Living with Λ

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The return of the cosmological constant Λ to explain the accelerated stage of the universe has brought back many puzzles and misteries surrounding this constant. These enigmas touch upon the scales set by Λ, the cosmological aspect, the astrophysical effects of Λ and its role in quantum gravity. We will briefly discuss all these aspects putting the emphasis on issues not commonly known.

Keywords: Cosmological Constant; Astrophysics; Black body radiation

1. THE COSMOLOGY WITH Λ

Observation of standard candles like type Ia Supernova and other key observations in relation with Baryon acoustic Oscillations, Cosmic Microwave Background radiation and Large Scale Structure [1] led us to the conclusion that the expansion of the Universe as compared to the standard Friedmann model is accelerated. All evidence is in agreement with a positive cosmological constant. Indeed, in a homogeneous and isotropic universe the equations governing the behavior of the Universe are the Friedmann's equations (including Λ)

$$\begin{aligned} H^2 &\equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2 R_0^2} \qquad (1) \\ \frac{\ddot{a}}{a} &= -\frac{4\pi G_N}{3}(\rho + 3p(\rho)) + \frac{\Lambda}{3} \end{aligned}$$

together with the energy conservation:

$$\dot{\rho} = -3(\rho + p(\rho))\frac{\dot{a}}{a} \qquad (2)$$

For a spatially flat universe i.e $k = 0$ we can re-write one of the equations in the form

$$\Omega_m + \Omega_\Lambda = 1 \qquad (3)$$

where $\Omega_m = \frac{\rho}{\rho_{\rm crit}}$ with $\rho_{\rm crit} = \frac{3H^2}{8\pi G_N}$, $\Omega_\Lambda = \frac{\rho_{\rm vac}}{\rho_{\rm crit}}$ and $\Lambda = 8\pi G_N \rho_{\rm vac}$ The acceleration means that $\ddot{a} > 0$ which would be impossible to achieve without modifications (in our case the modification enters in form of Λ) of either the Einstein's tensor of the cosmological energy-momentum tensor. The cosmological equations (1) imply then

$$\Omega_\Lambda > \frac{2}{3},\ \Omega_m < \frac{1}{3} \qquad (4)$$

Curiously, these limiting values are close to the observational ones, namely $\Omega_{\Lambda 0} \simeq 0.7$ (subscripts with 0 refer to values at the present epoch). An intuitive understanding of the accelerated expansion is given by the Newtonian Limit. For a spherically symmetric object with mass M we can use the Schwarzschild-de Sitter metric

$$ds^2 = -e^{\nu(r)}dt^2 + e^{-\nu(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \qquad (5)$$

with

$$g_{00} = e^{\nu(r)} = 1 - \frac{2r_s}{r} - \frac{r^2}{3r_\Lambda^2},\ r_s = G_N M,\ r_\Lambda = \frac{1}{\sqrt{\Lambda}}$$

Using the well known connection between the zero-zero component of the metric and the gravitational potential Φ

$$g_{00} \simeq -(1 + 2\Phi) \qquad (6)$$

one obtains

$$\Phi(r) = -\frac{r_s}{r} - \frac{1}{6}\frac{r^2}{r_\Lambda^2} \qquad (7)$$

The last term plays the role of a repulsive external force! The Galilean spacetime gets replaced by Newton-Hooke spacetime where each two space points go apart due to the cosmological constant (this is the part of the cosmological expansion which survives the Newtonian limit). Although, our universe is governed by a particular value of $\rho_{\rm vac}$ close to the critical density, it is illustrative to consider other cases. What kind of universes are possible with Λ or what type of universes can we encounter in a multi-verse (according to the anthropic principle which would, in principle, allow a variety of universes). To this end, let us consider universes with $k \neq 0$ [2]. In such a case, one of our cosmological equations becomes $\Omega_{m0} + \Omega_{\Lambda 0} - \Omega_k = 1$ with $\Omega_k = \frac{k}{a^2 R_0^2 H^2}$. Since k is a constant we can use this equation to write

$$k = R_0^2 H_0^2 [\Omega_{m0} + \Omega_{\Lambda 0} - 1] \qquad (8)$$

We look now for values of Λ which give us a quasi-static universe for zero pressure ($p = 0$). The latter implies $\rho = \rho_0 a^{-3}$. Together with (8) and $\dot{a} = \ddot{a} = 0$ this allows us to search for the critical scale factor a and the value of Λ. After some elementary algebra the equation we obtain for this critical value is

$$x^3 - 3x + \frac{2(\Omega_{m0} - 1)}{\Omega_{m0}} = 0 \qquad (9)$$

with $x = \left(\frac{2\Omega_{\Lambda 0}}{\Omega_{m0}}\right)^{1/3}$. The roots can be translated into $\Lambda_{\rm crit}$. If

- $\Lambda > \Lambda_{\rm crit}$ there is no Big Bang
- $\Lambda \sim \Lambda_{\rm crit}$ we have a 'semi-static' coasting universe.

