Short Wavelength Oscillations of a Magnetized Current-Carrying Plasma

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The linear wave propagation is analyzed for inhomogeneous collisionless cylindrical plasmas in a helical magnetic field. Using the geometric optics approximation the electron Landau damping of the kinetic Alfvén, fast Alfvén, fast magnetosonic and atmospheric whistler waves are studied in a current-carrying plasma with "hot" electrons and one kind of ions. The expressions obtained for real and imaginary parts of the radial refractive indexes can be used in the wide range of the wave phase velocities relative to the electron thermal velocity. The ion-cyclotron damping of fast magnetosonic waves is estimated taking into account the magnetic shear corrections.

I. Introduction

The problems of plasma heating and current drive by the Alfvén and ion-cyclotron waves in tokamaks have stimulated an interest renewed in a study of the waves excited in magnetized current-carrying plasmas. The theory of magnetohydrodynamic (MHD) waves, in homogeneous magnetic field plasmas, has been basically completed (see, for example, Ref. [1, 2] and the references therein). In the last years, using the kinetic models of high-temperature cylindrical and toroidal plasmas in a helical magnetic field, the MHD waves were intensively studied[3−7] in connection with laboratory experiments. It was shown that the properties of MHD waves depend strongly on the magnetic field structure, on the ratio of the wave phase velocity to the thermal velocity of particles, and on the density and temperature gradients of the background plasma.

As is well known, the theory of linear waves is based on the solution of Maxwell's equations for the components of the perturbed electromagnetic fields $\mathbf{E}, \mathbf{H}$ and current density $\mathbf{j}$. The set of Maxwell's equations will be closed if we know the relation between of $\mathbf{j}$ and $\mathbf{E}$. Usually, this connection is defined via the wave conductivity tensor $\hat{\sigma}_{ik}$: $j_i = \hat{\sigma}_{ik} E_k$, or via the dielectric tensor $\hat{\epsilon}_{ik}$: $\hat{\epsilon}_{ik} = \delta_{ik} + i4\pi\hat{\sigma}_{ik}/\omega$, where δ_{ik} are the Kroneker constants. Depending on the various plasma parameters, the elements of $\hat{\epsilon}_{ik}$ can have different forms, which is related to the nature of the wave phenomena observed in the plasma, in particular, by its oscillation spectra and by the magnetic field configuration. On the other hand, every plasma model needs to justify the dielectric tensor components valid at a given frequency range. For collisional plasmas, to derive the tensor $\hat{\epsilon}_{ik}$, it is possible to use the ideal MHD plasma model or two-fluids MHD equations[8]. Usually, the laboratory fusion plasma is collisionless, it means that the wave frequency ω is larger than the electron-ion collision frequency ν_{ei} and the mean free path of electrons is bigger than the wavelength along the magnetic field lines. The corresponding expressions for $\hat{\epsilon}_{ik}$ of a collisionless plasma can be derived solving the linearized Vlasov equation for the perturbed distribution functions of plasma particles. The solution of the Vlasov equation and the evaluation of the dielectric tensor components are shown in Ref. [9], which are applied for Alfvén heating and current drive problems in a current-carrying plasma.

Our purpose in this paper is to find the real and imaginary parts of the radial refractive index for the basic eigenmodes, using the geometric optics approximation in cylindrical current-carrying plasmas for the frequency range of Alfvén and ion cyclotron waves. In this range of the frequencies, it is possible the excitation of kinetic Alfvén waves[10], the fast Alfvén (see Refs. [1, 2], called sometimes as sheared Alfvén[4]) waves, and fast magnetosonic (called sometimes as compressional Alfvén[4]) waves. Analyzing the dissipation characteristics of these waves in collisionless plasmas, we should take into account two kinds of collisionless damping: the electron Landau damping (Alfvén heating problem); and the ion-cyclotron damping, when the resonant ions absorb the wave energy via the Doppler effect (ion-cyclotron resonant heating). In this paper we prolong the analysis of those phenomena taking into account a plasma inhomogeneity and extending results in Refs. [4-6],[9] .

The present paper is organized as follows. In section II, we describe the plasma model. The dispersion characteristics of the Alfvén and fast magnetosonic waves we analyze in section III, taking into account the Landau damping by the plasma electrons. In section IV, we present the contribution of the resonant ions, to the dielectric tensor, for wave frequencies near the ion-cyclotron frequency, and evaluate the ion-cyclotron damping of the fast magnetosonic wave in current-carrying plasmas.

II. Cylindrical plasma model

II.1. Equilibrium current and helical magnetic field

Here, we use a simple model of a cylindrical current-carrying plasma, when the equilibrium current $\mathbf{j}_0$ is parallel to the helical magnetic field $\mathbf{B}_0$. Under the equilibrium conditions, we assume that the ions have no directed current velocity, so that only electrons are the source of the current $\mathbf{j}_0$ in the plasma. This longitudinal current induces the poloidal magnetic field $B_{0\theta}(r)$. In this case, the magnetic surfaces may be represented by the circular and concentric cylinders. For this model we use cylindrical coordinates (r, θ, z), and the same type of a stationary magnetic field, as in Ref. [4]:

$$B_{0r} = 0, \qquad B_{0\theta} = h_\theta B_0, \qquad B_{0z} = h_z B_0, \qquad B_0 = \sqrt{B_{0\theta}^2 + B_{0z}^2}, \tag{1}$$

where $h_\theta = B_{0\theta}/B_0$ and $h_z = B_{0z}/B_0$ are the cylindrical projections of the unit vector, $\mathbf{h} = \mathbf{B}_0/B_0$. The magnetic field configuration (1) is defined by the parameters χ_1 and χ_2:

$$\chi_1 = r{h_z}^2 \frac{\partial}{\partial r}\left(\frac{h_\theta}{rh_z}\right) = -\frac{h_z h_\theta}{q}\frac{dq}{dr}, \qquad \chi_2 = \frac{{h_z}^2}{r}\frac{\partial}{\partial r}\left(\frac{rh_\theta}{h_z}\right) = 2\frac{h_z h_\theta}{r}\left(1 - \frac{r}{2q}\frac{dq}{dr}\right). \tag{2}$$

