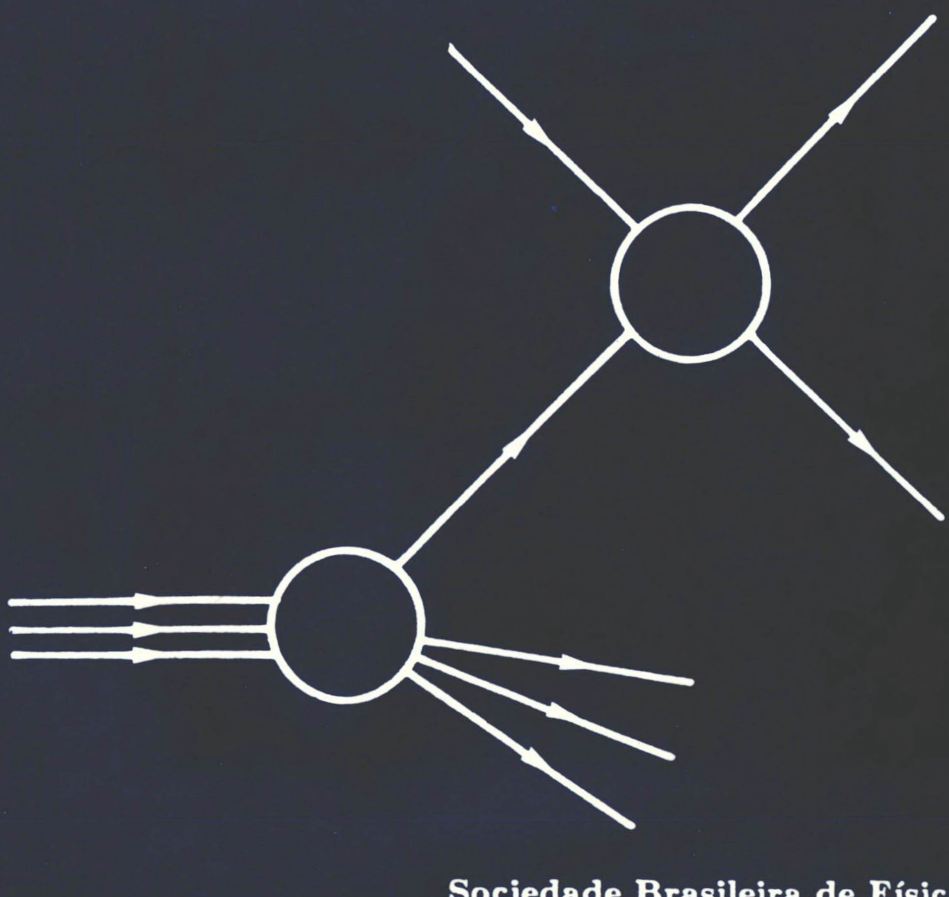
1991

XII Encontro Nacional

Partículas e Campos
Particles and Fields



Sociedade Brasileira de Física

1991 Brasil
XII Encontro Nacional

Partículas e Campos
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XII ENCONTRO NACIONAL DE FÍSICA DE PARTÍCULAS E CAMPOS

Caxambu, 18-22 de setembro de 1991

Esta publicação contém os trabalhos apresentados durante o XII Encontro Nacional de Física de Partículas e Campos, realizado em Caxambu, MG, entre 18 e 22 de setembro de 1991.

Como aconteceu nos anos anteriores, a reunião contou com cerca de 150 participantes, entre eles pesquisadores estrangeiros convidados especialmente a participar na reunião com palestras de revisão.

Também contamos nesta oportunidade com o financiamento da FAPESP e do CNPq. Aproveitamos a oportunidade para agradecer o apoio destas instituições de fomento à pesquisa. Agradecemos em particular à FAPESP pela concessão de auxílio para aquisição do material usado na confecção destes anais e ao Instituto de Física da UNICAMP que possibilitou sua impressão.

A realização destes Encontros e a posterior publicação dos anais são parte de uma tradição estabelecida nos anos anteriores e, por ser uma amostra representativa do esforço da produção científica no pais, esperamos que tenha continuidade nos anos seguintes.

No final deste volume apresentamos a relação nominal dos participantes aos quais estendemos nossos agradecimentos.

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XII NATIONAL MEETING ON PARTICLE PHYSICS AND FIELDS

Caxambu, 18-22 Setember 1991

This volume collects most of the material presented in the XII National Meeting on Particle Physics and Fields, which was held in Çaxambu, MG, from 18 to 22 Setember 1991.

As in previous occasions there were about 150 participants, some of them visiting scientists invited to give review lectures.

The Meeting was sponsored by brazilian financial agencies FAPESP and CNPq and we express here our gratitude to them. We are grateful to FAPESP (for financial support) and to the Instituto de Física , Universidade Estadual de Campinas (printing facilities), whose support made possible the publication of this volume.

The Meeting and the publication of its Proceedings are already a tradition that, we hope, will continue in the future.

The full list of the participants appears at the end of this book. To all of them we are very grateful.

ORGANIZING COMMITEE

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Fixing the Gauge at Future Null Infinity

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ABSTRACT

After a review of the fundamental concepts around the notion of isolated systems in curved spacetimes, we analyze the problem of the ambiguities of the supertranslations at future null infinity. We propose a tool that provides a clean way to fix the gauge problem; namely the notion of "nice sections". We conclude with some resent results on "nice sections" on a particular class of radiating spacetimes.

---

* Member of Consejo Nacional de Investigaciones Científicas y técnicas CONICET.

1 INTRODUCTION

Among the interactions defined by physicists, gravitation appears as the weakest of them. And since the source of gravitational interactions is the presence of matter, physically interesting gravitating systems normally involve large concentration of it.

There are two types of physical systems where the use of the general relativistic picture of the gravitational interactions is essential, they are the systems that involve very compact objects and the systems that involve all known matter, namely the universe. We will next be concerned with the first type of systems.

In the study of very compact objects it is generally the case that we can neglect the influence of the rest of the universe on them. In this case we say that the system can be modeled by an isolated system. In the Newtonian picture an isolated system is normally represented by a gravitational potential that goes to zero at large distance as $1/r$. In a relativistic theory of gravity one expects that at large distances from the central object the spacetime will approach more and more the characteristics of Minkowski space; more precisely, the discussion of isolated systems is made in terms of the class of spacetimes which have their curvature going to zero when one recedes to infinity, these are called *asymptotically flat spacetimes*.

The notion of asymptotically flat spacetimes (that we will discuss in the second part of our talk) introduces a partial boundary for the spacetime, that is very convenient for studying the asymptotic physical fields. However a new ingredient appears (in comparison with what happens in Minkowski spacetime), now the gauge group at infinity is much larger than the Poincaré one; in fact it has infinite dimension. This has as a consequence that it is much more difficult to handle the physical fields in this case. We will later propose a solution for the gauge problem.

2 ASYMPTOTIC FLATNESS

2.1 WHAT SHOULD WE LOOK FOR?

Since isolated systems in relativistics theories of gravity should be modeled by asymptotically flat spacetimes, we should have a precise idea of the meaning of this concept. A definition of an asymptotically flat spacetime should include somehow the statement that the curvature tensor tends to zero when one recedes from the central region where the sources are placed. However in a Lorentzian spacetime there are three types of distinct direction which one could take to move away from the central region; besides, one should be very careful with the idea of a tensor tending to zero since in principle, the notion of the limit of a tensor is not geometrically clear. Comparison of components with respect to some particular chart will not work. Another problem that should also be considered is the one of having to expressed the conditions *in the limit to infinity*.

Since we would like to take into account the effects of radiation, it turns out that it is more convenient to discuss these phenomena when one recedes along null directions; therefore we should search for the notion of asymptotic flatness at null infinity.

Although the notion of isolated systems is very frequent in physics, we note that there is a great difference whether the theory one is considering involves the very structure of the spacetime or not. For example in Maxwell theory in Minkowski spacetime, the notion of isolated system does not introduce any problem. In this case if one have a localized distribution of charges, and the electromagnetic field goes to zero at infinity (in an appropriate way) one says to have an isolated system. In contrast in general relativity the spacetime structure is not given *a priori*, in fact using Einstein equation, it depends on the matter distribution. In this case instead it is much more difficult to figure out which are the appropriate boundary conditions to be imposed on the spacetime and which are also consistent with the field equation.

In order to deal with the difficulties of taking the limits of tensor along asymptotic regions, it has been convenient the use of the conformal techniques, by which one introduces a conformal metric

$$\tilde{g}_{ab} = \Omega^2 g_{ab}$$

Since to get to infinity one needs to cover an infinite distance, with respect to the metric g_{ab}, one may think that if one makes an appropriate choice of Ω, infinity will be at a finite distance with respect to $\tilde{g}_{ab}$. The function Ω should go to zero as we approach infinity. Also in these techniques one usually attaches new points to the spacetime manifold, so that the region where $\Omega = 0$ is included in a new enlarged manifold. By doing this one could replace limits to infinity by statements on these boundary points.

In the definitions of spacetimes representing isolated systems, one has to specify the precise asymptotic behavior of the geometry as Ω goes to zero. The choice of appropriate asymptotic conditions for a spacetime is a delicate one, since conditions too strong will rule out solutions that clearly represent isolated physical systems, and conditions too weak will allow for too many cases in which useful aspects of the asymptotic behavior of physical fields are messed up with unnecessary bad behavior.

It is interesting to consider the possibility of separating the notion of isolated system in general relativity from a particular field equation. This is suggested by the following fact. Taking into account that any physically meaningful gravitational theory has to have Newton theory as a weak field limit, one can deduce from the geodesic hypothesis that in such a limit the time component of the metric should have the following asymptotic behavior:

$$g_{00} = 1 + \frac{m}{r} + O(\frac{1}{r^2}) \quad .$$

Then, assuming some *uniform smoothness condition* (that we will not discuss here), one could expect that all physically meaningful theories of gravity should admit the notion of asymptotic flatness.

However, in studying this kind of ideas it is good to recall a statement appearing in the literature[1]: "so far no firm arguments have been presented either in favour of or against the conjecture that nonstationary isolated systems can really be described by asymptotically flat spacetimes, in the sense in which this concept has been made precise up to now".

