

# Magnetic Nanomaterials.

## Some Biomedical Applications: Magnetofection, Magnetic Hyperthermia, and Ferrogels for Drug Delivery

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*Physics Building, 1905*



<http://www.fisica.unlp.edu.ar/Members/sanchez/escola-de-magnetismo-vitoria-es-brasil-03-11-13-al-08-11-13>

Why magnetic nanomaterials for biomedicine?

?

# Bibliography

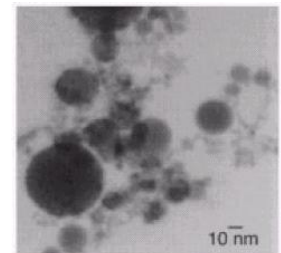
*Introduction to Magnetic Materials* B.D. Cullity,  
(Massachusetts, Addison-Wesley, 1972).

*Introduction to the Theory of Ferromagnetism*, Amikam  
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**Nanomedicine: design and applications of magnetic  
nanomaterials, nanosensors and nanosystems**  
**Vijay K. Varadan, Linfeng Chen, Jining Xie, 2008**  
**John Wiley & Sons, Ltd**



Selected articles

<http://www.fisica.unlp.edu.ar/Members/sanchez/curso-de-posgrado-nanomateriales-magneticos-intema/bibliografia>

<http://www.fisica.unlp.edu.ar/>

Actividad Académica

Docentes

Profesores

[Sánchez, Francisco Homero](#)

Two links:

Escola de Magnetismo - Vitoria, ES,  
Brasil

Curso de Posgrado "Nanomateriales  
Magnéticos"

# Index

## Class 1

Brief revision: contributions to energy in magnetic nanoparticles

Stoner – Wohlfarth model

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Paramagnetism

Superparamagnetism

Interacting superparamagnets

Demagnetizing factor  $N_{Def}$  in samples with disperse magnetic NPs

## **Class 2a**

Understanding SAR and searching for high performance nanomaterials  
zinc-doped magnetite nanoparticles and ferrofluids for hyperthermia  
applications

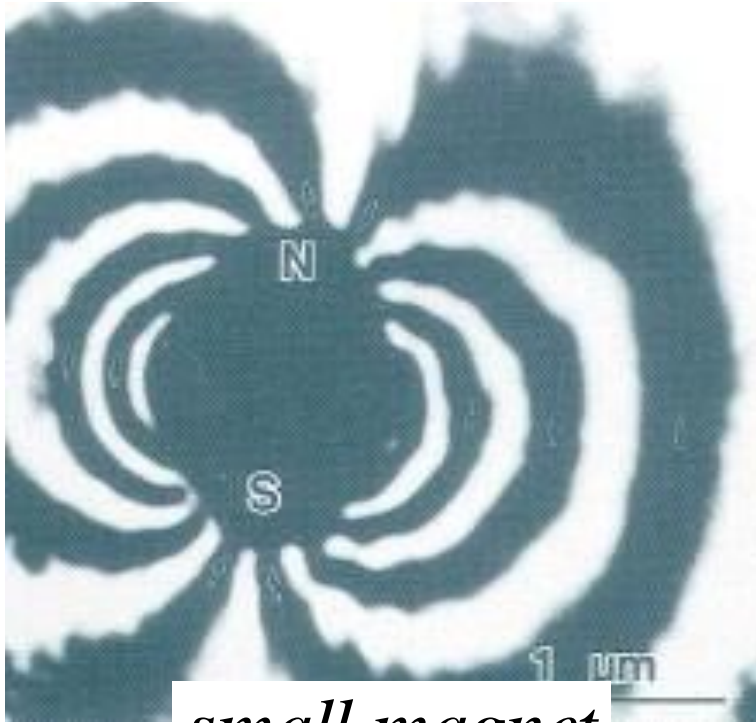
Citric Acid Coated Magnetite Nanoparticles for Magnetic Hyperthermia  
In vitro experiments

## **Class 2b**

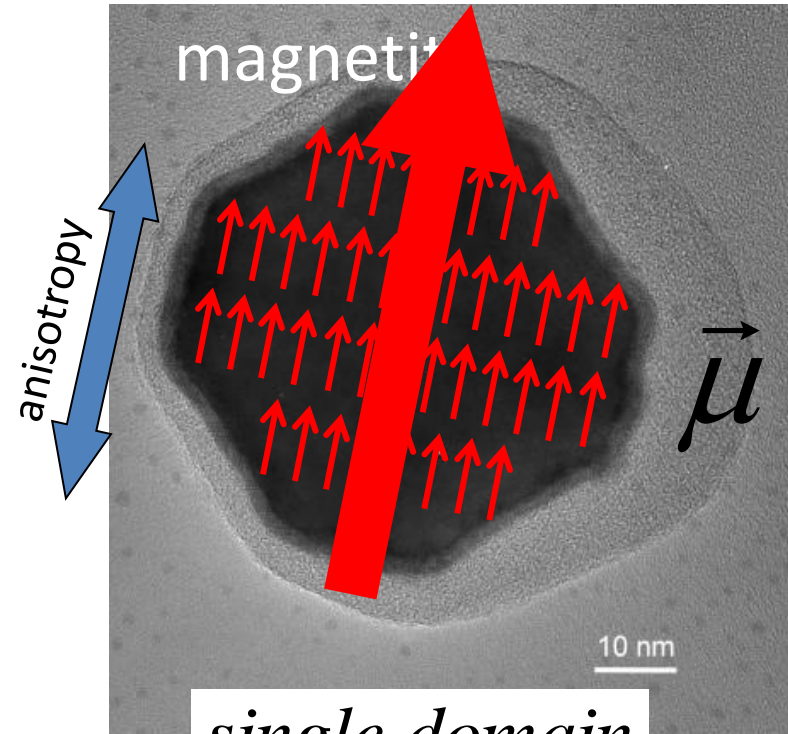
In Vitro Magnetofection: Magnetic Force Influence

Ferrogels PVA/Fe oxide

# Magnetic nanomaterials



*small magnet*



*single domain*

Magnetic Nanocomposites.

Examples:

- Ferrofluids
- Virus/NPs complexes
- Ferrogels: magnetic hydrogels

# Magnetic nanomaterials

A brief introduction to nanomaterials magnetic state:

Exchange interaction

Magnetic anisotropy

Magnetostatic energy: dipolar interaction

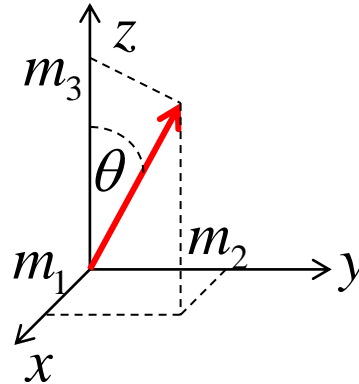
Zeeman interaction: response to an applied field



# Magnetic Anisotropy – phenomenological description

$$m_{1,2,3} = \frac{M_{x,y,z}}{M} \dots$$

$m_i$ :  
magnetization  
director cosines



$e_K$ : anisotropy energy  
per volume unity

$$e_K = \sum_i K_i m_i^2 + \sum_{ij} K_{ij} m_i^2 m_j^2 + K_{123} m_1^2 m_2^2 m_3^2 + \sum_i K_i m_i^4 + \dots$$

$E_K$ : anisotropy energy

$$E_K = \int e_K dV$$

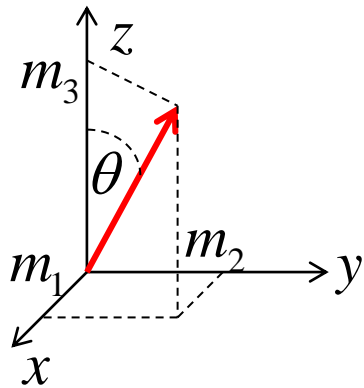
# Magnetocrystalline Anisotropy in Cubic Crystals

$$e_K = \sum_{ij} K_1 m_i^2 m_j^2 + K_2 m_1^2 m_2^2 m_3^2$$

Material	$K_1$ ( $10^5$ J/m <sup>3</sup> )	$K_2$ ( $10^5$ J/m <sup>3</sup> )	Eje fácil
Ni	-0.045	-0.023	(111)
Fe	0.480	0.05	(100)

# Magnetocrystalline Anisotropy in Hexagonal Crystals

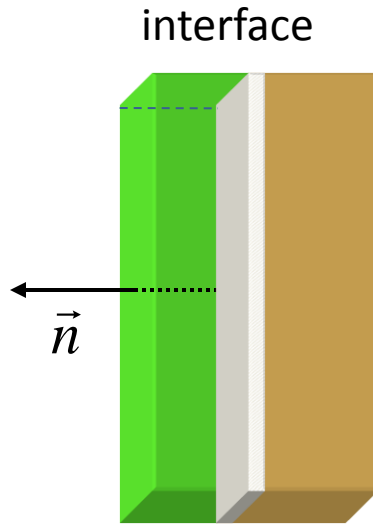
$$e_K = K_1 \sin^2 \theta + K_2 \sin^4 \theta$$



Material	$K_1$ ( $10^5$ J/m <sup>3</sup> )	$K_1$ ( $10^5$ J/m <sup>3</sup> )	Easy axis
Co	4.1	1.0	hexagonal
SmCo <sub>5</sub>	1100	-	hexagonal

# Surface Anisotropy

Interface anisotropy

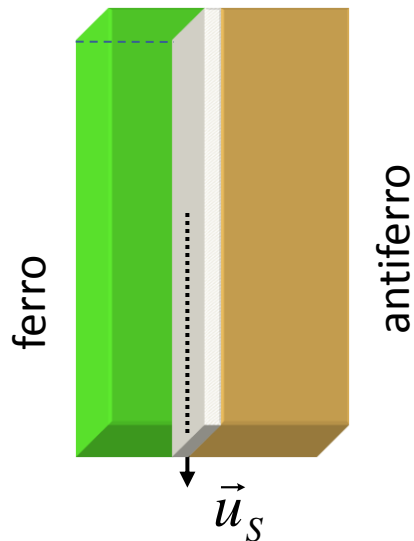


$$e_K = K_S [1 - (\vec{m} \cdot \vec{n})^2]$$

$$K_S > 0 \Rightarrow \vec{m} // \text{sup}$$

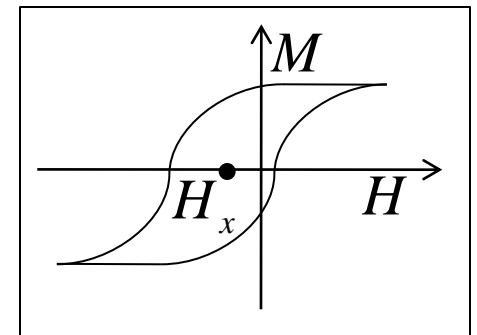
$$K_S < 0 \Rightarrow \vec{m} \perp \text{sup}$$

Exchange anisotropy\*



$$e_K = K_S \vec{m} \cdot \vec{u}_S = \frac{H_x}{2} \vec{m} \cdot \vec{u}_S$$

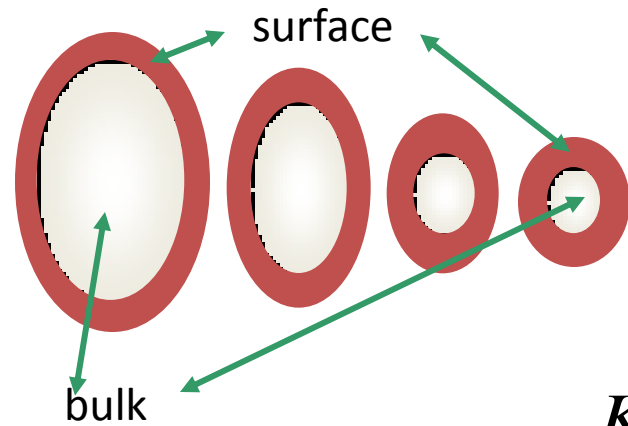
$$e_K = \frac{H_x}{2} m \cos \varphi$$



Exchange bias field

\*called also unidirectional

# Surface anisotropy in nanoparticles



$K_S = 10^{-5} - 10^{-4} \text{ J/m}^2$ ,  
anisotropy energy  
per surface area

unit

$$K_{ef} = K_B + K_{V_S}^{ef}$$

$$K_{V_S}^{ef} \approx \frac{SK_S}{V} \approx \frac{4\pi r^2}{\frac{4}{3}\pi r^3} K_S = \frac{3}{r} K_S = \frac{6}{d} K_S \quad \text{spherical NP}$$

$$K_{ef} = K_B + \frac{6K_S}{d}$$

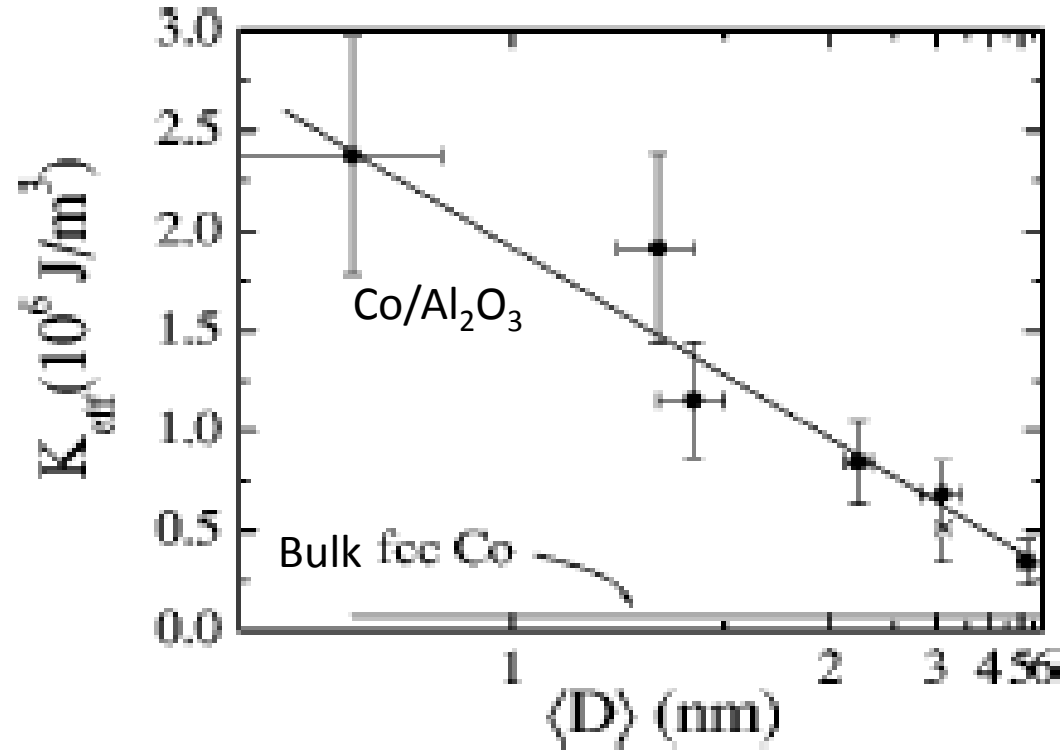
$$K_{ef} = K_B + \gamma \frac{K_S}{d}$$

Bødker et. Al (1994)

surfaces/interfaces:

- Compositional and configurational discontinuity
- Large anisotropic effect

# Surface anisotropy in nanoparticles



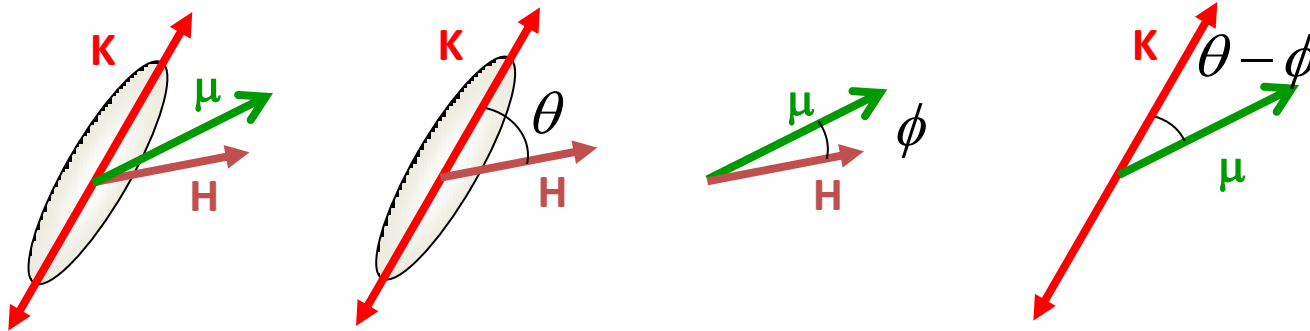
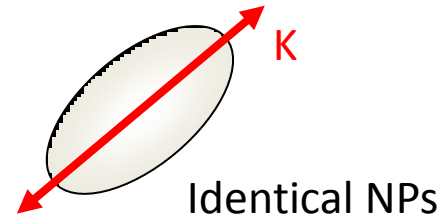
F. Luis, J.M. Torres, L.M. Gracia, J. Bartolomé, J. Stankiewicz, F. Petroff, F. Fettar, J. L. Maurice and A. Vaurés. Phys. Rev B, **65** (2002) 094409

# Stoner – Wohlfarth Model

Single domain NPs

Uniaxial anisotropy

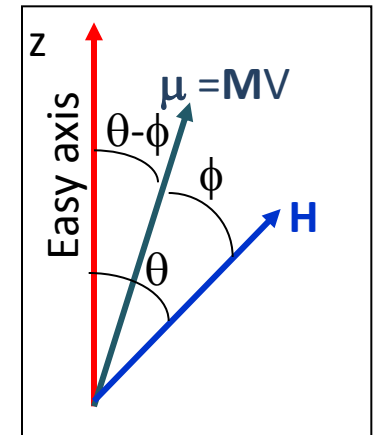
no interactions among NPs



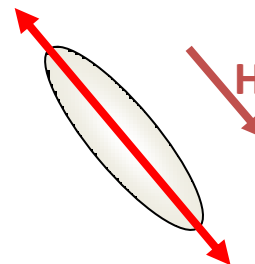
$$E = E_K + E_H = KV \sin^2(\phi - \theta) - \mu_0 H M_s V \cos \phi$$

anisotropy

Zeeman

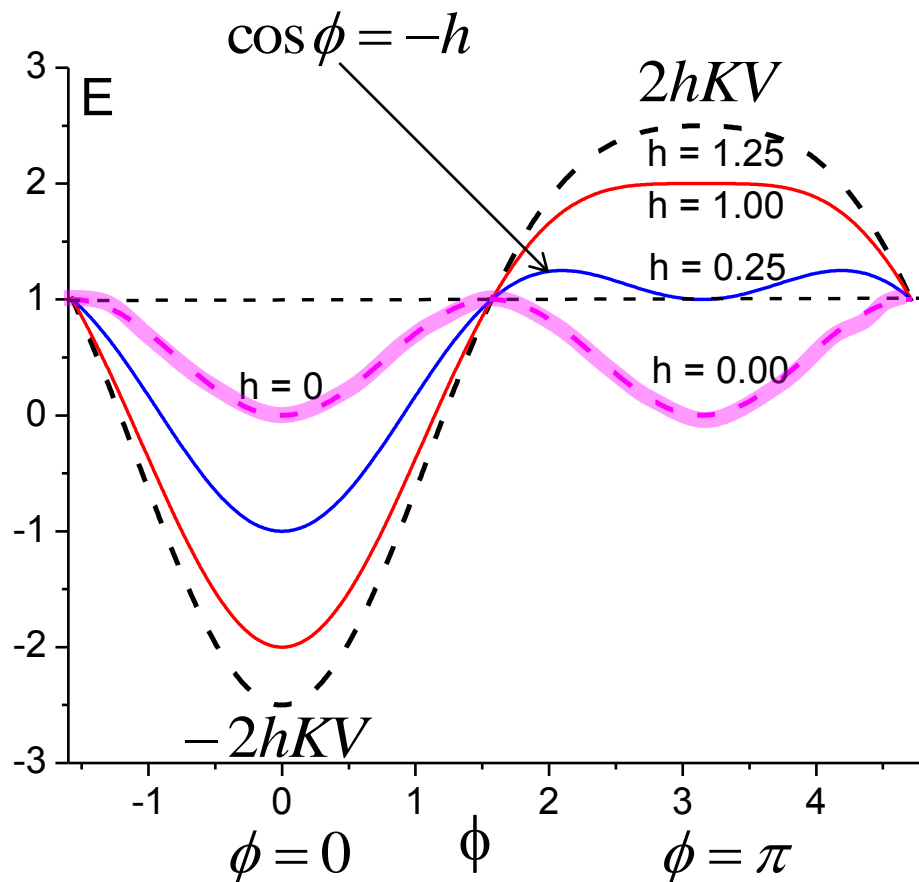


# Field in the easy direction



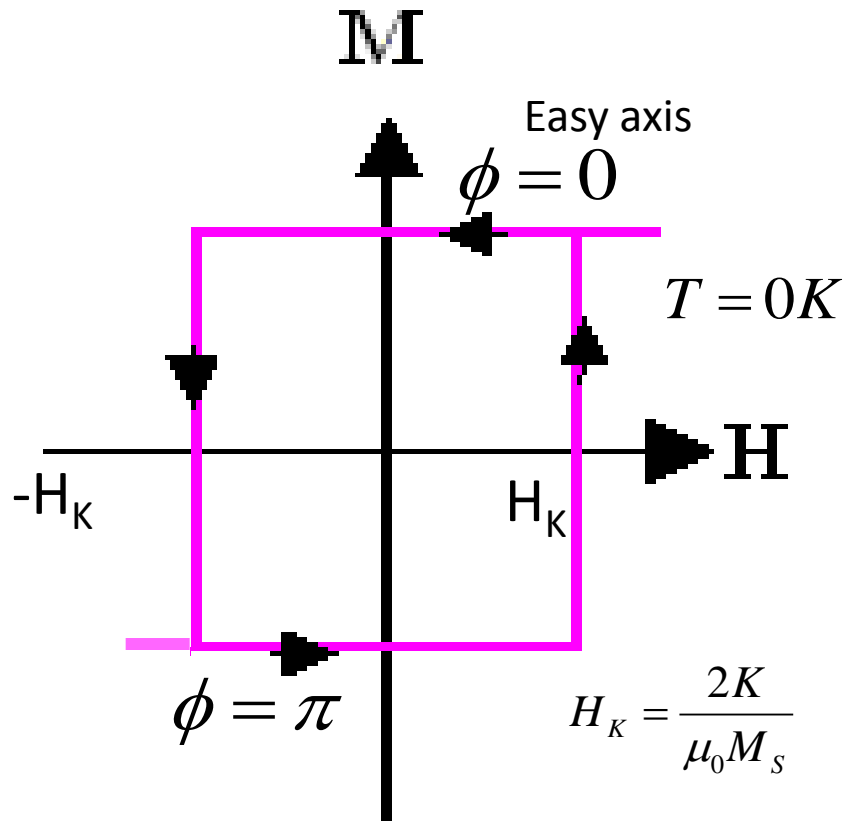
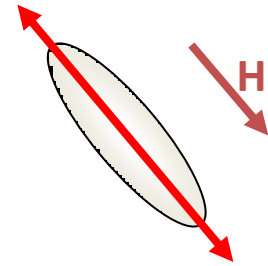
$$E = KV(\sin^2 \phi - 2h \cos \phi)$$

$$h = \frac{H}{H_K} \quad H_K = \frac{2K}{\mu_0 M_S}$$



# Field in the easy direction

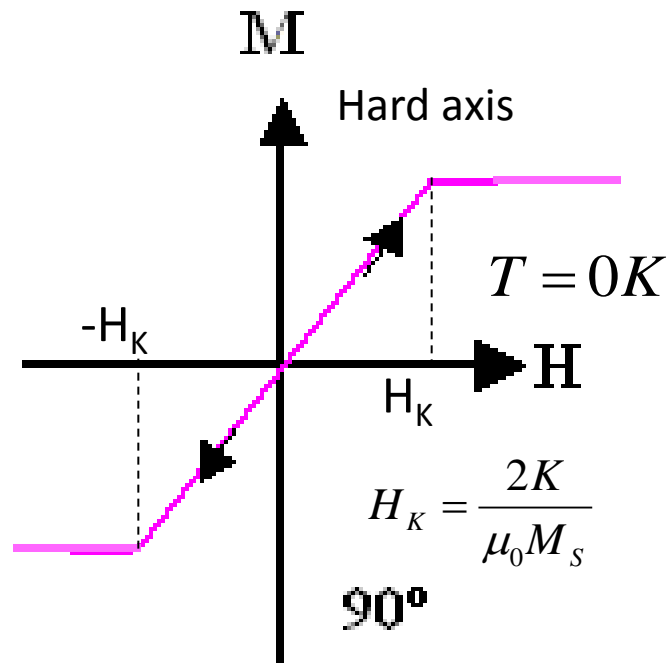
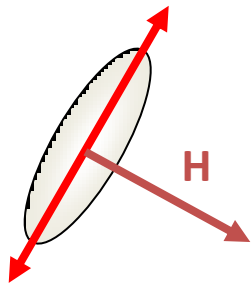
$$M_z = M_s \cos \theta$$