Here, $q = rh_z/Rh_\theta$ corresponds to the tokamak safety factor, where R is the tokamak major radius. Keeping in the mind that many of our results are valid for toroidal systems, we shall consider the plasma cylinder of length $2\pi R$ as an approximation of a large aspect ratio torus where the trapped particle effect will be neglected. Furthermore, we assume that the poloidal magnetic field is much smaller than the axial magnetic field, $B_{0\theta} << B_{0z}$. In the used plasma model, we also take into account the radial inhomogeneity of the plasma density $n_0(r)$ via the parameter, $\chi_n = \partial \ln n_0/\partial r$. This model is reasonable for plasma devices with low pressure, $\beta = c_s^2/c_A^2 \ll 1$, where $c_s = \sqrt{T_{0e}/M_i}$ is sound velocity and $c_A = \sqrt{B_0^2/4\pi n_0 M_i}$ is Alfvén velocity. The current velocity v_0 is assumed to be much smaller than the electron thermal velocity, $v_0 << v_{Te} = \sqrt{T_{0e}/m_e}$, where T_{0e} is the plasma (electron) temperature and M_i and m_e are mass of ions and electrons, respectively.

Using the Maxwell's equation, for equilibrium current density, and magnetic field,

$$(rot\mathbf{B}_0)_{\|} = \frac{4\pi}{c}j_0 = -\frac{4\pi}{c}en_0v_0, \tag{3}$$

we can find the following expression for the current velocity v_0:

$$v_0 = -\frac{\chi_2 {c_A}^2}{\omega_{ci}} \tag{4}$$

where $\omega_{ci} = eB_0/M_ic$ is the ion cyclotron frequency. This velocity should be taken into account (see Ref. [9]) to describe correctly the steady-state distribution function of plasma electrons.

Using this model, we can consider three different limits ("submodels"):

a) when the current velocity is equal to zero, the poloidal magnetic field is absent, and we have a plasma confined in a straight magnetic field;

b) when the plasma current is uniform, $j_0 = const$, the safety factor q is constant too, $B_{0\theta} \sim r$, $dq/dr = 0$, and we have a helical magnetic field without shear;

c) when the equilibrium current is nonuniform, the radial derivative of q is not equal to zero, $dq/dr \neq 0$, which is the characteristic of a sheared magnetic field, and we have general model.

II.2. Maxwell's equations in the current-carrying plasma

We use the normal A_1, binormal A_2 and parallel A_3 components, relative to the $\mathbf{B}_0$ field line, for the set of vector values $\mathbf{A} = \{\ \mathbf{E},\ \mathbf{H},\ \mathbf{j}\}$, which are related to the cylindrical components A_r, A_θ, and A_z, through the following relationships:

$$A_1 = A_r, \qquad A_2 = A_\theta h_z - A_z h_\theta, \qquad A_3 = A_z h_z + A_\theta h_\theta. \tag{5}$$

Since the equilibrium parameters are independent of time t, angle θ and variable z, the dependence of oscillating values on these variables may be represented by $exp(-i\omega t + im\theta + ik_z z)$ (one plane wave approximation). Here, ω is the wave frequency, k_z is the axial projection of the wave vector, m is the azimuthal wave number. It means that we deal with waves (m and k_z are arbitrary), propagating in the nonuniform current-carrying low β plasma. For such waves, the Maxwell's equations can be rewritten as

$$\begin{aligned} H_1 &= \frac{c}{\omega}(k_b E_3 - k_{\|} E_2); \\ H_2 &= \frac{c}{\omega}(k_{\|} E_1 - i\chi_1 E_2 + i\frac{\partial E_3}{\partial r} + i\frac{h_\theta^2}{r}E_3); \\ H_3 &= \frac{c}{\omega}(-k_b E_1 - i\frac{1}{r}\frac{\partial}{\partial r}rE_2 + i\frac{h_\theta^2}{r}E_2 - i\chi_2 E_3); \end{aligned} \tag{6}$$

$$\begin{aligned} k_b H_3 - k_{\|} H_2 &= -\frac{\omega}{c}\hat{\epsilon}_{1j}E_j; \\ k_{\|} H_1 - i\chi_1 H_2 + i\frac{\partial H_3}{\partial r} + i\frac{h_\theta^2}{r}H_3 &= -\frac{\omega}{c}\hat{\epsilon}_{2j}E_j; \\ k_b H_1 + \frac{i}{r}\frac{\partial}{\partial r}rH_2 - i\frac{h_\theta^2}{r}H_2 + i\chi_2 H_3 &= -\frac{\omega}{c}\hat{\epsilon}_{3j}E_j; \end{aligned} \tag{7}$$

where $k_{\|} = k_z h_z + h_\theta m/r$ and $k_b = h_z m/r - k_z h_\theta$ are the parallel and binormal components of the wave vector relative to $\mathbf{B_0}$. Here, we have taken into account the magnetic shear effect via the dependence of $k_{\|}$ on r, which is the principal effect in our model. If the expressions for dielectric tensor elements are known, the equations (6) and (7) are basic to study the eigenmodes of current-carrying plasmas in the helical magnetic field. In this paper, we use these equations to evaluate the dispersion characteristics of fast magnetosonic, fast Alfvén, kinetic Alfvén, atmospheric whistler waves.

