Removing the Wess Zumino Fields in the BFFT Formalism

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In this paper we give a method that removes the Wess Zumino fields of the BFFT formalism. Consequently, we derive a gauge invariant system written only in terms of the original second class phase space variables where important physical properties can be raised. Here, the Wess Zumino fields are considered only as auxiliary variables that permit us to reveal the underlying symmetries present in a second class system. We apply our formalism in three important and nontrivial constrained systems which are the Abelian Proca model, the Chern Simons Proca theory and the reduced SU(2) Skyrme model.

I. INTRODUCTION

The BFFT formalism[1, 2] converts second class constrained systems into first class ones by enlarging the original second class phase space variables with the Wess Zumino (WZ) fields. In order to guarantee that the same degrees of freedom are maintained with the original second class system, the WZ fields are introduced in equal number to the number of second class constraints. The introduction of the WZ fields modifies the second class constraints and the second class Hamiltonian in order to satisfy a first class algebra. Thus, the presence of the WZ fields allows us to obtain a gauge invariant model where symmetries are revealed from the original second class system. The symmetries permit us to describe the physical properties in a more general way. For this reason we can disclose important and interesting physical results. As an example, we can cite the case of a noncommuting second class algebra resulting from a nonstandard gauge condition[3, 4].

The purpose of this paper is to give a method in order to remove the WZ fields of the BFFT formalism and, consequently, to obtain a gauge invariant system written only in terms of the original second class phase space variables. In our formalism, the WZ fields are treated as auxiliary variables that permit us to build a first class system from the second class one, and, consequently, to enforce symmetries. As an additional step, we replace the WZ fields by convenient functions that lead us to derive a first class system written only in terms of the initial second class phase space variables. As we will see, we can choose gauge symmetry generators and, consequently, gauge fixing conditions that allow us to reveal interesting physical properties. Since many important constrained systems have only two second class constraints, so, in this paper, we describe our formalism only for systems with two second class constraints without any loss of generality.

In order to clarify the exposition of the subject, this paper is organized as follows: In Section II we give a short review of the BFFT formalism. In Section III, we present the formalism. In Section IV, we apply the formalism to the Abelian Proca model[5], the Chern Simons Proca theory (CSP)[6]

and the collective coordinates expansion of the SU(2) Skyrme model[7, 8]. These three physical systems are important nontrivial examples of the second class constrained systems. The Abelian Proca model is a four dimensional field theory which describes electromagnetism with massive photon field. The Chern Simons Proca theory concerns with the interaction of a charged particle with magnetic field and it is known that this model exhibits a noncommutative algebra[9]. The Skyrme model is a nonlinear effective field theory which describes hadrons physics and its quantization is obtained with quantum mechanics on a curved space. Here, we would like to remark that, using our formalism (embedding techniques), we have obtained a noncommutative Skyrmions system, a new result which is derived from a particular gauge condition. In Section V, we make our concluding remarks.

II. A BRIEF REVIEW OF THE BFFT FORMALISM

As we have mentioned in the introduction, the BFFT formalism converts second class system into first class one by adding WZ fields to the original second class system. All the second class constraints and the second class Hamiltonian are changed in order to satisfy a first class algebra.

Consider the original phase space variables as (q_i, p_i) where a constrained system has two second class constraints, T_α, $\alpha = 1, 2$, obeying the algebra

$$\{T_\alpha, T_\beta\} = \Delta_{\alpha\beta}, \tag{1}$$

where the matrix $\Delta_{\alpha\beta}$ has a nonvanishing determinant. First, in the BFFT formalism, the two first class constraints are constructed by the following expansion

$$\tilde{T}_\alpha(q_i, p_i, \Phi_\alpha) = T_\alpha + \sum_{m=1}^{\infty} T_\alpha^{(m)}, \tag{2}$$

where Φ_α are the WZ fields satisfying the algebra

$$\{\Phi_\alpha, \Phi_\beta\} = \omega_{\alpha\beta}, \tag{3}$$

being $\omega_{\alpha\beta}$ an antisymmetric matrix. $T_\alpha^{(m)}$ are the correction terms which are powers of Φ_α, i.e., $T_\alpha^{(m)} \sim \Phi_\alpha^{(m)}$. The first class constraints must satisfy the boundary condition