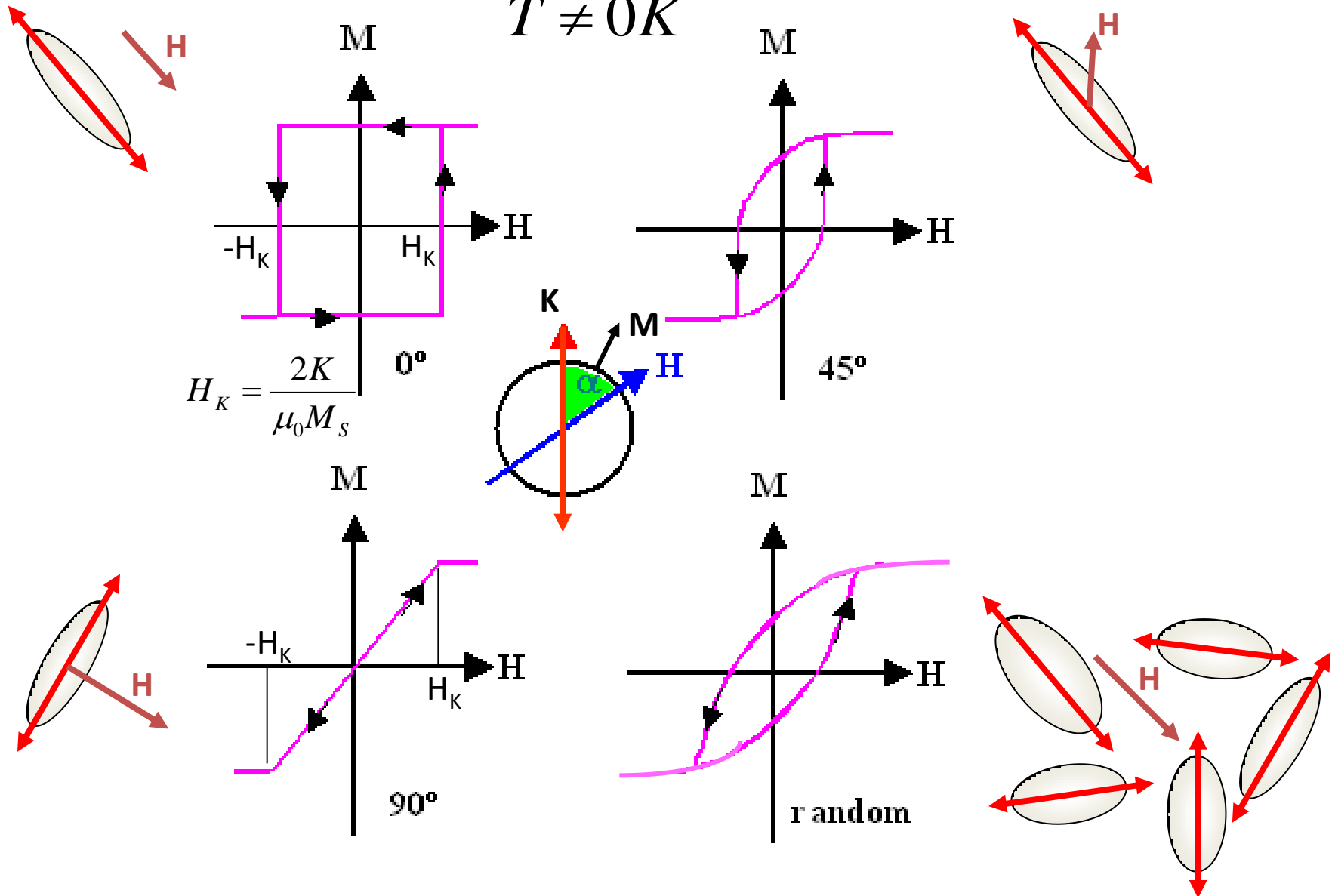
# Field in the hard direction

$$M_z = \frac{M_S}{H_K} H; \quad |h| < 1$$



# Stoner – Wohlfarth Model

$T \neq 0K$



# Stoner – Wohlfarth Model

[ 599 ]

## A MECHANISM OF MAGNETIC HYSTERESIS IN HETEROGENEOUS ALLOYS

BY E. C. STONER, F.R.S. AND E. P. WOHLFARTH  
*Physics Department, University of Leeds*

*(Received 24 July 1947)*

VOL. 240. A. 826 (Price 10s.)

74

[Published 4 May 1948

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# Stoner – Wohlfarth Model

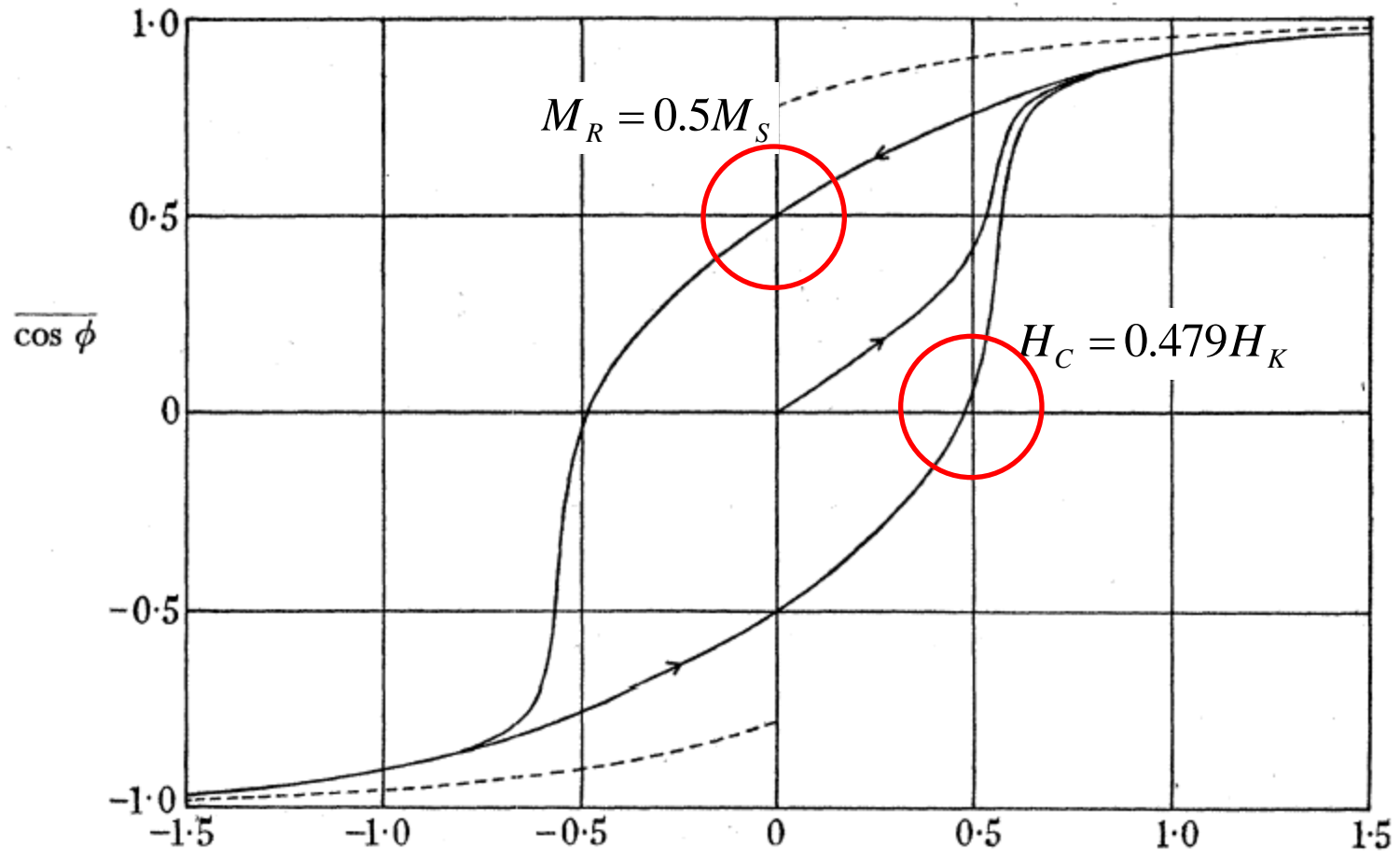
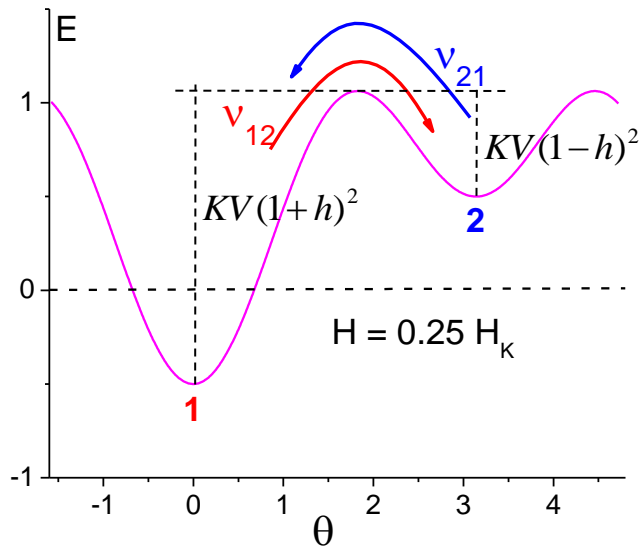


FIGURE 7. Magnetization curves for prolate (full curves) and oblate (broken curves) spheroids orientated at random. The curves refer to similar prolate (or oblate) spheroids orientated at random.  $\overline{\cos \phi}$  is proportional to the mean resolved magnetization per spheroid in the positive field direction, or to the resultant magnetization in this direction of the assembly.  $H = (|N_a - N_b|) I_0 h$ .

# Extending the Stoner – Wohlfarth Model

$$T \neq 0K$$



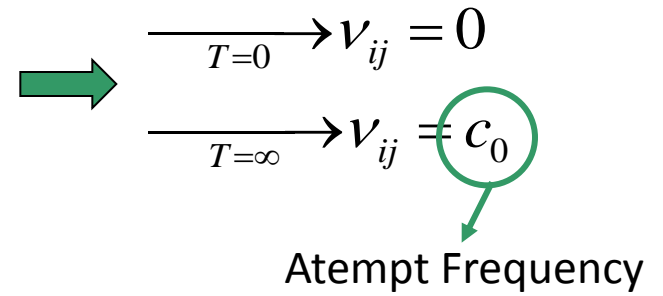
$$\Delta E_{ij} = KV(1+h)^2$$

$$v_{ij} = c_0 e^{-\frac{\Delta E_{ij}}{kT}}$$

Jump Frequency

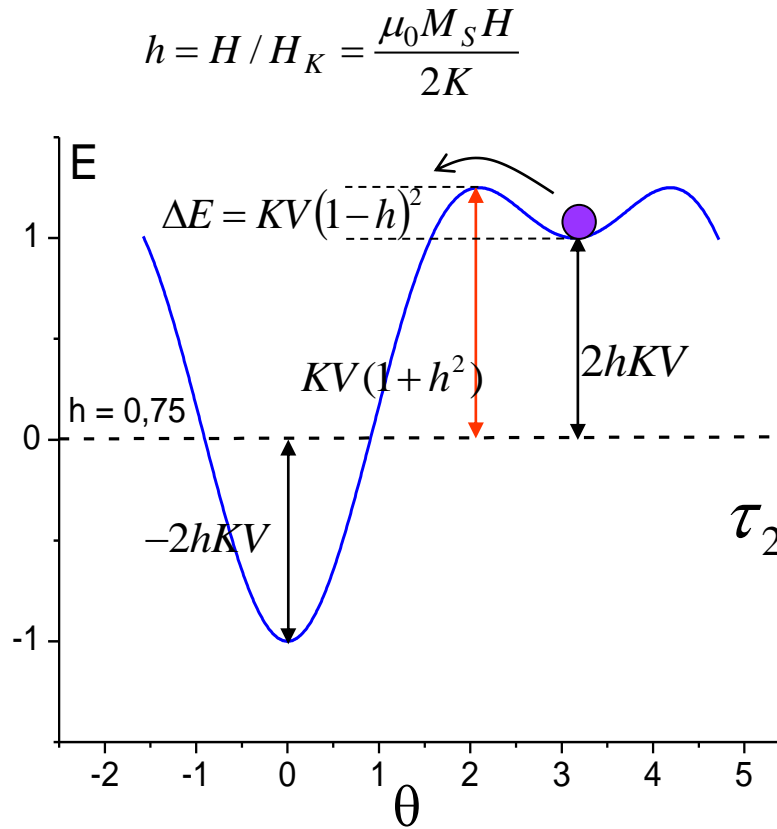
$$\tau_{ij} = c_0^{-1} e^{\frac{\Delta E_{ij}}{kT}}$$

Relaxation Time



# Extending the Stoner – Wohlfarth Model

## Coercive Field Temperature Dependence

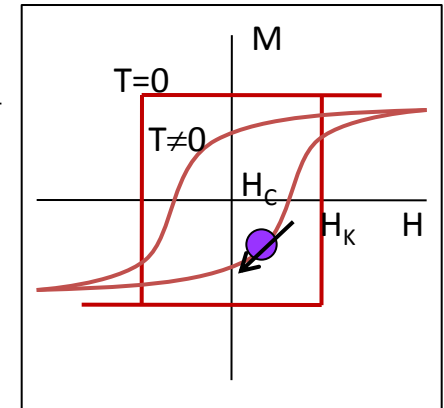


$$H_C = H_K = \frac{2K}{\mu_0 M_S}$$

$$\tau_{21} = \tau_0 e^{\frac{KV(1-h)^2}{kT}}$$

moment inversion occurs when

$$\tau_{21} \approx \tau_{\text{exp}}$$



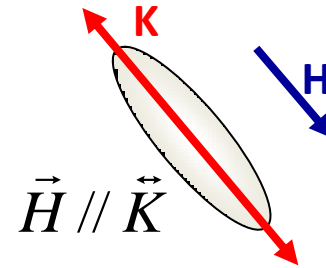
# Extending the Stoner – Wohlfarth Model

## Coercive Field Temperature Dependence

$$KV(1-h)^2 \approx kT \ln(\tau_{\text{exp}} / \tau_0)$$

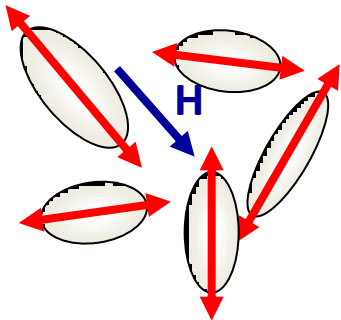
$$h \approx 1 - \sqrt{\frac{kT}{KV} \ln(\tau_{\text{exp}} / \tau_0)}$$

$$H_C(T) \approx H_K \left( 1 - \sqrt{\frac{kT}{KV} \ln(\tau_{\text{exp}} / \tau_0)} \right)$$

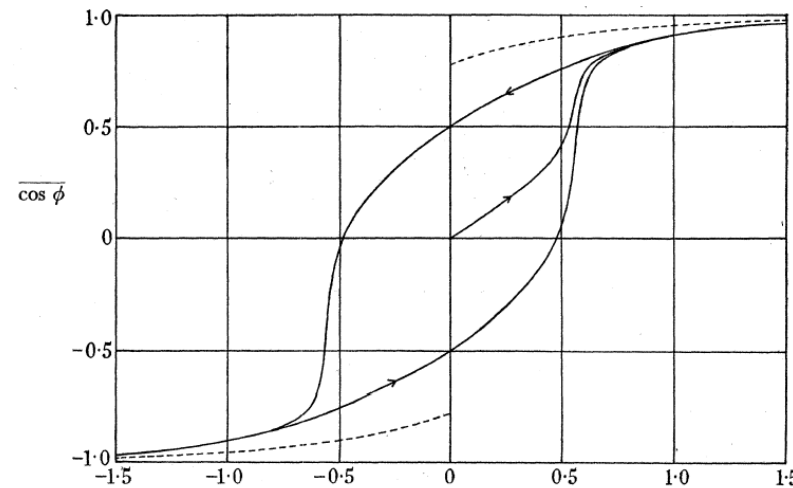


## NPs Random Orientation

$$H_C(T) \approx 0.48H_K \left( 1 - \sqrt{\frac{kT}{KV} \ln(\tau_{\text{exp}} / \tau_0)} \right)$$



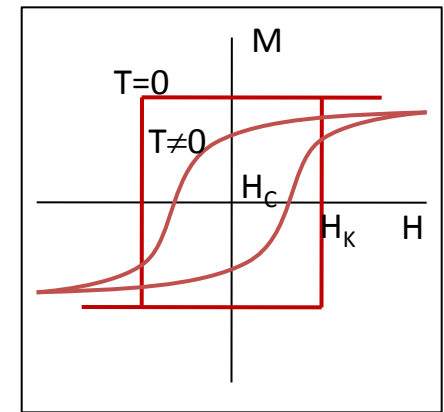
$$H_C(0) = 0.479H_K$$



# Temperature Dependent Magnetic Properties of Barium-Ferrite Thin-Film Recording Media

Yingjian Chen, *Member, IEEE*, and Mark H. Kryder, *Fellow, IEEE*

IEEE TRANSACTIONS ON MAGNETICS, VOL. 34, NO. 3, MAY 1998



$$H_c(t') = H_k \left\{ 1 - \left[ \frac{k_B T}{K_u V_{sw}} \ln \left( \frac{A t'}{0.693} \right) \right]^n \right\}$$

the easy axis orientation. In a system with uniaxially aligned easy axes,  $n$  is 1/2 [29], and in a system with random easy axis orientations,  $n$  is 2/3 [30]. The fitting parameters  $V_{sw}$

[29] M. P. Sharrock and J. T. McKinney, *IEEE Trans. Magn.*, vol. MAG-17, p. 3020, 1981.

[30] R. H. Victora, "Predicted time dependence of the switching field for magnetic materials," *Phys. Rev. Lett.*, vol. 63, pp. 457-460, 1989.

Uso extendido de la expresión

$$H_c = \alpha \frac{2K}{M_s} \left[ 1 - \left( \frac{T}{T_B} \right)^{1/2} \right]$$

Magnetic Interactions in ferromagnetic nanotubes of LaCaMnO and LaSrMnO,

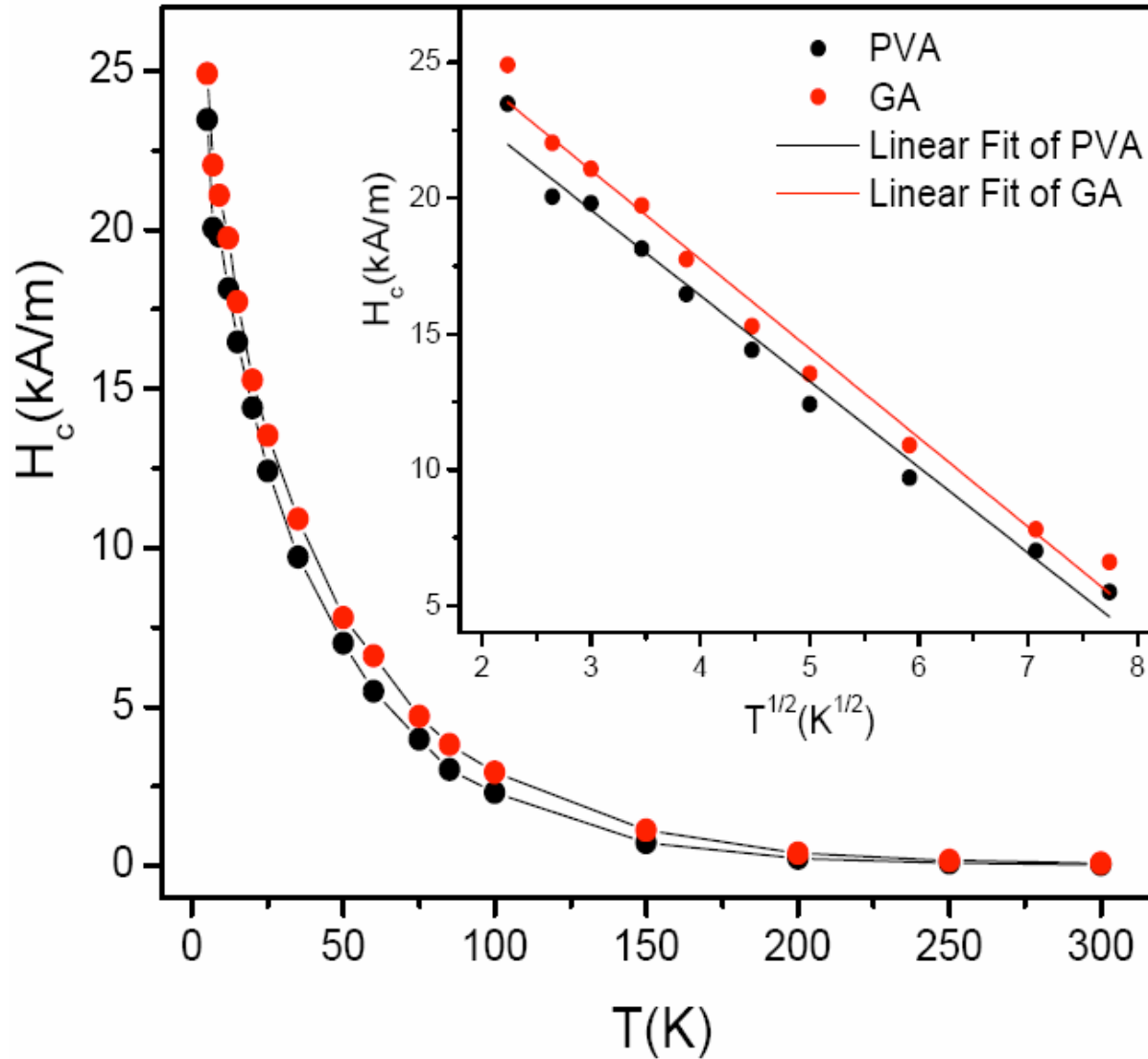
J. Curiale et al., AFA 2006

Marina Tortarola, Thesis, IB, 2008



# Extending the Stoner – Wohlfarth Model

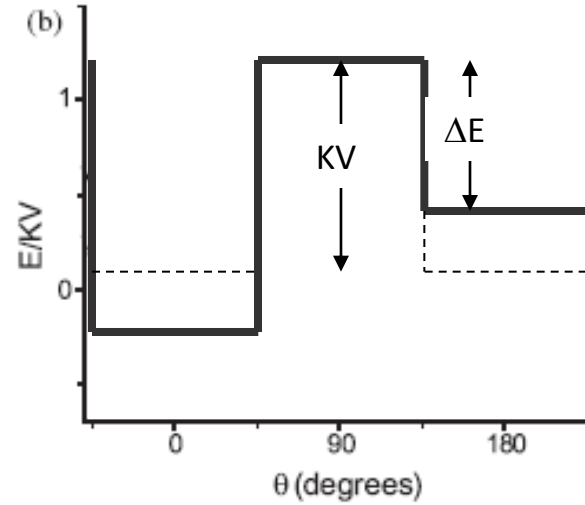
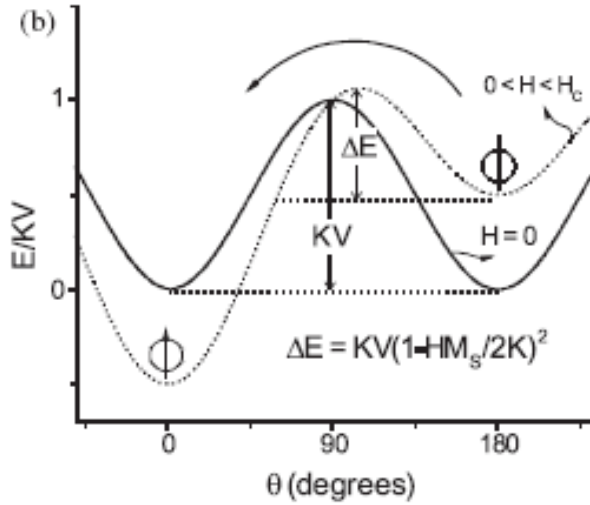
## Coercive Field Temperature Dependence



Ferrogel of maghemite NP (8 nm) in PVA hydrogel, Mendoza Zélis et al.