Indeed, if there is a multi-verse with different values of Λ, some of it members do not have an initial singularity.

2. THE SCALES OF Λ

The physical scales in a theory are of two kinds: the (fixed) parameters entering the theory and initial values. Often the order of magnitude of a result within the theory is a combination of both. For instance, the order of magnitude of a bound orbit in Newtonian theory is

$$R_{\rm orbit} \sim \frac{r_l^2}{r_s} \tag{10}$$

where r_l is proportional to the conserved angular momentum squared (determined by initial values) and $r_s = G_N M$ is half the Schwarzschild radius. If, as discussed in the next section, there is maximum angular velocity given in terms of the parameter of the theory, $R_{\rm orbit}$ becomes the maximal possible extension of abound orbit. Before concentrating on this issue in the next section, it is of some importance to first discuss the scales set by Λ itself or in combination with G_N (other scales are considered in [3]).

- Density:

$$\rho_{\rm vac}^{\rm obs} \simeq 0.7\rho_{\rm crit} \sim \rho_{\rm crit} \sim 2h_0^2 \times 10^{-29}{\rm gcm}^{-3}$$

$$\rho_{\rm Pl} = G_N^{-2} \simeq 5\times 10^{93}{\rm gcm}^{-3}$$

$$\rho_{\rm vac}/\rho_{Pl} \sim 10^{122}$$

- Length:

$$r_\Lambda = \frac{1}{\sqrt{\Lambda}} = \frac{1}{\sqrt{3}}\left(\frac{\rho_{\rm vac}}{\rho_{\rm crit}}\right)^{-1/2} H_0^{-1} \sim H_0^{-1}$$

$$r_{\rm Pl} = G_N^{1/2} \sim 1.5\times 10^{-33}{\rm cm}$$

$$\frac{r_\Lambda}{r_{\rm Pl}} \sim 10^{61}$$

- Large Mass:

$$M_\Lambda = \frac{1}{G_N\sqrt{\Lambda}} = 3.6\times 10^{22} h_0^{-1}\left(\frac{\rho_{\rm vac}}{\rho_{\rm crit}}\right)^{-1/2} M_\odot$$

$$m_{\rm Pl} = G_N^{-1/2} \sim 10^{19}{\rm GeV} \sim 2\times 10^{-5}{\rm g}$$

$$M_\Lambda/m_{\rm Pl} \sim 10^{60}$$

- Small Mass:

$$m_\Lambda = \sqrt{\Lambda} \sim 3\times 10^{-42}{\rm GeV}$$

$$m_{\rm Pl}/m_\Lambda \sim 10^{60}$$

$$M_\Lambda \gg m_{\rm Pl} \gg m_\Lambda$$

The above examples clearly demonstrate that gravity with Λ becomes, at least effectively, a two-scale theory with the two scales far apart from each other. What makes the exercise with the scales interesting is: (i) the fact that the small mass scale set by Λ is tiny; therefore questions regarding the mass of a black hole remnant which is usually assumed to be of the order of Planck mass are justified [4], (ii) a number of coincidences associated with these scales [5] and, last but not least, (iii) the question whether a meaningful combination of two scales appears in physics (see next section) [6]. Let us first look at the coincidences.

- Cosmological coincidence No.1: $\rho_{\rm vac} \sim \rho_{\rm crit}$.
- Cosmological coincidence No.2: r_Λ ca. extension of the visible Universe.
- Cosmological coincidence No.3: M_Λ ca. mass (energy) of the visible Universe.

These coincidences might be correlated, but it makes sense quote them separately. The coincidence is that we are living right now in a Universe whose mass, length and density scales are dominated by a constant Λ. After all, we could have been living in a different Universe or at an earlier/later epoch in the same Universe. Then neither the radius nor the density will be dominated by Λ. Ω_Λ which is now of order one, was different in the past and will deviate from one in the future. The transitions in both directions are quite steep [7]. This does not exhaust the list of coincidences. The next two are associated with acceleration (recall that Λ has been re-introduced in order to explain yet another acceleration crisis, namely of the whole universe).