$$\tilde{T}_\alpha(q_i, p_i, 0) = T_\alpha^{(0)} = T_\alpha. \tag{4}$$

From the Abelian first class algebra

$$\{\tilde{T}_\alpha, \tilde{T}_\beta\} = 0, \tag{5}$$

we obtain recursive equations which determine the correction terms $T_\alpha^{(m)}$. As an example, we have a basic equation in the lowest order

$$\Delta_{\alpha\beta} + X_{\alpha\gamma}\omega^{\gamma\lambda}X_{\lambda\beta} = 0, \tag{6}$$

and the first order correction term written as

$$T_\alpha^{(1)} = X_{\alpha\beta}(q_i, p_i)\Phi^\beta. \tag{7}$$

The matrices $\omega_{\alpha\beta}$ and $X_{\alpha\beta}$ in Eqs.(3) and (6), which are the inherent arbitrariness of the BFFT formalism, can be chosen with the aim of obtaining algebraic simplifications in the determination of the correction terms $T_\alpha^{(m)}$.

In a similar way, the gauge invariant Hamiltonian is obtained by the expansion

$$\tilde{H} = H_c + \sum_{m=1}^{\infty} H^{(m)}, \tag{8}$$

where H_c is the canonical second class Hamiltonian and the correction terms, $H^{(m)}$, are powers of Φ_α, i.e., $H^{(m)} \sim \Phi_\alpha^{(m)}$. Also, from the Abelian first class algebra

$$\{\tilde{H}, \tilde{T}_\alpha\} = 0, \tag{9}$$

we have recursive equations which determine the correction terms $H^{(m)}$ and, consequently, the gauge invariant Hamiltonian.

III.   REMOVING THE WESS ZUMINO FIELDS

Our formalism begins by choosing, as example, $\tilde{T}_1$, one of the two first class constraints, Eq.(2), to be the extended gauge symmetry generator of the theory

$$\tilde{T} = \tilde{T}_1. \tag{10}$$

The other first class constraint, $\tilde{T}_2$, will be discarded. To eliminate the WZ auxiliary fields, Φ_α, we must find functions(an appropriate functional) for the WZ fields written only in terms of the original second class phase space variables (q_i, p_i), namely

$$\Phi_\alpha = F_\alpha(q_i, p_i). \tag{11}$$

At this stage, two conditions must be satisfied: the first one determines that the algebraic form of the functions $F_\alpha(q_i, p_i)$ must have the same infinitesimal gauge transformations given by Φ_α, i.e.

$$\delta\Phi_\alpha = \delta F_\alpha(q_i, p_i), \tag{12}$$

where

$$\delta\Phi_\alpha = \varepsilon\{\Phi_\alpha, \tilde{T}\}, \tag{13}$$

and

$$\delta F_\alpha = \varepsilon\{F_\alpha, T_1\}, \tag{14}$$

being ε an infinitesimal parameter and T_1 the second class constraint that builds the extended gauge symmetry generator; the second condition imposes that when we make the constraint surface $T_2 = 0$, where T_2 is the original second class constraint that builds the discarded first class constraint, the function $F_\alpha(q_i, p_i)$ must vanish, i.e.

$$T_2 = 0 \Rightarrow F_\alpha(q_i, p_i) = 0. \tag{15}$$

With this condition we must recover the second class Hamiltonian, H_c. The relation (15) is the boundary condition of the formalism or the gauge fixing constraint that reduces our gauge invariant model to the second class one. This condition ensures the equivalence of the gauge invariant model obtained by our prescription and the original second class theory that has been embedded by the BFFT formalism[10].

It is important to mention that we have arbitrariness in our prescription because we need to select one of the two first class constraints, Eqs.(2), to be the extended gauge symmetry generator. In addition, the two conditions exposed above, at first, do not determine completely the algebraic form of the function $F_\alpha(q_i, p_i)$. However, arbitrariness, in principle, occurs in all methods that embed second class constrained systems and can be useful to unveil important physical properties of the models.