We can deal partially with this situation by working with a notion of isolated system which is independent of a particular field equation; in this way one should see later whether the theory one likes admits this notion. We will next present a definition of an asymptotically flat spacetime which does not refer to any field equation.

2.2 GENERAL FUTURE ASYMPTOTICALLY FLAT SPACETIMES[2]

Let M be a C^∞ manifold and g a C^3 metric in M. We define the orientable spacetime (M,g) to be *general future asymptotically flat* (GeFAF) if there exists a manifold $\bar{M}$ with boundary $\mathscr{I}^+$, metric $\bar{g}_{ab}$ and a function Ω on $\bar{M}$ such that $\bar{M}$ is diffeomorphic to (and therefore identified with) $M \cup \mathscr{I}^+$ and

a) on M: Ω is C^∞, $\Omega > 0$ and $\bar{g}_{ab} = \Omega^2 g_{ab}$;

b) at $\mathscr{I}^+$: $\Omega = 0$ and Ω is C^0; $\mathscr{I}^+$ is diffeomorphic to $S^2 \times \mathbb{R}$; at every point of $\mathscr{I}^+$ there end future directed null geodesics of $\bar{M}$, and $\bar{g}$ is non-degenerate; and finally

c) the leading behavior of the Riemann tensor for $\Omega \to 0$ can be expressed by

$$R_{abc}{}^{d} = f(\Omega)\, \hat{R}_{abc}{}^{d} + \delta R_{abc}{}^{d} \qquad (2.1)$$

where there exists $\Omega_0 > 0$ such that for $\Omega < \Omega_0$, $\frac{df}{d\Omega} > 0$, $\lim_{\Omega \to 0} f = 0$; $\hat{R}_{abc}{}^{d}$ is a regular tensor at $\mathscr{I}^+$, and $\delta R_{abc}{}^{d}$ is a tensor that goes to zero faster than $f(\Omega)$ for $\Omega \to 0$. Condition (2.1) must be understood as saying that every component of the Riemann tensor $R_{abc}{}^{d}$ with respect to an orthogonal tetrad of $\tilde{g}_{ab}$, which is regular at $\mathscr{I}^+$, behaves like eq. (2.1).

Note that from condition b) one can see that we are implicitly requiring $\bar{g}_{ab}$ to be C^1 at $\mathscr{I}^+$, since we can write and solve the geodesic equation up to $\mathscr{I}^+$ itself†. Also observe that in general the tensor $\hat{R}$ is unrelated to the curvature of the metric $\bar{g}$.

This definition of asymptotic flatness along null directions is clearly more general that former ones; in particular it is clear that implies a flatness condition, and also has the property that it does not refer to any field equation.

2.3 DISCUSSION OF GEFAF SPACETIMES

Since the definition of GeFAF spacetimes involves metric conditions along with curvature conditions, it seems that a direct way of obtaining information is to introduce a tetrad, if naturally available, in order to study everything with respect to it. The use of null tetrads for the study of gravitational radiation has proved to be quite useful over the years. Here we will use the G.H.P. notation[3] for the spin coefficients.

Using these technic one can prove for example that future null infinity is a null hypersurface. Also that the components Ψ_4 and Ψ_3 of the curvature tensor (in the above notation) behave like radiation field, although one has not mentioned any field equations yet!

---

† $\mathscr{I}^+$ is pronounced "scri plus".

In what follows we give the explicit asymptotic behavior of the spacetime implied by the definition of asymptotic flatness. We make use of the fact that the conformal factor Ω can be taken as the inverse of a radial coordinate that measures affine distance along outward null geodesics; that is:

$$\Omega = \frac{1}{r} \quad ;$$

and we also take the physical asymptotic behavior described by:

$$f(\Omega) = \Omega \quad .$$

For completeness we write down next all the quantities that can be defined in our formalism.

In the framework we are considering, the most basic object is the tetrad. Each vector of the tetrad can be expressed in terms of the coordinate system ($x^0 = u$, $x^1 = r$, x^2, x^3) by the equations

$$l^a = \left(\frac{\partial}{\partial r} \right)^a$$

$$m^a = \left(\xi^{\underline{i}} \frac{\partial}{\partial x^{\underline{i}}} \right)^a \qquad \underline{i} = 2,3$$

$$n^a = \left(\frac{\partial}{\partial u} + U \frac{\partial}{\partial r} + X^{\underline{i}} \frac{\partial}{\partial x^{\underline{i}}} \right)^a \quad ;$$

where m^a is a complex vector, and it is satisfied that

$$l^a n_a = 1$$

$$m^a \bar{m}_a = -1 \quad ,$$

and all other contractions give zero.

The torsion free metric conditions on the connection provide equations that relate the spin coefficients, as defined by G.H.P., and the tetrad components[2]. These equations can be used to express the spin coefficients in terms of the tetrad components as described in ref. [2].

The definition of asymptotic flatness impose conditions on the behavior of the metric and the curvature tensor explicitly. Using these conditions and after a long calculation[4] one can obtain the leading behavior of the different fields.

The spin coefficients with spin-boost weight have the following asymptotic behavior

$$\rho = -\frac{1}{r} + 0 - \frac{\sigma^0 \bar{\sigma}^0}{r^3} + O(r^{-4})$$

$$\sigma = \frac{\sigma^0}{r^2} + 0 + O(r^{-4})$$

$$\tau = \frac{\bar{\eth}_0\, \sigma^0}{r^2} - \frac{(\psi_1^0 + 2\, \sigma^0\, \eth_0 \bar{\sigma}^0)}{r^3} + O(r^{-4})$$

$$\kappa = 0$$

$$\rho' = \frac{1}{2\, r} + \frac{(\psi_2^0 + \sigma^0\, \dot{\bar{\sigma}}^0 + \eth_0^2\, \bar{\sigma}^0)}{r^2} + O(r^{-3})$$

$$\sigma' = -\frac{\dot{\bar{\sigma}}^0}{r} + \frac{(\bar{\eth}_0\, \eth_0\, \bar{\sigma}^0 - \frac{1}{2}\bar{\sigma}^0)}{r^2} + O(r^{-3})$$

$$\tau' = -\, \bar{\tau}$$

$$\kappa' = O(r^{-2}) \quad .$$

The spin coefficients ρ and σ express also the expansion, shear an twist of the congruence of null geodesics generated by the vector field l^a.

The leading behavior of the curvature components is given by

$$\psi_4 = -\frac{\ddot{\bar{\sigma}}^0}{r} + O(r^{-2})$$

$$\psi_3 = -\frac{\eth_0 \dot{\bar{\sigma}}^0}{r^2} + O(r^{-3})$$

$$\psi_2 = \frac{\psi_2^0}{r^3} + O(r^{-4})$$

$$\psi_1 = \frac{\psi_1^0}{r^4} + O(r^{-5})$$

$$\psi_0 = \frac{\psi_0^0}{r^5} + O(r^{-6})$$

$$\Phi_{22} = \quad + O(r^{-2})$$

$$\Phi_{21} = \quad + O(r^{-3})$$

$$\Lambda, \Phi_{11}, \Phi_{20} = \quad + O(r^{-4}) \quad ;$$

$$\Phi_{01} = \quad + O(r^{-5})$$

$$\Phi_{00} = \frac{\Phi_{00}^0}{r^5} + O(r^{-6})$$

where the ψ's are the components of the Weyl tensor, and Λ and the Φ's are the components of the Ricci tensor.

One often deals with the case of Einstein vacuum field equation, that is the case of zero Ricci tensor. The behavior of the Weyl tensor in this case is usually called peeling behavior.

In analogy with the case of Maxwell field in Minkowski spacetime one associates the component ψ_4 to the notion of radiation. So $\dot{\psi}_4 = 0$ means absence of radiation. In fact, neglecting divergent asymptotic behavior of the Weyl tensor, one can actually prove that $\psi_4 = 0$ implies that all the other components are constant in time. This reinforce the interpretation of ψ_4 as the gravitational radiation content of the spacetime.

It is also observed that ψ_3 refer to radiation content, since when there is no radiation $\psi_3 = 0$.

We will later refer to the physical meaning usually attached to the components ψ_2, ψ_1 and ψ_0.

3 ASYMPTOTIC SYMMETRIES

3.1 THE BMS GROUP

The asymptotic structure of an asymptotically flat spacetime singles out a preferred set of coordinate systems and tetrads at null infinity. This is analogous to the case of Minkowski space in which the metric gives preference to the orthogonal Cartesian coordinates along with their associated tetrads.

A set of coordinates at future null infinity are said to be of Bondi type if the restriction of the conformal metric to the boundary of the spacetime is given by the metric of the unit sphere; that is:

$$\tilde{g}\Big|_{\mathscr{I}^+} = - dS^2 = - \frac{4\, d\zeta\, d\bar{\zeta}}{(1+\zeta\bar{\zeta})^2} \quad ;$$

where we have used stereographic coordinates.

Transformations among these coordinates system form a group; the so called BMS[5,6] group.

The following is a representation of the Lie algebra of the BMS group in terms of the generators acting on $\mathscr{I}^+$:

$$R_z = i\left(\zeta \frac{\partial}{\partial\zeta} - \bar{\zeta}\frac{\partial}{\partial\bar{\zeta}}\right) \qquad B_z = \frac{(\zeta\bar{\zeta}-1)}{(1+\zeta\bar{\zeta})}\, u\frac{\partial}{\partial u} - \zeta\frac{\partial}{\partial\zeta} - \bar{\zeta}\frac{\partial}{\partial\bar{\zeta}}$$

$$R^+ = -\left(\zeta^2\frac{\partial}{\partial\zeta} + \frac{\partial}{\partial\bar{\zeta}}\right) \qquad B_z = \frac{2\zeta}{(1+\zeta\bar{\zeta})}\, u\frac{\partial}{\partial u} + \zeta^2\frac{\partial}{\partial\zeta} - \frac{\partial}{\partial\bar{\zeta}}$$

$$R^- = \overline{R^+} \qquad B^- = \overline{B^+}$$

$$p_{lm} = Y_{lm}\frac{\partial}{\partial u} \quad ;$$

where Y_{lm} are the spherical harmonics, and l as usual is any non-negative integer, while $|m| \le l$. In Minkowski space one can chose the Bondi frame so that the generators R_z, R^+ and R^- coincide with the Killing rotations, the generators B_z, B^+ and B^- coincide with the boosts symmetries, and the generators $p_{l_1 m}$ with $l_1 \le 1$ coincide with the generator of translations. The rest of the infinite family of generators $p_{l_2 m}$ with $l_2 > 1$ do not have a Minkowskian analog and are associated to the notion of the so called *supertranslations.*

The appearance of the supertranslations constitutes the main difference between the asymptotic symmetries of an isolated system and the symmetries of Minkowski space; and because of this it is difficult in general to extend the physical concepts of flat space to the boundary $\mathscr{I}^+$ of an asymptotically flat spacetime.

