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A Kaluza-Klein Interpretation of an Extended Gauge Theory

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Abstract A possible geometric justification for the inclusion of three vector potentials transforming under a common compact and simple gauge group is presented in terms of the spontaneouscornpactification of a higher-dimensional theory of coupled Yang-Mills-gravity with non-trivial torsion.

Taking for granted all the benefits stemming from the requirement of local gauge invariance in the formulation of interacting theories for massless and massive spin-1 particles, and adopting the widespread viewpoínt of basing the description of the fundamental forces of Nature on local gauge symmetries, our efforts in refs.1,2 have been an attempt to extend the notion of gauge principle through the introduction of more than just one gauge potential in association with a single compact and sirnple gauge group.

An immediate consequence following from such an approach would be the possibility of having a unified description of massless and massive gauge bosons without the need of introducing families of scalar particles to spontaneously break the gauge symmetry. Issues like the proof of the renormalisability and unitary for this kind of extended gauge theories have been completed for the U(1)-case. The non-Abelian version is still lacking a better understanding of the structure of the globally conserved currents that are present in the theory along with the local gauge current. This would be an essential step in the programme of proving that possible unphysical modes carried by the extra gauge potentials effectively decouple from the physical amplitudes. In ref. 3, the Hamiltonian quantisation and the construction of a relativistic functional integral are discussed for a massive Yang-Mills model where two gauge potentials are associated with a single non- compact gauge group, which the author takes to be $SL(2,C)$.

In the case where three gauge potentials are present, we start by postulating the following transformation laws

$$A_\mu \to U A_\mu U^{-1} + i(\partial_\mu U)U^{-1} \tag{1a}$$

$$B_\mu \to U B_\mu U^{-1} + i(\partial_\mu U)U^{-1} \tag{1b}$$

$$F_\mu \to U F_\mu U^{-1} + i(\partial_\mu U)U^{-1} \quad , \tag{1c}$$

where the fields A_μ, B_μ and F_μ are all valued in the Lie algebra of a given compact and simple gauge group G with generic transformation law represented by U.

It is worthwhile to stress here that, though the three potentials undergo exactly the same transformations, they do not differ from one another by a pure gauge. Actually, by analysing the possible gauge-invariant Lagrangeans built up from A_μ, B_μ and F_μ, one can check that they can be distinguished by the facts that they obey different equations of motion, are classified by different quantum numbers (such as mass, for example) and carry different numbers of degrees of freedom, as discussed in ref. 1.

Aside from these considerations of a more physical nature, one can also try to realise the distinction between the potentials A_μ, B_μ and F_μ on more geometrical grounds. Indeed, as already discussed in ref. 5, two among the three potentials (let us take A_μ and B_μ) can be shown to appear in the four-dimensional world with the transformations eqs.(1a) and (1b) upon spontaneous compactification of a higher-dimensional matter-gravity interacting theory. By studying the purely gravitational sector of such a theory, it can be shown that for a non-vanishing torsion, the vielbein and spin-connection fields give rise in Minkowski space to the potentials A_μ and B_μ, both transforming under the action of the local isometry group (the gauge group G) of the internal compact homogeneous space which emerges as a solution of the coupled higher-dimensional equations of motion.

Motivated by what has been exposed in the previous paragraph, we would like in this work to try to understand how we could trace back the origin of the third potential, F_μ, at the level of the coupled matter-

-gravity theory in more than four dimensions. Since we have exhausted the geometrical structures that potentially contain the possible gauge fields in four dimensions, namely, the vielbein and spin-connection, the appearance of the field F_μ can only be understood in terms of the Yang-Mills sector of higher dimensions. After all, Yang-Mills fields have to be already present in the initial theory if one has to invoke the mechanism of spontaneous compactification to justify the internal symmetries of the four-dimensional world.