Substituting Eq.(11) in the BFFT first class Hamiltonian, Eq.(8), we obtain a gauge invariant Hamiltonian, $\tilde{H}$, written only as a function of the original second class phase space variables (q_i, p_i), satisfying the first class algebra

$$\{\tilde{H}, T_1\} = 0, \tag{16}$$
$$\{T_1, T_1\} = 0, \tag{17}$$

where now the second class T_1 becomes the only gauge symmetry generator of the theory. The relations (16) and (17) show that, in some situations,due to the specific arbitrariness of our prescription we can achieved similar results of the gauge unfixing formalism[5, 10, 11] .

IV.   APPLICATIONS OF THE FORMALISM

A.   The Abelian Proca model

The Abelian Proca model is a four dimensional field theory with the corresponding Lagrangian density given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A^\mu A_\mu, \tag{18}$$

where $g_{\mu\nu} = diag(+,-,-,-)$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The explicit mass term breaks the gauge invariance and, consequently, we have a second class constrained system. The primary constraint is

$$T_1 = \pi_0 \approx 0. \tag{19}$$

By using the Legendre transformation we obtain the canonical Hamiltonian written as

$$H_c = \int d^3x \ \mathcal{H}_c =$$

$$\int d^3x \left[\frac{1}{2}\pi_i\pi_i + \frac{1}{4}F_{ij}F_{ij} - \frac{m^2}{2}(A_0^2 - A_i^2) + A_0(\partial_i\pi_i) \right], \tag{20}$$

with $\pi_i = \frac{\partial L}{\partial \dot{A}_i} = -F_{0i}$. From the temporal stability condition of the primary constraint, Eq.(19), we get the secondary constraint

$$T_2 = -\partial_i\pi_i + m^2 A_0 \approx 0. \tag{21}$$

We observe that no further constraints are generated via this iterative procedure. Then, T_1 and T_2 are the total second class constraints of the Abelian Proca model.

Using the BFFT formalism to convert this second class system into first class one, we obtain the two first class constraints and the gauge invariant Hamiltonian written as[5]

$$\tilde{T}_1 = T_1 + m^2\theta, \tag{22}$$
$$\tilde{T}_2 = T_2 + \pi_\theta, \tag{23}$$
$$\tilde{H} = H_c + \int d^3x \left[\frac{\pi_\theta^2}{2m^2} + \frac{m^2}{2}(\partial_i\theta)^2 - m^2\theta\partial_i A_i \right], \tag{24}$$

where the extra canonical pair of fields θ and π_θ satisfy the algebra $\{\theta(x), \pi_\theta(y)\} = \varepsilon\delta(x-y)$. The first class constraints and the first class Hamiltonian obey the following Poisson brackets

$$\{\tilde{T}_1, \tilde{T}_2\} = 0, \tag{25}$$
$$\{\tilde{T}_1, \tilde{H}\} = \tilde{T}_2, \tag{26}$$
$$\{\tilde{T}_2, \tilde{H}\} = 0. \tag{27}$$

In order to apply our formalism, we choose the first class constraint, Eq.(23), to be the extended gauge symmetry generator

$$\tilde{T} = \tilde{T}_2 = T_2 + \pi_\theta = -\partial_i\pi_i + m^2 A_0 + \pi_\theta. \tag{28}$$

The infinitesimal gauge transformations of the WZ fields generated by the extended gauge symmetry generator $\tilde{T}$ are

$$\delta\theta = \varepsilon\{\theta, -\partial_i\pi_i + m^2 A_0 + \pi_\theta\} =$$

$$\varepsilon\{\theta(x), \pi_\theta(y)\} = \varepsilon\delta(x-y), \tag{29}$$

$$\delta\pi_\theta = \varepsilon\{\pi_\theta, -\partial_i\pi_i + m^2 A_0 + \pi_\theta\} = \varepsilon\{\pi_\theta(x), \pi_\theta(y)\} = 0. \tag{30}$$

From the infinitesimal gauge transformations, Eq.(30), we can choose a representation for π_θ as

$$\pi_\theta = 0. \tag{31}$$

A representation for θ can be determined by imposing the first class strong equation, Eq.(22),

$$T_1 + m^2\theta = 0 \Rightarrow \theta = -\frac{1}{m^2}T_1 = -\frac{1}{m^2}\pi_0. \tag{32}$$