III. Electron Landau damping of short wavelength oscillations in a current-carrying plasma

III.1. Dielectric tensor-operator for MHD waves

In a first step, we are going to analyze the waves of a small scale wavelength in the direction perpendicular to $\mathbf{B}_0$, in a nonuniform current-carrying plasma for the frequency range, $\omega_{dr} << \omega << \omega_{ci}$, where $\omega_{dr} \sim k_b \chi_n c_s^2/\omega_{ci}$ is the diamagnetic (or drift) frequency. The assumption of a small scale wavelength permits us to use the geometric optics approximation to describe analytically the waves in a broad frequency range for an arbitrary plasma density inhomogeneity. Here, we use the general expressions $\hat{\epsilon}_{ij}$ given in Ref. [9] and the approximation of cold plasma ions, $k_\parallel^2 v_{Ti}^2/\omega_{ci}^2 << 1$. After the summation over electrons and ions, we can obtain the following expressions for the operator representing the dielectric tensor of current-carrying plasmas:

$$\hat{\epsilon}_{11} = \epsilon_1 + i\tilde{\epsilon}\frac{k_b^2 c^2}{\omega^2}; \qquad \hat{\epsilon}_{12} = ig - \tilde{\epsilon}\frac{k_b c^2}{r\omega^2}\frac{\partial}{\partial r}(r...); \qquad \hat{\epsilon}_{13} = i\frac{k_b}{k_\parallel}\epsilon_\perp;$$

$$\hat{\epsilon}_{21} = -ig - \tilde{\epsilon}\frac{k_b c^2}{\omega^2} r\frac{\partial}{\partial r}\left(\frac{1}{r}...\right); \quad \hat{\epsilon}_{22} = \epsilon_2 - i\tilde{\epsilon}\frac{c^2}{\omega^2}\frac{\partial}{\partial r} r\frac{\partial}{\partial r}\left(\frac{1}{r}...\right); \quad \hat{\epsilon}_{23} = -\frac{\epsilon_\perp}{k_\parallel}\left(\frac{\partial}{\partial r} + \chi_n\right);$$

$$\hat{\epsilon}_{31} = -i\frac{k_b}{k_\parallel}\epsilon_\perp; \qquad \hat{\epsilon}_{32} = \frac{\epsilon_\perp}{k_\parallel r}\frac{\partial}{\partial r}(r...); \qquad \hat{\epsilon}_{33} = \epsilon_3 + \epsilon_\perp\frac{\chi_n k_b}{k_\parallel^2}, \tag{8}$$

where

$$\epsilon_1 = \frac{c^2}{c_A^2}\left(1 + \frac{\omega^2}{\omega_{ci}^2}\right); \qquad \tilde{\epsilon} = 2\beta\sqrt{\pi} Z_e W_e;$$

$$\epsilon_2 = \epsilon_1 + \frac{c_2}{c_A^2}\frac{\chi_1 v_0 \omega_{ci}}{\omega^2}; \qquad g = \frac{c^2}{c_A^2}\left(\frac{\omega}{\omega_{ci}} + \frac{k_\parallel v_0 \omega_{ci}}{\omega^2}\right);$$

$$\epsilon_3 = \frac{c^2}{c_A^2}\frac{\omega_{ci}^2}{k_\parallel^2 c_S^2}(1 + i\sqrt{\pi} Z_e W_e); \qquad \epsilon_\perp = \frac{c^2\omega_{ci}}{c_A^2\omega}\left(1 + i\sqrt{\pi} Z_e W_e\right).$$

In (8), W_e is the plasma dispersion function of electrons

$$W_e = W_e(Z_e) = \exp(-Z_e^2)\left[1 + \frac{2i}{\sqrt{\pi}}\int_0^{Z_e}\exp(t^2)dt\right], \qquad Z_e = \frac{\omega - k_\parallel v_0}{\sqrt{2}k_\parallel v_{Te}}. \tag{9}$$

Note that, in the expressions $\hat{\epsilon}_{ik}$, we keep only the main terms, which are proportional to the electron current velocity v_0 and the radial gradient of plasma density χ_n. The terms in $\hat{\epsilon}_{ik}$, which are proportional to the χ_1 and χ_2 parameters, are of the same order as the shear induced terms in the Maxwell's equations (6) and (7). Furthermore, we take into account the Doppler effect via the term, $k_\parallel v_0$, in the argument of the plasma dispersion function (9). Supposing that $v_0 = 0$ (or $h_\theta = 0$), we get the known result for the dielectric permeability, of a nonuniform cylindrical plasma, in the straight magnetic field (see Ref. [11]).

Using (8), we can see that the off-diagonal elements $\hat{\epsilon}_{13}, \hat{\epsilon}_{23}, \hat{\epsilon}_{31}$, $\hat{\epsilon}_{32}$ are not small, and allow to evaluate the so called "transit-time magnetic pumping" dissipation for MHD waves[1,2]. Moreover, these elements of $\hat{\epsilon}_{ik}$ are important and give a contribution to the Alfvén current drive via ponderomotive forces[9].

III.2. Dispersion equation

To obtain the dispersion equations for Alfvén and fast magnetosonic waves, we should solve the equations

(6) and (7). The corresponding solutions can be represented in the form:

$$E_{1,2,3}(r)\exp\left(i\int_0^r k_r(\rho)d\rho\right),$$

where $E_{1,2,3}(r)$ are the slowly varying amplitudes, and k_r is the normal (radial) component of the wave vector, which satisfies the condition $k_r(r)/\chi_n >> 1$. In this paper, we do not consider the mode conversion effects because the geometric optics approximation is not valid[2] at the conversion zones where $\epsilon_1 \approx k_\parallel^2 c^2/\omega^2$.

