete set of eigenstates of the (QCD) Hamiltonian and $Z = Tr(\exp(-(H/T))$ is the partition function. We use here the quark-gluon basis, which allows for a smooth extension of the QCD sum rule program to non-zero temperature [3]. At $T = 0$ and to leading order in PQCD [10]

$$\frac{1}{\pi} Im\,\psi_5(x,0) = \ \frac{3}{8\pi^2}\, m_Q^4\, \frac{(1-x)^2}{x} , \tag{3}$$

where $x \equiv m_Q^2/s$, with $s \geq m_Q^2$, and $0 \leq x \leq 1$. At finite temperature contribute both the so called scattering term (q^2 space-like), and the annihilation term (q^2 time-like) [2] to the correlator. We find the former to be exponentially suppressed, so that it can be safely neglected, while the latter is given by

$$\frac{1}{\pi} Im\,\psi_5(x,T) = \frac{1}{\pi} Im\,\psi_5(x,0) \tag{4}$$
$$\left\{1 - n_F\left[\frac{\omega}{2T}(1+x)\right] - n_F\left[\frac{\omega}{2T}(1-x)\right]\right\},$$

where $Im\,\psi_5(x,0)$ is given by Eq.(3), $n_F(z) = (1+e^z)^{-1}$ is the Fermi thermal function, and in the rest frame ($\mathbf{q} = 0$) of the thermal bath $x = m_Q^2/\omega^2$. The first thermal function is exponentially suppressed and can be safely neglected for temperatures of order $O(100-200\,\mathrm{MeV})$, but the second one does contribute near threshold.

Up to dimension $d = 6$ the non-perturbative expansion of the correlator at $T = 0$ is given by [8]

$$\psi_5(q^2)|_{NP} = \alpha C_4 < O_4 > + \beta C_5 < O_5 > + \gamma C_6 < O_6 > \tag{5}$$

where $\alpha = m_Q^2/(m_Q^2-q^2)$, $\beta = m_Q^3 q^2/4(m_Q^2-q^2)^3$ and

$$\gamma = \frac{m_Q^2}{6}\left[\frac{2}{(m_Q^2-q^2)^2} - \frac{m_Q^2}{(m_Q^2-q^2)^3} - \frac{m_Q^4}{(m_Q^2-q^2)^4}\right]$$

Our condensates are

$$C_4 < O_4 >= \frac{1}{12\pi} < \alpha_s G^2 > - m_Q < \bar{q}q >, \tag{6}$$

$$C_5 < O_5 >=< g_s \bar{q}\, i\sigma_{\mu\nu} G^a_{\mu\nu} \lambda^a q > \equiv 2\, M_0^2\ < \bar{q}q > , \tag{7}$$

$$C_6 < O_6 >= \pi\alpha_s < (\bar{q}\gamma_\mu\lambda^a q)\sum_q \bar{q}\gamma^\mu\lambda^a q > \tag{8}$$
$$\stackrel{VS}{\Longrightarrow} -\frac{16}{9}\pi\alpha_s\rho| < \bar{q}q > |^2 ,$$

and $m_c(m_c) \simeq 1.3\,\mathrm{GeV}$, $m_b(m_b) \simeq 4.*\,\mathrm{GeV}$, $< \bar{q}q > \simeq (-250\,\mathrm{MeV})^3$, $< \alpha_s G^2/12\pi > \simeq 0.003\,\mathrm{GeV}^4$, $M_0^2 \simeq 0.4 - 0.6\,\mathrm{GeV}^2$, and $\rho \simeq 3-5$ accounts for deviations from vacuum saturation. Use of these values in Hilbert moment sum rules reproduce the pseudoscalar meson masses at $T = 0$. Changes in these parameters would only affect the normalization at $T = 0$.

For the light-quark condensate at finite temperature we use the result of [11], obtained in the composite operator formalism, valid for the whole range of temperatures $T = 0 - T_c$, where T_c is the critical temperature for chiral symmetry restoration. There is lattice evidence [12] as well as analytical evidence [6] for this critical temperature to be the same as that for deconfinement. The ratio $R(T) =<< \bar{q}q >> / < \bar{q}q >$ from [11] as a function of T/T_c is shown in Fig. 1 .