As we can observe, the function for θ satisfies the infinitesimal gauge transformation, Eq.(29),

$$\delta\theta = \varepsilon\{\theta, \tilde{T}\} = \varepsilon\left\{ -\frac{1}{m^2}\pi_0, T_2 \right\} =$$

$$\varepsilon\left\{ -\frac{1}{m^2}\pi_0, -\partial_i\pi_i + m^2 A_0 \right\} = \varepsilon\delta(x-y). \tag{33}$$

Substituting the WZ formulas, Eqs.(31) and (32), in the extended first class Hamiltonian, Eq.(24), we get a first class Hamiltonian written only in terms of the original second class fields

$$\tilde{H} = H_c + \int d^3x \left[\pi_0\partial_i A_i + \frac{1}{2m^2}(\partial_i\pi_0)^2 \right], \tag{34}$$

or

$$\tilde{H} = H_c + \int d^3x \left[\pi_0\partial_i A_i - \frac{1}{2m^2}\pi_0\partial_i^2\pi_0 \right], \tag{35}$$

being the only gauge symmetry generator

$$T_2 = -\partial_i\pi_i + m^2 A_0, \tag{36}$$

which satisfies an Abelian first class algebra

$$\{T_2, T_2\} = 0, \tag{37}$$
$$\{\tilde{H}, T_2\} = 0. \tag{38}$$

Here, we would like to comment that the first class Hamiltonian, Eq.(35), is identical to the gauge invariant Hamiltonian which was derived by using the GU formalism[5]. Then, this result confirms the validity of our formalism.

B. The Chern Simons Proca theory

The Chern Simons Proca theory (CSP) describes a charged particle constrained to move on a two dimensional plane, interacting with a constant magnetic field B which is orthogonal to the plane. In the vanishing mass limit (infrared limit), the Lagrangian that governs the dynamics is

$$L = \frac{B}{2} q_i \varepsilon_{ij} \dot{q}_j - \frac{k}{2} q_i q_i, \tag{39}$$

where k is a constant and $\varepsilon_{12} = 1$. The CSP model is a second class constrained system with the two constraints given by

$$T_i = p_i + \frac{B}{2} \varepsilon_{ij} q_j, \ \ i = 1, 2 \tag{40}$$

where p_i are the canonical momenta ($p_i = \frac{\partial L}{\partial \dot{q}_i}$), and the Poisson brackets between the second class constraints read as

$$\{T_i, T_j\} = B \varepsilon_{ij}. \tag{41}$$

From the Legendre transformation we obtain the second class Hamiltonian

$$H_c = p_i \dot{q}_i - L = \frac{k}{2} q_i q_i. \tag{42}$$

Using the BFFT formalism to convert this second class system into first class one, we get the two first class constraints and the gauge invariant Hamiltonian written as[12]

$$\tilde{T}_1 = T_1 + \sqrt{B} c_1, \tag{43}$$
$$\tilde{T}_2 = T_2 + \sqrt{B} c_2, \tag{44}$$
$$\tilde{H} = \frac{k}{2} \left[q_i q_i + \frac{2}{\sqrt{B}} \varepsilon_{ij} c_i q_j + \frac{1}{B} c_i c_i \right], \tag{45}$$

where c_1 and c_2 are the WZ variables. By construction, we have a first class algebra

$$\{\tilde{T}_i, \tilde{T}_j\} = 0, \tag{46}$$
$$\{\tilde{H}, \tilde{T}_i\} = 0, \tag{47}$$

where the WZ variables satisfy the following Poisson brackets

$$\{c_i, c_j\} = \varepsilon_{ji}. \tag{48}$$

At this point, we begin our formalism by choosing the first class constraint, Eqs.(43), to be the extended gauge symmetry generator

$$\tilde{T} = \tilde{T}_1 = T_1 + \sqrt{B} c_1 = p_1 + \frac{B}{2} q_2 + \sqrt{B} c_1. \tag{49}$$