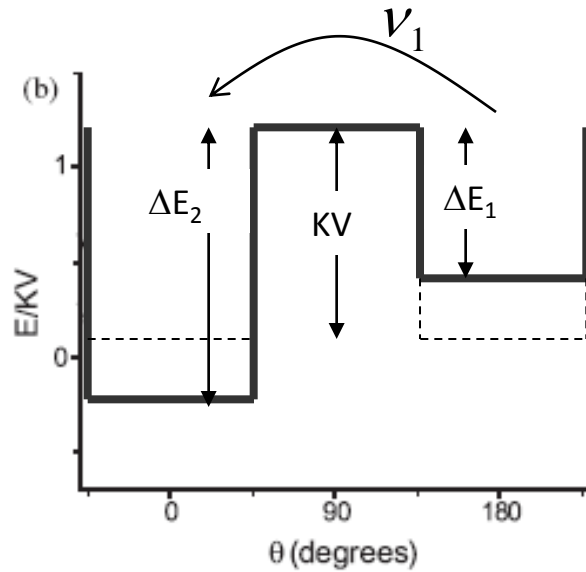
# Two levels model

Field in easy direction



Simplification: 2 levels

# Two levels model

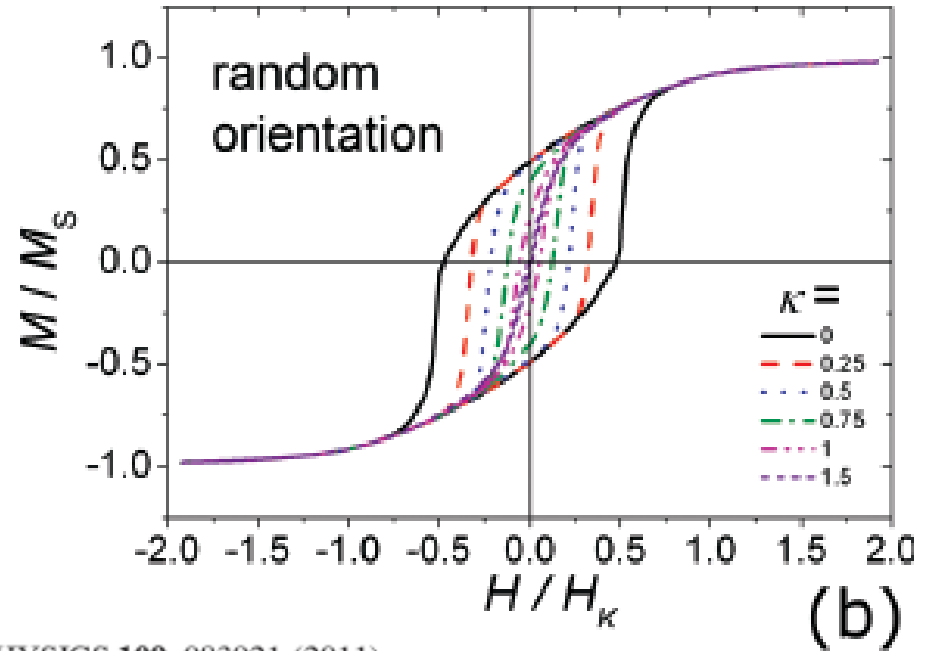
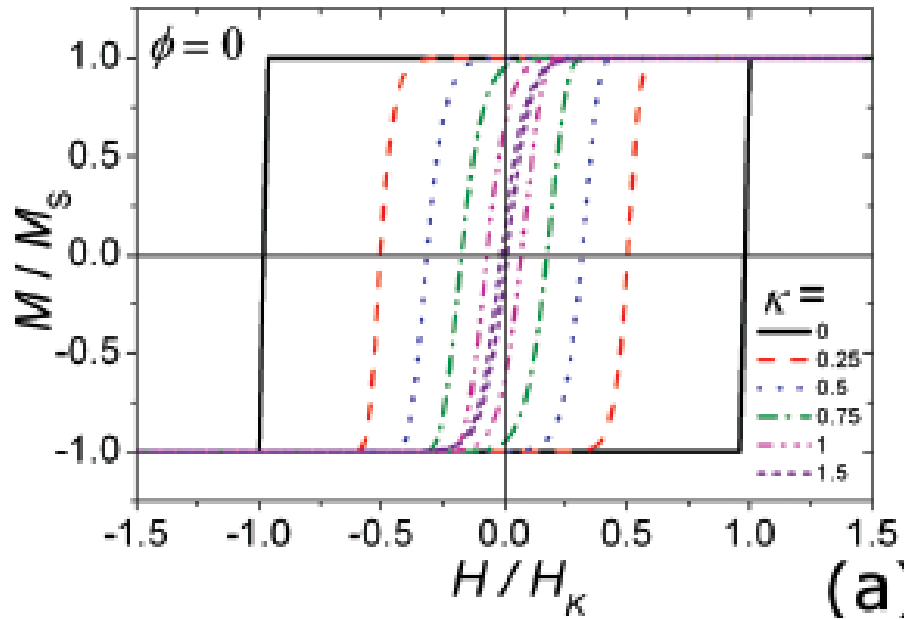


$$v_1 \approx v_0 e^{\Delta E_1 / kT} \quad v_2 \approx v_0 e^{\Delta E_2 / kT}$$

$$\frac{\partial p_1}{\partial t} \approx (1 - p_1)v_2 - p_1v_1$$

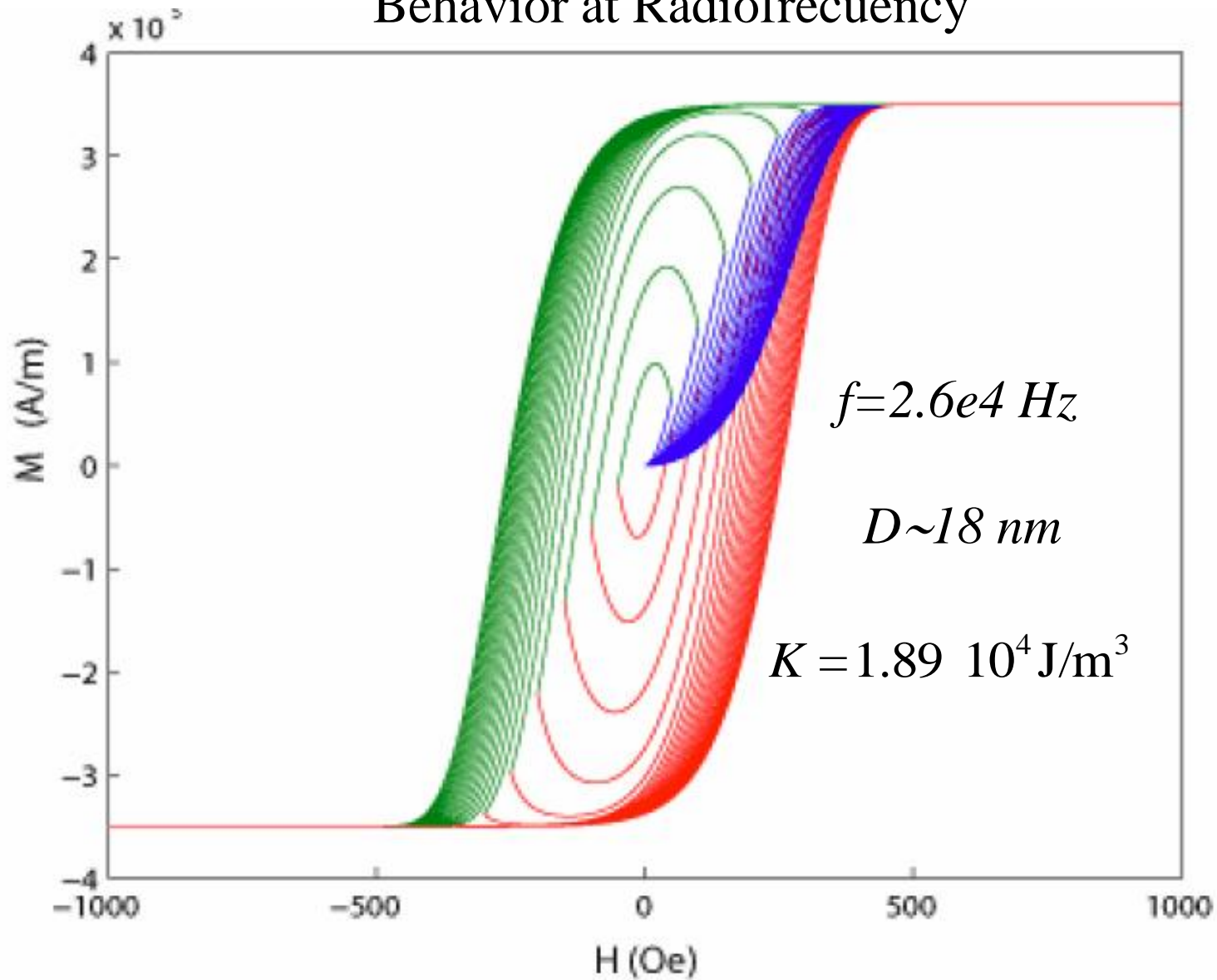
$$\begin{cases} M = M_s(2p_1 - 1) \\ \frac{\partial H}{\partial t} = H_0 f_H(t) \end{cases} \quad \rightarrow \quad dM = \frac{M_s}{H_0} \left( 1 - \frac{M}{M_s} \right) \frac{(v_2 - v_1)}{f_H(t)} dH$$

# Two levels model



# Two levels model

## Behavior at Radiofrequency



# Magnetostatic Energy (dipolar interactions)

**M** uniform  $\xrightarrow{\text{General Case}}$  **H<sub>int</sub>** **NO** uniform

2nd degree surface (conics)  $\xrightarrow{\hspace{10em}}$  **H<sub>int</sub>** uniform

ellipsoid

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

**M** uniform

$$\vec{H}_D = -\hat{N}\vec{M}$$

Demagnetizing field      Demagnetizing Tensor

Diagonal if coordinate axis coincide with ellipsoid ones

Unit Trace

$$N_x + N_y + N_z = 1$$

Number if  $\vec{M} = \begin{cases} M_s \vec{i} \\ M_s \vec{j} \\ M_s \vec{k} \end{cases}$

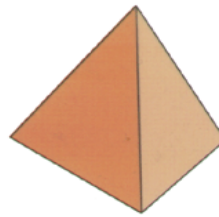
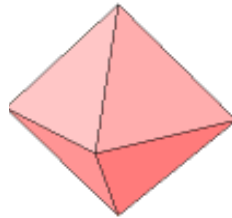
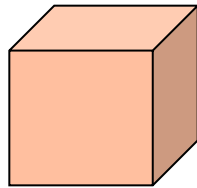
# Non quadratic surfaces

$$E_M = \frac{\mu_0 V}{2} (N_x M_x^2 + N_y M_y^2 + N_z M_z^2)$$

Valid also for bodies with non quadratic surfaces: cubes, prisms, cylinders, octahedra, etc.

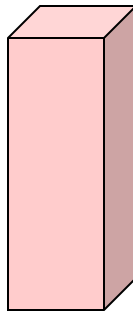
(Brown-Morrish theorem)

Cube,  
octahedra,  
tetrahedra



$$N_x = N_y = N_z = 1/3$$

Regular  
prism,  
cylinder



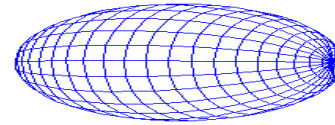
$$N_x = N_y \neq N_z$$

Limit case,  $z \rightarrow \infty$

$$N_x = N_y = \frac{1}{2}; \quad N_z = 0$$

# Shape anisotropy: ellipsoidal NPs

$$a = b < c \Rightarrow N_x = N_y > N_z$$



$$E_M = \frac{\mu_0 V}{2} (N_x M_x^2 + N_y M_y^2 + N_z M_z^2) \xrightarrow{N_y=N_x} \frac{\mu_0 V}{2} (N_x (M_x^2 + M_y^2) + N_z M_z^2)$$

$$M_S^2 = M_x^2 + M_y^2 + M_z^2$$

$$E_M = \frac{\mu_0 V}{2} (N_z - N_x) M_z^2 + const = \frac{\mu_0 V}{2} (N_z - N_x) M_S^2 \cos^2 \theta + const$$

$$E_M = -\frac{\mu_0 V}{2} (N_z - N_x) M_S^2 \sin^2 \theta + const = K_{ME} V \sin^2 \theta + const$$

$$E_M = K_{ME} V \sin^2 \theta$$

$$K_{ME} = \frac{\mu_0}{2} (N_x - N_z) M_S^2$$



# Demagnetizing factors

## Demagnetizing factors for rectangular ferromagnetic prisms

Amikam Aharoni<sup>a)</sup>

*Department of Electronics, Weizmann Institute of Science, 76100 Rehovoth, Israel*

JOURNAL OF APPLIED PHYSICS

VOLUME 83, NUMBER 6

15 MARCH 1998

TABLE I. The demagnetizing factor,  $D_z^s$ , of a prolate spheroid and the magnetometric demagnetizing factor,  $D_z^p$ , of a square prism, for an aspect ratio,  $p$ .

$p$	$D_z^s$	$D_z^p$
2.0	0.17356	0.19832
3.0	0.10871	0.14036
4.0	0.075407	0.10845
5.0	0.055821	0.088316
6.0	0.043230	0.074466
7.0	0.034609	0.064363
8.0	0.028421	0.056670
9.0	0.023816	0.050617
10.0	0.020286	0.045731
11.0	0.017515	0.041705

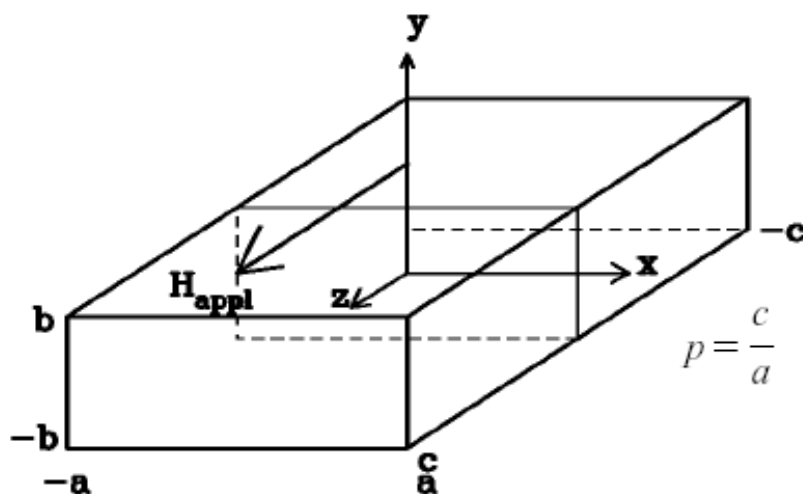


FIG. 1. The coordinate system used in the calculations. Its origin is at the center of the rectangular prism. The field  $H_{\text{appl}}$  is applied along the  $z$  axis.

# Demagnetizing Factors– references

Formulae, tables y graphs for demagnetizing factors, Chen et al. IEEE Trans. Magnetics **27**, 3601-19 (1991)

Demagnetizing Field y Magnetic Measurements, J.A. Brug y W.P. Wolf, J.Appl.Phys. **57**, 4685-701 (1985)

Demagnetizing factors calculations,

<http://magnet.atp.tuwien.ac.at/dittrich/?http://magnet.atp.tuwien.ac.at/dittrich/content/tools/magnetostatics/streufeld.htm>

# Paramagnetism

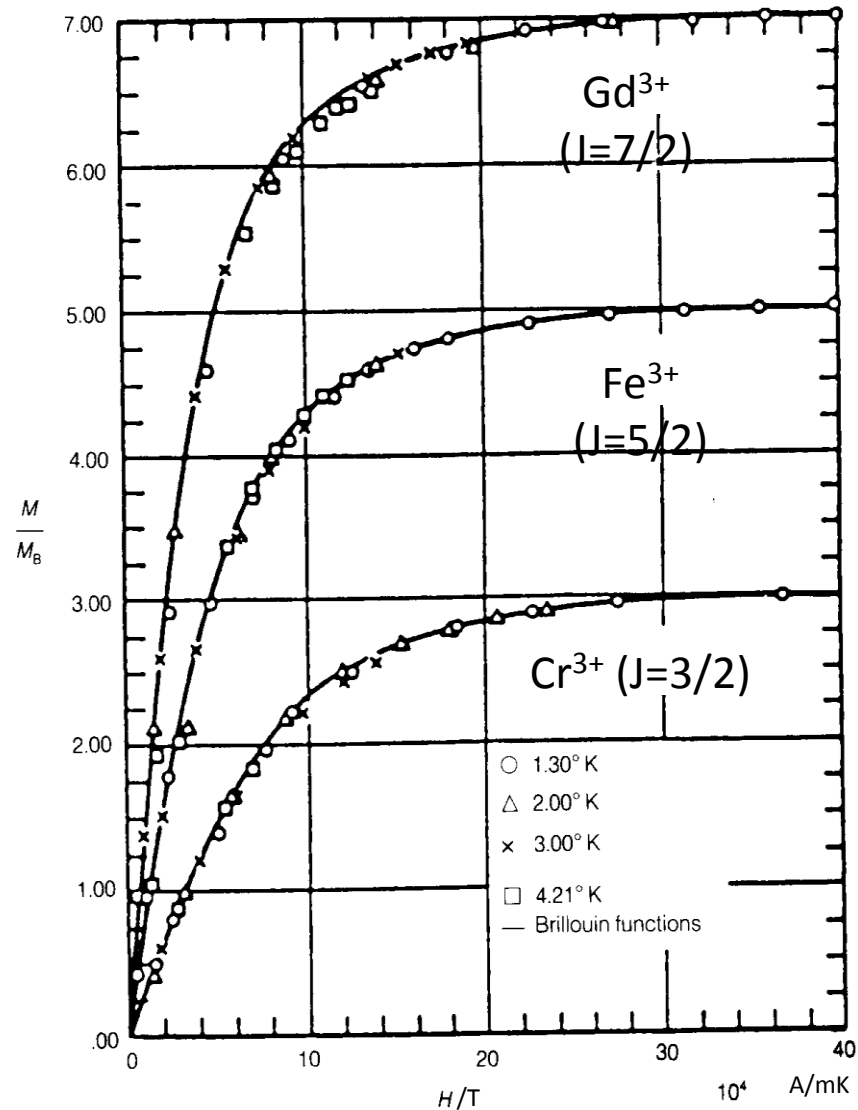
$$M(H, T) = M_s(T) B_J(x) = M_s(T) \left\{ \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} x\right) - \frac{1}{2J} \coth\left(\frac{x}{2J}\right) \right\}$$

$$x = \mu_0 \mu H / kT$$

$$\mu = gJ\mu_B$$

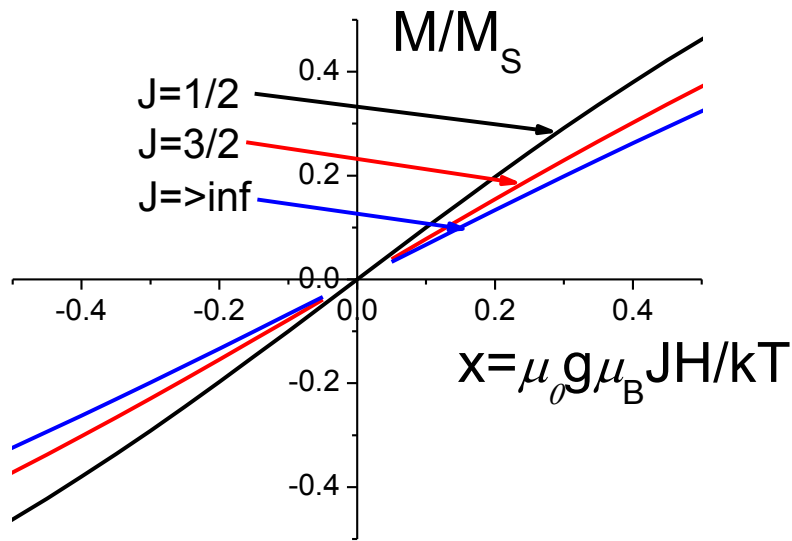
$J$  Ion angular momentum

Agreement with  
experimental  
results

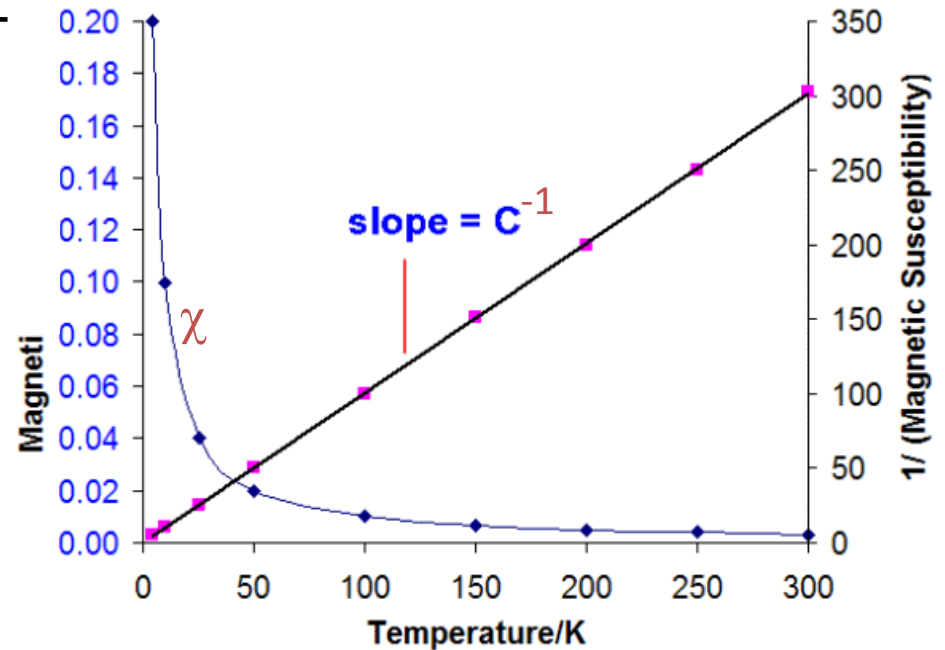


# Paramagnetism

$$x \rightarrow 0 \quad \chi = \frac{M}{H} \approx \frac{N\mu_0 \mu^2}{3kT} = \frac{C}{T}$$