- Observed Pioneer10 anomalous acceleration [8]:

$$a_0 \sim 10^{-8}{\rm cm\,s}^{-2} \sim (\Lambda/3)^{1/2} \tag{11}$$

(using $\hbar = c = 1$) which is another strange coincidence.

- MOND (Modified Newtonian Dynamics) of Milgrom as a rival theory to Dark Matter [9]:

$$\begin{aligned} a^2/a_0 &= MG_N r^{-2},\ a_0 \gg a \\ a = a_N &= MG_N r^{-2},\ a \gg a_0 \end{aligned} \tag{12}$$

where by yet another coincidence a_0 is the same number as in the anomalous acceleration of Pioneer10 above. An interpolating function $\mu(x)$ with $x = a/a_0$, and $\mu(x)a = a_N$ ca be for example $\mu(x) = x/(1+x)$. It follows

$$a = G_N M/2r^2 + \sqrt{G_N^2 M^2/4r^4 + a_0 G_N M/r^2} \tag{13}$$

or due to the above coincidence

$$a = G_N M/2r^2 + \sqrt{r_s^2/4r^4 + (r_s/R_\Lambda)/r^2},\ R_\Lambda \sim r_\Lambda \tag{14}$$

which for $r \gg \sqrt{r_s R_\Lambda}$ becomes $a \simeq \frac{\sqrt{r_s/R_\Lambda}}{r} = v^2/r \to v(r) \simeq {\rm const.}$ This way, a theory of scales of acceleration got converted into a theory of lengths scales where, by an coincidence, the scale of Λ enters.. Indeed, notice that $R_\Lambda \sim r_\Lambda$ and also that $(r_s r_\Lambda)^{1/2}$ is indeed of astrophysical order of magnitude since r_s is small compensating the large number of r_Λ. A similar combination of different scales (invoking Λ, of course) will appear in the next section and address the point (iii).

- The last coincidence is related to well-known number coincidences called Dirac's Large Numbers [10]:

$$10^{41} \sim m_p H_0^{-1} \sim m_{\rm Pl}^2/(m_p m_e)$$

With a non-zero cosmological constant we can add to this

$$10^{41} \sim \frac{m_p}{m_\Lambda}$$

which adds one more piece to the puzzle.

3. Λ IN ASTROPHYSICS

We will discuss here the motion of a test body in a Schwarzschild-de Sitter metric [6]. One would suspect that the inclusion of Λ is irrelevant in this astrophysical setting. Note, however, that there are now three length scales involved which, as already demonstrated, can combine to give a relevant physical scales. These scales are: r_s, r_Λ and $r_l = L$. The equation of motion of a test body with proper time τ in the Schwarzschild-de Sitter metric is given by

$$\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + U_{\rm eff} = \frac{1}{2}\left(\mathcal{E}^2 + \frac{L^2\Lambda}{3} - 1\right) \equiv C = \text{constant} \quad (15)$$

where $\mathcal{E}$ and L are conserved quantities defined by

$$\mathcal{E} = e^{\nu(r)}\frac{dt}{d\tau}, \; L = r^2\frac{d\Phi}{d\tau} \quad (16)$$

where Φ is the azimuthal angle. $U_{\rm eff}$ given by

$$U_{\rm eff}(r) = -\frac{r_s}{r} - \frac{1}{6}\frac{r^2}{r_\Lambda^2} + \frac{L^2}{2r^2} - \frac{r_s L^2}{2r^3} \quad (17)$$

is the analog of the effective potential potential in classical mechanics. It is clear that at a certain distance, the terms $-r_s/r$ and $-r^2/r_\Lambda^2$ will become comparable leading to a local maximum. This local maximum at $r_{\rm max}$ is a new feature due to Λ. Consider first the radial motion with $L = 0$. In such a case the 'new' maximum is located at

$$r_{\rm max} = \left(3 r_s r_\Lambda^2\right)^{1/3} \simeq 10^{-4}\left(\frac{M}{M_\odot}\right)^{1/3}\left(\frac{\rho_{\rm crit}}{\rho_{\rm vac}}\right)^{1/3} h_{70}^{-2/3}\text{Mpc} \quad (18)$$