Using expressions (8) and (6), the equations (7) can be written as

$$\begin{aligned}
&\left[\epsilon_1 - N_\parallel^2 - N_b^2(1-i\tilde{\epsilon})\right]E_1 + \left[N_b N_r(1-i\tilde{\epsilon}) + ig + iN_\parallel N_1\right]E_2\\
&\qquad + \left[N_\parallel N_r + iN_b\left(\frac{\epsilon_\perp}{N_\parallel} - N_2\right)\right]E_3 = 0;\\
&\left[N_b N_r(1-i\tilde{\epsilon}) - ig - iN_\parallel N_1 + iN_\parallel N_2\right]E_1 + \left[\epsilon_2 - N_\parallel^2 - N_r^2(1-i\tilde{\epsilon})\right]E_2\\
&\qquad + \left[N_\parallel N_b - iN_r\left(\frac{\epsilon_\perp}{N_\parallel} - N_1 - N_2\right) - \frac{N_n}{N_\parallel}\epsilon_\perp\right]E_3 = 0;\\
&\left[N_\parallel N_r - iN_b\left(\frac{\epsilon_\perp}{N_\parallel} - N_2 + N_1\right)\right]E_1 + \left[N_\parallel N_b + iN_r\left(\frac{\epsilon_\perp}{N_\parallel} - N_1 - N_2\right)\right]E_2\\
&\qquad + \left(\epsilon_3 - N_r^2 - N_b^2\right)E_3 = 0,
\end{aligned} \tag{10}$$

where $N_{r,b,\parallel} = c\, k_{r,b,\parallel}/\omega$ are corresponding to the normal (radial), binormal and parallel refractive index components relatively to $\mathbf{B}_0$, $N_n = c\,\chi_n/\omega$, $N_{1,2} = c\,\chi_{1,2}/\omega$. Using the condition when the determinant of (10) is equal to zero, we obtain the dispersion relation for Alfvén and fast magnetosonic waves. These two wave branches are coupled by the ω/ω_{ci} terms and by the effects connected with equilibrium current, magnetic shear, and density gradient. To simplify this equation, we assume that $\epsilon_\perp >> N_\parallel N_1,\ N_\parallel N_2$. In this case, we can derive the following equation, for the complex radial refractive index:

$$\begin{aligned}
&\epsilon_3(\epsilon_1 - N_\parallel^2)\left[\epsilon_2 - N_\parallel^2 - (N_r^2 + N_b^2)(1-i\tilde{\epsilon}) - i\frac{N_b N_r N_\parallel N_2}{\epsilon_1 - N_\parallel^2}\right.\\
&\qquad\left. + \frac{N_b^2 N_1 N_2 - (g + N_\parallel N_1)(g + N_\parallel N_1 - N_\parallel N_2)}{\epsilon_1 - N_\parallel^2}\right]\\
&+ \left(N_r^2 + N_b^2\right)\left[\epsilon_1(N_r^2 + N_b^2) - \epsilon_2(\epsilon_1 - N_\parallel^2) + (g + N_\parallel N_1)(g + N_\parallel N_1 - N_\parallel N_2)\right.\\
&\qquad\left. - N_1 N_2\frac{N_r^2 N_b^2 + N_b^4 - N_\parallel^2 N_r^2}{N_r^2 + N_b^2} + iN_r N_b N_\parallel N_2\right]\\
&-(N_r^2 + N_b^2)\frac{\epsilon_\perp^2}{N_\parallel^2}(\epsilon_1 - N_\parallel^2) + iN_n N_r\frac{\epsilon_\perp^2}{N_\parallel^2}(\epsilon_1 - N_\parallel^2)\left[1 - \frac{N_\parallel^2(g + N_\parallel N_1)}{\epsilon_\perp(\epsilon_1 - N_\parallel^2)}\right] = 0.
\end{aligned} \tag{11}$$

Using (11), it is possible to analyze some interesting limits for the wave vector in parallel and perpendicular directions to the ambient magnetic field (the cold plasma limit, final Larmor radius effects and so on).

III.3. Fast waves

Retaining in zero approximation the largest terms proportional to ϵ_3 (i.e., assuming that $\epsilon_3 \to \infty$), we can get from (11) the equation for the real part of the radial refractive index, ReN_r for the fast waves. In the next approximation (by ϵ_3^{-1}), we obtain the damping coefficient $\nu = ImN_r/ReN_r$, where $N_r^2 = (ReN_r)^2(1+2i\nu)$ and $\nu << 1$. As a result, for the fast waves we have:

$$(ReN_r)^2_{FW} = \epsilon_2 - N_\parallel^2 - N_b^2 + \frac{N_b^2 N_1 N_2 - (g+N_\parallel N_1)(g+N_\parallel N_1 - N_\parallel N_2)}{\epsilon_1 - N_\parallel^2}; \tag{12}$$

$$\nu_{FW} = \frac{1}{2}\left\{\left(1+\frac{N_b^2}{(ReN_r)^2}\right)\beta\sqrt{\pi}Z_e\ exp(-Z_e^2) - \frac{1+N_b^2/(ReN_r)^2}{\epsilon_3(\epsilon_1 - N_\parallel^2)}\frac{\sqrt{\pi}Z_e\ exp(-Z_e^2)}{\mid 1+i\sqrt{\pi}Z_e W_e\mid^2}\right.$$
$$\times\left[\epsilon_1\left((ReN_r)^2+N_b^2\right) - \epsilon_2(\epsilon_1 - N_\parallel^2) + (g+N_\parallel N_1)(g+N_\parallel N_1 - N_\parallel N_2)\right. \tag{13}$$
$$\left.\left. - N_1N_2\frac{N_b^2 + N_b^4/(ReN_r)^2 - N_\parallel^2}{1+N_b^2/(ReN_r)^2}\right] - \frac{N_b N_\parallel N_2}{ReN_r(\epsilon_1 - N_\parallel^2)} + \beta\frac{N_n}{ReN_r}\left[1 - \frac{k_\parallel^2 c_A^2(g+N_\parallel N_1)}{\omega\omega_{ci}(\epsilon_1 - N_\parallel^2)}\right]\right\}.$$