Curie Law Plots



# Superparamagnetism

$$\mu \gg \mu_B \quad (NP)$$

$$\tau < \tau_{\text{exp}}$$

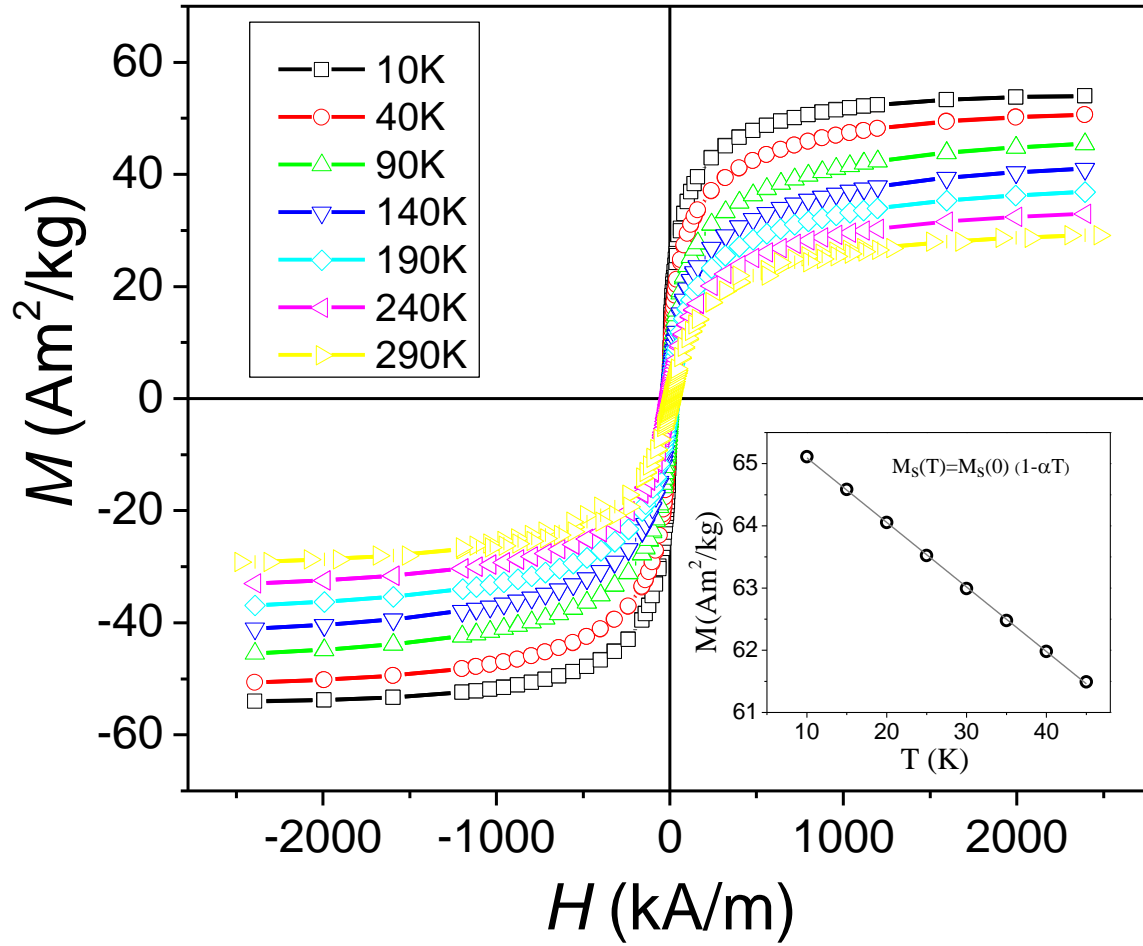
$$M(H, T) = M_S(T)L(x) = M_S(T) \left( \coth(x) - \frac{1}{x} \right)$$

For a NP moment distribution

$$M(H, T) = N \int_0^\infty \mu f(\mu) L\left(\frac{\mu_0 \mu H}{kT}\right) d\mu \quad f(\mu) d\mu = \frac{1}{\mu \sqrt{2\pi\sigma}} e^{-\frac{\ln^2(\mu/\mu_0)}{2\sigma^2}} d\mu$$

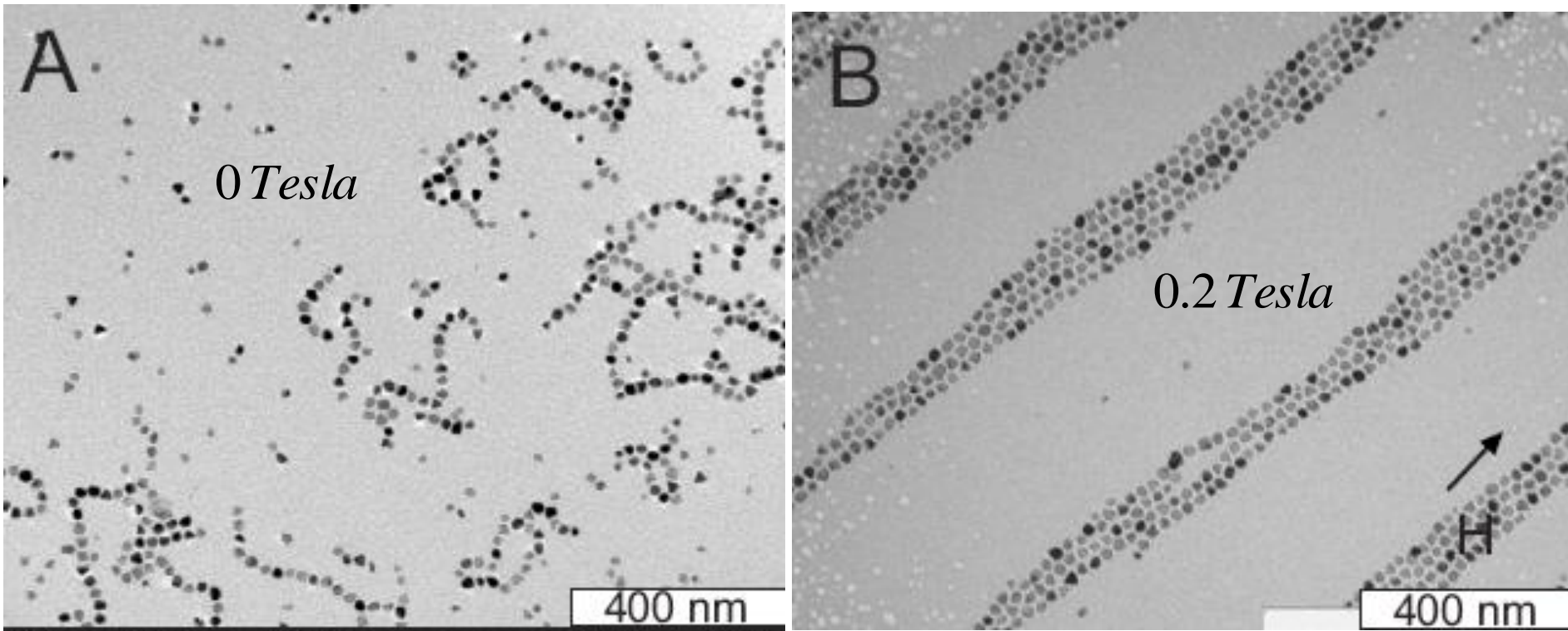
$$M_S(T) = N \int_0^\infty \mu f(\mu) d\mu = N \langle \mu \rangle(T)$$

# Superparamagnetism



# “Interacting Superparamagnets” Dipolar Interactions

$$\mu = 1.55 \times 10^{-18} \text{ Am}^2 \quad V_{\text{max}} = \left( \mu_0 \mu^2 / 2\pi d^3 \right) / kT = 9 \quad \xi = -\mu_0 \mu H / kT = 380$$



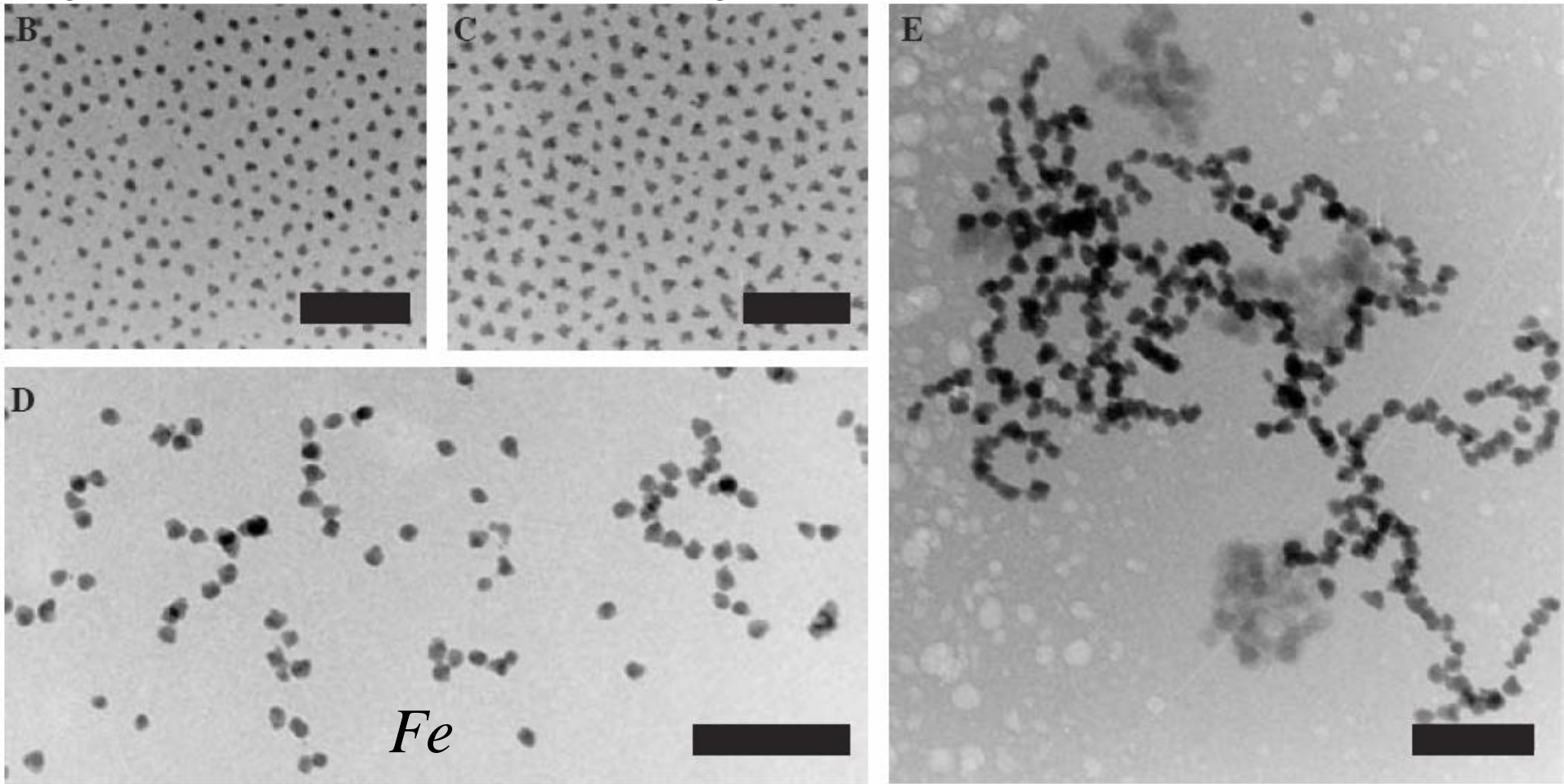
magnetita  $d = 18.4 \text{ nm}$  en  $C_{10}H_{18}$

FIG. 3. (a) Typical *in situ* cryo-TEM images of vitrified films of magnetite dispersion  $C$  in zero field ([24]). (b) In a homogeneous magnetic field (0.2 T), a transition occurs to equal-spaced columns that exhibit hexagonal symmetry [8].



# Iron(oxide) ferrofluids: synthesis, structure and catalysis

Karen Butter  
20 oktober 2003



**Figure 1.** Cryo-TEM pictures of ferrofluids consisting of metallic iron particles with a 7 nm thick organic surface layer dispersed in decalin [9-11]. The radius of the iron core gradually increases from ferrofluid B (6 nm) to ferrofluid E (8 nm). The scale bars are 100 nm.

# Magnetic interactions between nanoparticles

Steen Mørup<sup>\*1</sup>, Mikkel Fougth Hansen<sup>2</sup> and Cathrine Frandsen<sup>1</sup>

*Beilstein J. Nanotechnol.* **2010**, *1*, 182–190.

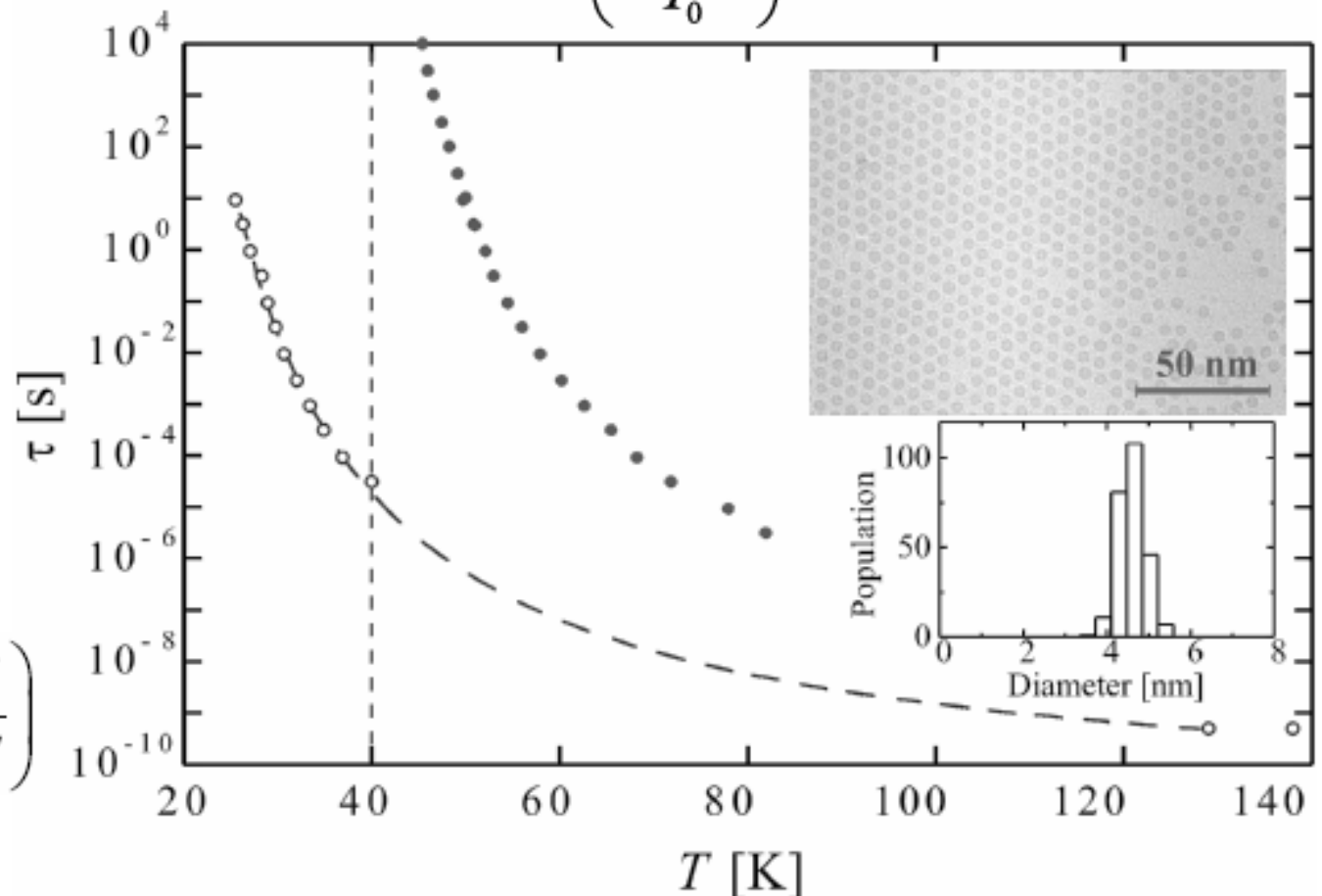
$$E_a = KV \sin^2 \theta$$

$$E_d \approx \frac{\mu_0 \mu^2}{4\pi d^3}$$

$$T_0 \approx \frac{E_d}{k_B}$$

$$\tau = \tau_0 \exp\left(\frac{KV}{k_B T}\right)$$

$$\tau = \tau^* \left(\frac{T - T_0}{T_0}\right)^{-zV}$$



## Other propositions

Vogel-Fulcher law

$$\tau = \tau_0 \exp[E_a/k(T_B - T_0)]$$

Shtrikman S and Wohlfarth E P 1981 *Phys. Lett.* **85A** 467

$$\tau = \tau_0 [T_f / (T_f - T^*)]^\alpha$$

Hohenberg P C and Halperin B I 1977 *Rev. Mod. Phys.* **49** 435

# A dynamic study of small interacting particles: superparamagnetic model and spin-glass laws

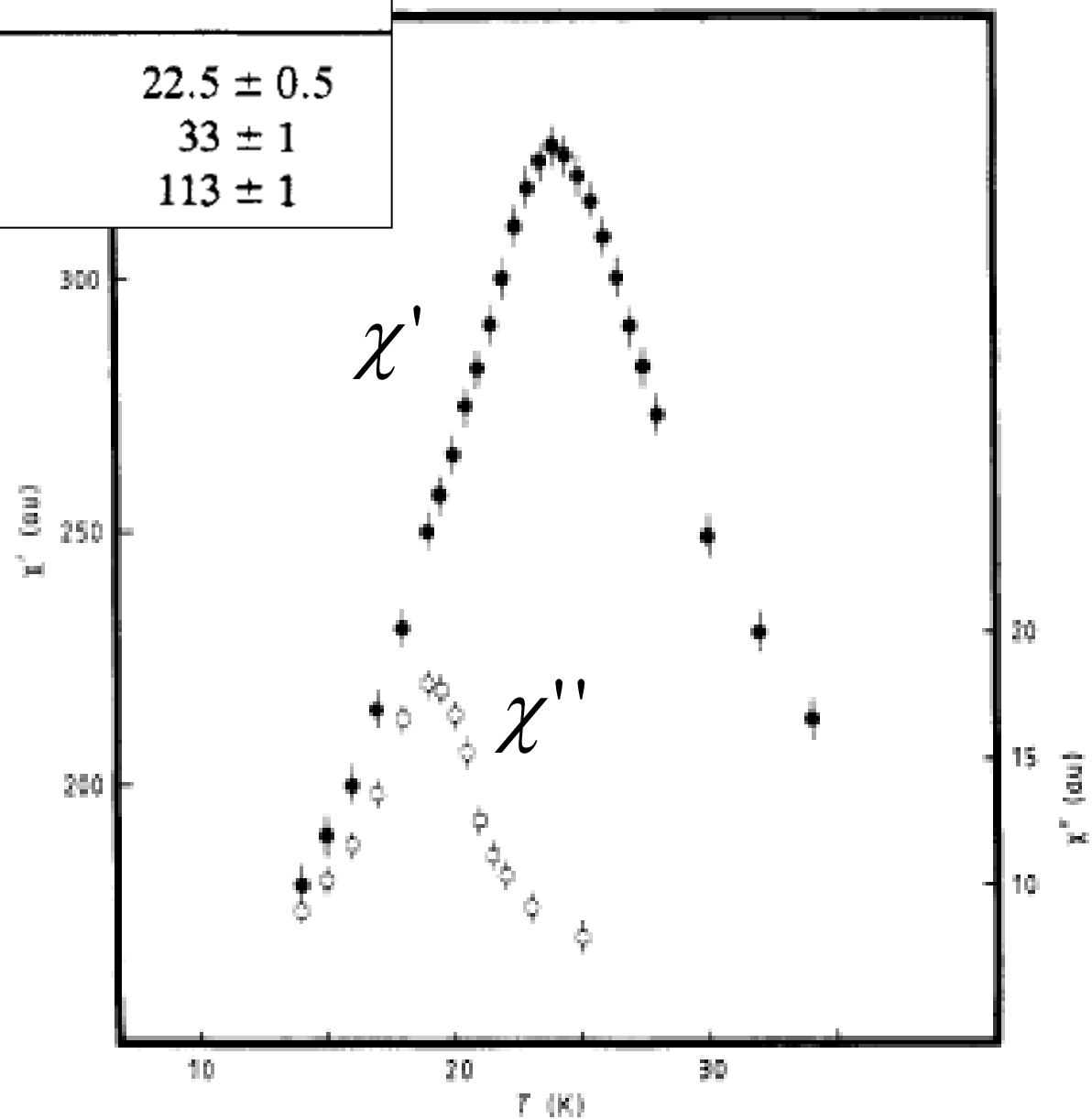
J L Dormann<sup>†</sup>, L Bessaïst<sup>†</sup> and D Fiorani<sup>‡</sup>

Received 3 July 1987, in final form 6 October 1987

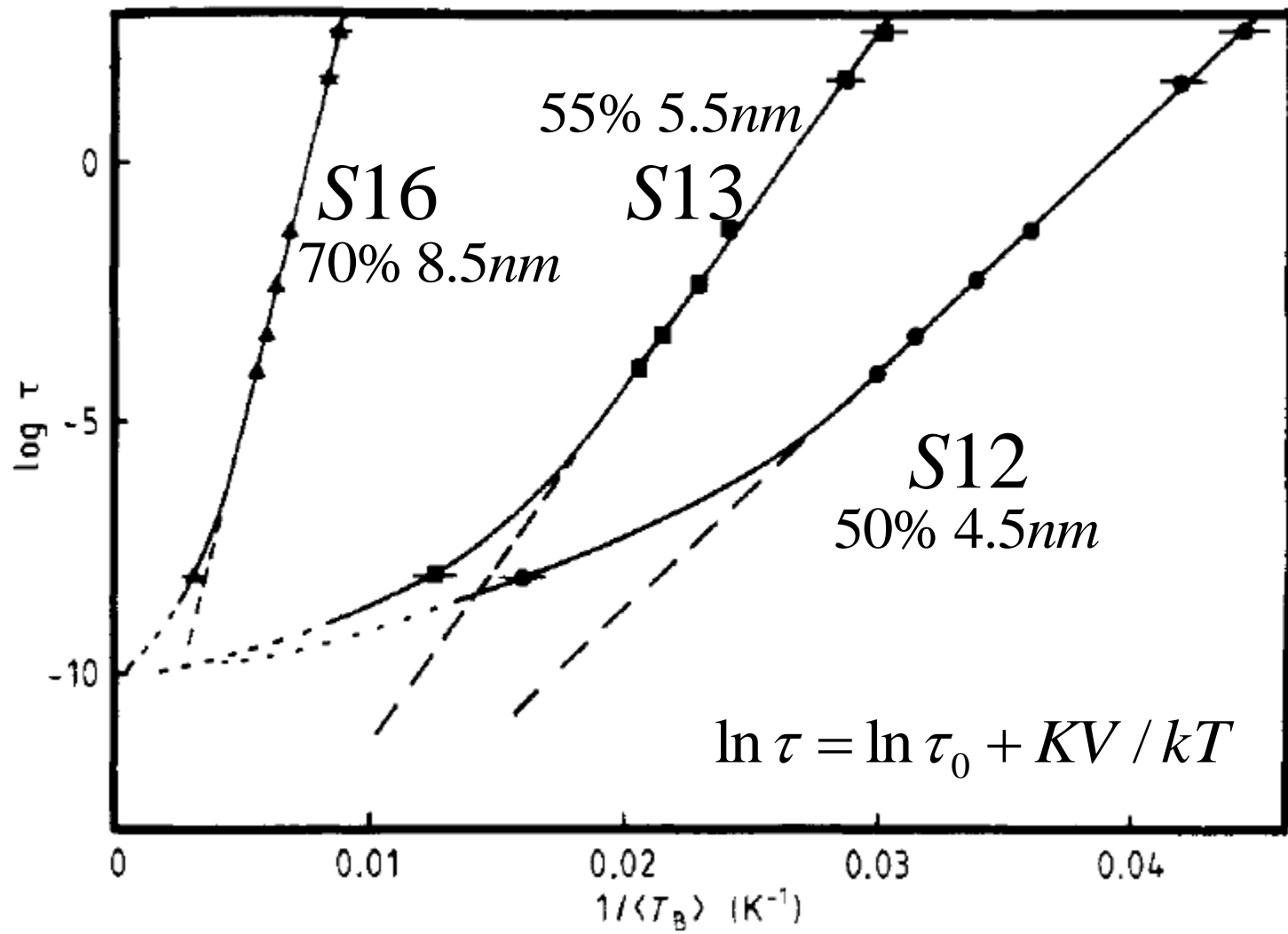
**Table 1.** Percentage weight of iron  $p$ , mean particle diameter  $\Phi$ , and atomic percentages of metallic iron ( $\text{Fe}^0$ ),  $\text{Fe}^{3+}$  and  $\text{Fe}^{2+}$  for different samples.