Beyond $r_{\rm max}$, $U_{\rm eff}$ is a continuously decreasing function. This implies that $r_{\rm max}$ is the maximum value within which we can find bound solutions for the orbit of a test body. Therefore we would expect that $r_{\rm max}$ sets a relevant astrophysical scale. Of course, we are talking here about scales neglecting dynamical aspects of many body interactions, but no doubt $r_{\rm max}$ is roughly the scale to be set for bound systems. Indeed, for M equal one million solar masses (the mass of the black hole in the center of our galaxy) $r_{\rm max}$ comes out of the order of 10kpc which is the order of magnitude of the size of the galaxy. What happens in the case of $r_l = L \neq 0$?. To settle this issue at least non-relativistically, we look for a saddle point i.e.

$$\frac{dU_{\rm eff}}{dr} = \frac{d^2U_{\rm eff}}{dt^2} = 0 \quad (19)$$

This is to say, the local standard minimum merges with the local maximum to form a saddle point. Its worth noting that with $\Lambda = 0$ this is not possible The two conditions lead to the position of the saddle point and a condition on one parameter, say $x_l \equiv \frac{r_l^2}{(r_\Lambda)^2}$. For the latter one obtains

$$x_l^4 - \left(\frac{3r_s}{4r_\Lambda}\right)^4 x_l - 12\left(\frac{3r_s}{4r_\Lambda}\right)^6 = 0 \quad (20)$$

After handling hyperbolic functions and their inverses, going through complex numbers and their roots, one arrives at a simple expression

$$r_l^{\rm max} = r_l^{\rm crit} = 0.8(r_s^2 r_\Lambda)^{1/3} \quad (21)$$

Going back now to equation (10) from non-relativistic mechanics the expression for the order of magnitude of a bound orbit is

$$R_{\rm orbit} \sim \frac{r_l^2}{r_s} \to R_{\rm orbit}^{\rm max} \sim 0.5 r_{\rm max} \quad (22)$$

which is a satisfying result as it does not change the order of magnitude of the estimate with zero angular momentum. The scales $(r_s R_\Lambda)^{1/3}$, $(r_s r_\Lambda^2)^{1/3}$ and $(r_s^2 r_\Lambda)^{1/3}$ discussed above have a well-defined meaning within astrophysics and are, in spite of the large value of r_Λ of astrophysical order of magnitude due to the relative smallness of r_s. Hence Λ has also an impact on astrophysics. Other examples of effects of Λ on astrophysics have been discussed in [11]. We mention here the case of gravitational equilibrium. The new virial theorem which accounts for the cosmological constant takes the form

$$\frac{d^2 I_{jk}}{dt^2} = 4K_{jk} + 2W_{jk} + \frac{2}{3}I_{jk}\Lambda \quad (23)$$

It is very often more convenient to handle the scalar part of the above equation with I, K and W denoting the corresponding traces of the inertial, kinetic and gravitational tensor, respectively. Assuming equilibrium and, for simplicity, constant density, we can solve for the average velocity entering the kinetic part.

$$\langle v^2\rangle = \frac{|W|}{M} - \frac{8\pi}{3}G_N\frac{\rho_{\rm vac}}{M}I \quad (24)$$

To appreciate the effect of Λ let us assume a shape of the astrophysical object to be an ellipsoid. The mean velocity can be now written as

$$\begin{aligned}\langle v^2\rangle_{\rm ellipsoid} &= \frac{32\pi}{45M}\rho\rho_{\rm vac}a_1a_2a_3\left(a_1^2+a_2^2+a_3^2\right)\\ &\times \left(\frac{3}{4}\frac{\rho}{\rho_{vac}}\Gamma_{\rm ellipsoid} - 1\right) \quad (25)\end{aligned}$$

The prolate case ($a_1 = a_2 < a_3$, $e = \sqrt{1 - a_1^2/a_3^2}$) gives

$$\Gamma_{\text{prolate}} = \frac{\left(\frac{a_1}{a_3}\right)^3}{1+2\left(\frac{a_1}{a_3}\right)^2} \frac{\ln\left(\frac{1+e}{1-e}\right)}{e} \simeq \left(\frac{a_1}{a_3}\right)^3 \ln\left(\frac{1+e}{1-e}\right) \tag{26}$$

We can conclude that if the constant ρ/ρ_{crit} is, say, 10^3, it suffices for the ellipsoid to have the ratio $a_1/a_3 \sim 10^{-1}$ in order that the mean velocity of its components approaches zero which is an effect of Λ.