It should be noted that the dispersion equation (12) is corresponding to the well known dispersion relations derived in Ref. [4] for the compressional and shear Alfvén waves if we omit the terms, which are proportional to the current density gradients in equation (12) (i.e., $N_1, N_2 \to 0$). Using equations (12) and (13), we can estimate the radial refractive index, and the damping coefficient of the fast magnetosonic (FMS) and fast Alfvén (FA) waves. In the frequency range $\omega << \omega_{ci}$, for the fast magnetosonic wave with $\omega^2 \approx (k_r^2 + k_b^2 + k_\parallel^2)c_A^2$, these expressions can be simplified to:

$$(ReN_r)^2_{FMS} = \epsilon_1 - N_\parallel^2 - N_b^2 - N_1N_2; \tag{14}$$

$$\nu_{FMS} = \left(1+\frac{N_b^2}{(ReN_r)^2}\right)\frac{\beta}{2}\sqrt{\pi}Z_e\ exp(-Z_e^2) - \frac{N_bN_\parallel N_2}{2ReN_r(\epsilon_1 - N_\parallel^2)} + \frac{\beta N_n}{2ReN_r}. \tag{15}$$

For the fast Alfvén wave with $\omega \approx k_\parallel\ c_A$, we have

$$(ReN_r)^2_{FA} = \frac{N_b^2 N_1 N_2 - (g+N_\parallel N_1)(g+N_\parallel N_1 - N_\parallel N_2)}{\epsilon_1 - N_\parallel^2} - N_b^2 - N_1N_2; \tag{16}$$

$$\nu_{FA} = \left(1+\frac{N_b^2}{(ReN_r)^2}\right)\frac{\beta}{2}\left[\sqrt{\pi}Z_e\ exp\left(-Z_e^2\right) - \frac{k_\parallel^2 c_A^2}{\omega_{ci}^2(\epsilon_1 - N_\parallel^2)}\frac{\sqrt{\pi}Z_e\ exp\left(-Z_e^2\right)}{\mid 1+i\sqrt{\pi}Z_eW_e\mid^2}\right]$$
$$- \frac{N_bN_\parallel N_2}{2ReN_r(\epsilon_1 - N_\parallel^2)} + \frac{\beta N_n}{2ReN_r}\left(1 - \frac{\omega(g+N_\parallel N_1)}{\omega_{ci}(\epsilon_1 - N_\parallel^2)}\right). \tag{17}$$

The dispersion characteristics of the fast Alfvén waves in the current-carrying plasma depend on the relation between two small parameters $k_\parallel v_0/\omega$ and ω^2/ω_{ci}^2. In particular, for $k_\parallel v_0/\omega << \omega^2/\omega_{ci}^2$, we have the following expression of $(ReN_r)^2_{FA}$, to consider the FA-wave radial structure:

$$(ReN_r)^2_{FA} = \frac{\omega^2\epsilon_1^2}{\omega_{ci}^2(N_\parallel^2 - \epsilon_1)} - N_b^2 - N_1N_2. \tag{18}$$

For $k_\parallel v_0/\omega >> \omega^2/\omega_{ci}^2$ for $(ReN_r)^2_{FA}$, we have

$$(ReN_r)^2_{FA} = \frac{N_b^2N_1N_2 - N_\parallel^2(2N_2^2 - 3N_2N_1 + N_1^2)}{\epsilon_1 - N_\parallel^2} - N_b^2 - N_1N_2. \tag{19}$$

From equation (18), assuming $v_0 \to 0$, we obtain the well-known condition of the FA-wave propagation in the plasma with a straight magnetic field, $N_\parallel^2 > \epsilon_1$. In the case, $v_0/c_A >> \omega^2/\omega_{ci}^2$, the FA-waves can be excited in the plasma with a strong nonuniform current ($dq/dr \neq 0$) under the condition, $N_\parallel^2 < \epsilon_1$. For the waves with $k_b = 0$ (that usually corresponds to the wave excitation with a poloidal wave number $m = 0$), the propagation condition becomes the usual one. Note that the radial wavelength of the fast Alfvén waves, in the current-carrying plasma, can be smaller than it is in the plasma in the straight magnetic field under the same values of ω, m, and k_z. These values can be given, for example, by the antenna/generator system.

The resonant conditions of the fast wave excitation substantially depend on the distribution of the equilibrium current. If we assume that the current velocity is equal to zero in equations (12) and (13), the real and imaginary parts of the radial refractive index, respectively, will be the same as the well-known results for the homogeneous magnetic field [1,4,7]. For plasmas with an uniform equilibrium current (a nonsheared magnetic field, $N_1 = 0$) it is necessary to take into account the current velocity in the dielectric tensor elements $\hat{\epsilon}_{12}$ and $\hat{\epsilon}_{21}$, which are important to derive the dispersion relation for the fast waves. For plasmas with a nonuniform current ($N_1 \neq 0$ and $dq/dr \neq 0$) the frequencies of the fast magnetosonic and fast Alfvén waves will depend also on the terms driven by shear in $\hat{\epsilon}_{22}$ and g.

The correction related to the density gradient in (15) and (17) is important to evaluate the damping of the fast magnetosonic and fast Alfvén waves in the nonuniform plasma. Moreover, the third term in equation (15), which depends on the sign of ReN_r under the given sign of χ_n, can cause either decrease or increase of the wave dissipation. In laboratory plasmas, the density usually decreases to the boundary; it means that in this case $\chi_n < 0$. Using (15), we find that the waves propagating into the plasma are absorbed more effectively due to the density gradient than it is in an uniform plasma. On the other hand, the waves propagating in opposite direction ($ReN_r > 0$, $\chi_n < 0$, and $\mid \sqrt{2}\chi_n N_\parallel v_{Te}/\sqrt{\pi} ReN_r\omega \mid > 1$) may become unstable.