Sample	$p$ (%)	$\Phi$ (Å)	Percentage		
			$\text{Fe}^0$	$\text{Fe}^{3+}$	$\text{Fe}^{2+}$
S12	$50 \pm 2$	$45 \pm 5$	$73 \pm 2$	$11 \pm 2$	$16 \pm 2$
S13	55	55	73	12	15
S16	70	85	65	11	24

Sample	$T_B$ (K)	
	$\nu = 0.02$ Hz	$\nu = 0.002$ Hz
S12	$24 \pm 0.5$	$22.5 \pm 0.5$
S13	$35 \pm 1$	$33 \pm 1$
S16	$121 \pm 2$	$113 \pm 1$



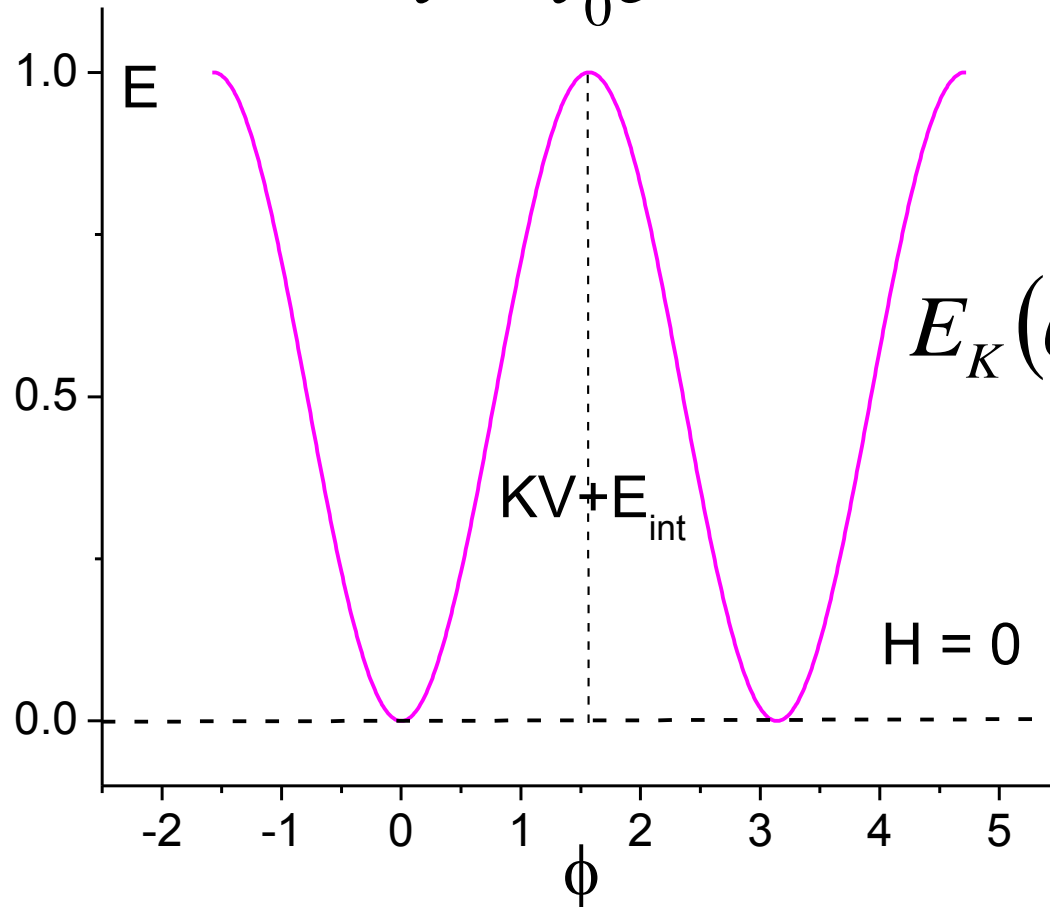
$$T_B = \frac{K\langle V \rangle}{k} \frac{1}{\ln(1/\nu\tau_0)}$$



# DFB proposition

$$E_{\text{Btot}} = K_u V + E_{\text{Bint}} \quad \text{for} \quad E_{\text{Bint}} \ll E_{\text{Btot}}$$

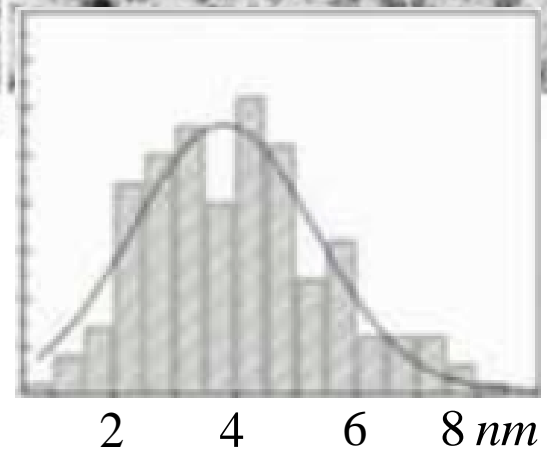
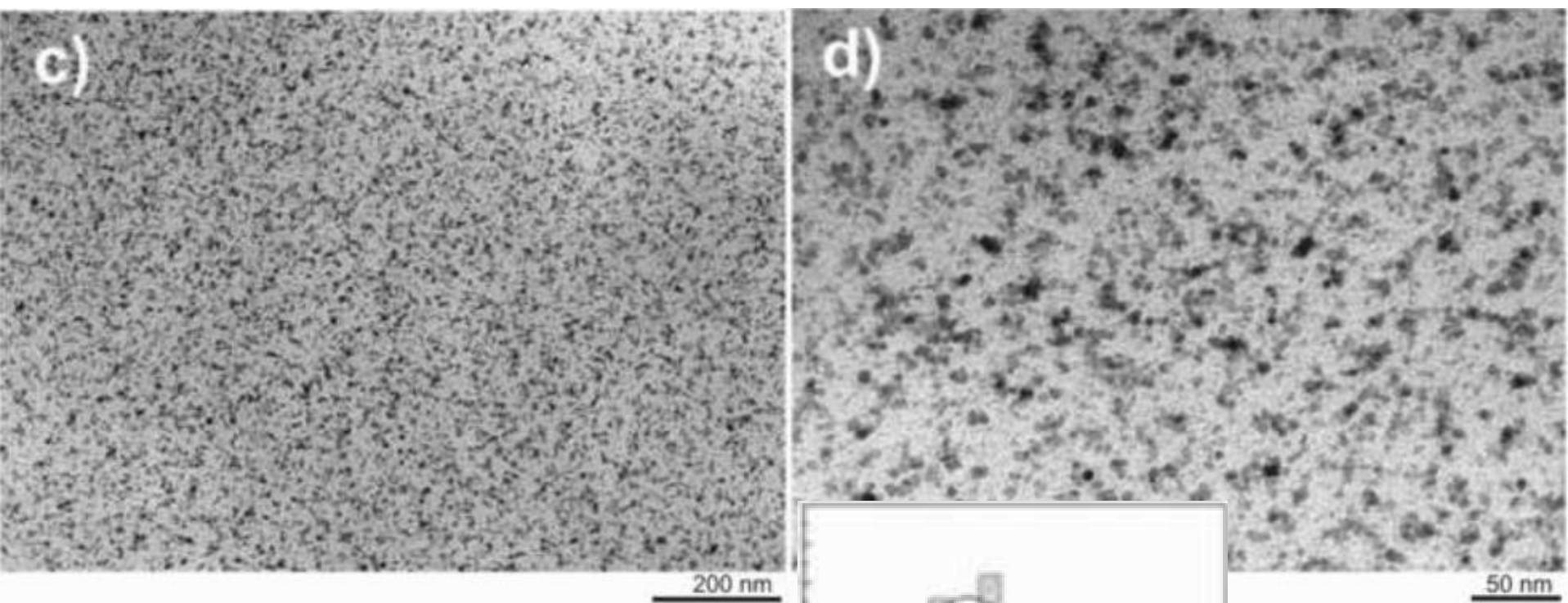
$$\tau \approx \tau_0 e^{E_B/kT}$$



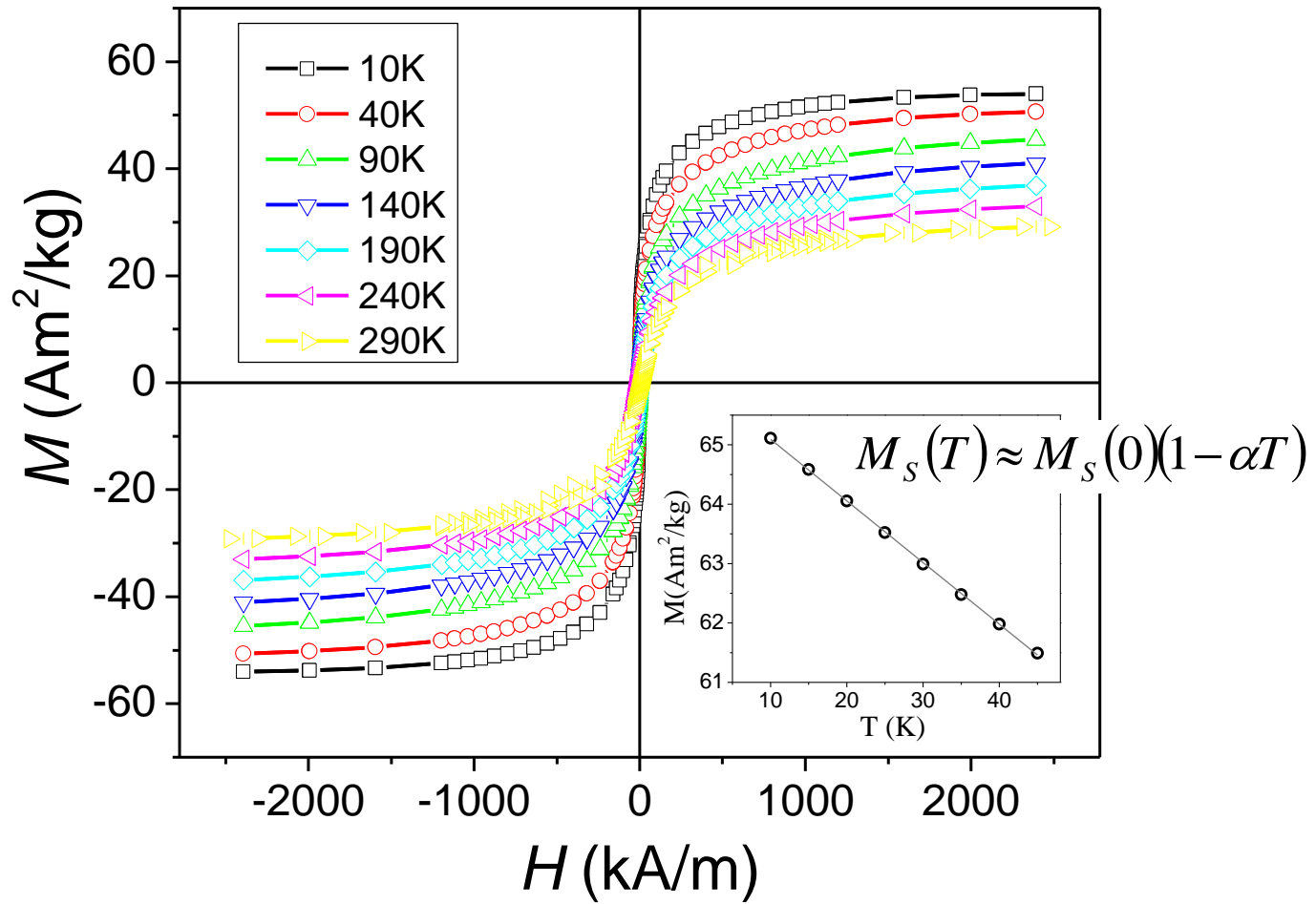
$$E_K(\theta) \approx E_B \sin^2 \theta$$

# A SIMPLE AND EFFICIENT PROCEDURE FOR THE SYNTHESIS OF FERROGELS BASED ON PHYSICALLY CROSSLINKED PVA

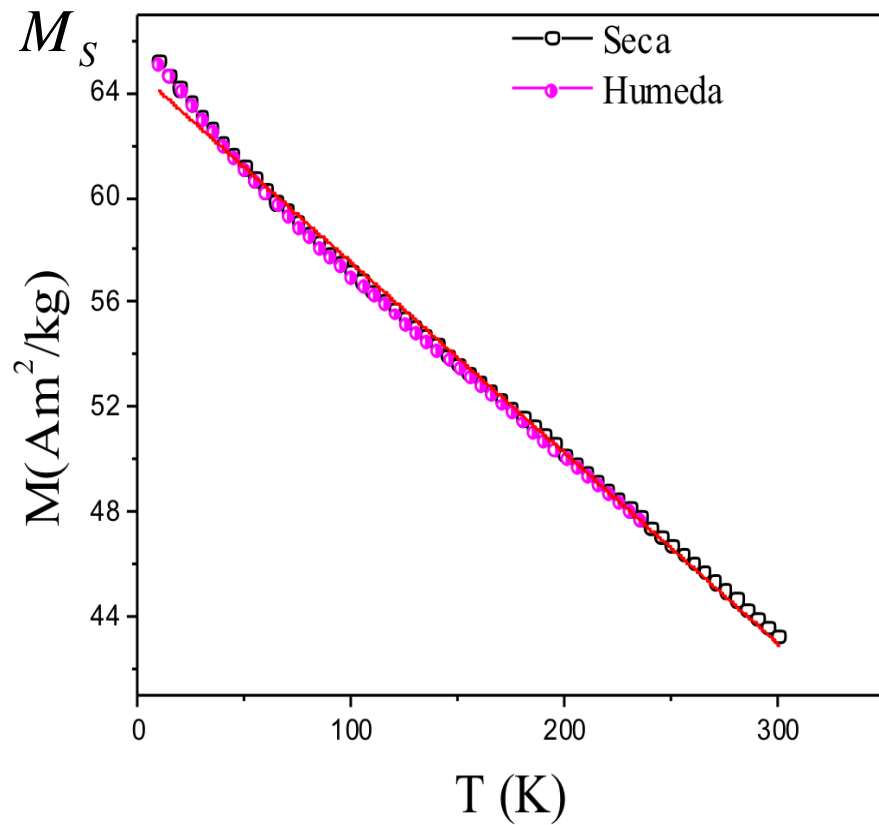
Jimena S. Gonzalez<sup>a\*</sup>, Cristina E. Hoppe<sup>a</sup>, Pedro Mendoza Zélis<sup>b</sup>, Lorena Arciniegas<sup>b</sup>, Gustavo A. Pasquevich<sup>b</sup>, Francisco H. Sánchez<sup>b</sup> Vera A. Alvarez<sup>a</sup>





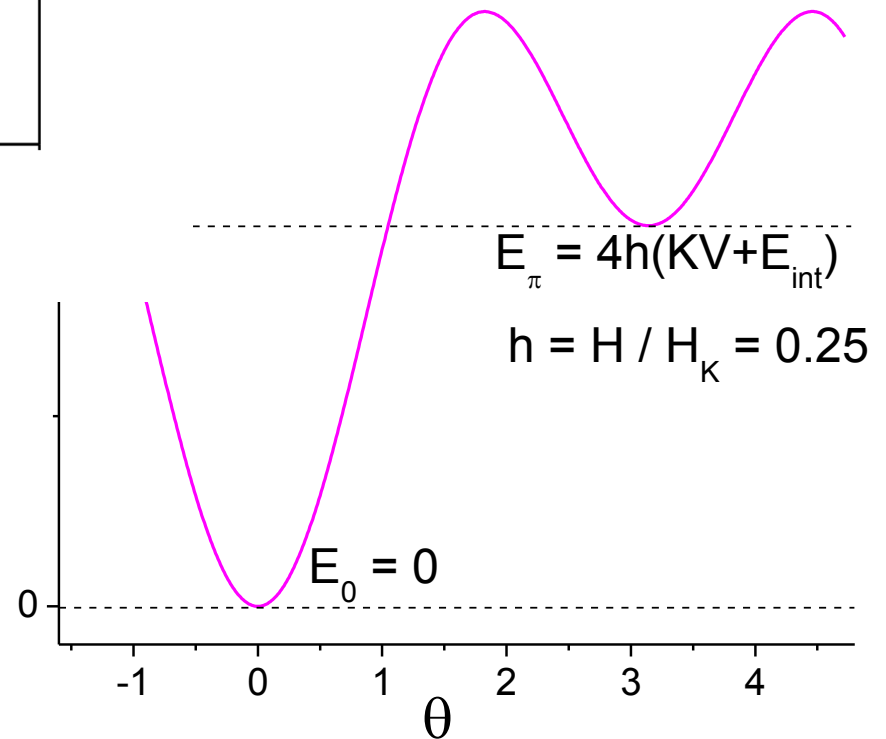


$$H_c = 0 \quad \text{for} \quad T \geq 40\text{K}$$



$$M_S(T) \approx M_S(0)(1 - \alpha T)$$

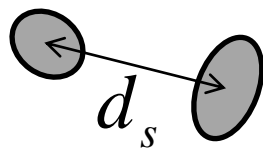
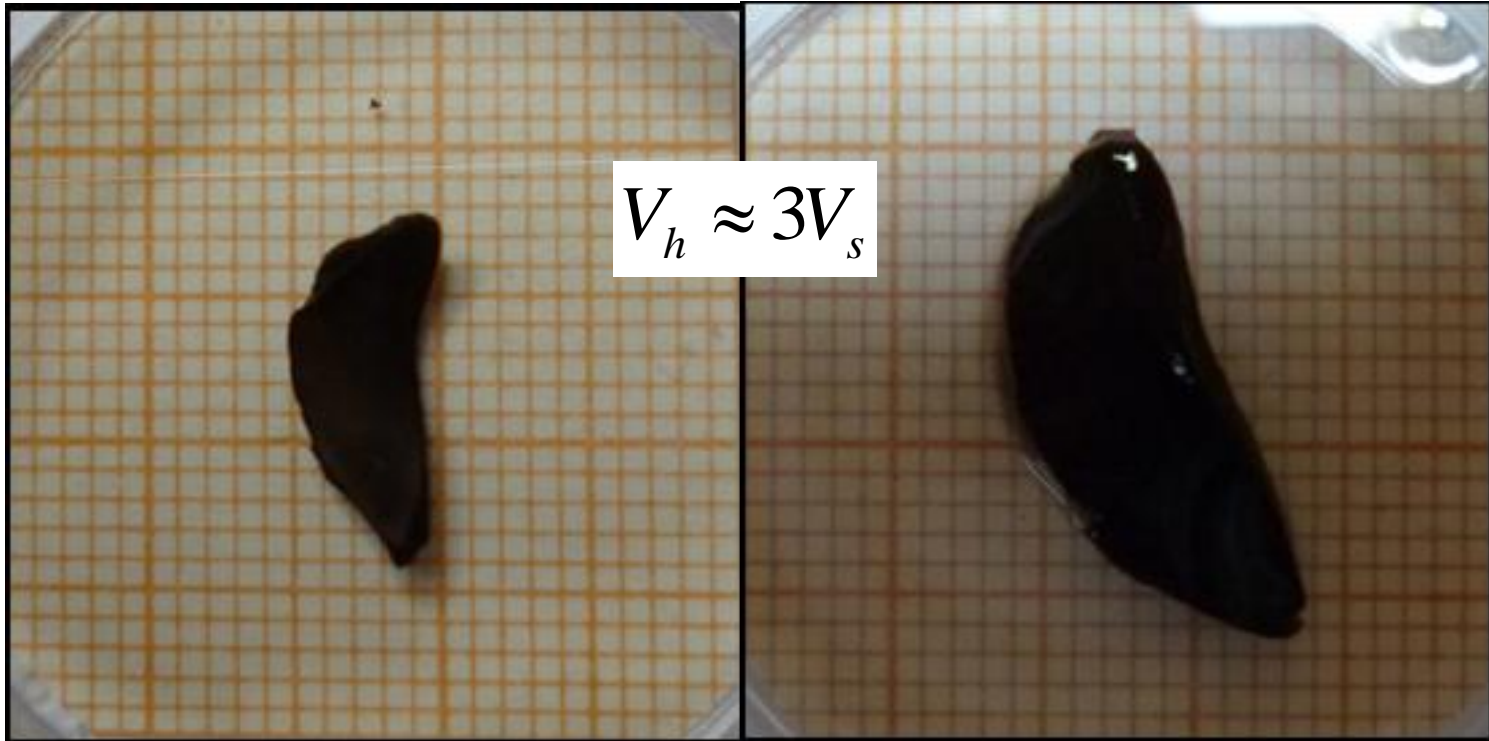
$$\alpha \approx k / 8(KV + E_{\text{int}})$$