4. Λ IN GENERALIZED UNCERTAINTY PRINCIPLE

The name "Cosmological Constant" has instilled in physicists and cosmologists the impression that Λ is a variable which is relevant for cosmology and nothing else. As shown above, this is an erroneous conclusion. Once Λ is accepted as an integral part of the Einstein tensor, it will affect not only cosmology. Gravity becomes a two-scale theory. If so, we should not be prejudiced in asking if Λ will play a role in quantum gravity. Recall that Λ sets a mass scale m_Λ which is much smaller than the Planck mass which eventually could play affect the black hole remnant emerging at the end of Hawking radiation. One way to establish a black hole remnant is to consider a Generalized Uncertainty Principle (GUP) which we first briefly discuss for the case $\Lambda = 0$ [4]. Let $E = p$ be the photon's energy, then the acceleration of a test particle is

$$a_G = \frac{G_N E}{r^2} \tag{27}$$

As an order of magnitude estimate, we can write

$$\Delta x_G \simeq \frac{G_N E}{r^2} L^2 \simeq G_N E = G_N p \tag{28}$$

where we used $r \sim L$. Setting $\Delta p \sim p$ we arrive at the GUP relation

$$\Delta x \geq \frac{1}{2\Delta p} + G_N \Delta p \tag{29}$$

which generalizes the Heisenberg uncertainty relation by introducing gravity effects within. Identifying

$$\Delta x \sim 2r_s = 2G_N M \tag{30}$$

and

$$\Delta p \sim E \sim T \tag{31}$$

we can establish a relation between T and M via the GUP relation. We obtain

$$2G_N M = 2\frac{M}{m_{\text{Pl.}}^2} = \frac{1}{2T} + \frac{T}{m_{\text{Pl.}}^2} \tag{32}$$

Solving this equation for $T = T(M)$ and introducing a calibration factor $(2\pi)^{-1}$ (we do not expect to get all factors right by invoking arguments from quantum mechanical uncertainty relation alone) gives

$$T = \frac{1}{2\pi}\left(M - \sqrt{M^2 - m_{\text{Pl.}}^2/2}\right) \tag{33}$$

Two conclusions are in order:

- Equation (33) reduces to Hawking's radiation formula $T = 1/(8\pi G_N M)$ for large M. To derive it via the GUP relation is a nice and economic way displaying also the main quantum issues involved.

- There is, however, a difference as compared with the standard Hawking formula, namely the existence of a black hole remnant to ensure the existence of a positive T:

$$M > \mathcal{M}_{\text{min}} = \frac{m_{\text{Pl.}}}{\sqrt{2}}$$

Based on equation (7), we can repeat the very same steps used above for the $\Lambda \neq 0$ case. The result is [12]

$$\Delta x \geq \frac{1}{2\Delta p} + \frac{\Delta p}{m_{\text{pl}}^2} - \frac{1}{3}\frac{m_\Lambda^2}{\Delta p^3} \tag{34}$$

As before, we can also use (34) to analyze the black hole radiation. The steps involved are conceptually equivalent to the ones described above and we quote only the final result

$$M(T) = \frac{m_{\text{Pl}}^2}{4}\frac{1}{T} + \frac{T}{2} - \frac{1}{6}\frac{m_{\text{Pl}}^2 m_\Lambda^2}{T^3} \tag{35}$$

If the right hand side is a positive function, there exist a $\mathcal{T}_{\text{min}}$ to ensure that. Indeed, the right hand side of equation (35) will be positive if defining $s = T^2$ the following function is bigger than zero

$$F(s) \equiv s^2 + \frac{m_{\text{Pl}}^2}{2}s - \frac{1}{3}m_{\text{Pl}}^2 m_\Lambda^2 \geq 0 \tag{36}$$

Hence the zero of $F(s)$ can be identified with the minimum temperature. On the other hand for temperature much larger than $\mathcal{T}_{\text{min}}$ we can go back to equation (33) where $\mathcal{M}_{\text{min}}$ corresponds to $\mathcal{T}_{\text{max}}$ We can summarize this in one equation, namely

$$\mathcal{T}_{\text{max}} \sim m_{\text{Pl}} \geq T \geq \mathcal{T}_{\text{min}} \sim m_\Lambda \tag{37}$$

These limiting values of the temperature apply to black body radiation strictly speaking in black hole evaporation. It is amazing, however, that we can derive them also a different context of black body radiation (see next section). Indeed, $\mathcal{T}_{\text{max}}$ has been derived by Sakharov forty years ago using a different method [13]. The agreement tells us also that the generalized uncertainty relation with gravity and Λ is correct.