III.4. Kinetic Alfvén wave

Another solution of the dispersion equation (11) corresponds to the so called kinetic Alfvén (KA) wave. In this case, the refractive index $N_{r_{KA}}^2$ of KA waves is a large value proportional to ϵ_3 and we have respectively:

$$(ReN_r)_{KA}^2 + N_b^2 = \frac{\epsilon_3}{\epsilon_1}\left(\epsilon_1 - N_\parallel^2\right); \tag{20}$$

$$\nu_{KA} = \left(1 + \frac{N_b^2}{(ReN_r)^2}\right)\frac{\sqrt{\pi}}{2} Z_e\, exp\left(-Z_e^2\right) + \frac{N_b N_\parallel N_2}{2ReN_r(\epsilon_1 - N_\parallel^2)} - \frac{\beta N_n}{2ReN_r}\left(1 - \frac{\omega(g + N_\parallel N_1)}{\omega_{ci}(\epsilon_1 - N_\parallel^2)}\right). \tag{21}$$

Analyzing these expressions, we see that the equilibrium current does not make a substantial effect on the dispersion characteristics of the kinetic Alfvén wave in the weakly nonuniform low beta plasma, if $\beta << N_{r_{KA}} c_A/N_n v_{Te}$. As is in the uniform plasma[10], the damping rate of these waves is defined by the $Im\epsilon_3$ with a shifted argument for the plasma dispersion function, $Z_e = (\omega - k_\parallel v_0)/\sqrt{2}k_\parallel v_{Te}$.

Note that the influence of the current on the radial attenuation of the kinetic Alfvén, fast Alfvén and fast magnetosonic waves with $k_b = 0$ (or $m = 0$) will be smaller by a factor β than the corresponding damping rates, ν, in equations (15), (17) and (21). The effect of the shear terms in these expressions shows us that FA and FMS waves, which have the small damping coefficients, $\nu_{FW} \sim \beta Z_e$, can be unstable in the nonuniform current-carrying plasma.

The exact solutions for E_z, H_z, and $\partial(rE_r)/\partial r + imE_\theta$ components of the perturbed electromagnetic field are proportional to $\sim J_m(k_\perp r)exp(-i\omega t + im\theta + ik_z z)$ (see Ref. [11]) in homogeneous plasmas confined in the homogeneous magnetic field. Here, $J_m(k_\perp r)$

is the Bessel function. It means that the expressions obtained for real and imaginary parts of the considered waves become corresponding to the expressions for cylindrical waves[11] in the uniform plasma, under the following conditions: $k_\perp^2 = k_r^2 + k_b^2$ and $k_r^2 >> k_b^2$.

III.5. Atmospheric whistlers

Now we are going to analyze the high-frequency fast magnetosonic waves in the frequency range, $\omega_{ci} << \omega << \omega_{ce} = eB_0/m_e c$. These waves are called as atmospheric whistlers (AtW). To obtain the real and imaginary parts of the AtW's radial refractive index, we should take into account that for these frequencies the values of ϵ_1 and g in (11) are given by expressions:

$$\epsilon_1 = -\frac{\omega_{pi}^2}{\omega^2}\left(1+\frac{\omega_{ci}^2}{\omega^2}\right), \qquad g = -\frac{c^2\omega_{ci}}{c_A^2\omega}\left(1-\frac{k_\parallel v_0}{\omega}+\frac{\omega_{ci}^2}{\omega^2}\right), \tag{22}$$

where $\omega_{pi} = \sqrt{4\pi n_0 e^2/M_i}$ is the plasma ion frequency, and the other components of the dielectric tensor will be defined by the electron contributions to $\hat{\epsilon}_{ik}$ in (8). In this case, the expressions for dispersion characteristics of atmospheric whistlers, $\omega_{AtW}^2 \approx (k_r^2+k_b^2+k_\parallel^2)c_A^2(\omega_{ci}^2+k_\parallel^2 c_A^2)/\omega_{ci}^2$, are given by:

$$(ReN_r)_{AtW}^2 = \epsilon_1 - N_\parallel^2 - N_b^2 + \frac{g^2}{N_\parallel^2-\epsilon_1};$$

$$\nu_{AtW} = \left(1+\frac{N_b^2}{(ReN_r)^2}\right)\frac{\beta}{2}\left[\sqrt{\pi}Z_e exp\left(-Z_e^2\right) + \frac{k_\parallel^4 c_A^4\sqrt{\pi}Z_e exp\left(-Z_e^2\right)}{(\omega_{ci}^2+k_\parallel^2 c_A^2)^2 \mid 1+i\sqrt{\pi}Z_e W_e \mid^2}\right] + \frac{N_2 k_b k_\parallel c_A^2}{2ReN_r(\omega_{ci}^2+k_\parallel^2 c_A^2)} + \frac{\beta N_n}{2ReN_r}\frac{\omega_{ci}^2}{(\omega_{ci}^2+k_\parallel^2 c_A^2)}. \tag{23}$$

One can see above that the radial damping rate of atmospheric whistlers have the order of the magnitude as higher as the damping rate (15) for FMS waves in the low-frequency range, $\omega, k\|c_A << \omega_{ci}$. When $\omega_{ci}^2 << k_\parallel^2 c_A^2$ (or $k_\parallel \approx k_r$ and $\mid Z_e \mid << 1$), the damping rate of the high-frequency fast magnetosonic wave is two times higher than the damping rate of the low-frequency magnetosonic waves.