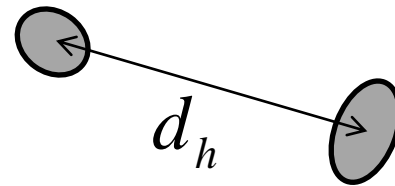
# Ferrogel swelling

dry

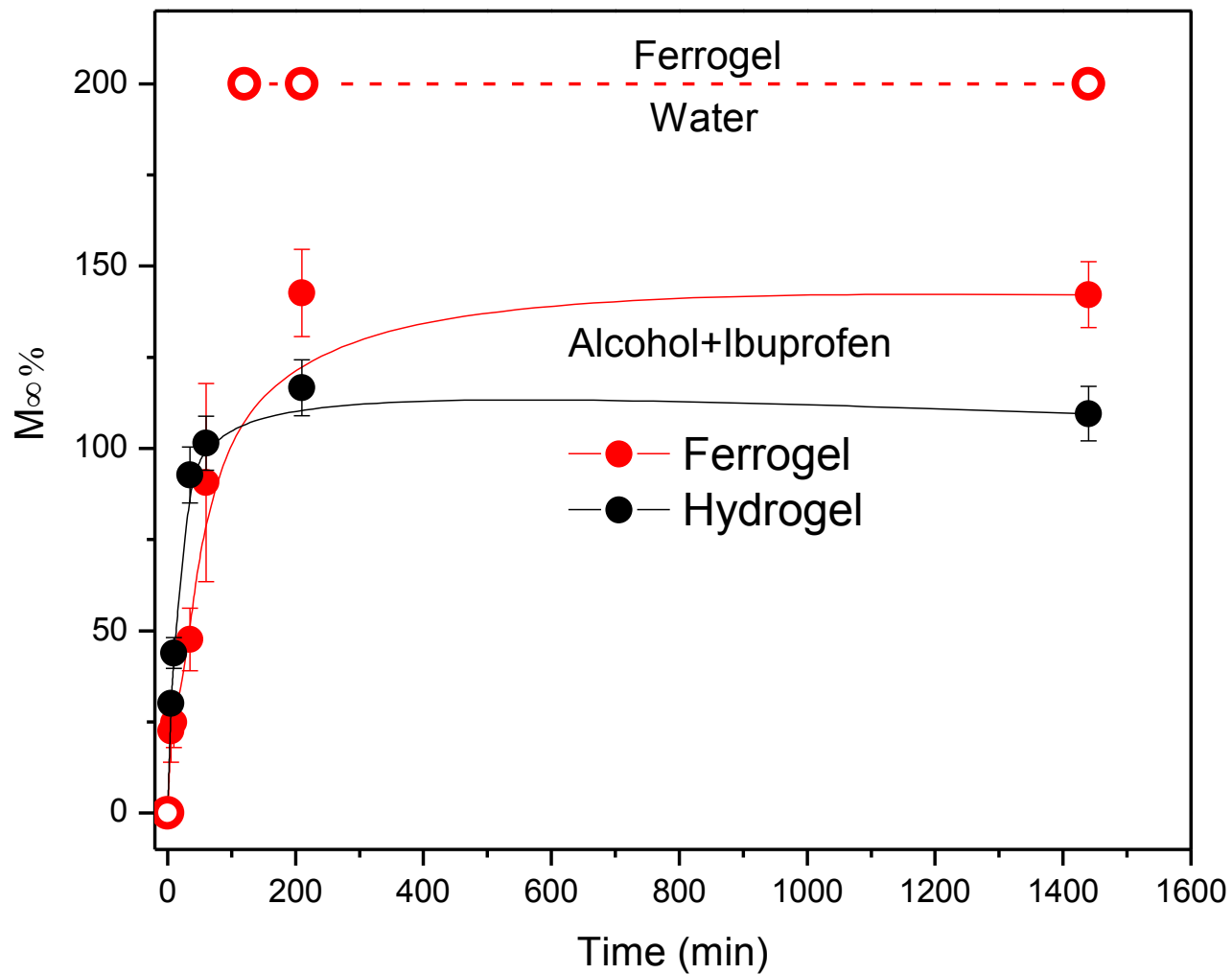
hydrated



$$d_h \approx 1.44d_s$$



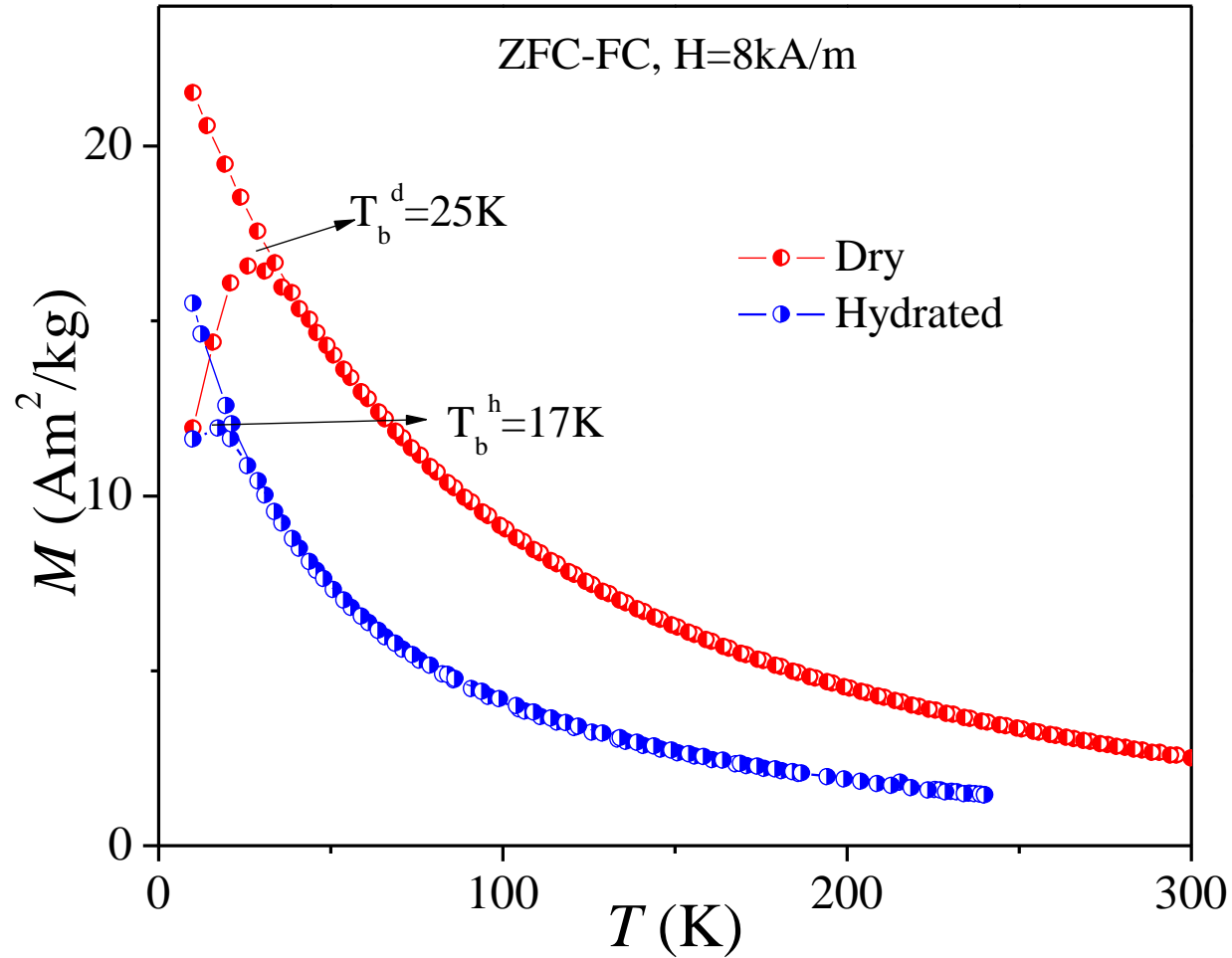
# Ferrogel swelling



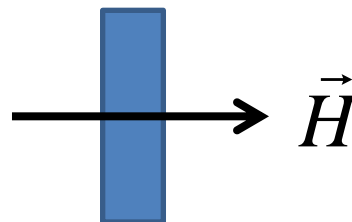
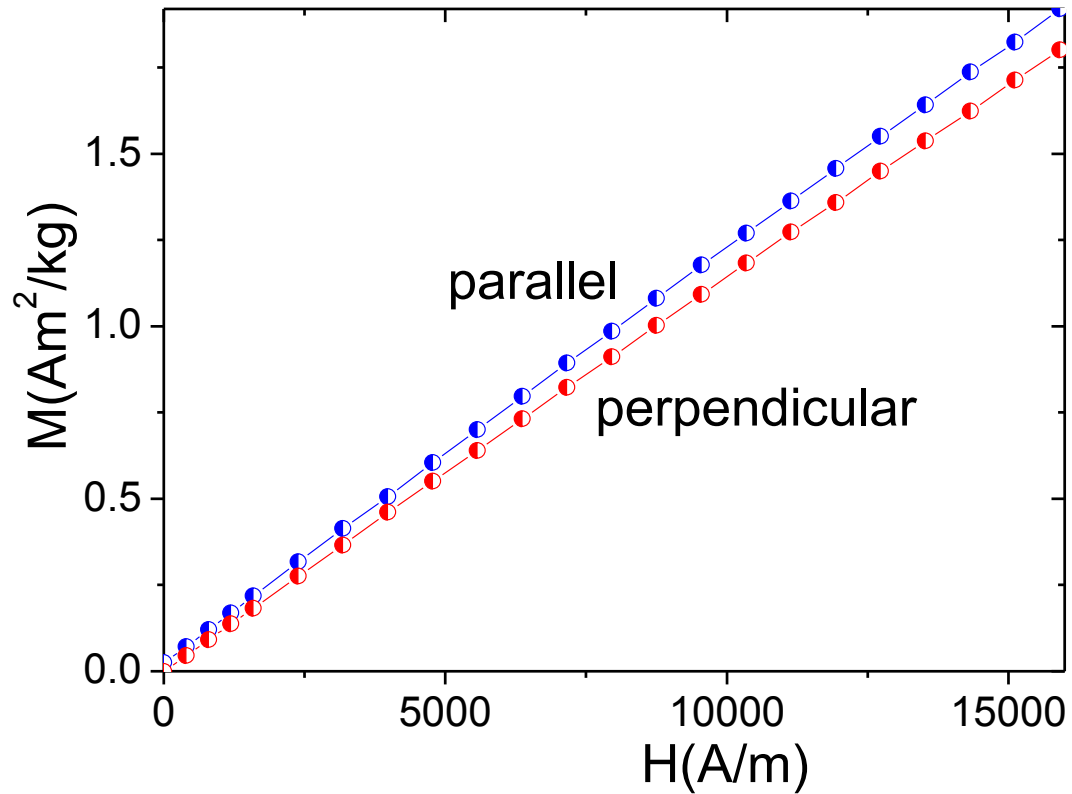
$$\tau_{dry} \approx \tau_0 e^{(KV + E_{int}^{dry})/kT}$$

$$\tau_{hyd} \approx \tau_0 e^{(KV + E_{int}^{hyd})/kT}$$

$$E_{int}^{dry} > E_{int}^{hyd} \Rightarrow T_B^{dry} > T_B^{hyd}$$



# Shape effects



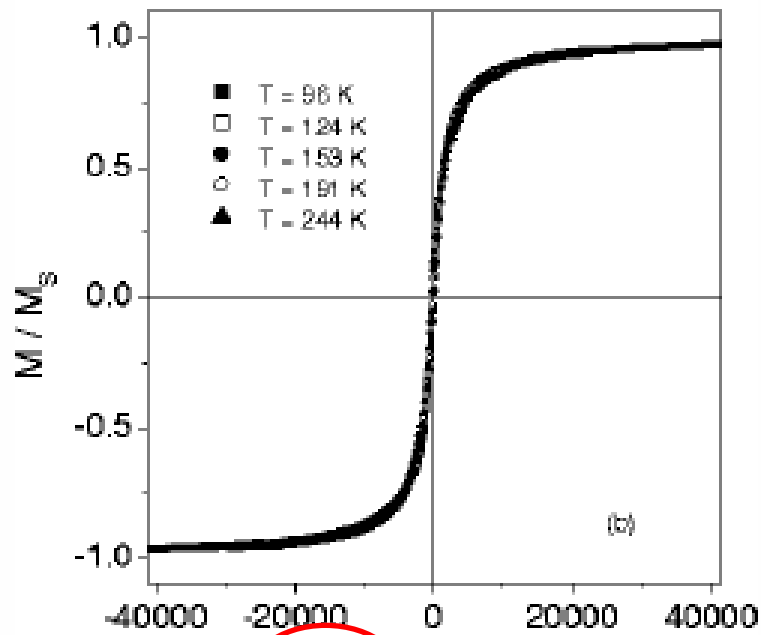
# Granular Cu-Co alloys as interacting superparamagnets

Paolo Allia,<sup>1</sup> Marco Coisson,<sup>2</sup> Paola Tiberto,<sup>3</sup> Franco Vinai,<sup>3</sup> Marcelo Knobel,<sup>4</sup> M. A. Novak,<sup>5</sup> and W. C. Nunes<sup>5</sup>

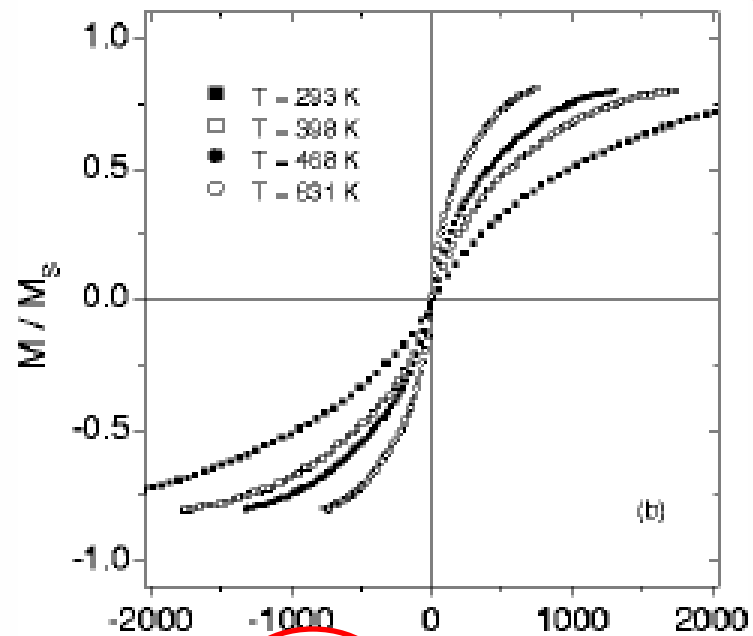
PHYSICAL REVIEW B, VOLUME 64, 144420

$$M(H, T) = M_s L\left(\frac{\mu_0 \mu H}{kT}\right) = M_s L\left(\frac{\mu_0 V M_s H}{kT}\right) = M_s L\left(\frac{\mu_0 V}{k} z\right)$$

$$z = \frac{M_s H}{T}$$



$M_s(H/T)$  (erg cm<sup>-3</sup> K<sup>-1</sup>)



$M_s(H/T)$  (erg cm<sup>-3</sup> K<sup>-1</sup>)

Analysis of an interacting superparamagnet with theoretical expressions valid for non interacting systems (Langevin)

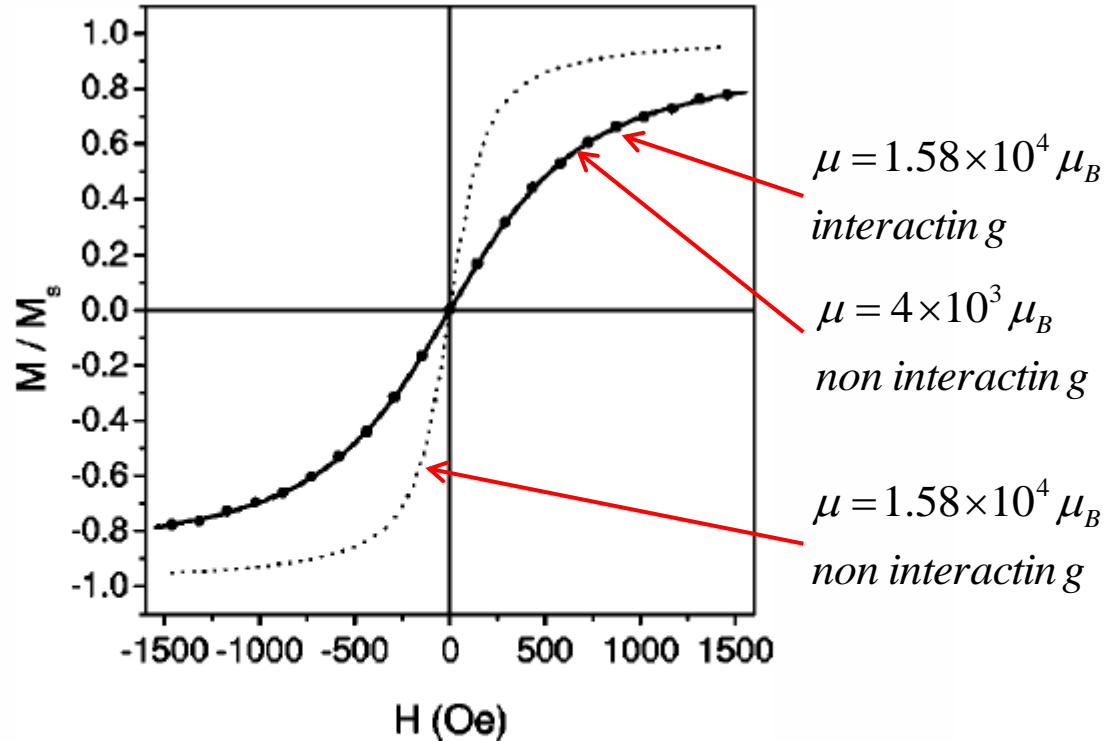
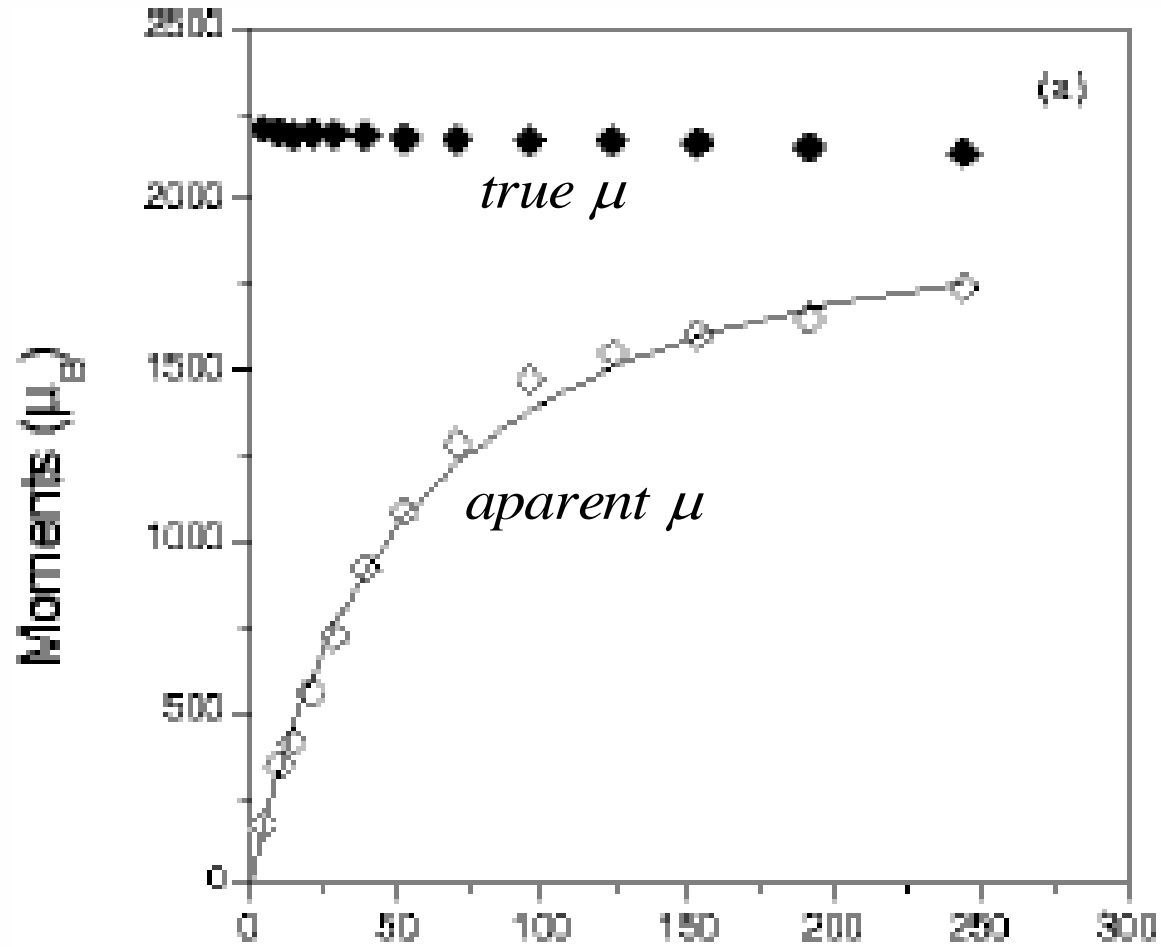


FIG. 6. Solid symbols: simulation of the anhysteretic magnetization behavior for an assembly of identical interacting Co moments ( $\mu = 1.58 \times 10^4 \mu_B$  at  $T = 82$  K, from Ref. 31). Dotted line: Langevin function for  $\mu = 1.58 \times 10^4 \mu_B$  at  $T = 82$  K. Solid line: Langevin function for  $\mu = 4.0 \times 10^3 \mu_B$  at  $T = 82$  K.



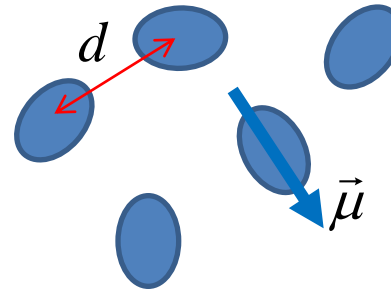
Analysis of an interacting superparamagnet with theoretical expressions valid for non interacting systems (Langevin)



## Dipolar interactions

$$\varepsilon_D = \alpha \mu_0 \frac{\mu^2}{d^3}$$

$$\alpha \approx 1$$



Hypothesis: dipolar interactions give rise to an apparent higher temperature

$$T_a = T + T^*$$

$$\text{con } \varepsilon_D = kT^* \quad \longrightarrow \quad T^* = \frac{\mu_0 \alpha}{k} \frac{M_S^2}{N}$$

$$\frac{M(H, T)}{M_S} = L\left(\frac{\mu_0 \mu H}{kT}\right)$$

Dipolar interactions



$$\frac{M(H, T)}{M_S} = L\left(\frac{\mu_0 \mu H}{k(T + T^*)}\right)$$

$$M(H, T) = N_a \mu_a L \left( \frac{\mu_0 \mu_a H}{kT} \right) = N \mu L \left( \frac{\mu_0 \mu H}{k(T + T^*)} \right)$$

$$\mu_a = \frac{1}{1 + T^*/T} \mu$$

$$N_a \mu_a = N_a \frac{1}{1 + T^*/T} \mu = N \mu \Rightarrow N_a = (1 + T^*/T) N$$

when  $T \ll T^*$

$$\mu_a \approx \frac{T}{T^*} \mu \approx \frac{kT}{\varepsilon_D} \mu \approx \frac{kTd^3}{\alpha \mu_0 \mu} \approx \frac{kTd^3}{\alpha \mu_0 M_S d^3} = \frac{kT}{\alpha \mu_0 M_S} \xrightarrow{T \rightarrow 0} 0$$

$$\varepsilon_D = kT^* \quad \varepsilon_D = \alpha \mu_0 \frac{\mu^2}{d^3}$$

Low field susceptibility

$$\chi = \frac{N\mu_0\mu^2}{3k(T + T^*)}$$

$$T^* = \frac{\mu_0\alpha M_S^2}{k N}$$

$$\frac{1}{\chi} = \frac{3kT}{N\mu_0\mu^2} + \frac{3kT^*}{N\mu_0\mu^2} = \frac{3kN^2T}{N\mu_0M_S^2} + \frac{3kN^2}{N\mu_0M_S^2} \frac{\mu_0\alpha M_S^2}{k N} = \frac{3kN}{\mu_0} \left( \frac{T}{M_S^2} \right) + 3\alpha$$

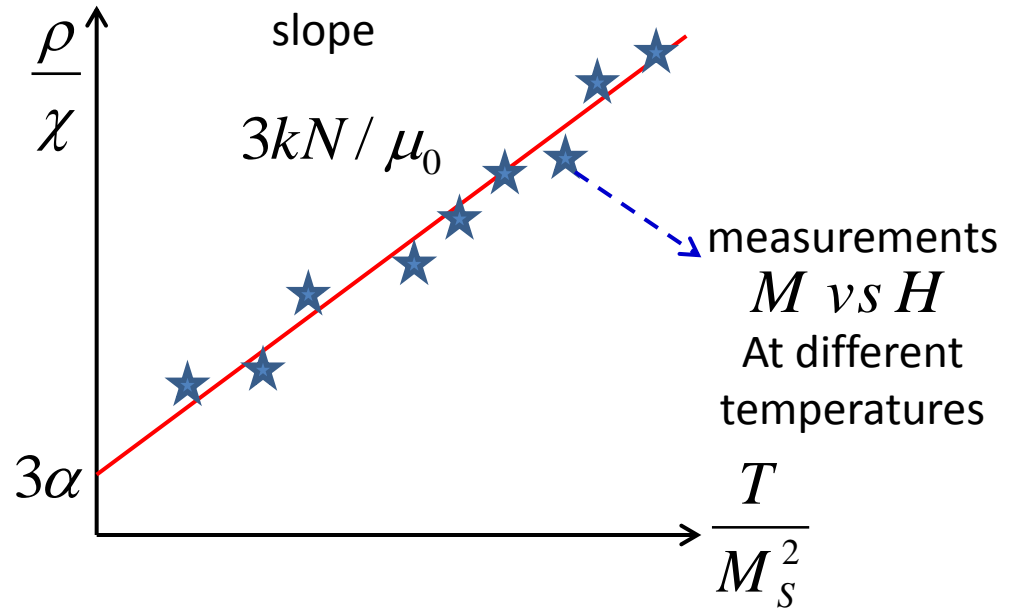
$$\boxed{\frac{1}{\chi} = \frac{3kN}{\mu_0} \left( \frac{T}{M_S^2} \right) + 3\alpha}$$