The D-dimensional ($D=4+K$) gravity-matter coupled theory is described by the following set of fields:

the vielbein, $E_M^{\;A}(z)$,

the spin-connection, $B_M^{\;AB}(z)$,

the Yang-Mills field, $A_M^{\;\hat{i}}(z)$,

with Lagrangean given by

$$L = E\left[\frac{1}{\kappa^2} E_A^{\;M} E_B^{\;N} R_{MN}^{\;\;\;AB} + \Lambda - \frac{1}{4g^2} F_{MN}^{\;\;\hat{i}} F^{MN\hat{i}}\right] , \tag{2}$$

where $E = \det\{E_M^{\;A}(z)\}$, κ and g are respectively the gravitational and gauge coupling constants, Λ is the D-dimensional cosmological constant and

$$R_{MNAB} = \partial_M B_{NAB} + B_{NAC} B_{MCB} - (M \leftrightarrow N) , \tag{3a}$$

$$F_{MN}^{\hat{i}} = \nabla_M A_N^{\hat{i}} - \nabla_N A_M^{\hat{i}} + f^{\hat{i}\hat{j}\hat{k}} A_M^{\hat{j}} A_N^{\hat{k}} , \tag{3b}$$

$$\nabla_M A_N^{\hat{i}} = \partial_M A_N^{\hat{i}} - \Gamma_{MN}^{P} A_P^{\hat{i}} , \tag{3c}$$

$$\Gamma_{MN}^{P} = \{{}^{\;P}_{MN}\} + K_{MN}^{P} \tag{3d}$$

$\{{}^{\;P}_{MN}\}$ stands for the Christoffel symbol and K_{MN}^{P} for the contorsion tensor.

It would be wortwhile to give now some explanation of our conventions:

- z^M denotes the coordinates of the D-dimensional space-time, $z^M = (x^\mu, y^m)$, where x^μ stands for the coordinates of the four-dimensional Minkowski space and y^m parametrises the internal compact space $(m = 1,2,\ldots,K)$;

- the middle alphabet letters $M,N,P,\ldots = 1,2,\ldots,D$ are world indices;

- A, B, C, ... = 1,2,..., D are local frame labels;

- $\hat{i}$, $\hat{j}$, $\hat{k}$ are the indices of the adjoint representation of the Yang-Mills gauge group $\hat{G}$.

Notice that in our definition eq.(3b) of the Yang-Mills field strength, we have written the covariant derivative, despite the antisymmetry in the indices M and N. The reason is that we are going to consider the general case of a non-vanishing torsion tensor, $T_{MN}{}^A$

$$T_{MN}{}^A = \partial_M E_N{}^A + E_N{}^e B_M{}^A{}_e - (M \leftrightarrow N) \quad , \tag{4}$$

so that the connection coefficient do not exhibit symmetry in their two lower indices,

Now, suppose that the matter fields (here, the Yang-Mills fields) coupled to the gravitational degree of freedom induce the mechanism of spontaneous compactification[6,7], that is, the equations of motion of the coupled system factorise the ground state geometry according to $M^4 \times B^K$, where M^4 is the four-dimensional Minkowski space and B^K is a K-dimensional compact Riemannian manifold. For example, in the case in which B^K is a compact homogeneous space of the form $B^K = G/H$, it is sufficient to consider an H-invariant Yang-Mills background configuration to ensure spontaneous compactification on G/H [8]. In such a case, B^K is invariant under the left-action of the compact Lie group G and such that any of its points can be transformed into any other by the action of the elements of G.

The compactification ansatz actually consists in taking our background field configurations given by:

$$\langle A_\mu^{\hat{i}}(z)\rangle = 0 \quad , \tag{5a}$$

$$\langle A_m^{\hat{i}}(t)\rangle = \bar{A}_m^{\hat{i}}(x;y) \tag{5b}$$

and

$$\langle E_M{}^A(z)\rangle = \begin{pmatrix} \delta_\mu^\alpha & 0 \\ 0 & \bar{e}_m^\alpha(x;y) \end{pmatrix} , \qquad (5c)$$

where $\bar{e}_m^\alpha(x;y)$ denotes the vielbein of the internal manifold B^K. Here, a and a label the frame indices of the external non-compact and internal compact spaces respectively.

