Nonlinear Resonance in Bouncing Universes

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The dynamics of closed LFRW universes with a massive inflaton field is examined where Friedmann equations are corrected by the introduction of a potential term arising from quantum gravity corrections to cosmological scenarios near the singularity. This extra term implements nonsingular bounces in the early evolution of the universe. For certain windows in the parameter space (labeled by the scalar field mass and the conserved Hamiltonian), phenomena of nonlinear resonance take place. Nonlinear resonance may induce the destruction of KAM tori that trap the inflaton, leading to a rapid growth of the scale factor and consequent escape of the universe into inflation. We make a complete analysis of the nonlinear resonance phenomena and show that windows of parametric resonance, characterized by an integer $n \geq 2$, are the ones that strongly favour inflation in the system. We discuss how generic is this behaviour for inflationary models.

The initial conditions of our present expanding Universe must have been fixed when the early Universe emerged from the semiclassical Planckian regime and started its classical evolution. Evolving back the initial conditions using classical equations of motion, the system is driven towards a neighborhood of a singular point, where the classical regime is no longer valid and must be substituted by the quantum regime. Here we consider quantum gravity corrections to cosmological scenarios in Friedmann equations on the brane, due to the influence of a bulk geometry, that implement nonsingular bounces in the scale factor[1]. We are then led to consider the introduction of an extra potential term in Friedmann's equations when treating a class of closed FRW models with a conformally coupled massive scalar field ϕ (the inflaton field) plus radiation. We assume the potential $U(\phi) = \Lambda + m^2\phi^2/2$ where the cosmological constant term Λ is the vacuum energy of the inflaton field and ϕ its spatially homogeneous expectation values. For perfect fluid cosmologies this correction term may be assumed to be of the form A/a^{α}, where A is a constant, a the scale factor and the integer $\alpha \geq 2$. For simplicity we will fix $\alpha = 6$ in the present paper, but the features of the dynamics may be substantially different for other choices of α and deserve a future examination[2]. In the conformal time gauge, the dynamics of the model may then be derived from the Hamiltonian constraint

$$H = -\frac{p_{\varphi}^2 + \varphi^2}{2} + \frac{p_a^2}{12} + V(a) - \frac{m^2}{2} a^2 \varphi^2 - E_0 = 0, \qquad (1)$$

with a the scale factor and E_0 the total conserved energy of the model. p_{φ} and p_a are the momenta canonically conjugated to $\varphi = a\phi$ and a, respectively. In the above $V(a)$ is the effective potential, given by

$$V(a) = 3a^2 - \Lambda a^4 + \frac{A}{a^2}. \qquad (2)$$

The parameter space is labelled by (m, E_0), and here we will treat dynamical configurations for which the universe undergoes a series of bounces before the universe enters an inflationary regime. The initial conditions for these configurations will be fixed by *resonance windows* in the parameter space since the resonances destroy KAM tori that trap the inflaton about the origin ($\varphi = 0, p_{\varphi} = 0$). We can show that resonance windows are approximated by the curve in parameter space $R = 2/n$, where $n \geq 2$ and R is a transcendental function of (m, E_0)[3]. The nonlinear resonance of KAM tori produces complex dynamical phenomena, as long time diffusion through Cantori, a phenomena that may have some striking consequences for structure formation [3]. In the following we fix $E_0 = 0.8$.

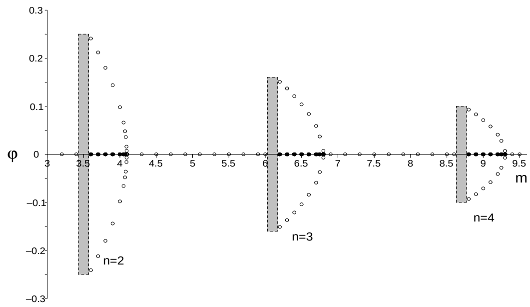

FIG. 1: Diagram (m, φ_0) of parametric bifurcation of the periodic orbit at the origin ($\varphi = 0, p_{\varphi} = 0$), between the resonances $n = 2$ and $n = 4$. The shaded regions correspond to values of m connected to disruptive resonances for initial condition values near the invariant plane ($\varphi = 0, p_{\varphi} = 0$). On their left is the region of parametric stability for the periodic orbit. Further to the left, a value of m is reached beyond which the periodic orbit at the origin bifurcates. The stable and unstable branches are shown by white and black dots, respectively, marking the parametric domain of the resonance.

In Fig.1 we show a bifurcation diagram connected to the resonances $n = 2$ to $n = 4$. There it is seen that, in the parametric domain of a resonance, the origin ($\varphi = 0, p_{\varphi} = 0$) becomes a saddle and allows the inflaton to escape to the DeSitter configuration at infinity.