3.2 PHYSICAL QUANTITIES AT FUTURE NULL INFINITY

One of the main reasons for introducing the notion of isolated systems is that one would like to have access to physical concepts, like total momentum or total angular momentum of the system, in order to simplify the description of the system.

Therefore having defined the notion of asymptotically flat spacetime, we would like now to know what is the appropriate notion of *total momentum* at null infinity.

Let us recall that in flat spacetime the total momentum is given as an integral over a spacelike hypersurface, where the integrand contains the translational Killing vectors as argument. We also know that this integral is equivalent to an integral on the boundary of the hypersurface, that is on a 2-dimensional surface.

To each generator of Bondi transformations one can associate an integral on a section of $\mathscr{I}^+$. Following the approach of Geroch and Winicour[7], as described by Walker[8], we define de components of the supermomentum with respect to a section u=constant of scri, by the equation

$$P_{lm}(u) \equiv -\frac{1}{\sqrt{4\pi}} \int_u Y_{lm} \left(\psi_2^0 + \sigma^0 \dot{\bar{\sigma}}^0 + \eth_0^2 \bar{\sigma}^0\right) dS^2 \quad .$$

Only the P_{lm} with $l \leq 1$ have an invariant meaning since only the four generator of translation generate an invariant subgroup of the BMS group. It is because of this reason that the *Bondi momentum*, defined by:

$$(P^0,P^1,P^2,P^3) = \left(P_{00}, \frac{1}{\sqrt{6}}(P_{11} - P_{1-1}), -\frac{i}{\sqrt{6}}(P_{11} + P_{1-1}), \frac{1}{\sqrt{3}} P_{10}\right)$$

is a physically meaningful object.

The *Bondi mass* is given by P_{00}; from which it can be seen that when we take a frame for which the spacelike components of the Bondi momentum are zero, the Bondi mass gives the total energy content of the spacetime.

The fact that translations form a normal subgroup of the BMS group, permits to relate the Bondi momentum defined on two different sections of scri. In fact one can express a flux law.

Since there is no Poincaré subgroup of the BMS group, it is not simple to extend the definition of total angular momentum to asymptotically flat spacetimes. In fact, there are several inequivalent definitions of angular momentum at null infinity. The usual problem with these definitions is that they depend too much on the section in which they are calculated; and so it is very difficult to relate the corresponding angular momentum values which belong to two different cuts. A further difficulty in standard approaches is the supertranslation ambiguity, since even if one had succeeded in relating a definition for two different cuts, one is still left with the supertranslation gauge freedom. The only definition of angular momentum which is free of supertranslation ambiguities is the one introduced in reference [9]. In order to get rid of the supertranslation gauge dependence, a unique Bondi system was defined by imposing some conditions in the limit of the retarded time u going to $-\infty$. This kind of construction has advantages[9] and disadvantages. One natural criticism is that we think of an astrophysical observer as residing at future null infinity, which we assume has complete information on the local properties of the spacetime. This observer, using the local information, should be able to make a physical description of the system. If we were forced to define a Universal center of mass system by using the properties of the spacetime at the retarded time $u = -\infty$, then, this would imply going against the idea of local information description.

Since the definition of angular momentum at future null infinity is a difficult task, one can imagine that the definition of multipole moments will be even worse. As usual the difficulty has to do with the supertranslation problem, since one does not know in general what to do with it. In $\mathscr{I}^+$ Janis and Newman[10] have argued on a interpretation of data at null infinity and multipole moment structure of the sources. In relation to this, a personal interpretation is that: the leading behavior of ψ_2 is associated with the monopole structure which in turn has to do with the mass aspect of the sources, the leading behavior of ψ_1 is associated with the dipole structure which in turn has to do with the angular momentum aspect of the sources, and the leading behavior of ψ_0 is associated with the higher multipole structure which in first order would describe the quadrupole aspect of the sources. For the cases of static and stationary spacetimes Geroch[11] and Hansen[12] have introduced a definition of multipole moments; however their construction is done at spacelike infinity. In other words, there is still lacking a systematic study of multipole moments at future null infinity.

4. SUPERCENTER OF MASS SYSTEM

4.1 "NICE" SECTIONS OF FUTURE NULL INFINITY

The fact that the group of symmetries of null infinity, of an asymptotically flat spacetime, is not the Poincaré group but the infinite dimensional BMS group, has been a difficulty in the physical understanding of the geometric asymptotic fields.

Over the years a number of trials have been made in order to restrict this infinite dimensional freedom to a more convenient one. Some of these efforts included conditions of a global character, as has been mentioned, in which a unique Bondi system was singled out by imposing conditions in the limit for the retarded time u going to $+\infty$, or $-\infty$.

Let us recall that in Minkowski space, every point singles out a Lorentz group, which leaves that point intact (those are the Lorentz rotations around that point). Analogously, in a general future asymptotically flat spacetime, any section S of $\mathscr{I}^+$ singles out a set of fix generators of the BMS group that leave S intact. For a general space, S will not be the intersection of the future null cone of a point with $\mathscr{I}^+$.

Is there any invariant way we can fix a family of sections at future null infinity?

We present here a choice of retarded time which is local in character, in contrast to the previous ones, and which has a clear geometrical meaning.

We define[13] a section S to be *nice* if the G-W supermomentum P_{lm} satisfies

$$P_{lm} = 0 \qquad \text{for} \qquad l \neq 0 \; . \tag{4.I}$$

Let us study next this definition in the simple case of a stationary isolated system.

4.2 THE CASE OF STATIONARY ASYMPTOTICALLY FLAT SPACETIMES

When there is no radiation content in the spacetime, we can prove[13] that it is possible to find a section $\bar{S}$ that satisfies

$$\bar{P}_{lm}(\bar{S}) = 0 \qquad \text{for} \qquad l \neq 0 \; ;$$

which is our condition of *nice* section.

If we now make a translation from $\tilde{S}$, we will still get a *nice* section. In other words, there is a 4-parameter family of *nice* sections in stationary isolated systems.

In order to determine a unique set of sections, we select a family of them that *follows* the system, as we now explain. Using the construction of reference [9], we can define an asymptotic section for the retarded time $u \longrightarrow -\infty$, by the requirement that the angular momentum coincides with the intrinsic angular momentum. Then, if we allow only for translations that are parallel to the Bondi momentum, we will get a unique set of sections on $\mathscr{I}^+$ which, in this particular case, agrees with the set of sections given by u = constant of the Center of Mass Bondi system[9].

The question arises: can we carry out this construction in the presence of radiation?

4.3 THE CASE OF "NICE" SECTIONS IN RADIATING ISOLATED SYSTEMS

It was pointed out in ref. [9] that in any asymptotically flat spacetime admitting the notion of angular momentum, one could single out a Center of Mass Bondi system, which in particular contains an asymptotic sphere in the limit $u \to -\infty$ which satisfy the property of *nice* spheres. As was mentioned in the previous section, when there is no radiation content, we can obtain a unique set of *nice* sections by performing timelike translations which are parallel to the Bondi momentum. Since we know that physically reasonable astronomical systems will radiate gravitational energy very slowly, we expect to be able to find a consecutive section from the original one, which will still be *nice* even in the general radiating case.

More concretely in ref.[13] it was shown that if the time derivative of

$$\psi \equiv \psi_2^0 + \sigma^0 \dot{\bar{\sigma}}^0 + \eth_0^2 \bar{\sigma}^0 \quad ,$$

is less than one; that is

$$\dot{\psi} < 1 \quad ;$$

then given an initial *nice* section S, there exists a local family of *nice* sections around S.

At this point two questions remain open: a) is the condition $\dot{\psi} < 1$ physically reasonable?, and b) can we find an original *nice* section S in a non-trivial radiating spacetime?

Question a) was answered in ref.[13]. It turns out to be a reasonable condition; since even studies on systems including collapsing black holes have

$$\dot{\psi} < 10^{-6} \quad .$$

In the next subsection we refer to question b).

4.4 RESULTS ON "NICE" SECTIONS IN THE ROBINSON-TRAUTMAN METRICS

A very important example of radiative spacetimes is the one of Robinson-Trautman metrics[14] (R-T). These are spacetimes which are solutions of the vacuum Einstein equation, and which contain a congruence of null geodesics, with vanishing shear and twist, but diverging.

We will specialize our study to those (R-T) spacetimes whose null congruence reaches future null infinity and has no angular singularities.

From the assumption that the twist is zero, it can be deduced that the congruence is hypersurface-orthogonal; that is, by hypothesis, there exists a family of null hypersurfaces which contain shear-free null geodesics. This fact allows us to introduce a coordinate system as follows. Let u be a parameter which labels these null hypersurfaces with u=const. We can associate an affine parameter r for the null geodesics of the congruence.

To complete the coordinate system we introduce a pair of complex stereographic coordinates ζ and $\bar{\zeta}$ for the 2-surfaces $S_{u,r}$ defined by u=const., r=const., which are topologically 2-spheres. The pair $(\zeta,\bar{\zeta})$ labels the geodesics in the hypersurface u=const.