The infinitesimal gauge transformations of the WZ variables generated by the extended gauge symmetry generator $\tilde{T}$ are

$$\delta c_1 = \varepsilon \{c_1, \tilde{T}\} = \varepsilon \{c_1, p_1 + \frac{B}{2} q_2 + \sqrt{B} c_1\} = 0, \tag{50}$$
$$\delta c_2 = \varepsilon \{c_2, \tilde{T}\} = \varepsilon \{c_2, p_1 + \frac{B}{2} q_2 + \sqrt{B} c_1\} = \varepsilon \sqrt{B}. \tag{51}$$

From the infinitesimal gauge transformations, Eq.(50), we can choose a representation for c_1 as

$$c_1 = 0. \tag{52}$$

A representation for c_2 can be determined by imposing the first class strong equation, Eq.(44)

$$T_2 + \sqrt{B} c_2 = 0 \ \Rightarrow \ c_2 = -\frac{1}{\sqrt{B}} T_2. \tag{53}$$

As we can see, the function for c_2 satisfies the infinitesimal gauge transformation, Eq.(51),

$$\delta c_2 = \varepsilon \{c_2, \tilde{T}\} = \varepsilon \left\{ -\frac{1}{\sqrt{B}} T_2, \tilde{T} \right\} =$$
$$\varepsilon \left\{ -\frac{1}{\sqrt{B}} T_2, T_1 \right\} = \varepsilon \sqrt{B}. \tag{54}$$

Then, substituting the functions for c_1 and c_2, Eqs.(52) and (53), in the first class Hamiltonian, Eq.(45), we obtain a gauge invariant Hamiltonian written only in terms of the original second class phase space variables

$$\tilde{H} = \frac{k}{2} q_i q_i + \frac{k}{B} q_1 T_2 + \frac{k}{2B^2} T_2^2 =$$
$$\frac{k}{2} \left[q_2 q_2 + \left(q_1 + \frac{T_2}{B} \right)^2 \right], \tag{55}$$

being the only gauge symmetry generator

$$T_1 = p_1 + \frac{B}{2} q_2, \tag{56}$$

which satisfies an Abelian first class algebra

$$\{T_1, T_1\} = 0, \tag{57}$$
$$\{\tilde{H}, T_1\} = 0. \tag{58}$$

We can observe that when we make $T_2 = p_2 - \frac{B}{2}q_1 = 0$ (the second class constraint that builds the discarded first class constraint, condition two of the formalism) the first class Hamiltonian, Eq.(55), reduces to the CSP second class Hamiltonian, Eq.(42). This result guarantees the equivalence of our CSP gauge invariant system and the original CSP second class model.

C. The reduced Skyrme model or the Skyrme model expanded in terms of the SU(2) collective coordinates

The Skyrme model describes baryons and their interactions through soliton solutions of the nonlinear sigma model type Lagrangian given by

$$L = \int d^3x \left[\frac{f_\pi^2}{4} Tr(\partial_\mu U \partial^\mu U^+) + \frac{1}{32e^2} Tr[U^+ \partial_\mu U, U^+ \partial_\nu U]^2 \right], \tag{59}$$

where f_π is the pion decay constant, e is a dimensionless parameter and U is a SU(2) matrix. Performing the collective semiclassical expansion[8] just substituting $U(r,t)$ by $U(r,t) = A(t)U_0(r)A^+(t)$ in Eq. (59), being A a SU(2) matrix, we obtain

$$L = -M + \lambda Tr[\partial_0 A \partial_0 A^{-1}], \tag{60}$$

where M is the soliton mass and λ is the moment of inertia[8]. The SU(2) matrix A can be written as $A = a_0 + ia \cdot \tau$, where τ_i are the Pauli matrices, and satisfies the spherical constraint relation

$$T_1 = a_i a_i - 1 \approx 0, \; i = 0, 1, 2, 3. \tag{61}$$

The Lagrangian (60) can be read as a function of a_i as

$$L = -M + 2\lambda \dot{a}_i \dot{a}_i. \tag{62}$$

Calculating the canonical momenta

$$\pi_i = \frac{\partial L}{\partial \dot{a}_i} = 4\lambda \dot{a}_i, \tag{63}$$

and using the Legendre transformation, the canonical Hamiltonian is computed as

$$\begin{aligned} H_c = \pi_i \dot{a}_i - L &= M + 2\lambda \dot{a}_i \dot{a}_i \\ &= M + \frac{1}{8\lambda} \sum_{i=0}^{3} \pi_i \pi_i. \end{aligned} \tag{64}$$