The low temperature expansion of the gluon condensate is proportional to the trace of the energy-momentum tensor, and it starts only at order T^8 [13]. To a good approximation it can be written as

$$<< \frac{\alpha_s}{12\pi} G^2 >>=< \frac{\alpha_s}{12\pi} G^2 > \left[1 - \left(\frac{T}{T_c}\right)^8\right] . \tag{9}$$

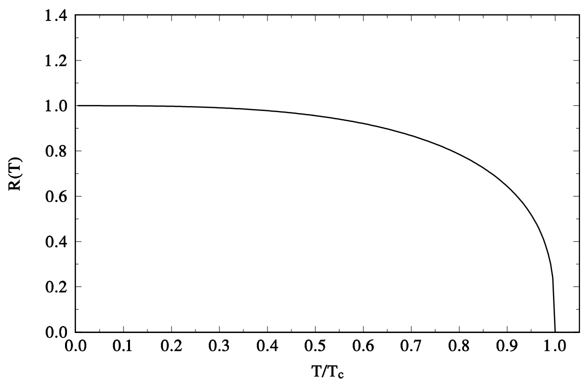

FIG. 1: The light-quark condensate ratio $R(T) =<< \bar{q}q >> / < \bar{q}q >$ as a function of T/T_c from [11].

Because of this T- dependence, the gluon condensate remains essentially constant up to temperatures very close to T_c. Hence, the thermal non-perturbative QCD correlator is basically driven by the quark condensate. Concerning the dimension $d = 6$ condensate, it has been argued that the vacuum saturation approximation breaks down at finite temperature [14]. This is based on the comparison between the slopes of the low temperature expansion (chiral perturbation theory) with and without assuming vacuum saturation. They are in fact numerically different. However, this result is only valid at very low temperatures ($T << f_\pi$); hence it cannot be extrapolated to $T \simeq T_c$. In fact, both the quark condensate and the four-quark condensate should vanish at the same temperature $T = T_c$. In any case, numerically, at temperatures of order $T \simeq 100\,\text{MeV}$ the quark condensate dominates over the gluon condensate, the dimension $d = 5$ condensate is comparable to $<< \bar{q}q >>$, and the dimension $d = 6$ condensate is almost two orders of magnitude smaller. Hence, potential violations of vacuum saturation can be safely ignored. Finally, at finite temperature it is possible, in principle, to have non-zero values of non-diagonal (Lorentz non-invariant) vacuum condensates. There is one example discussed in the literature [15] with enough detail to make a numerical estimate of their importance, and it refers to operators of spin-two (quark and gluon energy momentum tensors). The low temperature expansion of these terms starts at order $O(T^4)$, in contrast to a T^2 dependence of the diagonal condensates. We find that at temperatures of order $T \simeq 100\,\text{MeV}$ both non-diagonal condensates are three orders of magnitude smaller than the corresponding diagonal equivalents. We shall then ignore non-diagonal condensates in the sequel.

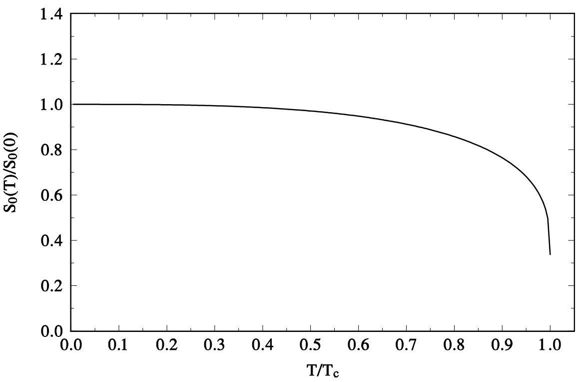

FIG. 2: The ratio $s_0(T)/s_0(0)$, Eq.(14), as a function of T/T_c for $m_Q = m_c$.