With this choice of coordinates, the Robinson-Trautman line element takes the form:

$$ds^2 = \left(-2\,H\,r + K + 2\,\frac{\Psi_2^0(u)}{r}\right) du^2 + 2\,du\,dr - \frac{r^2}{P^2}\,d\zeta\,d\bar{\zeta} \quad ;$$

where P is a function of u, ζ and $\bar{\zeta}$, and the functions H and K are related to P through:

$$H = \dot{P}/P$$
$$K = \Delta\,\ell n P \quad ,$$

where $(\dot{\ })$ stands for $\partial/\partial u$, and Δ is the 2-dimensional Laplacian for the 2-surfaces $S_{u,r}$ with line element

$$dS^2 = \frac{1}{P^2}\, d\zeta\, d\bar{\zeta} \quad .$$

The function Ψ_2^0 is the coefficient of the leading term in an expansion in powers of (1/r) of Ψ_2, which represents a component of the Weyl tensor in the spin coefficient formalism.

The vacuum condition becomes an equation for P as follows,

$$-2\,\Psi_2^0 + 6\,\Psi_2^0\, H = \frac{1}{2}\,\Delta K \quad ;$$

which is called the Robinson-Trautman equation.

An immediate solution to this equation is V = constant, which when V = 1 characterizes the Schwarzschild metric. In several works[15,16,17,18] it has been indicated that the R-T metrics of the spherical type tend asymptotically to the Schwarzschild form. More concretely, in ref.[16] it was established that the Schwarzschild solution is asymptotically stable in the Lyapunov sense.

This means that when the retarded time u tends to ∞ the R-T spaces cease to radiate, since they tend to the Schwarzschild spacetime which is static. Then this suggests that probably in this asymptotic regime, one can find *nice* sections (which we know exists in stationary spacetimes).

In fact in the Appendix it is proved that in the R-T spaces one can find *nice* sections in the asymptotic region of $\mathscr{I}^+$ for $u \longrightarrow \infty$.

In this way we answer question b) of section 4.3, on the existence of *nice* section for non-trivial radiating spacetimes.

5 FINAL COMMENTS

Whether one is interested in asymptotically flat spacetimes because one wants to tackle problems of celestial relativistic mechanics, or the quantization of the gravitational degrees of freedom, one is always faced with the gauge problem at null infinity. We have here presented, by a clear geometric and physical construction, a way of fixing this gauge problem.

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APPENDIX

The Robinson-Trautman equation

$$- 2\, \dot{\Psi}^0_2 + 6\, \Psi^0_2\, H = \frac{1}{2}\, \Delta K \quad ;$$

can be simplified by noting that the function Ψ^0_2 can be made constant by redefining u without involving other coordinates; more concretely, by a transformation of the form $u' = f(u)$.

It is convenient to make use of the GHP notation[3], where the differential operators edth and edth bar are defined; and which in our case, for a function f of spin weight s, become*

$$\eth f = \frac{\sqrt{2}}{r}\, P^{1-s}\, \frac{\partial}{\partial \zeta}(P^s f)$$

and

$$\bar{\eth} f = \frac{\sqrt{2}}{r}\, P^{1+s}\, \frac{\partial}{\partial \bar{\zeta}}(P^{-s} f) \quad ,$$

respectively. Furthermore, we define the function $V(u,\zeta,\bar{\zeta})$ and P_0 by

$$P_0 \equiv \frac{(1 + \zeta\bar{\zeta})}{2} \quad ,$$

and

$$P \equiv V\, P_0 \quad .$$

With this conventions, the Robinson-Trautman equation can be put in the following form:

$$- \mu\, \dot{V} = V^4\, \eth^2\bar{\eth}^2 V - V^3\, \eth^2 V\, \bar{\eth}^2 V \quad , \tag{1}$$

where

$$\mu \equiv -3\, \Psi^0_2 > 0$$

and $\eth$ and $\bar{\eth}$ are defined with respect to the unit sphere; that is

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* Originally the edth operator is denoted by $\eth$, which however we are going to use to represent the edth operator for the unit sphere, since it will appear frequently. For this reason we here denote the original edth operator by $\eth_{*}$.

$$ðf = \sqrt{2}\, P_0^{1-s} \frac{\partial}{\partial\zeta}(P_0^{s} f)$$

and

$$\bar{ð}f = \sqrt{2}\, P_0^{1+s} \frac{\partial}{\partial\bar{\zeta}}(P_0^{-s} f) \quad .$$

The natural coordinate system adapted to the R-T family of spacetimes, which we have already used in the last section to express the line element of the Robinson-Trautman metrics, does not coincide with a Bondi system; instead, it belongs to a more general class of coordinate systems that we could call NU (Newman-Unti) type.

The induced conformal metric on scri in terms of a NU system will be

$$d\tilde{s}^2 = -\, 2du\,d\omega - \frac{1}{P^2}\, d\zeta d\bar{\zeta} \quad ,$$

where one has taken $\omega = 1/r$ and $P = P(u,\zeta,\bar{\zeta})$ is smooth and positive. Among NU coordinates there exists the freedom in the choice of the coordinate u at future null infinity, given by

$$u^* = G(u,\zeta,\bar{\zeta}) \quad ;$$

where one should also change accordingly the conformal factor $\Omega = r^{-1}$ and the radial affine coordinate r by

$$r^* = \dot{G}^{-1}\, r$$

and

$$\Omega^* = \dot{G}\, \Omega \quad ,$$

for some smooth function G such that $\dot{G}$ is also smooth and positive. In this way one obtains another coordinate system $(u^*,r^*,\zeta,\bar{\zeta})$ in a neighborhood of future null infinity which is also of the NU type; for which

$$P^* = \dot{G}^{-1}\, P \quad .$$

A Bondi coordinate system can be characterized in these terms by those which have the property that

$$P = P_0 = \frac{(1 + \zeta\bar{\zeta})}{2} \quad .$$

It follows that a coordinate system $(u,r,\zeta,\bar{\zeta})$, adapted to the Robinson-Trautman metrics, is related to a Bondi coordinate system $(u^B,r^B,\zeta^B,\bar{\zeta}^B)$ by a transformation for which

$$\dot{G} = V \quad .$$

More explicitly the relating transformation has the asymptotic form

$$u^B = \int_{u_0}^{u} V(u', \zeta, \bar{\zeta})\, du' + u_0^B(\zeta,\bar{\zeta}) + o(1/r)$$

$$r^B = V^{-1}\, r + o(1)$$

$$\zeta^B = \zeta \quad ;$$

where $u_0^B(\zeta,\bar{\zeta})$ is an arbitrary smooth function. Note that u^B and u have the same origin, that is they define the same section $u = u^B = 0$, if one chooses $u_0^B(\zeta,\bar{\zeta}) \equiv 0$ and $u_0 = 0$.

In ref. [19] it was shown that in the asymptotic future one can expressed the function V by

$$V = 1 + \left(\sum_{\substack{l=1 \\ -l<m<l}}^{\infty} \delta_n^{lm}(u) \right) Y_{lm}(\zeta,\bar{\zeta}) \quad ;$$

where the δ's have the following asymptotic behavior

$$\delta_n^{lm}(u) = \exp^{\left(\frac{-6}{\mu}\, n\, u\right)}\, q_n^{lm}(u) \quad ,$$

in which the q_n^{lm} are polynomials of order s, with $s < n$.

Therefore one can write the transformation from RT coordinates to Bondi coordinates in terms of this expansion

$$u^B = \int_{u_0}^{u} \left(1 + \sum \delta_n^{lm}(u')\, Y_{lm}(\zeta,\bar{\zeta})\right) du' + u_0^B(\zeta,\bar{\zeta})$$

$$= u - u_0 + \left(\sum \int_{u_0}^{u} \delta_n^{lm}(u')\, du'\right) Y_{lm}(\zeta,\bar{\zeta}) + u_0^B(\zeta,\bar{\zeta}) \quad ;$$

and carrying out the integration, one obtains

$$u^B = u - u_0 + \left(\sum \exp^{\left(\frac{-6}{\mu} n\, u\right)} p_n^{lm}(u)\right) Y_{lm} -$$

$$\left(\sum \exp^{\left(\frac{-6}{\mu} n\, u_0\right)} p_n^{lm}(u_0)\right) Y_{lm} + u_0^B(\zeta,\bar{\zeta}) \qquad (4.*)$$

where again $p_n^{lm}(u)$ is a polynomial of degree $s < n$.

It is observed in the last expression that the departure of the RT coordinate system from a Bondi coordinate system is given in terms of an asymptotic expansion of the form

$$\Delta = \sum_{\substack{l=1 \\ -l<m<l}}^{\infty} \varepsilon_n^{lm}(u) \;\; Y_{lm}(\zeta,\bar{\zeta}) \quad ;$$

where the $\varepsilon_n^{lm}(u)$ have similar behavior as the $\delta_n^{lm}(u)$; in particular they are governed by the exponential $\exp(-6nu/\mu)$. We can then carry out a sum, up to certain order $n=N$ to make an approximation of this expression with error of order $N+1$. All the discussion on the asymptotic behavior of the δ_n of ref. [19] are applicable to this expansion also.

The first order calculation

Let us assume that the section $u^B = 0$ coming from the above transformation does not coincide with a *nice* section; then we can try to reach one of them by a further Bondi transformation

$$\tilde{u}^B = K\,(u^B - \gamma)$$

$$\zeta^B = \frac{a\,\zeta^B + b}{c\,\zeta^B + d} \quad .$$

Then the supermomentum in the new section $\tilde{u}^B = 0$, with respect to the new Bondi system, is given by

$$\tilde{P}_{lm}(\tilde{u} = 0) = -\frac{1}{\sqrt{4\pi}} \int_{\tilde{S}=(\tilde{u}=0)} \tilde{Y}_{lm}(\zeta,\bar{\zeta})\; \Psi(\tilde{u} = 0)\; d\tilde{S}^2 \quad ,$$

where we are using the definition

$$\Psi \equiv \Psi_2 + \sigma\,\dot{\bar{\sigma}} + \eth^2\bar{\sigma} \quad ,$$

in order to simplify the expression.