From the temporal stability condition of the spherical constraint, Eq.(61), we get the secondary constraint

$$T_2 = a_i \pi_i \approx 0. \tag{65}$$

We observe that no further constraints are generated via this iterative procedure. T_1 and T_2 are the second class constraints with

$$\{T_1, T_2\} = 2a_i a_i. \tag{66}$$

Using the BFFT formalism we obtain the first class constraints written as[13]

$$\tilde{T}_1 = T_1 + b_1 = a_i a_i - 1 + b_1, \tag{67}$$
$$\tilde{T}_2 = T_2 - a_i a_i b_2 = a_i \pi_i - a_i a_i b_2, \tag{68}$$

which satisfy an Abelian first class algebra

$$\{\tilde{T}_1, \tilde{T}_2\} = 0, \tag{69}$$

with the WZ variables obeying the following Poisson bracket relation

$$\{b_i, b_j\} = 2\varepsilon_{ij}, \; i, j = 1, 2. \tag{70}$$

The first class Hamiltonian is given by

$$\begin{aligned} \tilde{H} &= M + \frac{1}{8\lambda} \frac{a_i a_i}{a_i a_i + b_1} \pi_j \pi_j - \frac{1}{4\lambda} \frac{a_i a_i b_2}{a_i a_i + b_1} a_j \pi_j + \frac{(a_i a_i)^2 (b_2)^2}{a_i a_i + b_1} \\ &= M + \frac{1}{8\lambda} \frac{a_i a_i}{a_i a_i + b_1} [\pi_j - b_2 a_j]^2, \end{aligned} \tag{71}$$

which also satisfies an Abelian first class algebra

$$\{\tilde{H}, \tilde{T}_\alpha\} = 0, \ \ \alpha = 1, 2. \tag{72}$$

The first class Lagrangian is written as

$$L = -M + 2\lambda \frac{\dot{a}_i \dot{a}_i}{a_j a_j} - \frac{\lambda}{2} \frac{\dot{b}_1 \dot{b}_1}{(a_j a_j)^2}. \tag{73}$$

At this stage, we are ready to apply our formalism. We begin by choosing the first class constraint, Eq.(67), to be the extended gauge symmetry generator

$$\tilde{T} = \tilde{T}_1 = T_1 + b_1 = a_i a_i - 1 + b_1. \tag{74}$$

The infinitesimal gauge transformations of the WZ variables generated by the extended gauge symmetry generator $\tilde{T}$ are

$$\delta b_1 \ = \ \varepsilon\{b_1, \tilde{T}\} = \varepsilon\{b_1, a_i a_i - 1 + b_1\} = 0, \tag{75}$$
$$\delta b_2 \ = \ \varepsilon\{b_2, \tilde{T}\} = \varepsilon\{b_2, a_i a_i - 1 + b_1\} = -2\varepsilon. \tag{76}$$

From the infinitesimal gauge transformations, Eq.(75), we can choose a representation for b_1 as

$$b_1 = 0. \tag{77}$$

A representation for b_2 can be determined by imposing the first class strong equation, Eq.(68)

$$a_i \pi_i - a_i a_i b_2 = 0 \Rightarrow b_2 = \frac{a_i \pi_i}{a_j a_j}. \tag{78}$$

As we can see, the function for b_2 satisfies the infinitesimal gauge transformation, Eq.(76),

$$\delta b_2 = \varepsilon\{b_2, \tilde{T}\} = \varepsilon\{\frac{a_i \pi_i}{a_j a_j}, T_1\} = \varepsilon\{\frac{a_i \pi_i}{a_j a_j}, a_i a_i - 1\} = -2\varepsilon. \tag{79}$$