If we go now into the hadronic sector,, the spectral function at $T = 0$ can be written as

$$\frac{1}{\pi} Im\,\psi_5(s)|_{HAD} = 2 f_P^2 M_P^4\,\delta(s - M_P^2) + \theta(s - s_0)\,\frac{1}{\pi} Im\,\psi_5(s)|_{PQCD}\,, \tag{10}$$

where M_P and f_P are the mass and leptonic decay constant of the pseudoscalar meson, and the continuum, starting at some threshold s_0, is modeled by perturbative QCD. With this normalization, $f_\pi \simeq 93\text{MeV}$. Our previous experience with thermal evolution of two point functions according to finite energy sum rules suggests us strongly to anticipate the pseudoscalar mesons to develop a sizable width $\Gamma_P(T)$ at finite temperature (particle absorption in the thermal bath), and using a Breit-Wigner parametrization, the following replacement will be understood

$$\delta(s - M_P^2) \Longrightarrow const\ \frac{1}{(s - M_P^2)^2 + M_P^2\Gamma_P^2}\,, \tag{11}$$

where the mass and width are $T-$dependent, and the constant is fixed by requiring equality of areas, e.g. if the integration is in the interval $(0-\infty)$ then $const = 2M_P\Gamma_P/\pi$. The continuum

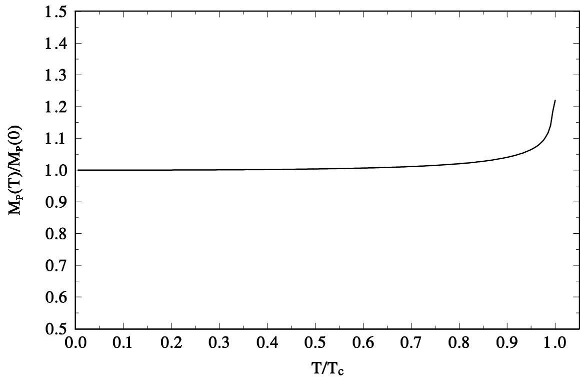

FIG. 3: The ratio $M_P(T)/M_P(0)$ as a function of T/T_c.

threshold s_0 above also depends on temperature; to a good approximation it scales universally as the quark condensate [16], i.e.

$$\frac{s_0(T)}{s_0(0)} \approx \frac{<< \bar{q}q >>}{< \bar{q}q >}\,, \tag{12}$$

where $s_0(0)$ is clearly channel dependent. At the critical temperature we expect $s_0(T_c) = m_Q^2$, in which case Eq. (13) can be rewritten as

$$\frac{s_0(T)}{s_0(0)} \approx \frac{<< \bar{q}q >>}{< \bar{q}q >}\left[1 - \frac{m_Q^2}{s_0(0)}\right] + \frac{m_Q^2}{s_0(0)}\,, \tag{13}$$

This is shown in Fig. 2 for the case $m_Q = m_c$ and $s_0(0) = 5\text{GeV}^2$; a qualitatively similar behaviour is obtained for $m_Q = m_b$ and $s_0(0) \simeq (1.1 - 1.3)M_B^2$.

The correlation function $\psi_5(q^2, T)$, Eq.(1), satisfies a twice subtracted dispersion relation. To eliminate the subtractions one can use Hilbert moments at $Q^2 \equiv -q^2 = 0$, i.e.

$$\varphi^{(N)}(T) \equiv \frac{(-)^{N+1}}{(N+1)!}\left(\frac{d}{dQ^2}\right)^{N+1} \psi_5(Q^2, T)|_{Q^2=0} = \frac{1}{\pi}\int_{m_Q^2}^{\infty} \frac{ds}{s^{N+2}}\, Im\,\psi_5(s, T)\,, \tag{14}$$

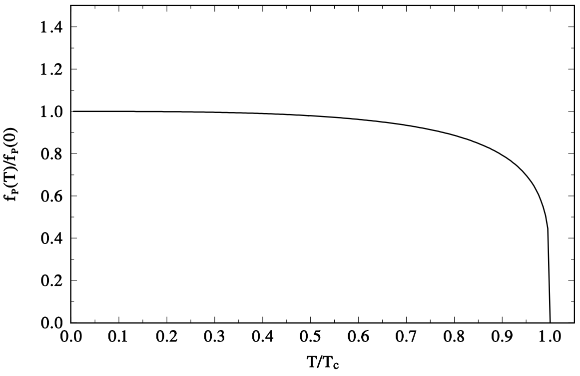

FIG. 4: The ratio $f_P(T)/f_P(0)$ as a function of T/T_c.