Allia et al. show that when a NP moments distribution exists former expression becomes:

$$\frac{\rho}{\chi} = \frac{3kN}{\mu_0} \left( \frac{T}{M_S^2} \right) + 3\alpha$$

where

$$\rho = \frac{\langle \mu^2 \rangle}{\langle \mu \rangle^2} \equiv \frac{\langle \mu_a^2 \rangle}{\langle \mu_a \rangle^2}$$



$$\mu = \frac{M_S}{N}$$

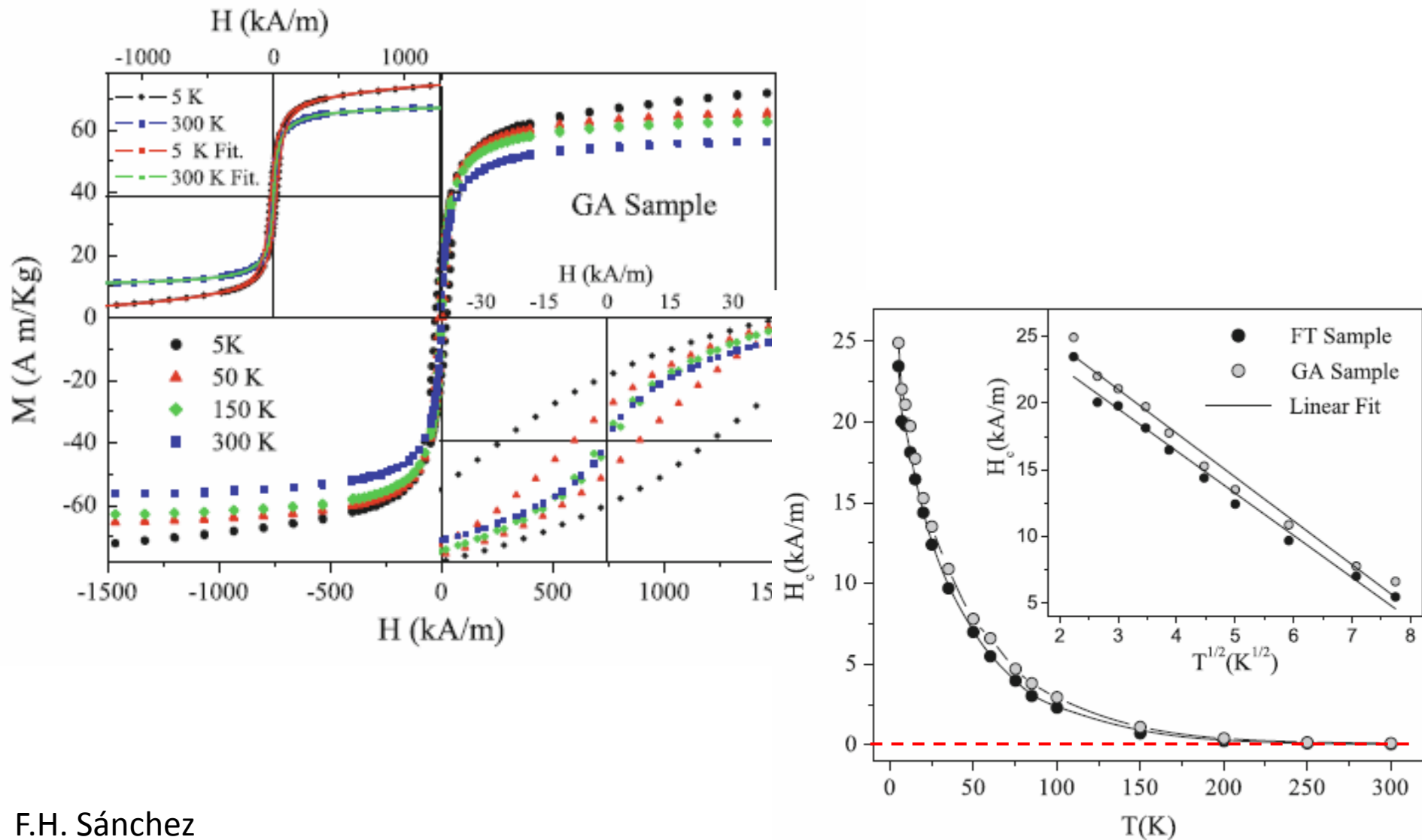
$$T^* = \frac{\mu_0 \alpha}{k} \frac{M_S^2}{N}$$

$$\varepsilon_D = kT^*$$

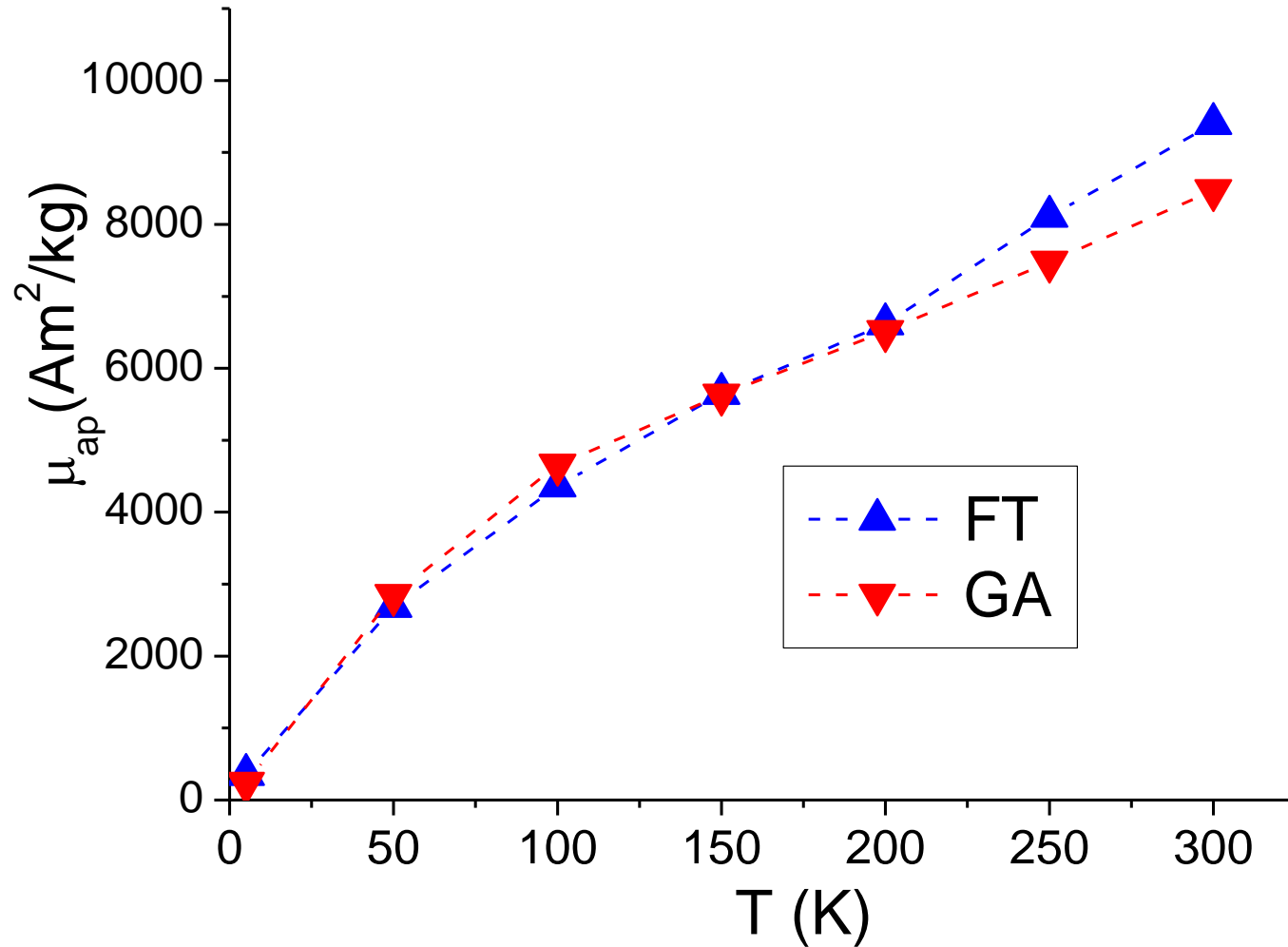
# Magnetic properties study of iron-oxide nanoparticles/PVA ferrogels with potential biomedical applications

P. Mendoza Zélis · D. Muraca · J. S. Gonzalez ·  
G. A. Pasquevich · V. A. Alvarez · K. R. Pirola ·  
F. H. Sánchez

J Nanopart Res (2013) 15:1613



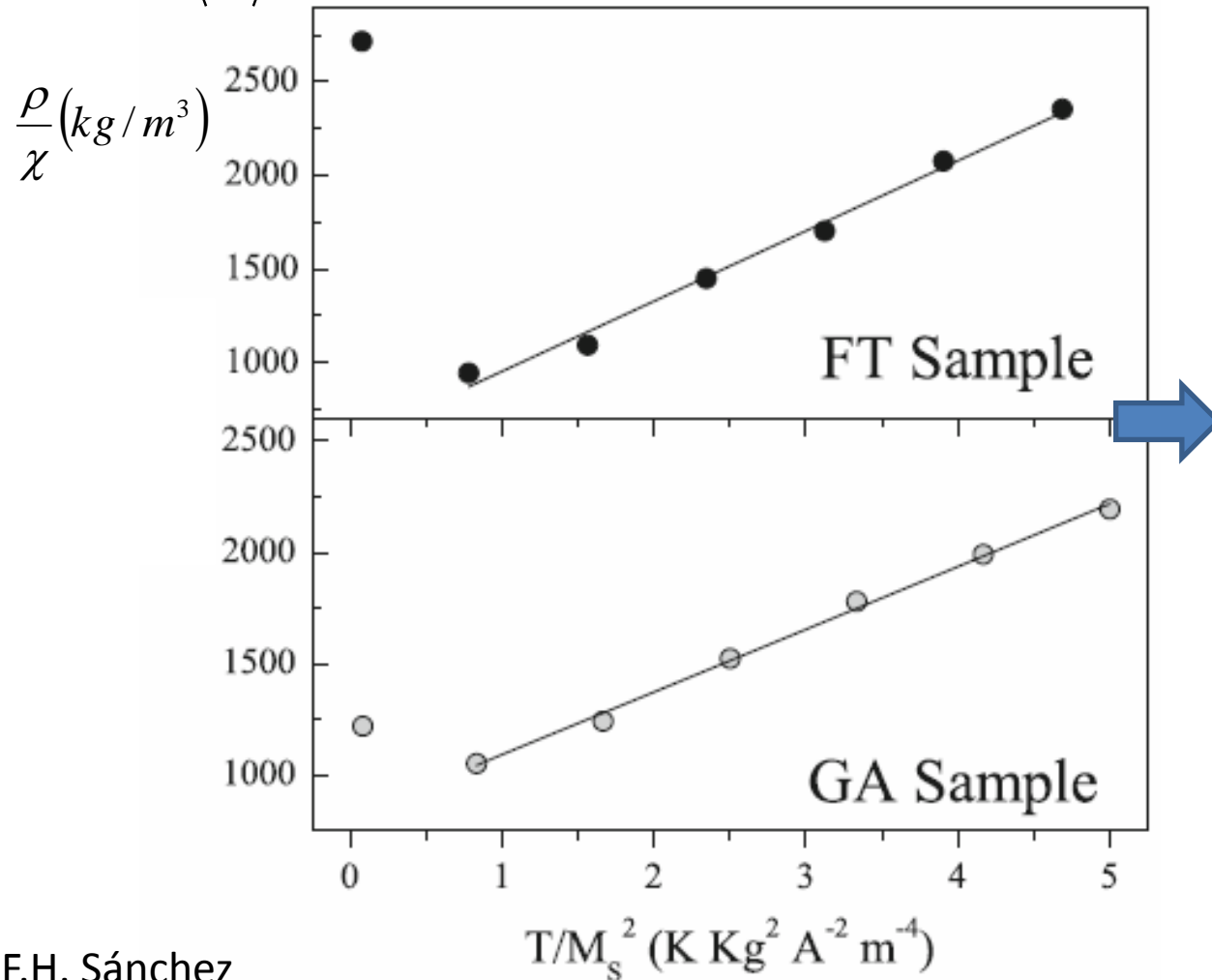
# Aparent moment variation with temperature



$$\rho = \frac{\langle \mu^2 \rangle}{\langle \mu \rangle^2}$$

$$\frac{\rho}{\chi} = \frac{3kN}{\mu_0} \left( \frac{T}{M_s^2} \right) + 3\phi\alpha$$

$$\phi = \frac{mass_{NPs}}{mass_{FG}}$$



*FT*

$$\mu = 9500 \mu_B$$

$$\varepsilon_D = 1.37 \times 10^{-21} J$$

$$D_p = 8.1 nm$$

$$d = 26 nm$$

*GA*

$$\mu = 12600 \mu_B$$

$$\varepsilon_D = 2.38 \times 10^{-21} J$$

$$D_p = 9.1 nm$$

$$d = 32 nm$$



# Demagnetizing factor $N_{\text{Def}}$ in samples with disperse magnetic NPs

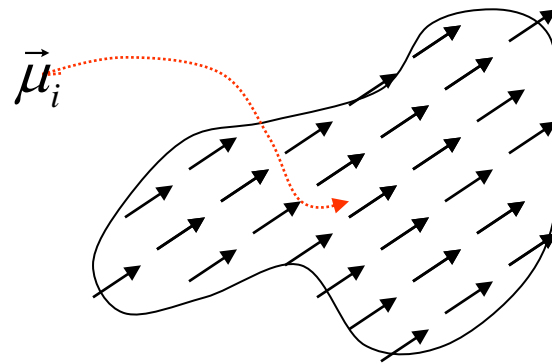
*F.H. Sánchez, unpublished*

# Demagnetizing factor $N_{Def}$ in samples with disperse magnetic NPs

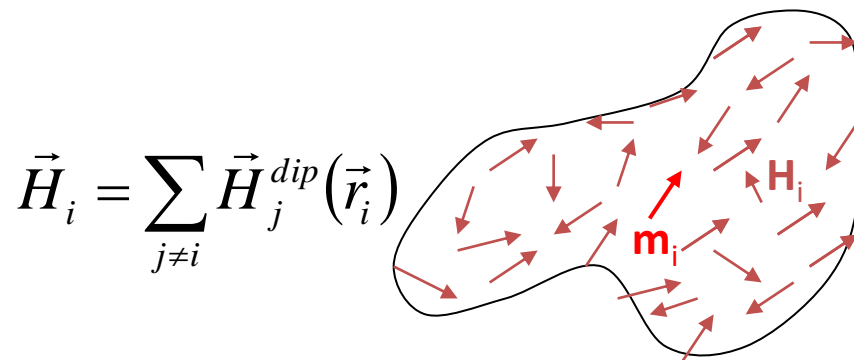
Magnetostatic energy or magnetic self energy

Case 1: continuous material

Interacción among materials dipoles

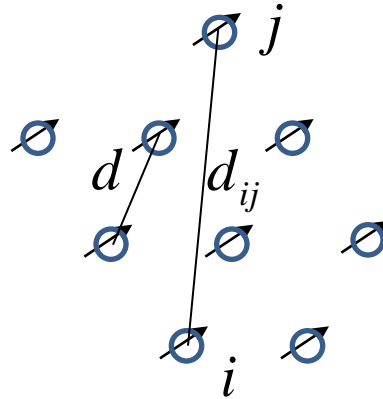
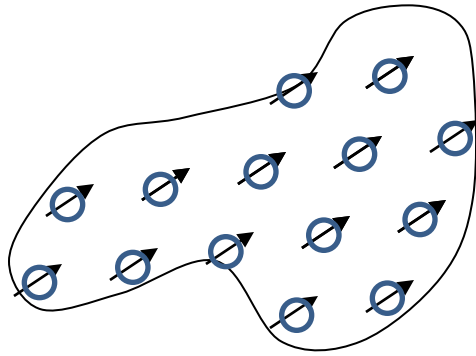


$$E_M = -\frac{1}{2} \sum_i \vec{\mu}_i \cdot \vec{B}_i = -\frac{\mu_0}{2} \sum_i \vec{\mu}_i \cdot \vec{H}_i = -\frac{\mu_0}{2} \sum_i \vec{M}_i \cdot \vec{H}_i V_i \approx -\frac{\mu_0}{2} \int \vec{M} \cdot \vec{H} dV$$



$$\vec{H} \approx -N_D \vec{M}$$

## Case 2: magnetic NPs disperse in non magnetic media



$d$ : distance between neighbors

$$d_{ij} = \gamma D e_{ij}$$

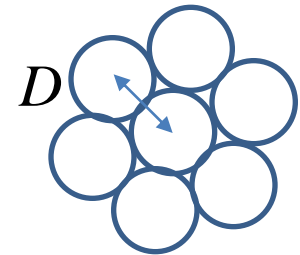
$e_{ij}$  is a normalized array describing the geometry of NPs arrangement

*i.e. square*  $\Rightarrow e_{ij} = \{1, \sqrt{2}, 2, \sqrt{5} \dots\}$

*i.e. bcc*  $\Rightarrow e_{ij} = \{1, 2/\sqrt{3}, 2\sqrt{2}/\sqrt{3}, 2 \dots\}$

If NPs are in contact:  $\gamma = 1$   
(case of a continuous magnetic material)

$\gamma$  "dilution" factor



Identical NPs:

$$V = \beta D^3; \beta = 1, \pi / 6, \text{etc.}$$

$$E_M = -\frac{\mu_0}{2} \sum_i \vec{\mu}_i \cdot \vec{H}_i = \frac{1}{2} \sum_i \varepsilon_i \quad \varepsilon_i = -\mu_0 \vec{\mu}_i \cdot \vec{H}_i$$

Dipolar field on NP i:

$$\vec{H}_i = \frac{\kappa\mu}{\gamma^3 D^3} \sum_j \frac{1}{e_{ij}^3} (3(\vec{v}_j \cdot \vec{u}_{ij})\vec{u}_{ij} - \vec{v}_j) = \frac{\kappa\mu}{\gamma^3 D^3} \sum_j \frac{\vec{s}_{ij}}{e_{ij}^3} \quad \kappa = 1/4\pi$$

$$\vec{\mu}_i = \mu\vec{v}_i \quad \vec{s}_{ij} = 3(\vec{v}_j \cdot \vec{u}_{ij})\vec{u}_{ij} - \vec{v}_j$$

defining  $\vec{\lambda}_i = \sum_j \frac{\vec{s}_{ij}}{e_{ij}^3} \longrightarrow \vec{H}_i = \frac{\kappa\mu\vec{\lambda}_i}{\gamma^3 D^3}$

$\vec{\lambda}_i$  Just depends on the geometry of the NPs array, on the relative orientations between moments and relative orientations between them and the segment joining them.  $\vec{s}_{ij}$  may take different orientations.

Volume per particle

$$\mu = V_{pp} M_S \approx \gamma^3 V M_S$$

“global” sample magnetization

using

$$V = \beta D^3; \beta = 1, \pi/6, \text{etc.}$$

$$H_i = \kappa \beta \lambda_i M_S$$

$\kappa, \beta, \gamma, \lambda_i,$   
adimensional

Saturated sample,  $\lambda_i$  corresponds to the saturation ( $s_{ij}$ ) configuración  $\lambda_i^S$

$$H_i^S = \kappa\beta\lambda_i^S M_S$$

Assumption for non saturated sample: same proportionality constant between H and M holds.

$$H_i \approx \kappa\beta\lambda_i^S M$$

Aproximation done:

$$\lambda_i M_S \approx \lambda_i^S M$$

Averaging i on sample

$$H_D \approx \kappa\beta\langle\lambda_S\rangle M = -N_D M$$



$$N_D \approx -\kappa\beta\langle\lambda_S\rangle$$

Using magnetization of magnetic phase (NPs)  $M_p$

$$M_p = \gamma^3 M$$

hence

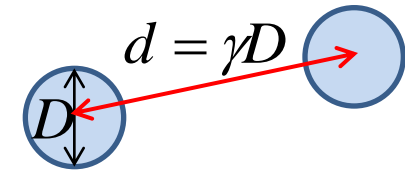
$$H_i \approx \frac{\kappa \beta \lambda_i^s}{\gamma^3} M_p = -N_{Def} M_p$$

Determined by shape and  
NP dilution

$$N_{Def} = \frac{N_D}{\gamma^3}$$

Determined by shape

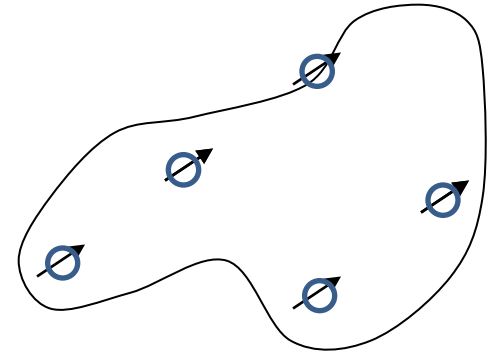
if  $\gamma > 3.16 \Rightarrow N_{Def} < 0.1 N_D$



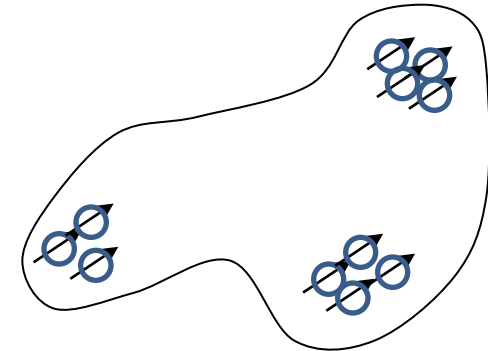
or 
$$N_{Def} = \left(\frac{D}{d}\right)^3 N_D$$

# Cases of interest

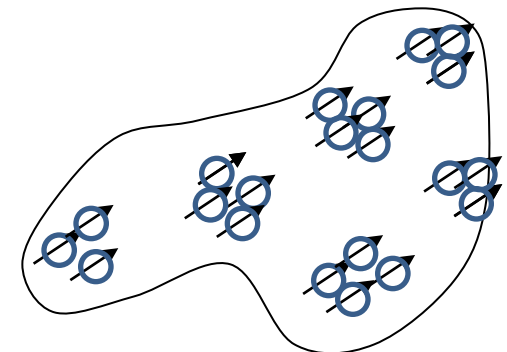
1. NPs homogeneously distributed.  $N_D$  is the demagnetizing factor which corresponds to sample shape.



2. NPs are aggregated in clusters, and clusters do not interact among them.  $N_D$  is the demagnetizing factor which corresponds to cluster average shape.



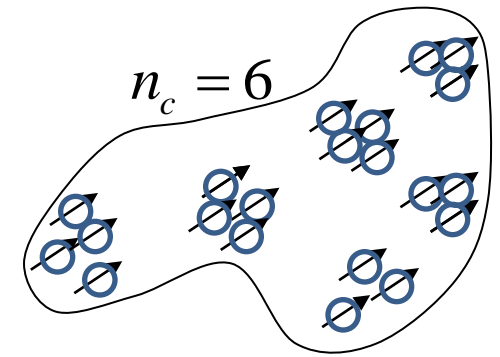
3. NPs are aggregated in clusters, and clusters interact among them.  $N_D$  is the demagnetizing factor which corresponds both to sample shape and to cluster average shape.



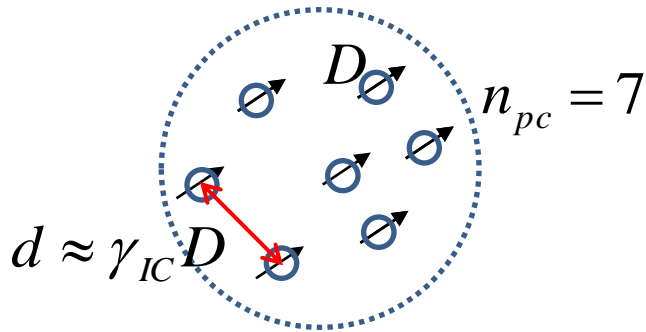


## Case of interest 3

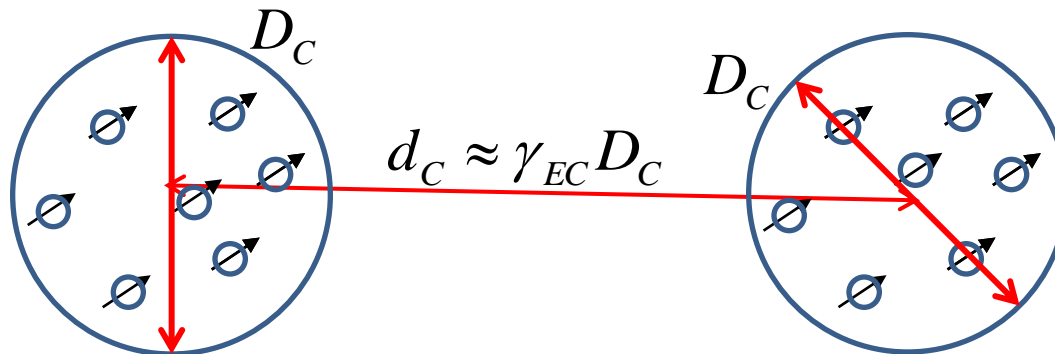
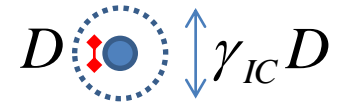
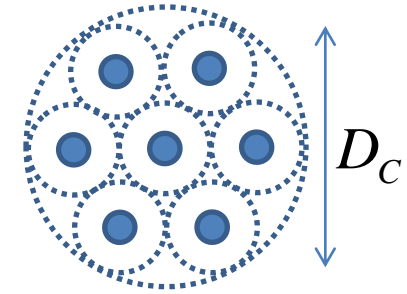
3. NPs are aggregated in clusters, and clusters interact among them.  $N_D$  is the demagnetizing factor which corresponds both to sample shape and to cluster average shape.