Then, substituting the functions for b_1 and b_2, Eqs.(77) and (78), in the first class Hamiltonian, Eq.(71), we obtain a gauge invariant Hamiltonian written only in terms of the original second class phase space variables

$$\tilde{H} = M + \frac{1}{8\lambda} \left[\pi_j \pi_j - \frac{(a_i \pi_i)^2}{a_j a_j} \right] =$$

$$M + \frac{1}{8\lambda} \left[\pi_j \pi_j - \frac{(T_2)^2}{a_j a_j} \right], \tag{80}$$

with the only gauge symmetry generator of the theory

$$T_1 = a_i a_i - 1, \tag{81}$$

which satisfies an Abelian first class algebra

$$\{T_1, T_1\} = 0, \tag{82}$$
$$\{\tilde{H}, T_1\} = 0. \tag{83}$$

Note that when we make the second class constraint equal to zero, $T_2 = a_i \pi_i = 0$, we observe that the first class Hamiltonian, Eq.(80), reduces to the original second class Hamiltonian, Eq.(64). This result ensures the equivalence of our gauge invariant model and the original second class system.

The gauge invariant Hamiltonian, Eq.(80), can be written as

$$\tilde{H} = M + \frac{1}{8\lambda} \pi_i M^{ij} \pi_j, \tag{84}$$

where the phase space metric M^{ij} given by

$$M^{ij} = \delta^{ij} - \frac{a^i a^j}{a_i a_i}, \tag{85}$$

is a singular matrix which has a_i as an eigenvector with null eigenvalue, namely,

$$a_i M^{ij} = 0. \tag{86}$$

Then, due to the fact that the matrix M is singular, in principle, it is not possible to obtain the first class Skyrmion Lagrangian written only in terms of the original second phase space variables with the gauge symmetry generator being T_1, Eq.(81).

Now we choose the other first class constraint, Eq.(68), to be the extended gauge symmetry generator of theory

$$\tilde{T} = \tilde{T}_2 = T_2 - a_i a_i b_2 = a_i \pi_i - a_i a_i b_2. \tag{87}$$

The infinitesimal gauge transformations of the WZ variables generated by this extended gauge symmetry generator $\tilde{T}$ are

$$\delta b_1 \ = \ \varepsilon\{b_1, \tilde{T}\} = \varepsilon\{b_1, a_i \pi_i - a_i a_i b_2\} = -2\varepsilon a_i a_i, \tag{88}$$
$$\delta b_2 \ = \ \varepsilon\{b_2, \tilde{T}\} = \varepsilon\{b_2, a_i \pi_i - a_i a_i b_2\} = 0. \tag{89}$$

From the infinitesimal gauge transformations, Eq.(89), we can choose a representation for b_2 as

$$b_2 = 0. \tag{90}$$

The use of the condition (90) in the extended gauge symmetry generator, Eq. (87), ensures that the infinitesimal gauge transformations of the original phase space variables (a_j, π_j) are given by $T_2 = a_i \pi_i$. A representation for b_1 can be obtained by imposing the first class strong equation, Eq.(67)

$$a_i a_i - 1 + b_1 = 0$$
$$\Rightarrow b_1 = 1 - a_i a_i. \tag{91}$$

The function for b_1 satisfies the infinitesimal gauge transformation, Eq.(88),

$$\delta b_1 = \varepsilon\{b_1, \tilde{T}\} = \varepsilon\{1 - a_i a_i, a_i \pi_i\} = -2\varepsilon. \tag{92}$$

Substituting the functions for b_1 and b_2, Eqs.(90) and (91), in the first class Hamiltonian, Eq.(71), we get a gauge invariant Hamiltonian written only in terms of the original second class phase space variables

$$\tilde{H} = M + \frac{1}{8\lambda} a_i a_i \pi_j \pi_j, \tag{93}$$

with the only gauge symmetry generator of the theory

$$T_2 = a_i \pi_i, \tag{94}$$

which satisfies an Abelian first class algebra

$$\{T_2, T_2\} = 0, \tag{95}$$
$$\{\tilde{H}, T_2\} = 0. \tag{96}$$

Again, when we make the second class constraint equal to zero, $T_1 = a_i a_i - 1 = 0$, the gauge invariant Hamiltonian Eq.(93) reduces to the original second class Hamiltonian, Eq.(64).