Furthermore, the existence of a maximum and minimum temperature guarantees automatically not only a minimum (remnant) black hole mass, but also a maximum value via

$$\mathcal{M}_{\min} = M(\mathcal{T}_{\max}) \sim m_{\rm Pl} \leq M_{\rm BH} \leq \mathcal{M}_{\max} = M(\mathcal{T}_{\min}) \sim M_\Lambda \quad (38)$$

We can ensure the existence of $\mathcal{M}_{\max}$ independently of $\mathcal{T}_{\min}$. For small temperature equation (35) can be approximated as

$$f(T) \equiv T^3 - \frac{m_{\rm Pl}^2}{4M}T^2 + \frac{m_{\rm Pl}^2 m_\Lambda^2}{3M} = 0 \quad (39)$$

Establishing the zeros of $f(T)$ means that we can construct the function $T(M)$. By inspection $f(T)$ has a local maximum at $T_\times = 0$ and a local minimum at $T_* = m_{\rm Pl}^2/6M$. The function $f(T)$ will have positive zeros only iff $f(T_*) \geq 0$. This is a condition on M which results in $M \leq \mathcal{M}_{\max} \sim M_\Lambda$ in agreement with (38).

5. Λ IN BLACK BODY RADIATION

As already mentioned in the preceding section, in 1966 Andrei Sakharov found a maximum temperature of black body radiation to be of the order of Planck mass [13]. He based his results on very general arguments and we could re-derive it via the Generalized Uncertainty Principle in equation (38). This result bears a certain importance. Combined with Hawking's formula for black hole evaporation $T = 1/(8\pi G_N M)$, it implies independently of GUP the existence of a black hole remnant of the order of Planck mass. Indeed, the value of the maximal temperature is $\sim 10^{32}\, K$ and has only a physical relevance in black hole evaporation. We can show yet a third way, to establish this important result. This method is then also suitable to include Λ. Because of the definition of proper time in General relativity, the g_{00} component of the metric should be positive definite [14]. We can regard also the mass M entering the Schwarzschild metric as energy which, in turn, can be replaced by energy density ρ i.e.

$$0 < g_{00} = 1 - \frac{2G_N M}{R} = 1 - (8\pi/3)G_N \rho R^2 \quad (40)$$

Hence

$$\rho < \frac{3}{8\pi}\frac{1}{G_N R^2} \quad (41)$$

Using the Stefan-Boltzmann law $\rho = \sigma T^4$ gives [15]

$$T^4 < \frac{3}{8\pi}\frac{1}{\sigma G_N R^2} \quad (42)$$

Finally, to get rid of the radius R we employ the quantum mechanical result for black body radiation, $R > 1/T$ [16]. The maximal temperature obtained this way, namely

$$T < T_{\max} = \sqrt{\frac{45}{8\pi^3}}m_{\rm Pl.} \quad (43)$$

is of the same order of magnitude as $\mathcal{T}_{\max}$ in equation (37). Repeating the same steps $\Lambda \neq 0$ i.e. for the Schwarzschild-de Sitter metric we can write

$$0 < \rho < \frac{3}{8\pi}\frac{m_{\rm Pl.}^2}{R^2} - \frac{1}{3}\frac{3}{8\pi}m_{\rm Pl.}^2 m_\Lambda^2 \quad (44)$$

These inequalities can be translated into

$$\frac{1}{\sqrt{3}}m_\Lambda = \mathcal{T}_{\min} < T < T_{\max} \quad (45)$$

confirming the existence of a minimal temperature in a different, more general, way. We draw the reader's attention that we established the results regarding

- The existence black hole remnant of the order of Planck mass.
- The existence of maximal black hole mass (due to Λ).
- The existence of minimal (due to Λ) and maximal temperature in black body radiation

in different ways and, as far as the order of magnitude is concerned all the results are consistent with each other.

6. CONCLUSION

We have discussed in this paper the different facets of the Cosmological Constant Λ. It has its origin (or rather motivation) in Cosmology to explain the acceleration of the Universe, but its effects are not limited to Cosmology alone. Indeed, as a second fundamental constant of gravity (at least effectively) it affects astrophysical results through combination of scales or enhancement mechanisms. Even, regarding semiclassical aspects, Λ limits the minimal temperature of black body radiation and the maximal mass of a black hole.

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