where $N = 1, 2, \ldots$. Invoking quark-hadron duality

$$\varphi^{(N)}(T)|_{HAD} = \varphi^{(N)}(T)|_{QCD} \;, \tag{15}$$

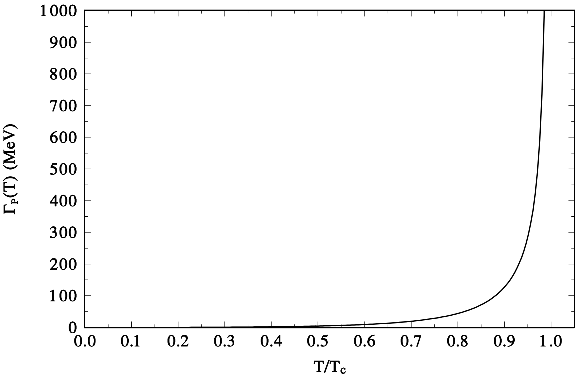

FIG. 5: The width $\Gamma_P(T))$ as a function of T/T_c, with $\Gamma_P(0) = 0$.

and combining the continuum contribution in the hadronic spectral function with the PQCD piece of the QCD counterpart leads to the finite energy Hilbert moments

$$\frac{1}{\pi}\int_0^{s_0(T)} \frac{ds}{s^{N+2}}\, Im\,\psi_5(s,T)|_{POLE} =$$
$$\frac{1}{\pi}\int_{m_Q^2}^{s_0(T)} \frac{ds}{s^{N+2}}\, Im\,\psi_5(s,T)|_{PQCD} + \varphi^{(N)}(T)|_{NP} \;, \tag{16}$$

where $Im\,\psi_5(s,T)|_{POLE}$ is given by the first term in Eq.(11) modified according to Eq.(12), the PQCD spectral function corresponds to Eq.(4), and

$$\varphi^{(N)}(T)|_{NP} = \frac{A}{m_Q^{2N+2}}\left[1 - \frac{1}{12\pi}\frac{<<\alpha_s G^2>>}{A} - \frac{1}{4}(N+2)(N+1)\frac{M_0^2}{m_Q^2} - \frac{4}{81}(N+2)(N^2+10N+9)\,\pi\,\alpha_s\,\rho\,\frac{<<\bar{q}q>>}{m_Q^3}\right] \tag{17}$$

where we introduced $A = -m_Q << \bar{q}q >>$. Using the first three moments one obtains the temperature dependence of the mass, the leptonic decay constant, and the width. Results from this procedure are shown in Figs.3-5 for the charm case; in the case of beauty mesons, results are qualitatively similar.

3. VECTOR MESONS

We consider the correlator of the heavy-light quark vector current

$$\Pi_{\mu\nu}(q^2,T) = i\int d^4x\, e^{iqx}\theta(x_0) << |[V_\mu(x), V_\nu^\dagger(0)]| >>$$
$$= -(g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi^{(1)}(q^2,T) + q_\mu q_\nu \Pi^{(0)}(q^2,T) \tag{18}$$

where $V_\mu(x) =: \bar{q}(x)\gamma_\mu Q(x) :$. In the sum rule we shall use the function $-Q^2\Pi^{(1)}(q^2,T)$, which is free of kinematical singularities. A straightforward calculation gives

$$\frac{1}{\pi} Im\,\Pi^{(1)}(x,T) = \frac{1}{8\pi^2}(1-x)^2(2+x) \times \left[1 - n_F\left(z_+\right) - n_F\left(z_-\right)\right] \;, \tag{19}$$

where $z_\pm \equiv \frac{\omega}{2T}(1\pm x)$. In the hadronic sector, we define the vector meson leptonic decay constant f_V through

$$< 0|V_\mu(0)|V(k) >= \sqrt{2}\, M_V\, f_V\, \varepsilon_\mu \;, \tag{20}$$

so that the pole contribution to the hadronic spectral function is $2f_V^2\delta(s - M_V^2)$. At $T = 0$ the vector Meson $D^*(2010)$ has a very small width in the keV range (96 ± 22) keV which we expect to increase with increasing T, so that the replacement in Eq.(12) will be made.