The background geometry specified by $\langle E_M{}^A(x;y)\rangle$ has the usual Poincaré symmetry. Its local G-invariance consists of x-dependent left-translations,

$$(x;y) \rightarrow (x;y' = F(g(x);y)) , \quad g(x) \in G , \qquad (6)$$

and local frame rotations in B^K. The combination of coordinate transforrnations and frarne rotations leaving the vielbein form-invariant is called an isometry.

Under the isornetries of the internal manifold, the following transformations hold[7]:

$$\bar{e}'^{\alpha}_m(x;y') = \frac{\partial y^n}{\partial y'^m} \bar{e}^\beta_n(x;y)\ \Lambda_\beta{}^\alpha(g(x);y) \qquad (7)$$

and

$$\bar{B}'_{m\alpha\beta} = \frac{\partial y^n}{\partial y'^m}\left[\bar{B}_{n\gamma\delta}(x;y)\Lambda^\gamma{}_\alpha(g(x);y)\ \Lambda^\delta{}_\beta(g(x);y) + (\Lambda^{-1}\partial_m\Lambda)_{\alpha\beta}\right], \qquad (8)$$

$g(x)$ being a transformation of the group G as the point x of Minkowski space. Infinitesirnally, the isometries of B^K are specified by the Killing vectors, K_m^i (i is the index of the adjoint representationof the isometry group G), and a certain number of parameters known as Killing angles.

Now, some care has to be taken in considering the background configurationsof the Yang-Mills field. They could, in principle, break the isometry group G of B^K, but this is exactly what we would like to avoid. The reason is that, as discussed in ref. 5, the four-dimensional potentials A_μ and B_μ of eqs. (1a) and (1b), which come from thevielbein and spin-connection, are gauge potentials of G, which appears as the four-dimensional gauge group by virtue of the spontaneous compactification the theory undergoes. Therefore, we should take the Yang-Mills background configuration in such a way to still guarantee that the whole G be the four-dimensional Yang-Mills gauge group.

In order that the configuration $\bar{A}_m^{\hat{i}}(x;y)$ respect the isometry group of B^K, one has to consider suitable gauge transformation of $\hat{G}$, which combined with the isometry transformations of B^K, leave $\bar{A}_m^{\hat{i}}(x;y)$ invariant in form. By considering the action of an isometry,

$$\bar{A}'^{\hat{i}}_m(x;y) = \frac{\partial y^m}{\partial y'^m} \bar{A}^{\hat{i}}_n(x;y) \quad , \tag{9}$$

in association with a $\hat{G}$-transformation,

$$\bar{A}'^{\hat{i}}_m(x;y) = \bar{A}^{\hat{j}}_m R_{\hat{j}}^{\ \hat{i}}(u(z)) + i\left[u(z)\partial_m u(z)^{-1}\right]^{\hat{i}} , \quad u(z) \in \hat{G} \quad , \tag{10}$$

it is possible to have a background $\bar{A}_m^{\hat{i}}(x;y)$ which does not break the isometry group. For this, one can find a gauge transformation u which is g-dependent (recall that one has $y \to y' = F(g(x);y)$) and compensates the isometry transformation of $\bar{A}_m^{\hat{i}}$. However, one should stress that $\bar{A}_m^{\hat{i}}$ breaks the Yang-Mills group $\hat{G}$.

Once the background geometry has been set, we can write that the fields of the theory are split according to

$$E_M^{\ A}(z) = \langle E_M^{\ A}\rangle + \delta E_M^{\ A} \tag{11a}$$

$$B_M^{\ AB}(z) = \langle B_M^{\ AB}\rangle + \delta B_M^{\ AB} \tag{11b}$$

and

$$A_M^{\hat{i}}(z) = \langle A_M^{\hat{i}}\rangle + \delta A_M^{\hat{i}} \tag{11c}$$

where the δ's indicate fluctuations around the respective background configuration. As concluded in ref. 5, the four-dimensional gauge potentials A_μ and B_μ are located in the space-time components $e_\mu^{\ \alpha\beta}(x;y)$ and $B_\mu^{\ \alpha\beta}(x;y)$ of the vielbein and spin-connection.