If we set $u^B_0 = 0$, then the section $u^B = 0$ coincide with the section $u = u_0$ of the original RT coordinate system. Then, since the RT metrics tends in the asymptotic future to the Schwarzschild space, for which the sections u = constant are *nice*, we expect that if we take u_0 very big the section $u^B = 0$ will be very close to a *nice* section. More concretely we expect γ to be small, in some appropriate measure, and K to be almost the identity.

We can also express the supermomentum in the new section $\tilde{S}$ with respect to the original Bondi system, giving

$$\tilde{P}_{lm}(\tilde{u}^B{=}0) = -\frac{1}{\sqrt{4\pi}}\,K_{lm}{}^{l'm'}\int_{\tilde{S}=(u=\gamma)} Y_{l'm'}(\zeta,\bar{\zeta})\,[\Psi^B(u^B{=}\gamma) - \eth^2\bar{\eth}^2\gamma]\,dS^2;$$

where the matrix $K_{lm}{}^{l'm'}$ is the transformation matrix of the generators of supertranslations, that is

$$\tilde{p}_{lm} = K_{lm}{}^{l'm'}\;p_{l'm'} \quad ;$$

and where the generators are given by

$$p_{lm} = Y_{lm}(\zeta,\bar{\zeta})\,\frac{\partial}{\partial u} \;.$$

The quantity Ψ^B can be expressed by

$$\Psi^B(u^B{=}\gamma) = \int_{\infty}^{u^B=\gamma} \dot{\Psi}^B\, d(u^{B\prime}) + \Psi^B_{\infty}$$

where Ψ^B_{∞} is the limit of Ψ^B for $u^B \longrightarrow \infty$ and $\cdot$ denotes now $\partial/\partial u^B$.

Calculating $\dot{\Psi}^B$ in terms of the Bondi quantities, it is obtained

$$\dot{\Psi}^B = \dot{\sigma}^B\,\dot{\bar{\sigma}}^B \quad ;$$

and $\dot{\sigma}^B$ can in turn be expressed in terms of the function V characterizing the RT metrics as follows

$$\dot{\sigma}^B = V^{-1}\, \eth^2 V$$

$$\sim (1 - \delta_1 + ...)\, (\eth^2\delta_1 + \eth^2\delta_2 + ...)$$

$$\sim \eth^2\delta_1 + ...$$

Therefore the first order of the asymptotic behavior of $\dot{\sigma}^B$ is given by

$$\dot{\sigma}^B = \sqrt{3}\; A_1^{2m} \exp^{\left(\frac{-6}{\mu} u\right)}\; {}_{-2}Y_{2m}(\zeta,\bar{\zeta}) + \mathcal{O}(n=2) \;\; ;$$

where the $A^{2m}{}_1$ are constants determining the space; and so the first order of the asymptotic expansion of Ψ^B is

$$\Psi^B = 3 \exp^{\left(\frac{-12}{\mu} u\right)}\; \Sigma_{m,m'}\, A_1^{2m}\, A_1^{2m'}\; {}_{-2}Y_{2m}\; {}_{-2}\bar{Y}_{2m'} + ...$$

$$= 3 \exp^{\left(\frac{-12}{\mu} u\right)}\; \Sigma_{L,M}\, B_2^{LM}\; Y_{LM} + ...$$

for $0 \leq L \leq 4$ y $-L \leq M \leq L$. The coefficients B_2^{LM} are given by:

$$B_2^{LM} = 5\; [4\pi(2l+1)]^{(-1/2)}\; \Sigma\; (-1)^{m'}\; A_1^{2m}\; A_1^{2(-m')}\; \langle 22mm'|LM\rangle$$

where the sum is over all values of $-2 < m$ and $m' < 2$ such that $m + m' = M$. Explicit calculation of B_2^{1m} and B_2^{3m} show that they are zero.

Working up to second order in the calculation of the supermomentum, we can replace du^B by du, since they differ by first order terms. In fact one has the relation

$$du^B = \left(1 + \sum \delta_a^{lm}(u)\; Y_{lm}(\zeta,\bar{\zeta})\right)\, du \quad .$$

Let us define the quantity

$$\omega = \exp^{\left(\frac{-6}{\mu} u_0\right)}$$

and assume that the function γ is $O(\omega)$; then we can express Ψ^B evaluated at the new section by

$$\Psi^B(u^B=\gamma) = \Psi^B_\infty + \int_\infty^{u_0+\gamma+\dots} \exp^{\left(\frac{-12}{\mu} u\right)} du \; 3 \; B_2^{lm} \; Y_{lm} + \dots$$

$$= \Psi^B_\infty + \exp^{\left(\frac{-12}{\mu} u_0\right)} \left[1 - 12\frac{\gamma}{\mu}\right] \left[-\frac{\mu}{4} B_2^{lm} \; Y_{lm}\right] + \dots$$

$$= \Psi^B_\infty + \exp^{\left(\frac{-12}{\mu} u_0\right)} \left[-\frac{\mu}{4} B_2^{lm} \; Y_{lm}\right] + O(\omega^3) \quad .$$

Let us define

$$\chi \equiv \exp^{\left(\frac{-12}{\mu} u_0\right)} \left[-\frac{\mu}{4} B_2^{lm} \; Y_{lm}\right] \quad ;$$

and let us express γ by

$$\gamma = \gamma_I + \gamma_{II} \quad ,$$

where γ_I contains only spherical harmonics with $l = 0, 1$ and γ_{II} is expressed in terms of spherical harmonics with $l \geq 2$. Then if γ_{II} satisfies

$$\eth^2\bar{\eth}^2\gamma_{II} = \chi \quad ,$$

one has

$$\Psi^B(u^B=\gamma,\zeta,\bar{\zeta}) - \eth^2\bar{\eth}^2\gamma(\zeta,\bar{\zeta}) = \Psi^B_\infty + C + o(\omega^3) \quad ;$$

where C is a constant term of order ω^2.

Therefore, for $l \neq 0$ the expression

$$\int_{S(u^B=\gamma)} [\Psi^B(u^B=\gamma,\zeta,\bar{\zeta}) - \eth^2\bar{\eth}^2\gamma(\zeta,\bar{\zeta})] \; Y_{lm}(\zeta,\bar{\zeta}) \; dS^2$$

vanishes up to order ω^2.

A Lorentz transformation of order ω^3 induces a transformation of the form

$$K = 1 + O(\omega^3) \quad ;$$

which does not alter the present result.

We conclude then that it is possible to determine a *nice* section in order $O(\omega^2)$ by finding γ_{II} from the above condition and choosing some γ_I (which should be $O(\omega)$).

It is important to note that in this order of approximation γ_{II} is independent form the proper translation part γ_I. This is so because an $O(\omega)$ γ_I induces variation in γ_{II} of $O(\omega^3)$.

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FINITENESS IN GAUGE FIELD THEORIES

by

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Abstract

Finiteness properties of gauge field theories are discussed by means of a functional differential equation which holds in the Landau-gauge and which allows to estabilish the non-renormalization of the ghost field c and of the composit operator (trc^3).

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1. Introduction

It is known since many years, mainly through direct inspection of Feynman graphs [1], that the Landau-gauge [2,3] exihibits remarkable finiteness properties.
Recently [4], a general renormalization scheme independent proof of these finiteness properties has been done by means of a functional differential equation which holds to all orders of perturbation theory.
This equation, which represents the integrated equation of motion of the ghost field, can be imposed (among the class of linear renormalizable covariant gauges) only in the Landau-gauge and turns out to be very powerful for studying the quantum properties of a large class of models as, for instance, the Yang-Mills theories and the recently proposed topological field theories in three and four space-time dimensions [4,5].
In these notes, which are close related to a work [4] done in collaboration with A. Blasi and O. Piguet, I will limit myself to discuss in details the example of the non-abelian gauge theories in four space-time dimensions.
We will see that, thanks to the ghost-equation, the model turns out to be described only by two independent parameters which can be associated with the renormalization of the gauge coupling constant and of the gauge field-amplitude; in other words the ghost field c is not renormalized.
A second important consequence of the ghost-equation is related to the proof of the finiteness of the gauge-invariant composit operator (trc^3), whose importance is due to its relation with the U(1) axial anomaly [1]. Indeed, as it is well known (see for instance [6]), the anomalies in a gauge theory can be characterized by means of a set of descent equations whose solutions are given by gauge invariant polynomials in the ghost-fields. It is not strange, then, that the finiteness of (trc^3) plays a crucial role for the non-renormalization theorem of the U(1) axial anomaly.

The work is organized as follows: in section 2. we establish the classical ghost-equation and the non-linear algebraic constraints which will be the starting point for the quantum analysis. In section 3. we discuss the quantum extension of the ghost-equation and we show the non-renormalization of the ghost field c. Finally, in section 4. we present the proof of the finiteness of the composite operator (trc^3).