The Hilbert moments at $Q^2 = 0$ of the function $-Q^2\Pi^{(1)}(Q^2)$ are given by

$$\varphi^{(N)}(T) \equiv \frac{(-)^{N+1}}{(N+1)!}\left(\frac{d}{dQ^2}\right)^{N+1}[-Q^2\Pi^{(1)}(Q^2,T)]|_{Q^2=0}$$
$$= \frac{1}{\pi}\int_{m_Q^2}^{\infty}\frac{ds}{s^{N+1}}\, Im\,\Pi^{(1)}(s,T) \;. \tag{21}$$

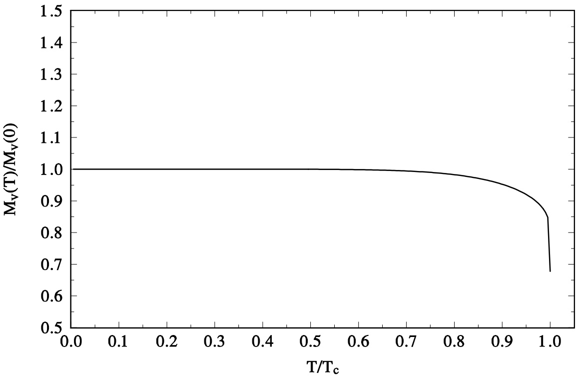

FIG. 6: The ratio $M_V(T)/M_V(0)$ as a function of T/T_c.

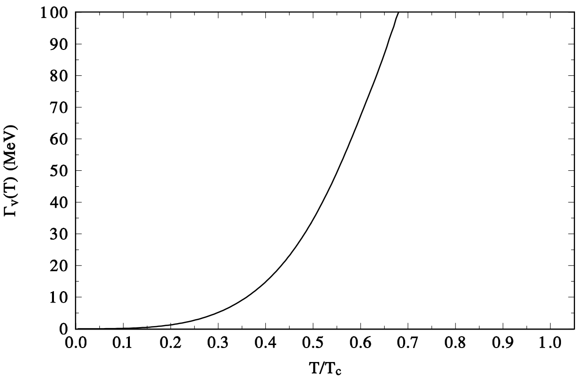

FIG. 7: The width $\Gamma_V(T)$ as a function of T/T_c.

Following the same procedure as for the pseudoscalar mesons (see Eq. (17)), the sum rules become

$$\frac{1}{\pi}\int_0^{s_0(T)} \frac{ds}{s^{N+1}}\, Im\,\Pi^{(1)}(s,T)|_{POLE} = \frac{1}{\pi}\int_{m_Q^2}^{s_0(T)} \frac{ds}{s^{N+1}}\, Im\,\Pi^{(1)}(s,T)|_{PQCD} + \varphi^{(N)}(T)|_{NP}\,, \tag{22}$$

where $\varphi^{(N)}(T)|_{NP}$ is given by

$$\varphi^{(N)}(T)|_{NP} = -\frac{A}{m_Q^{2N+4}}\Big[1 - \frac{<<\alpha_s G^2>>}{12\pi A} - \frac{(N+2)(N+3)}{4}\frac{M_0^2}{m_Q^2} + \frac{4}{81}(N+2)(20+N-N^2)\,\pi\alpha_s\rho\,\frac{<<\bar{q}q>>}{m_Q^3}\Big], \tag{23}$$

where we used once again $A = -m_Q << \bar{q}q >>$. Using the first three Hilbert moments to find the temperature dependence of the hadronic parameters, we obtain for the mass and the width of $D^*(2010)$ the results shown in Figs.6-7. The behaviour of the vector-meson leptonic decay constant is essentially the same as that of the pseudoscalar-meson shown in Fig.4. Similar results are found for the case of the beauty vector meson B^*.

4. CONCLUSIONS

The thermal behavior of pseudoscalar and vector meson decay constants, masses, and widths was obtained in the framework of Hilbert moment finite energy QCD sum rules. This behaviour is basically determined by the thermal light quark condensate on the QCD sector, and by the T-dependent continuum threshold on the hadronic sector. Normalizing the values at $T = 0$, and using the method of [9] for arbitrary masses, there follows a universal relation for the hadronic parameters as a function of T/T_c. Results show that the decay constants decrease with increasing temperature, vanishing at $T = T_c$, while the widths increase and diverge at the critical temperature. Such a behavior provides (analytical) evidence for quark-gluon deconfinement, and is in qualitative agreement with corresponding results obtained in the light-quark sector. Finally, pseudoscalar meson masses increase slightly with temperature by some $10-20\%$, while the vector masses decrease by $20-30\%$. Given the dramatic emergence of monotonically increasing widths $\Gamma(T)$, there is little if any significance of this temperature behaviour of the masses, i.e. the relevant signals for deconfinement are the vanishing of the leptonic decay constants and the divergence of the widths at $T = T_c$.

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