The higher-dimensional gauge potential $A_M^{\hat{i}}(z)$ can introduce another four-dimensional gauge potential through its component $A_\mu^{\hat{i}}(x;y)$. This can be seen from the fact that under an x-dependent $\hat{G}$-transformation, $u(x)$, it transforms as

$$A'^{\hat{i}}_\mu(z) = A^{\hat{j}}_\mu R_{\hat{j}}^{\ \hat{i}}(u(x)) + i\left[u(x)\partial_\mu u(x)^{-1}\right]^{\hat{i}} \qquad (12)$$

Under the same transforrnation, the cornponent $A_m^{\hat{i}}(x;y)$ changes homogeneously according to

$$A'^{\hat{i}}_m(z) = A^{\hat{j}}_m R_{\hat{j}}^{\ \hat{i}}(u(x)) \ , \qquad (13)$$

corresponding to scalar modes of the four-dimensional world. Therefore, the component of the higher-dimensional gauge field that is relevant for what we wish to do is $A_\mu^{\hat{i}}(x;y)$: it carries spin-1 and transforms inhomogeneously under $u(x)$, a gauge transformation of $\hat{G}$.

Now, the whole problem lies in the following question: could we extract out of $A_\mu^{\hat{i}}(x;y)$ a four-dimensional mode, $F_\mu^{i}(x)$, transforming under the irometry group G of B^K ? To be convinced that this ispossible, one should define the potential $A_\mu^{\hat{i}}(x;y)$ in terms of the following mode expansion:

$$A_\mu^{\hat{i}}(x;y) = F_\mu^{i}(x)\ K_i^m(y)\ \bar{A}_m^{\hat{i}}(x;y) \quad , \qquad (14)$$

where $F_\mu^{i}(X)$ represents a vector field propagating in four-dimensional space-time. With such a decomposition, one can check that the vector modes represented by $F_\mu^{i}(x)$ behave as additional gauge modes: they transform in the adjoint representation of the (left-) isometry group according to

$$F'^{i}_\mu(x) = F^{j}_\mu S_j^{i}(g(x)) - i\left[g(x)\ \partial_\mu g(x)^{-1}\right] \ , \qquad (15)$$

where $g(x)$ is a left-isometry transformation.

Bringing this conclusion together with the results of[5], one can finally state that a coupled gravity-Yang-Mills $(4+K)$-dimensional theory may, under special ansatze, compactify on a K-dimensional homogeneous space G/H and give rise to three families of four-dimensional gauge potentials all transforming under the action of a common compact gauge group, G, according to the transformation laws eqs. (1a), (1b) and (1c). The gauge potentials $A_\mu(x)$ and $B_\mu(x)$ originates from the gravitational sector, whereas the potentials $F_\mu(x)$ comes from the Yang-Mills sector of the higher-dimensional theory.

Though it has not been our purpose here to discuss the issue of how the initial gauge group $\hat{G}$ breaks through the compactification mechanism, one can say that the final gauge group of the effective four-dimensional theory will be of the form $G \times \tilde{G}$, where $\tilde{G}$ is the unbroken subgroup of $\hat{G}$. However, it is important to stress (and this is the true motivation of our work) that the three families of gauge potentials A_μ, B_μ and F_μ transform in the adjoint representation of the factor G and do not suffer the action of the remnant of the higher-dimensional gauge group $\hat{G}$.

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Resumo

Uma possível justificação geométrica para a inclusão de três potenciais vetoriais transformando-se sob o mesmo grupo de gauge compacto e simples é apresentado em termos da compactação espontânea de uma teoria em dimensões superiores que acopla Yang-Mills e gravidade em presença de torção.