2. The ghost-equation

Let us start with a purely massless gauge theory quantized in the Landau-gauge:

$$S = -\frac{1}{4g^2}\int d^4x\,(F^a_{\mu\nu}F^{a\mu\nu}) + \int d^4x\,(b^a\partial A^a + \bar{c}^a\partial^\mu(D_\mu c)^a)\ , \tag{2.1}$$

where $b, c, \bar{c}$ are respectively the Lagrangian multiplier, the ghost, the antighost and

$$(D_\mu c)^a = (\partial_\mu c^a + f^{abc}A^b_\mu c^c)$$

is the covariant derivative with f^{abc} the structure constant of a compact semisimple gauge group G. The action (2.1) is invariant under the nilpontent BRS transformations [7]:

$$\begin{aligned} sA^a_\mu &= -(D_\mu c)^a \\ sc^a &= \frac{f^{abc}c^bc^c}{2} \\ s\bar{c}^a &= b^a \quad , \quad sb^a = 0 \\ s^2 &= 0 \end{aligned} \tag{2.2}$$

To write down the Slavnov identity corresponding to the s-invariance we couple [7] the non-linear transformations of (2.2) to external sources Ω, L:

$$S_s = \int d^4x\left(-\Omega^{a\mu}(D_\mu c)^a + L^a\frac{f^{abc}c^bc^c}{2}\right) \tag{2.3}$$

Then, the complete action

$$\Sigma = S + S_e \tag{2.4}$$

obeys to the classical Slavnov identity

$$\mathcal{B}(\Sigma) = \int d^4x \left(\frac{\delta\Sigma}{\delta\Omega^{a\mu}} \frac{\delta\Sigma}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta\Sigma}{\delta c^a} + b^a \frac{\delta\Sigma}{\delta\bar{c}^a} \right) = 0 \tag{2.5}$$

Let us introduce, for further use, the linearized nilpotent operator $\mathcal{B}_\Sigma$

$$\mathcal{B}_\Sigma = \int d^4x \left(\frac{\delta\Sigma}{\delta\Omega^{a\mu}} \frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta A^a_\mu} \frac{\delta}{\delta\Omega^{a\mu}} + \frac{\delta\Sigma}{\delta L^a} \frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta c^a} \frac{\delta}{\delta L^a} + b^a \frac{\delta}{\delta\bar{c}^a} \right) \tag{2.6}$$

$$\mathcal{B}_\Sigma \mathcal{B}_\Sigma = 0$$

The dimensions and the ghost numbers of the fields and the sources are (see table 1):

	A	b	c	$\bar{c}$	Ω	L
dim	1	2	0	2	3	4
Φπ	0	0	1	−1	−1	−2

Table 1. Dimensions and Ghost numbers

The Landau-gauge, being linear in the Lagrangian-multiplier, allows us to impose [8] the equation of motion of the b-field:

$$\frac{\delta\Sigma}{\delta b^a(x)} = \partial A^a \tag{2.7}$$

Commuting (2.7) with the Slavnov identity (2.5) one obtains the usual constraint [8]:

$$\mathcal{G}^a(x)\Sigma = \left(\frac{\delta}{\delta\bar{c}^a(x)} + \partial^\mu \frac{\delta}{\delta\Omega^{a\mu}} \right) \Sigma = 0 \tag{2.8}$$

which is nothing but the equation of motion for the antighost field $\bar{c}(x)$.

The action (2.4) is also invariant under the rigid gauge transformations:

$$\mathcal{H}^a_{rig}\Sigma = 0 \tag{2.9}$$

where

$$\mathcal{H}^a_{rig} = \sum_{\varphi} \int d^4x\, f^{abc} \varphi^b \frac{\delta}{\delta \varphi^c} \tag{2.10}$$
$$\varphi = A_\mu, c, \bar{c}, b, L, \Omega$$

i. e., all the fields belong to the adjoint representation of the gauge group $\mathcal{G}$.

Let us look now at the equation of motion of the ghost field c:

$$\frac{\delta \Sigma}{\delta c^a} = -\partial^2 \bar{c}^a - \partial\Omega^a - f^{abc} L^b c^c + f^{abc} \Omega^{b\mu} A^c_\mu + f^{abc} \left(\partial^\mu \bar{c}^b\right) A^c_\mu \tag{2.11}$$

Integrating on space-time and using the gauge condition (2.7) we get the ghost functional equation:

$$\mathcal{G}^a \Sigma = \Delta^a \tag{2.12}$$

where

$$\overline{\mathcal{G}} = \int d^4x \left(\frac{\delta}{\delta c^a} + f^{abc} \bar{c}^b \frac{\delta}{\delta b^c} \right) \tag{2.13}$$

and

$$\Delta^a = \int d^4x\, f^{abc} \left(\Omega^{b\mu} A^c_\mu - L^b c^c \right) \tag{2.14}$$

The ghost-equation (2.12) is peculiar of the Landau gauge and, as we will see in the next sections, imposes strong constraints on the structure of the Slavnov-invariant counterterms. The breaking Δ^a, being linear in the quantum fields A_μ and c is a classical breaking and allows us to try the quantum extension of the ghost-equation.

The Slavnov identity (2.5), the gauge condition (2.7) and the ghost-equation (2.12) form a non-linear algebra whose relevant part takes the form:

$$\mathcal{B}_\gamma \mathcal{B}(\gamma) = 0$$

$$\overline{\mathcal{G}}^a \mathcal{B}(\gamma) + \mathcal{B}_\gamma \left(\overline{\mathcal{G}}^a \gamma - \Delta^a\right) = \mathcal{H}^a_{rig}\gamma$$

$$\frac{\delta \mathcal{B}(\gamma)}{\delta b^a(x)} - \mathcal{B}_\gamma \left(\frac{\delta \gamma}{\delta b^a(x)} - \partial A^a\right) = \mathcal{G}^a(x)\gamma$$

$$\frac{\delta}{\delta b^a(x)} \left(\overline{\mathcal{G}}^b \gamma - \Delta^b\right) - \overline{\mathcal{G}}^b \left(\frac{\delta \gamma}{\delta b^a(x)} - \partial A^a\right) = 0$$

$$\overline{\mathcal{G}}^a \mathcal{G}^b(x)\gamma + \mathcal{G}^b(x) \left(\overline{\mathcal{G}}^a \gamma - \Delta^a\right) = f^{abc} \left(\frac{\delta \gamma}{\delta b^c} - \partial A^a\right)$$

$$\left[\mathcal{H}^a_{rig}, \overline{\mathcal{G}}^b\right] = -f^{abc}\overline{\mathcal{G}}^c \tag{2.15}$$

where γ is a generic functional with even ghost number. It is interesting to note that the rigid gauge invariance is a consequence of the Slavnov identity and of the ghost-equation.

3. Stability and Renormalization

To promote the previous classical equations to the quantum level, let us begin by showing that the ghost-equation (2.12) holds to all orders of perturbation theory. The proof is based by assuming the existence of a quantum vertex functional

$$\Gamma = \Sigma + O(\hbar) \tag{3.1}$$

which obeys:

i) the Slavnov identity [7]:

$$\mathcal{B}(\Gamma) = 0 \tag{3.2}$$

ii) the gauge-condition (2.7) [8]:

$$\frac{\delta \Gamma}{\delta b^a} = \partial A^a \tag{3.3}$$

iii) the rigid gauge invariance [8]:

$$\mathcal{H}^a_{rig} \Gamma = 0 \tag{3.4}$$

Let us write, now, a broken ghost-equation:

$$\bar{\mathcal{G}}^a \Gamma = \Delta^a + \Xi^a \tag{3.5}$$

where Ξ^a represents the breaking induced by the radiative corrections. According to the Quantum Action Principle [9] the lowest-order nonvanishing contribution to the breaking - of order $\hbar$ at least - is a local integrated field functional of dimensions 4 and ghost number -1.

The most general expression for Ξ^a reads:

$$\Xi^a = \int d^4x \left(w^{abc} \Omega^{b\mu} A^c_\mu + \tau^{abc} L^b c^c + \sigma^{abc} \left(\partial^\mu \bar{c}^b \right) A^c_\mu + \lambda^{abcd} \bar{c}^b \bar{c}^c c^d + \xi^{abc} \bar{c}^b b^c \right) \tag{3.6}$$

where w^{abc}, τ^{abc}, σ^{abc}, λ^{abcd}, ξ^{abc} are arbitrary coefficients. From the non-linear algebra (2.15) it follows that the breaking Ξ^a must satisfy the consistency conditions:

$$\begin{aligned}
B_\Sigma \Xi^a &= 0 \\
\frac{\delta \Xi^b}{\delta b^a} &= 0 \\
\mathcal{G}^a(x) \Xi^b &= 0 \\
\mathcal{H}^a_{rig} \Xi^b &= -f^{abc} \Xi^c \\
\bar{\mathcal{G}}^a \Xi^b + \bar{\mathcal{G}}^b \Xi^a &= 0
\end{aligned} \tag{3.7}$$

from which it follows that:

$$m^{abc} = r^{abc} = \sigma^{abc} = \lambda^{abcd} = \xi^{abc} = 0 \tag{3.8}$$

Equation (3.8) proves the ghost-equation (2.12) at the order considered, hence to all orders by induction.

For what concerns the stability [4], let us perturb the classical action Σ by an integrated local functional $\tilde{\Sigma}$ of dimensions 4 and zero ghost number and let us impose that the perturbed action

$$\left(\Sigma + \epsilon\tilde{\Sigma}\right) \tag{3.9}$$

satisfies, to the order ϵ, the same equations of Σ, i. e.:

$$\begin{aligned} \mathcal{B}\left(\Sigma + \epsilon\tilde{\Sigma}\right) &= 0 + 0(\epsilon) \\ \frac{\delta\left(\Sigma + \epsilon\tilde{\Sigma}\right)}{\delta b^a} &= \partial A^a + 0(\epsilon) \\ \mathcal{G}^a(x)\left(\Sigma + \epsilon\tilde{\Sigma}\right) &= 0 + 0(\epsilon) \\ \overline{\mathcal{G}}^a(x)\left(\Sigma + \epsilon\tilde{\Sigma}\right) &= \Delta^a + 0(\epsilon) \end{aligned} \tag{3.10}$$

To the first order in ϵ one gets:

$$\mathcal{B}_\Sigma\tilde{\Sigma} = 0 \tag{3.11}$$

$$\frac{\delta\tilde{\Sigma}}{\delta b^a} = 0 \tag{3.12}$$

$$\mathcal{G}^a(x)\tilde{\Sigma} = 0 \tag{3.13}$$

$$\overline{\mathcal{G}}^a\tilde{\Sigma} = 0 \tag{3.14}$$

The conditions (3.12) and (3.13) imply that $\tilde{\Sigma}$ is b-independent and that the field $\bar{c}$ and the source Ω^μ enter only through the combination

$$\gamma^{a\mu} = \Omega^{a\mu} + \partial^\mu\bar{c}^a \ , \tag{3.15}$$

i. e.:

$$\hat{\Sigma} = \hat{\Sigma}(A, c, \gamma, L) .$$

From the condition (3.11) one has:

$$\hat{\Sigma} = -\frac{\zeta_g}{4g^2}\int d^4x\,(F^a_{\mu\nu}F^{a\mu\nu}) + B_{\hat{\Sigma}}\int d^4x\,(-\zeta_c L^a c^a + \zeta_A \gamma^{a\mu} A^a_\mu) \tag{3.16}$$

where

$$\hat{\Sigma} = \Sigma - \int d^4x\,(b^a \partial A^a) \tag{3.17}$$

and

$$B_{\hat{\Sigma}} = \int d^4x \left(\frac{\delta\hat{\Sigma}}{\delta A^a_\mu}\frac{\delta}{\delta\gamma^{a\mu}} + \frac{\delta\hat{\Sigma}}{\delta\gamma^{a\mu}}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\hat{\Sigma}}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta\hat{\Sigma}}{\delta c^a}\frac{\delta}{\delta L^a} \right) \tag{3.18}$$

is the restriction of the linearized operator B_Σ to b-independent functionals obeying to equation (2.8).