The first class Skyrmion Lagrangian can be deduced by performing the inverse Legendre transformation

$$L = \pi_i \dot{a}_i - \tilde{H}, \tag{97}$$

where the momentum π_i is eliminated by using the Hamilton equation of motion

$$\dot{a}_i = \{a_i, \tilde{H}\} = \frac{1}{4\lambda} a_j a_j \pi_i. \tag{98}$$

Using relation (98) in Eq.(97) we derive the first class Lagrangian written as

$$L = -M + 2\lambda \frac{\dot{a}_i \dot{a}_i}{a_j a_j}, \tag{99}$$

with the infinitesimal gauge variation given by $\delta a_i = \varepsilon a_i$, where ε is a constant. Notice that it is only possible to derive this first class Lagrangian, Eq.(99), if we adopt the symmetry generator of the theory as $T_2 = a_i \pi_i$, Eq.(94). Moreover, using the relations (91) and (98) and imposing the constraint surface $a_i \pi_i = 0$, we obtain $\dot{b}_1 = 0$. Consequently, we can observe that the BFFT first class Lagrangian (73) reduces to

the gauge invariant Lagrangian, Eq.(99). This important result also confirms the consistency of our formalism.

Along the text we have mentioned the property that we have only one gauge symmetry generator. Thus from this property we can obtain a second class system from the gauge condition

$$T_{1\theta} = a_i a_i - \theta \pi_i \pi_i - 1, \tag{100}$$

where θ is a constant. $T_{1\theta}$ is a deformed spherical constraint with the Poisson bracket

$$\{T_2, T_{1\theta}\} = -2a_i a_i - 2\theta \pi_i \pi_i. \tag{101}$$

It is not difficult to observe that no additional constraints are generated by imposing the deformed spherical condition relation (100). T_2 and $T_{1\theta}$ are now the total second class constraints of the model. Using the Dirac brackets formula [3, 14]

$$\{A, B\}_{DB} = \{A, B\} +$$

$$\frac{1}{\{T_2, T_{1\theta}\}} \left(\{A, T_2\}\{T_{1\theta}, B\} - \{A, T_{1\theta}\}\{T_2, B\} \right), \tag{102}$$

we obtain the commutation relations between the collective coordinates operators upon quantization

$$\{a_i, a_j\}_{DB} = \theta \frac{a_j \pi_i - a_i \pi_j}{a^2 + \theta \pi^2}, \tag{103}$$

$$\{a_i, \pi_j\}_{DB} = \delta_{ij} - \frac{a_i a_j + \theta \pi_i \pi_j}{a^2 + \theta \pi^2}, \tag{104}$$

$$\{\pi_i, \pi_j\}_{DB} = \frac{a_j \pi_i - a_i \pi_j}{a^2 + \theta \pi^2}. \tag{105}$$

Note that if we make $\theta = 0$ we recover the usual algebra of this collective coordinates operators[15]. Therefore, it is important to observe that using a specific embedding procedure, choosing a particular gauge condition and applying the Dirac bracket quantization we get a noncommuting collective coordinates operators, relation(103). This new result is only derived if we have used $T_2 = a_i \pi_i$ as the symmetry generator in the first class Skyrmion system.

V. CONCLUSIONS

In this paper, we give some prescriptions in order to eliminate the WZ fields of the BFFT formalism. The WZ variables are considered only as auxiliary tools that enforce symmetries in an initial second class constrained system. Then, after embedding a second class system by the BFFT formalism, we substitute the WZ fields by convenient functions and, consequently, we derive a gauge invariant Hamiltonian written only in terms of the original second class phase space variables. This first class system has one gauge symmetry generator. It is an advantage because we have the possibility to select one

gauge condition which can reveal important physical properties. In all first class conversion formalisms there are ambiguities in the construction of the first class constraints and the gauge invariant Hamiltonian[2] and this situation is not different in our prescription. For example, the choices of the extended gauge symmetry generator (and, consequently, the gauge symmetry generator of the theory) and the key function $F_\alpha(q_i, p_i)$, Eq.(11), are arbitrary. However, these different possible options can be used in order to unveil important physical results or some choices can be related to obtain benefits in the algebraic calculations.

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