IV. Ion-cyclotron damping of the fast magnetosonic waves in a magnetic shear plasma

IV.1. Contribution of ion-cyclotron resonance to the dielectric tensor

In the previous section, we have studied the wave dissipation via the electron Landau damping. In magnetized plasmas, besides of the electron damping, it is possible to obtain the ion cyclotron damping. The ion cyclotron damping occurs, for example, when the wave interacts with plasma ions in the resonant condition $\omega - k_\parallel v_\parallel \approx \omega_{ci}$. As is well known (see Ref. [7]), the presence of a small group of the resonant particles modifies strongly the plasma dielectric properties. To calculate the contribution of the resonant ions to the plasma conductivity, it is necessary to solve the Vlasov equations for an arbitrary parameter $k_\parallel v_{Ti}/(\omega-\omega_{ci})$. In this case we obtain the following expressions for contributions of the resonant ions to $\hat{\epsilon}_{jk}^{(i)}$, at the fundamental (first) harmonic of the ion cyclotron frequency:

$$\hat{\epsilon}_{11}^{(i)} = \hat{\epsilon}_{22}^{(i)} = -\frac{\omega_{pi}^2}{2\omega(\omega+\omega_{ci})}\left[1-\frac{\omega+\omega_{ci}}{\omega-\omega_{ci}}(\Lambda_i^- - 1)\right];$$

$$\hat{\epsilon}_{12}^{(i)} = -\hat{\epsilon}_{21}^{(i)} = \frac{i\omega_{pi}^2}{2\omega(\omega+\omega_{ci})}\left[1+\frac{\omega+\omega_{ci}}{\omega-\omega_{ci}}(\Lambda_i^- - 1)\right];$$

$$\hat{\epsilon}_{13}^{(i)} = -\frac{i\omega_{pi}^2}{2\omega^2}\frac{\Lambda_i^-}{k_\parallel + \chi}\left(\frac{\partial}{\partial r} - k_b\right); \qquad \hat{\epsilon}_{31}^{(i)} = -\frac{i\omega_{pi}^2}{2\omega^2}\frac{\Lambda_i^-}{k_\parallel + \chi}\left(\frac{1}{r}\frac{\partial}{\partial r}(r...) + k_b\right); \tag{24}$$

$$\hat{\epsilon}_{23}^{(i)} = -\frac{\omega_{pi}^2}{2\omega^2}\frac{\Lambda_i^-}{k_\parallel + \chi}\left(\frac{\partial}{\partial r} - k_b\right); \quad \hat{\epsilon}_{32}^{(i)} = \frac{\omega_{pi}^2}{2\omega^2}\frac{\Lambda_i^-}{k_\parallel + \chi}\left(\frac{1}{r}\frac{\partial}{\partial r}(r...) + k_b\right); \quad \hat{\epsilon}_{33}^{(i)} = -\frac{\omega_{pi}^2}{\omega^2},$$

where

$$\Lambda_i^- = 1 + i\sqrt{\pi}Z_i^- W_i(Z_i^-); \; Z_i^- = \frac{\omega - \omega_{ci}}{\sqrt{2} \mid k_\parallel + \chi \mid v_{Ti}}; \; \chi = 2\frac{h_\theta h_z}{r}\left(1 - \frac{r}{4q}\frac{dq}{dr}\right).$$

In these expressions we have omitted the gradient terms of the ion density, assuming that it will be not important for excitation of high-frequency waves in the frequency range, $\omega \sim \omega_{ci}$. As can be seen in (25), the characteristic peculiarity of $\hat{\epsilon}_{jk}^{(i)}$ is the gradient shift χ of the $k_\parallel$ wave number, in the plasma dispersion functions Λ_i^-. This shift can be related to the difference between the conditions of the cyclotron resonances in the plasmas with the helical and straight equilibrium magnetic fields. When plasmas are in straight magnetic field, the cyclotron resonance condition is $\omega - k_z v_\parallel = l\omega_{c\alpha}$, and the corresponding condition for current-carrying plasmas (in the helical magnetic field) is given by

$$\omega - k_\parallel v_\parallel = l\left[\omega_{c\alpha} + 2\frac{h_z h_\theta}{r}\left(1 - \frac{r}{4q}\frac{dq}{dr}\right)\right], \tag{25}$$

where $\alpha = e, i$ is a kind of the plasma particles, $l = \pm 1, \pm 2, ...$ is the number of resonant cyclotron harmonic. The similar current effect on the cyclotron resonance condition has been found for the passing particles in the toroidal plasmas (see Ref. [12]).

Putting together the expressions (25) and (8), it is possible to obtain the dielectric tensor-operator, which can be used to study the wave phenomena, in the frequency range $\omega_{dr} << \omega << \omega_{ce}$; that allows us to analyze the ion cyclotron resonance (ICR) dissipation of eigenmodes in the current-carrying plasma.

IV.2. Ion-cyclotron damping of FMS waves in the current-carrying plasma

To estimate the ion cyclotron dissipation of the fast magnetosonic wave, at the fundamental cyclotron frequency we use the conditions, $\omega \sim k_r c_A \sim \omega_{ci}$, and $Z_i^- << 1$. In this case, we have

$$\epsilon_{11}^{(i)} = \epsilon_{22}^{(i)} = -\frac{c^2}{4c_A^2}\left(1 - i\sqrt{\frac{\pi}{2}}\frac{\omega + \omega_{ci}}{\mid k_\parallel + \chi \mid v_{Ti}}\right),$$
$$\epsilon_{12}^{(i)} = -\epsilon_{21}^{(i)} = ig = -\frac{ic^2}{4c_A^2}\left(3 - i\sqrt{\frac{\pi}{2}}\frac{\omega + \omega_{ci}}{\mid k_\parallel + \chi \mid v_{Ti}}\right), \tag{26}$$

and using the conditions

$$N_\parallel^2 << N_r^2 \approx c^2/c_A^2 << \mid \epsilon_{11} \mid, \qquad k_\parallel v_0 << \omega_{ci}, \qquad N_\parallel N_1 << \mid \epsilon_{12} \mid,$$

the following dispersion equation for fast magnetosonic waves can be obtained

$$N_{r_{FMS}}^2 = \left(\epsilon_{11}^2 + \epsilon_{12}^2\right)/\epsilon_{11}.$$