As it is well known [8], the expression (3.16) shows that the most general local solution of the Slavnov identity (2.5) compatible with the gauge-condition (2.7) contains three arbitrary parameters $\zeta_g, \zeta_c, \zeta_A$ which, in the parameterization of the classical action (2.4), can be identified with a renormalization of the coupling constant g (given by ζ_g), a renormalization of the gauge field A (given by ζ_A) and a renormalization of the ghost field c (given by ζ_c).

Finally, from the ghost condition (3.14), it follows that

$$\zeta_c = 0 \tag{3.19}$$

which means that the ghost field c is not renormalized.

4. Non-renormalization of (trc^3)

To study the BRS invariant composite operator (trc^3) we couple it to an invariant external field ρ of dimension 4 and ghost number -3, i. e. we add to the classical equation (2.4) the term

$$S_\rho = \int d^4x \left(\frac{f^{abc}c^a c^b c^c}{6} \right) \rho(x) \tag{4.1}$$

It is not difficult to see that the action

$$\Sigma_\rho = S + S_s + S_\rho \tag{4.2}$$

satisfies:

i) the Slavnov identity

$$\mathcal{B}(\Sigma_\rho) = 0 \tag{4.3}$$

ii) the gauge-condition

$$\frac{\delta \Sigma_\rho}{\delta b^a} = \partial A^a \tag{4.4}$$

iii) the modified ghost-equation

$$\int d^4x \left(\frac{\delta \Sigma_\rho}{\delta c^a} + f^{abc}\bar{c}^b \frac{\delta \Sigma_\rho}{\delta b^c} + \rho \frac{\delta \Sigma_\rho}{\delta L^a} \right) = \Delta^a \tag{4.5}$$

The possible invariant counterterms allowed by the Slavnov identity (4.3) and by the gauge-condition (4.4) are the same as before (see expr. (3.16)) with in addition one local counterterm of the form

$$\alpha \int d^4x \left(\frac{f^{abc}c^a c^b c^c}{6} \right) \rho(x) \tag{4.6}$$

where α is an arbitrary parameter. However, preservation of the modified ghost-equation (4.5) implies that

$$\alpha = 0 \tag{4.7}$$

This means that the external field ρ is not renormalized or, in other words, the composite field (trc^3) is finite.

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A PRESENÇA DA CONSTANTE COSMOLÓGICA NA TEORIA DE BRANS-DICKE E A SOLUÇÃO GERAL PARA O VAZIO.

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1. Introdução.

A teoria da gravitação de Brans-Dicke, surgida em 1961, desfrutou de grande popularidade nos anos sessenta, ocasião em que foi considerada uma séria alternativa à Relatividade Geral [ref.1]. Tendo como principal característica a presença de um campo escalar ϕ na lagrangiana de ação acoplado não-minimalmente à geometria, a teoria de Brans-Dicke pertence à classe das teorias da gravitação com G variável [ref.2], sendo G a *constante* gravitacional Newtoniana. Embora ainda careçam de confirmação experimental, esta classe de teorias vêm apresentando recentemente grande interesse teórico, especialmente em conexão com a questão cosmológica.

As equações de campo da teoria de Brans-Dicke c/constante cosmológica provém da lagrangiana total

$$L = \sqrt{-g}\left(\phi R + \frac{w}{\phi}\phi_{,\mu}\phi^{,\mu} + 2\Lambda\phi\right) + L_m ,$$

onde w é um parâmetro adimensional, a ser determinado *a posteriori* e L_m é a lagrangiana da matéria. Dados observacionais impõem o limite inferior $w>500$ e se fizermos $w \rightarrow \infty$ e ϕ = const. as equações de campo se reduzem às equações de Einstein.

Em 1968, Dicke [ref.3] obteve uma solução cosmológica, a partir desta teoria, que representava um modelo de universo espacialmente homogêneo e isotrópico, com seção espacial euclidiana, e que evoluía a partir de uma singularidade inicial. Curiosamente, esta solução era a mesma que havia sido proposta em 1933 por Dirac [ref.4] através de argumentos heurísticos que partiam da hipótese de G ser variável. A generalização do modelo de Dicke foi posteriormente obtida por Narial (1968)[ref.5].

A inclusão da constante cosmológica na teoria original deu origem a diversos trabalhos na literatura, em particular, citamos os de Uehara e Kim (1982), Lorenz-Petzold (1984), os quais consideraram um fluido perfeito como fonte da curvatura [refs.6,7].

Por outro lado, soluções para o vazio de matéria foram obtidas por vários autores, destacando-se o trabalho de O'Hanlon e Tupper (1972) [ref.8], os quais encontraram a solução geral para no caso de geometria do tipo Friedman-Robertson-Walker com k=o, demostrando também a não-existência de soluções para $w < -3/2$. Os resultados de O'Hanlon e Tupper foram reobtidos num contexto bem mais geral por Romero, Oliveira e Mello Neto (1989) [ref.9], os quais aplicando métodos da teoria de sistemas dinâmicos, também investigaram exaustivamente as propriedades de modelos isotrópicos com homogeneidade espacial e k=0 para fluido perfeito com equação de estado $p = \lambda\rho$ na teoria de Brans-Dicke.

Em 1983, Cerveró e Estévez propuseram uma teoria na qual o termo cosmológico aparece modificado comparando-se com a lagrangiana usual e encontraram soluções para o vazio de matéria [ref.10].

Mantendo a lagrangiana original da teoria de Brans-Dicke e incluindo o termo cosmológico Λ da maneira usual, Romero e Barros (1991) [ref.11] abordaram o problema do vazio e obtiveram a solução para modelos isotrópicos espacialmente homogêneos com seção euclidiana.

Neste trabalho, mostramos como as propriedades destas soluções podem ser estudadas através dos chamados diagramas de fase definidos pelas equações de campo. As expressões analíticas das soluções estão contidas na ref.11.

II. Representação das soluções através dos diagramas de fase.

Partindo-se da métrica $ds^2 = dt^2 - R^2(t)[dx^2 + x^2(d\theta^2 + \sin^2\theta\, d\varphi^2)]$ isto é, tipo FRW com $k=0$, as equações de campo para o vazio de matéria e constante cosmológica Λ na teoria de Brans-Dicke são dadas por: (*)

$$R_{\mu\nu} = -2\Lambda[(\omega+1)/(2\omega+3)] + \frac{\omega}{\phi^2}\phi_{,\mu}\phi_{,\nu} + \frac{1}{\phi}\phi_{;\mu;\nu} \qquad (1.a)$$

$$\Box\phi = 2\Lambda\phi/(2\omega+3) \qquad (1.b)$$

Definindo $\psi = \dot{\phi}/\phi$, as equacões acima podem ser postas na seguinte forma:

$$\dot{\theta} = -\theta^2/3 - \dot{\psi} - (\omega+1)\psi^2 + 2\Lambda(\omega+1)/(2\omega+3), \qquad (2a)$$

$$\dot{\theta} = -\theta^2 - \psi\theta + 6\Lambda(\omega+1)/(2\omega+3), \qquad (2b)$$

$$\dot{\psi} = -\psi^2 - \psi\theta + 2\Lambda/(2\omega+3), \qquad (2c)$$

onde $\theta = 3\dot{R}/R$ descreve a expansão do modelo. Como na teoria de Brans-Dicke o campo escalar ϕ é identificado a G^{-1}, $\psi = \dot{\phi}/\phi$ é, na verdade, uma variável associada a variação no tempo da *constante* gravitacional Newtoniana G. Estas equações conduzem a uma relação algébrica entre as variáveis θ e ψ:

$$\theta^2/3 + \theta\psi - \frac{\omega}{2}\psi^2 = \Lambda,$$

que funciona como uma espécie de *vinculo* do sistema dinâmico definido por 2b e 2c.

Os diagramas que se seguem representam as soluções das equações de campo expressas nas variáveis θ e ψ. Podemos, assim ter uma visão da evolução dos modelos com relação à variação desses dois parâmetros, conforme o valor da constante de acoplamento ω.

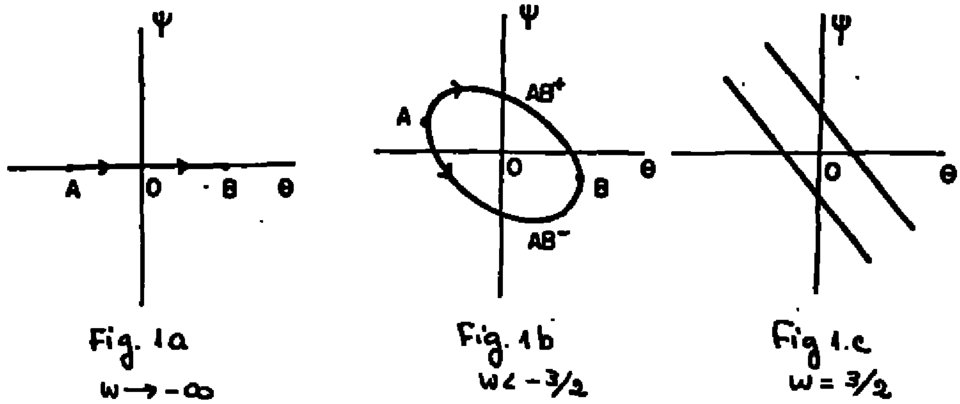

Fig. 1a $\omega \to -\infty$ Fig. 1b $\omega < -3/2$ Fig. 1.c $\omega = 3/2$

(*) No presente trabalho estamos considerando $\Lambda > 0$. Para o caso $\Lambda < 0$ ver ref.11.