NP number  $n_p = n_c n_{pc}$



$$D_C^3 = n_{pc} \gamma_{IC}^3 D^3$$



Leads to

$$N_{Def} \approx \frac{N_D^{cluster}}{\gamma_{IC}^3} + \frac{N_D^{sample} - N_D^{cluster}}{\gamma_{IC}^3 \gamma_{EC}^3}$$

This is the sought result, it gives the dependency of the effective demagnetizing factor in terms of cluster and sample ones, and the characteristic distances (dilution factors) of the problem.

$$N_D^{sample} = -\kappa\beta \left( \langle \lambda_{IC}^S \rangle + \frac{\langle \lambda_{EC}^S \rangle}{n_{pc}} \right) = N_D^s$$

$$N_D^{cluster} = -\kappa\beta \langle \lambda_{IC}^S \rangle = N_D^c$$

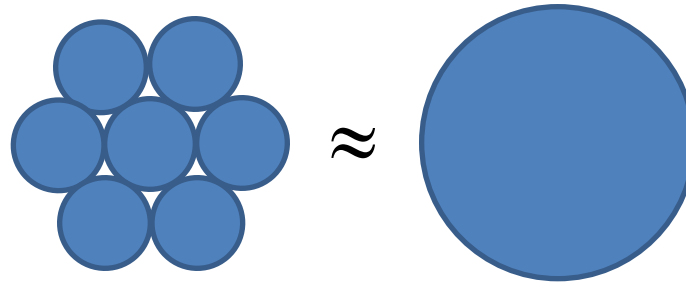
## Particular cases

### Continuous NPs distribution

Continuous Material 

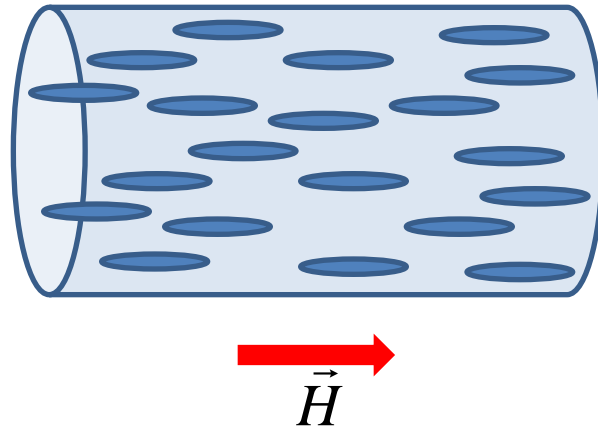
$$\gamma_{IC} = 1 \quad ; \quad \gamma_{EC} = 1$$

muestra



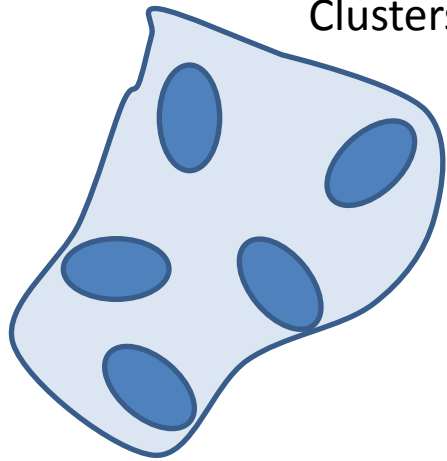
$$N_{Def} \approx \frac{N_D^c}{\gamma_{IC}^3} + \frac{N_D^s - N_D^c}{\gamma_{IC}^3 \gamma_{EC}^3} = N_D^s$$

Cluster with almost no interior demagnetizing field



$$N_{Def} \approx \frac{N_D^c}{\gamma_{IC}^3} + \frac{N_D^s - N_D^c}{\gamma_{IC}^3 \gamma_{EC}^3} = \frac{N_D^s}{\gamma_{IC}^3 \gamma_{EC}^3}$$

Clusters of random shape with orientations randomly distributed.



$$\langle N_D^c \rangle \approx 1/3$$

$$N_{Def} \approx \frac{N_D^c}{\gamma_{IC}^3} + \frac{N_D^s - N_D^c}{\gamma_{IC}^3 \gamma_{EC}^3} \approx \frac{1}{3\gamma_{IC}^3} \left( 1 + \frac{3N_D^s - 1}{\gamma_{EC}^3} \right)$$

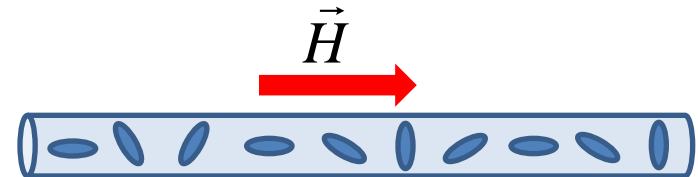
If clusters are far away from each other

$$N_{Def} \approx \frac{1}{3\gamma_{IC}^3}$$

If clusters are in contact

$$N_{Def} \approx \frac{N_D^s}{\gamma_{IC}^3}$$

If sample does not present demagnetizing effect



$$N_{Def} \approx \frac{1}{3\gamma_{IC}^3} \left( 1 - \frac{1}{\gamma_{EC}^3} \right)$$

Relationship between  $N_{Def}$  and  $N_D$

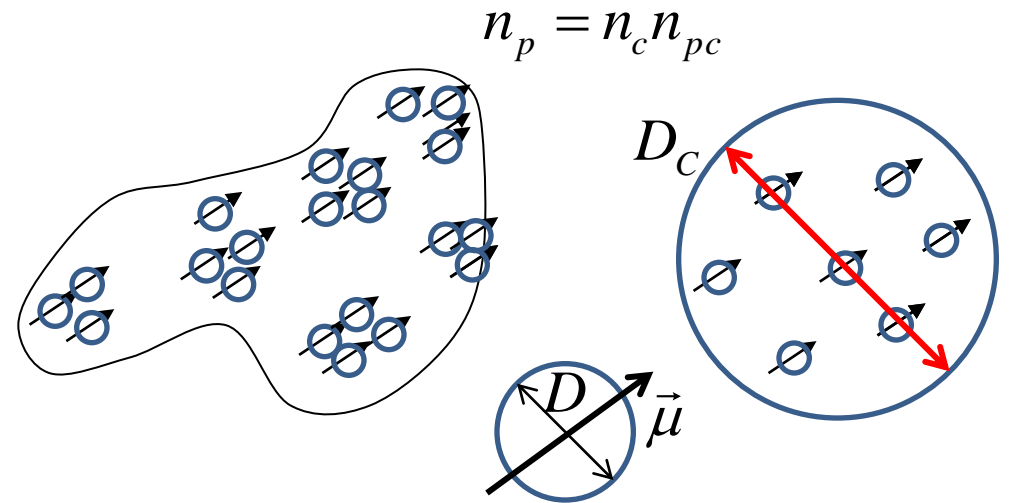
$$N_{Def} = N_D / \gamma_{IC}^3$$

Leads to

$$N_D \approx N_D^c + \frac{N_D^s - N_D^c}{\gamma_{EC}^3}$$

# Other considerations

NPs Magnetization  $M_p \approx \frac{\langle \mu_p \rangle}{D^3}$



cluster Magnetization  $M_c \approx \frac{M_p}{\gamma_{IC}^3}$

Sample Magnetization  $M \approx \frac{M_c}{\gamma_{EC}^3} = \frac{M_p}{\gamma_{IC}^3 \gamma_{EC}^3}$

$M_S$  y  $M_{pS}$  are measured. The dilution factors product can be estimated:

$$\frac{M_S}{M_{pS}} = \frac{M}{M_p} \approx \frac{1}{\gamma_{IC}^3 \gamma_{EC}^3}$$

$$N_D \approx \langle N_D^c \rangle + \frac{1}{\gamma_{EC}^3} (N_D^s - \langle N_D^c \rangle)$$

Lead to

$$\left\{ \begin{array}{l} \gamma_{EC}^3 \approx \frac{N_D^s - \langle N_D^c \rangle}{N_D - \langle N_D^c \rangle} \\ \gamma_{IC}^3 \approx \frac{M_{pS}}{M_S} \frac{N_D - \langle N_D^c \rangle}{N_D^s - \langle N_D^c \rangle} \end{array} \right.$$



For an isotropic clusters orientation distribution

$$\langle N_D^c \rangle \approx 1/3$$

$$N_D \approx \frac{1}{3} + \frac{1}{\gamma_{EC}^3} \left( N_D^s - \frac{1}{3} \right)$$

Leads to

$$\left\{ \begin{array}{l} \gamma_{EC}^3 \approx \frac{N_D^s - 1/3}{N_D - 1/3} \\ \gamma_{IC}^3 \approx \frac{M_{pS}}{M_S} \frac{N_D - 1/3}{N_D^s - 1/3} \end{array} \right.$$

# Analysis with Langevin model.

## Comparison with Allia's proposition

$$\vec{H} = \vec{H}^{ap} + \vec{H}^{dip}$$

$$M_p = M_{pS} L\left(\frac{\mu_0 \vec{\mu} \cdot \vec{H}}{kT}\right) = M_{pS} L\left(\frac{\mu_0 \mu (H^{ap} + H^{dip})}{kT}\right)$$

$$\vec{H}^{dip} = -N_{Def} \vec{M}_p$$

Assumptions  $\vec{M}_p \uparrow\uparrow \vec{H} \uparrow\downarrow \vec{H}^{dip}$

$$M_p(H, T) = M_{pS} L\left(\frac{\mu_0 \mu (H^{ap} - N_{Def} M_p)}{kT}\right) = \frac{\mu}{V_p} L\left(\frac{\mu_0 \mu (H^{ap} - N_{Def} M_p)}{kT}\right)$$

When Langevin function argument is  $\ll 1$

$$M_p = \frac{\mu_0 \mu^2 (H^{ap} - N_{Def} M_p)}{3kTV_p}$$

$$M_p = \frac{H^{ap}}{\frac{3kTV_p}{\mu_0 \mu^2} + N_{Def}}$$

$$\frac{1}{\chi_p} = \frac{3kT}{\mu_0 M_{pS}^2 V_p} + N_{Def}$$

$$\frac{1}{\chi_p} = \frac{3kTN_p}{\mu_0 M_{pS}^2} + N_{Def}$$

$$N_p = 1/V_p$$

 **Measured** susceptibility of magnetic phase.

$$\frac{1}{\chi_p} = \frac{3kN_p}{\mu_0} \left( \frac{T}{M_{pS}^2} \right) + N_{Def}$$

According to Allia

$$\frac{1}{\chi_p} = \frac{3kN_p}{\mu_0} \left( \frac{T}{M_{pS}^2} \right) + 3\alpha_p$$

It's concluded that  $N_{Def} = 3\alpha_p$

For a sample with a distribution of NP moment sizes, Allia shows

$$\frac{\rho}{\chi_p} = \frac{3kN_p}{\mu_0} \left( \frac{T}{M_{pS}^2} \right) + 3\alpha_p \qquad \rho = \frac{\langle \mu^2 \rangle}{\langle \mu \rangle^2} \equiv \frac{\langle \mu_a^2 \rangle}{\langle \mu_a \rangle^2}$$

$\mu_a$  and  $\mu$  are NP moments. The former are “aparent” values which are obtained when analyzing  $M(H, T)$  without considering dipolar interactions. The latter are values “corrected” from

$$\mu = \frac{M_{pS}}{N_p}$$

In the present case

$$\frac{\rho}{\chi_p} = \frac{3kN_p}{\mu_0} \left( \frac{T}{M_{pS}^2} \right) + N_{Def}, \qquad N_p = 1/V_p$$

As a function of sample “global” magnetization  $M_S$  for a homogeneous sample,

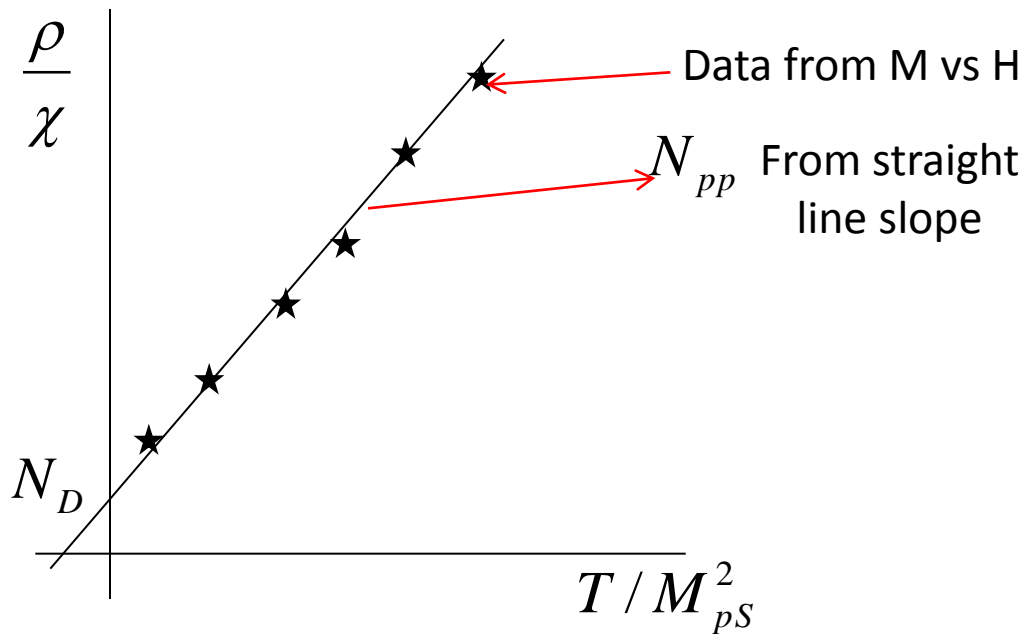
$$\frac{\rho}{\gamma^3 \chi} = \frac{3kT}{\mu_0 \gamma^6 M_S^2 V_p} + \frac{N_D}{\gamma^3} = \frac{\rho}{\gamma^3} \left( \frac{3kT}{\mu_0 M_S^2 \gamma^3 V_p} + N_D \right)$$

$$\frac{\rho}{\chi} = \frac{3kN_{pp}}{\mu_0} \left( \frac{T}{M_S^2} \right) + N_D \quad N_{pp} = 1/V_{pp} = 1/\gamma^3 V_p$$

 **Measured** sample susceptibility

$$N_D = 3\alpha \text{ (Allia)} \Rightarrow \alpha \leq 1/3$$

Measuring  $M$  vs.  $H$  at several temperatures, and plotting  $\rho/\chi$  vs.  $T/M_S^2$   $N_D$  and  $N_{pp}$  can be retrieved



$$\mu = \frac{M_S}{N_{pp}}$$

$$\varepsilon = \frac{\mu_0 N_D M^2}{2N_{pp}}$$

Max dipolar energy per NP  $\varepsilon = \frac{\mu_0 N_D M_S^2}{2N_{pp}}$

If there are clusters,

$$\rho = \frac{3k}{\mu_0 V_p} \left( \frac{T}{M_{pS}^2} \right) + N_{Def}$$

$M_S$



$$\frac{\rho}{\gamma_{IC}^3 \gamma_{EC}^3 \chi} = \frac{3k}{\mu_0 \gamma_{IC}^6 \gamma_{EC}^6 V_p} \left( \frac{T}{M_S^2} \right) + \frac{N_D}{\gamma_{IC}^3}$$

using

$$V_m \approx n_c \gamma_{EC}^3 n_{pc} \gamma_{IC}^3 V_p \Rightarrow \frac{1}{\gamma_{EC}^3 \gamma_{IC}^3 V_p} \approx \frac{n_c n_{pc}}{V_m} = \frac{n_p}{V_m} = \frac{1}{V_{pp}} = N_{pp}$$

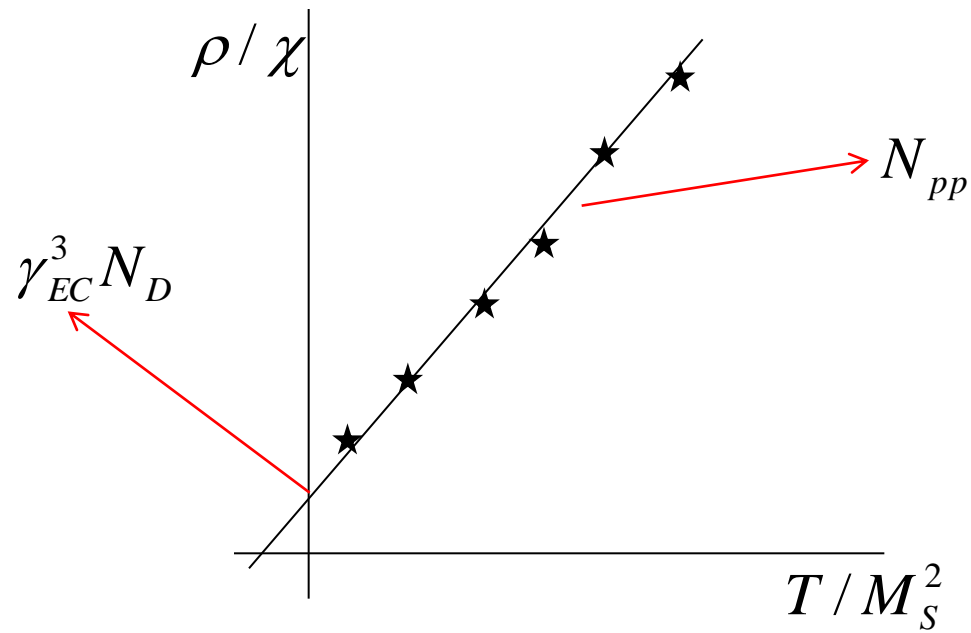
$$\frac{\rho}{\gamma_{IC}^3 \gamma_{EC}^3 \chi} = \frac{3k N_{pp}}{\mu_0 \gamma_{IC}^3 \gamma_{EC}^3} \left( \frac{T}{M_S^2} \right) + \frac{N_D}{\gamma_{IC}^3}$$



$$\frac{\rho}{\chi} = \frac{3k N_{pp}}{\mu_0} \left( \frac{T}{M_S^2} \right) + \gamma_{EC}^3 N_D$$

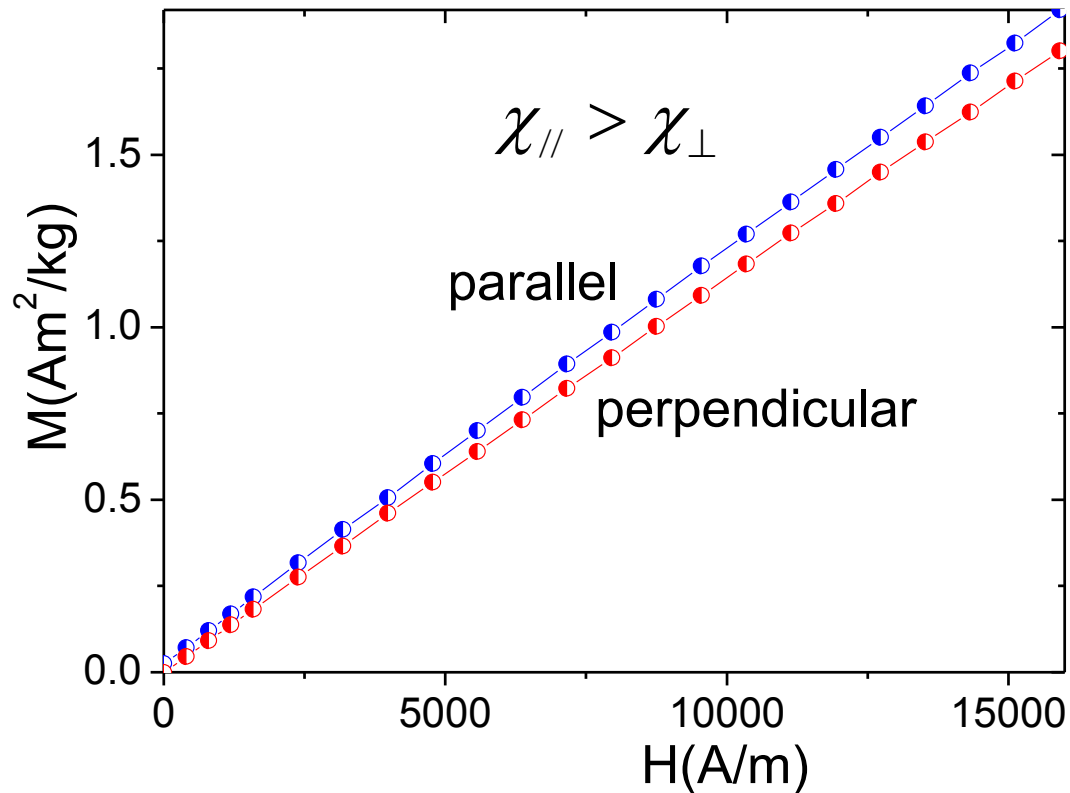
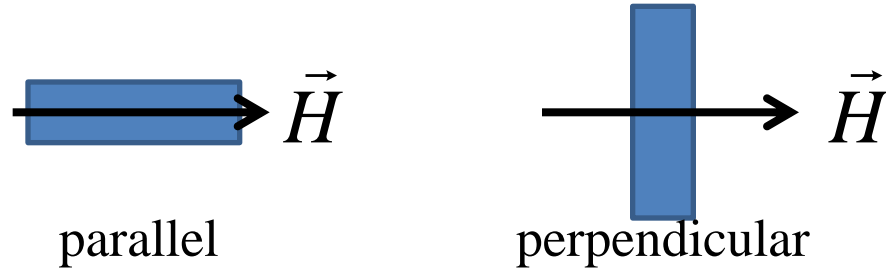


$$\frac{\rho}{\chi} = \frac{3kN_{pp}}{\mu_0} \left( \frac{T}{M_S^2} \right) + \gamma_{EC}^3 N_D$$



# Shape effects

Ferrogel PVA/maghemite 15.7% mass concentration



$$\chi_{//} \approx 1.13 \times 10^3 \text{ m}^3 / \text{kg}$$

$$\chi_{\perp} \approx 1.08 \times 10^3 \text{ m}^3 / \text{kg}$$

$$N_{Def \perp} - N_{Def //} \approx \frac{10^3}{4\pi\rho} \left( \frac{1}{\chi_{\perp}} - \frac{1}{\chi_{//}} \right)$$

Assuming a uniform distribution of NPs

$$\gamma^3 \approx \frac{N_{D\perp} - N_{D//}}{N_{Def \perp} - N_{Def //}}$$

De la forma de la muestra

## Demagnetizing factors for rectangular ferromagnetic prisms

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JOURNAL OF APPLIED PHYSICS

VOLUME 83, NUMBER 6

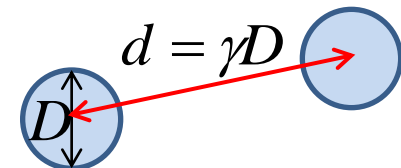
15 MARCH 1998

$$N_{//} = 0.06$$

$$N_{\perp} = 0.80$$



$$\gamma \approx 2.1$$



Estimation of  $\gamma$  from FG density and mass concentration of Fe oxide  $x$ , assuming a uniform distribution of NPs

$$x = \frac{m_{oFe}}{m_{FG}} \qquad \rho_{FG} = \frac{m_{FG}}{V_{FG}}$$

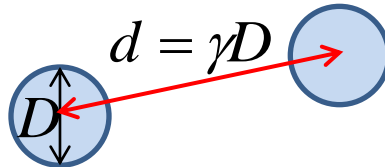
$$\rho_{FG} = \frac{m_{oFe}}{xV_{FG}} = \frac{N_{NP}m_{NP}}{xV_{FG}} = \frac{N_{NP}}{V_{FG}} \frac{\rho}{x} \frac{\pi D^3}{6} = \left(\frac{D}{d}\right)^3 \frac{\pi\rho}{6x} = \frac{\pi\rho}{6x\gamma^3}$$

$$\gamma = \left(\frac{\pi\rho}{6x\rho_{FG}}\right)^{1/3}$$

$$\gamma = \left( \frac{\pi\rho}{6x\rho_{FG}} \right)^{1/3}$$

For FG 10\_1

$$\text{using } \begin{cases} \rho \approx 5.18 \text{ g / cm}^3 \\ x = 0.157 \\ \rho_{FG} \approx 1.1 \text{ g / cm}^3 \end{cases} \Rightarrow \gamma \approx 2.5$$



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# Dipolar Energy

Energy per particle for a homogeneous saturated sample

$$\mathcal{E} = \frac{\mu_0}{2} N_{Def} M_{pS}^2 V_p$$

non saturated sample

$$\mathcal{E} = \frac{\mu_0}{2} N_{Def} M_p^2 V_p$$

In terms of sample global magnetization

$$\mathcal{E} = \frac{\mu_0}{2} \frac{N_D}{\gamma^3} \gamma^6 M^2 V_p = \frac{\mu_0}{2} N_D M^2 (\gamma^3 V_p) = \frac{\mu_0}{2} N_D M^2 V_{pp}$$

Using  $N = 1/V_{pp}$

$$\mathcal{E} = \frac{\mu_0 N_D M^2}{2N}$$