We are now ready to compare the dynamical behavior of the

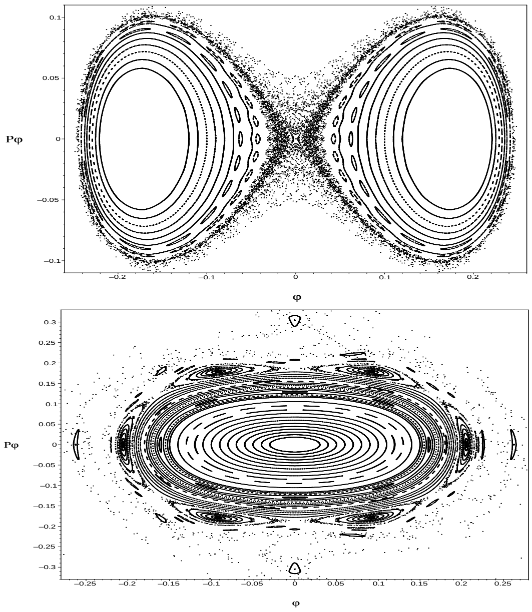

FIG. 2: Poincaré maps with surface of section $p_a = 0$ for $m = 3.80085$ in the domain of parametric resonance $n = 2$ (top), and $m = 4.29$ in the domain of parametric stability between the resonances $n = 2$ and $n = 3$ (bottom). The origin in the first map (top) is a saddle with two associated centers, consequence of the bifurcation of the periodic orbit due to the resonance. The structure of the stochastic sea differs in each case. The maps were constructed with $\tau \simeq 4{,}000$.

system in the parametric domain of a n resonance as opposed to the behavior in the region of parametric stability, as shown in Fig. 2. We consider the value $m = 3.80085$ corresponding to the parametric domain of the resonance $n = 2$, and $m = 4.29$ corresponding to the region of parameter stability between the resonances $n = 2$ and $n = 3$. In the map corresponding to $m = 3.80085$ the two main islands are centered about $(\varphi_0 \simeq \pm 0.18, p_{\varphi_0} = 0)$ and the origin is a hyperbolic point. Beyond the border of the main islands, we note the large region of phase space corresponding to a long time diffusion (before the orbits escape to inflation). The stochastic sea beyond the fuzzy border of the main islands presents the structure of a stochastic web through which diffusion takes place to large regions of phase space. No islands are seen in this region. This is opposed to the Poincaré map for $m = 4.29$ in the region of parametric stability, where the main island is centered about the origin and the stochastic sea beyond the border of the main island contains several secondary islands. The diffusion of orbits (with initial conditions beyond the border of the main island) in the stochastic sea towards the DeSitter infinity is extremely rapid.

The crucial point for the dynamics of inflation is the instability versus the stability of the origin $\varphi = 0,\ p_\varphi = 0$. The differences in the two situations will have a bearing in the dynamics of the spatially homogeneous expectation values $\varphi(\tau)$ of the inflaton field related to the escape into inflation. In this instance the initial conditions of the expectation values φ are assumed to be small, and are taken near the invariant plane of the dynamics $\varphi = 0,\ p_\varphi = 0$ – that corresponds to a neighborhood of the critical point of the map at the origin $(\varphi = 0, p_\varphi = 0)$. Therefore the region of parametric stability of the system will be unfavorable to the physics of inflation since the orbit (a configuration of the early universe) will be trapped in a stable state between two KAM tori near the center of the main island. However if the system is in the region of parametric resonance, initial conditions near the invariant plane would undergo a long time diffusion to large regions of phase space, and finally escape to the DeSitter configuration at infinity.

The above phenomena are generic for closed inflationary models, as far as bounces are included in the theory. Indeed the minimal ingredients in the inflationary paradigm are a FRW geometry plus a scalar field (the inflaton field) together with well formulated ideas of modern quantum field theory. The basic idea of inflation is that the vacuum energy of the inflaton field was the dominant component of the energy density of the universe in an early epoch of its evolution. The dynamics, ruled by Einstein's equations, has thus the cosmological constant-type term (connected to the vacuum energy of the inflaton field) and the two degrees of freedom $a(\tau)$ and $\varphi(\tau)$, respectively the scale factor and the spatially homogeneous expectation value of the inflaton field. In general the resulting potential for φ has always a minimum, providing then the condition for nonlinear resonance.

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[2] We note that the choice $n = 6$ corresponds to bulk-brane corrections in Friedmann's equations with dust.

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