As a result, the real and imaginary parts of the radial refractive index of these waves are given by

$$(ReN_r)^2_{FMS} = \frac{c^2}{c_A^2}, \qquad \nu_{FMS} = \mid k_\parallel + 2\frac{h_\theta h_z}{r}\left(1 - \frac{r}{4q}\frac{dq}{dr}\right) \mid \frac{v_{Ti}}{\sqrt{8\pi}\omega_{ci}}. \tag{27}$$

In this case, the ion cyclotron damping of the fast magnetosonic waves (28) and analogous damping for plasmas in an uniform magnetic field[1] are the same order of magnitude if the wavelength along the magnetic field is much smaller than the screw step of the helical magnetic field line, $k_\parallel = (m+nq)h_\theta/r >> h_\theta/r$. The main difference of the obtained result from the well-known[1] is: in the uniform magnetized plasma, the FMS waves, with $\omega = \omega_{ci}$ and propagating exactly perpendicular to a straight magnetic field ($k_z = 0$), are not absorbed by the plasma ions; in the helical magnetic field case, the absorption of FMS waves, with $k_\parallel = 0$ and $\omega = \omega_{ci}$, may be substantial due to the magnetic shear correction in the resonant condition (26). In this case, the ion-cyclotron damping rate of FMS waves propagating exactly across to the magnetic surfaces in the current-carrying plasmas is

$$\nu_{FMS}|_{k_\parallel=0} = \mid \frac{h_\theta}{r}\left(1 - \frac{r}{4q}\frac{dq}{dr}\right) \mid \frac{v_{Ti}}{\sqrt{2\pi}\omega_{ci}}. \tag{28}$$

This feature of the ion cyclotron absorption, of the fast magnetosonic waves, should be taken into account to analyze the wave dissipations and stability questions in the current-carrying plasmas near the so-called rational magnetic surfaces, where the $k_\parallel(r)$ change the sign.

V. Conclusions

To analyze the wave dispersion for eigenmodes, propagating in cylindrical current-carrying plasmas, we use the dielectric tensor elements obtained through the solution of the linearized Vlasov equation for plasmas in a helical magnetic field, taking into account the effects of the equilibrium current, magnetic shear, and the density gradient. To evaluate the dispersion equation for eigenmodes in a magnetized current-carrying plasma, we used all nine dielectric tensor components. Account of these components is necessary for the correct estimation of the damping rate of the basic MHD waves. This effect is related especially to the transit time magnetic pumping[1,2] dissipation of waves in the frequency range much smaller than the ion-cyclotron frequency, $\omega << \omega_{ci}$.

Using the geometric optics approximation, for small amplitude waves, we derived the analytical expressions for the electron Landau damping of fast magnetosonic waves (see the equations (14) and (15)), fast Alfvén waves (see the equations (16)- (19)), kinetic Alfvén waves (see the equations (20) and (21)), and atmospheric whistlers (see the equations (23) and (24)), in the plasma with "hot" electrons and one kind of ions, taking into account current, shear, and density inhomogeneity. It is shown that the radial gradients of plasma density and equilibrium magnetic field influence substantially on the damping rate of the eigenmodes. The damping rate of these modes contains three independent terms (see the equations (15), (17), (21) and (24)), which are associated to three "submodels" of plasmas in the helical magnetic field. The first term corresponds to the damping rate of eigenmodes in uniform magnetized plasmas without the equilibrium current; these expressions are corresponding to the well known results (see, for example, Refs. [1] and [7]). The second and third terms describe the influence of the radial gradients of the safety factor (by dq/dr) and the plasma density (by dn_0/dr) on the damping rate of eigenmodes in nonuniform current-carrying plasmas.

The expressions for real and imaginary parts of the radial refractive indexes of the considered waves, when the wave frequency, azimuthal, and longitudinal wave numbers are given, are valid in a wide range of the wave phase velocities relative to the electron thermal velocity in the argument of the dispersion plasma function, $Z_e = (\omega - k_\parallel v_0)/\sqrt{2}k_\parallel v_{Te})$. It allows us to analyze the "hot" and "cold" limits related to wave propagation, $Z_e << 1$ and $Z_e >> 1$, respectively.

The contributions of resonant ions to the dielectric tensor elements are presented in equation (25). The ion-cyclotron damping of the fast magnetosonic waves, in the frequency range $\omega \sim \omega_{ci}$, is evaluated taking into account the magnetic shear correction (see the equations (28) and (29)). It is shown that the ion-cyclotron damping of FMS waves, propagating perpendicular to the ambient helical magnetic field, is not equal to zero in contrast to the uniform magnetic field case. The magnetic shear corrections in the argument of the plasma dispersion function, $Z_i^- = (\omega - \omega_{ci})/\sqrt{2} \mid k_\parallel + \chi \mid v_{Ti}$, are also important to analyze the drift cyclotron instabilities. These instabilities can be excited around the rational magnetic surfaces where the longitudinal projection of the wave vector is equal to zero.

The results of this paper can be applied to study the Alfvén wave heating and current drive phenomena in tokamak plasmas with circular magnetic surfaces and a large aspect ratio. However, in a general case, the one-mode approximation for waves (typical for cylindrical plasmas) is not suitable for plasmas in toroidal geometry. Here, it is necessary to take into account the "non-local" effects, connected with the poloidal mode-coupling effect and the influence of trapped and untrapped particles on the wave dissipation [13].

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