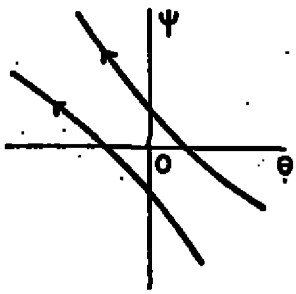

Fig. 1d
$-3/2 < w \leq -4/3$

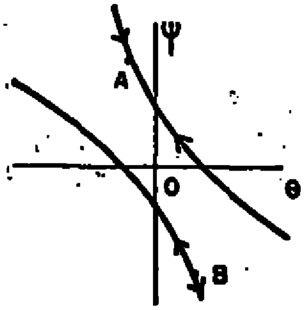

Fig. 1e
$-4/3 < w < -1$

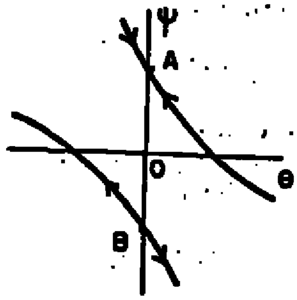

Fig. 1f
$w = -1$

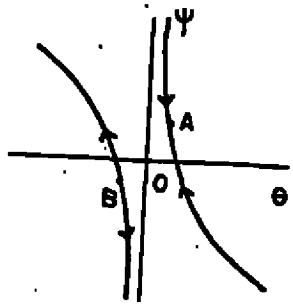

Fig. 1g
$w = 0$

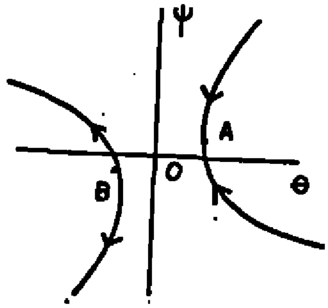

Fig. 1h
$w > 0$

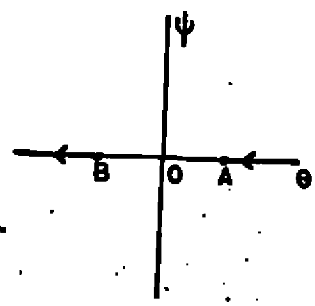

Fig. 1i
$w \rightarrow +\infty$

Comentemos brevemente alguns desses diagramas. Em primeiro lugar, com exceção do caso em que $-3/2 \leq w \leq -4/3$, verificamos em todos os diagramas a presença dos *pontos de equilíbrio* A e B, os quais correspondema a soluções $(\theta,\psi)=(\theta_o,\psi_o)=$ const., descrevendo, portanto, modelos cosmológicos do tipo *de Sitter*. Esses pontos de equilíbrio realizam uma rotação no plano de fase $\theta\psi$ à medida que w varia no intervalo $(-\infty,+\infty)$ (ver figuras 1a-1i). Quando $w=-1$, vemos que A e B representam duas soluções estáticas (θ=0), configuração que corresponde a uma geometria de Minkowski, porém com a constante gravitacional G variando no tempo (crescendo num caso e decrescendo no outro). Nestes modelos constatamos que a dinâmica de G é determinada unicamente pela presença da constante cosmológica Λ uma vez que não existem campos de matéria e devido ao fato de a geometria ser estática. Estas considerações nos levam naturalmente a indagar a respeito da existência de uma *relação cósmica* entre G e Λ, idéia que já foi levantada em contextos diferentes (ver refs. 12 e 13, por exemplo).

A conjectura formulada por Dirac de que G deveria decrescer em nosso Universo à medida que este expandisse pode ser encontrada nos diagramas de fase como uma propriedade exibida por algumas soluções desde que $w \geq 0$. Estas soluções são representadas pelo ponto A e pelas curvas que tendem a A com $\psi > 0$.

Com relação à existência de singularidades, uma simples inspeção dos diagramas nos mostra que não existem soluções singulares para $w < -3/2$. Por outro lado, quando $w > -4/3$ as únicas soluções não-singulares são as representadas pelos pontos de equilíbrio A e B.

Finalmente, no limite em que $w \rightarrow +\infty$ (ver diagrama) obtemos quatro soluções: os pontos de equilíbrio A e B, e, também, as duas curvas que tendem a estes pontos. A e B correspondem exatamente ao modelo de de Sitter p/ o vazio na Relatividade Geral com $G = 1/\phi$ = constante. As outras duas soluções, todavia, não satisfazem as equações de Einstein p/ o vazio com constante cosmológica, correspondendo, na verdade, a configurações da Relatividade Geral geradas por uma distribuição de matéria equivalente a um fluído perfeito com equação de estado $p = \rho$.

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ON GRAVITATIONAL WAVES, VORTICES AND SIGMA-MODELS

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We find that the existence of either vortices or cosmic strings solutions is not affected by the presence of gravitational plane fronted waves and that curvature singularities appear due to the interaction between the wave and either the string or the vortex.

The metric associated to a finite number of parallel cosmic strings and its generalization for a continuum of parallel cosmic strings was found by the author without making reference to its field theory origin.[1] Since cosmic strings are produced by symmetry breaking in early stages of the evolution of the Universe a consistent way to define cosmic strings is to consider the Einstein equations coupled to the Yang-Mills-Higgs field equations.[2] A solution to the previous equations that can be interpreted as a finite number of parallel vortex lines or a finite number of parallel cosmic strings was considered by Linet.[3] A similar solution was studied by Comtet and Gibbons[4] together with solutions to the Einstein equations coupled with σ-model type of field theories. The existence of the above mentioned solutions, as well as the multiple vortex solutions, relays on the fact that in a particular curve spacetime, albeit sufficiently general to contain the cosmic strings, the Bogomol'nyi equations[5] obtained in the Bogomol'nyi limit are essentially the same equations that in Minkowski spacetime[3,4].

The purpose of this communication is to study the Einstein equations coupled with either an Abelian gauge field interacting with a charged scalar field in presence of the usual symmetry breaking potential or a nonlinear σ-model type of field equations for the metric

$$ds^2 = Hdu^2 + 2dudv + 2Adudx + 2Bdudy - e^{-4V}(dx^2 + dy^2), \qquad (1)$$

where H, A and B, are functions of u, x, and y; V is a function of x, and y only. In particular, we shall be interested in the solutions that can be interpreted as cosmic strings. In Refs. 3 and 4 the existence of cosmic strings solutions were studied for the special case of a spacetime (1) with H=A=B=0. We shall consider the case in which the functions A, B, and H are restricted by

$$A_{,y} - B_{,x} = 0, \qquad A_{,x} + B_{,y} = 0, \qquad H_{,xx} + H_{,yy} = 0. \qquad (2)$$

When V =0 the metric (1) with the restrictions (2) represents a plane fronted wave[6] with a constant wave vector k^{μ}. The metric (1) is a particular case of the general metric that admits a null vector with zero covariant derivative.[7]

The Einstein tensor for the metric (1) with the restrictions (2) can be cast as

$$G_{\mu\nu} = -2e^{4V}(V_{,xx} + V_{,yy})(l_{\mu}k_{\nu} + l_{\nu}k_{\mu}), \qquad (3)$$

where $k_\mu=(H/2)\delta^u_\mu+\delta^v_\mu+A\delta^x_\mu+B\delta^y_\mu$, $l_\mu=\delta^u_\mu$, $m_\mu=e^{-2V}\delta^x_\mu$, and $n_\mu=e^{-2V}\delta^y_\mu$ is and orthonormal vierbein.

The Lagrangean for the U(1) gauge field that we shall consider is the the covariant generalization of the Ginzgurg–Landau model,

$$L=-(1/4)F^{\mu\nu}F_{\mu\nu}+(1/2)(\partial_\mu\phi-ieA_\mu\phi)(\partial^\mu\phi^*+ieA^\mu\phi^*)-\lambda(|\phi|^2-\eta^2)^2, \quad (4)$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$; e, λ, and η are three coupling constants. Assuming that $A_\mu=(0,0,A_i)$, and that $A_i\equiv(A_x,A_y)$, and $\phi\equiv\varphi_1+i\varphi_2$ are functions of $x^i=(x,y)$ only, we get

$$L=-(1/4)\gamma^{in}\gamma^{jm}F_{ij}F_{mn}-(1/2)\gamma^{ij}D_i\varphi_a D_j\varphi_a-\lambda(|\phi|^2-\eta^2)^2, \quad (5)$$

where $\gamma^{ij}=e^{4V}\delta^{ij}$, and $D_i\varphi_a\equiv\partial_i\varphi_a-e\varepsilon_{ab}A_i\varphi_b$, with $\varepsilon_{12}=-\varepsilon_{21}=1$, and $\varepsilon_{11}=\varepsilon_{22}=0$.

When the coupling constant are related by $e^2=8\lambda$ and the fields by

$$F_{ij}=e\,\eta_{ij}(|\phi|^2-\eta^2)/2,\quad D_j\varphi_a=\eta_{jk}\varepsilon_{ab}\gamma^{kl}D_l\varphi_b, \quad (6)$$

where $\eta_{jk}=e^{-4V}\varepsilon_{ij}$, the Lagrangean (5) is a total divergence and in consequence the solutions of the first order equations (6) (Bogomol'nyi equations) are solutions of the second order Euler–Lagrange field equations derived from (5). Furthermore, by direct substitution one can verify that $T_{ij}=0$. In this case we can cast the EMT associated to (5) as[8]

$$T_{\mu\nu}=-L(l_\mu k_\nu+l_\nu k_\mu), \quad (7)$$

with $-L\ . =(1/4)e^{4V}\delta^{ij}\partial_i\partial_j(-|\phi|^2+\eta^2 ln|\phi|^2)$. Defining the orthonormal vectors $\sqrt{2}t^\mu=k^\mu+l^\mu$, $\sqrt{2}z^\mu=k^\mu-l^\mu$, .we can put (7) in the form of the EMT that represents a cloud of strings,

$$T^{\mu\nu} = \rho(t^{\mu}t^{\nu}-z^{\mu}z^{\nu}), \tag{8}$$

with $\rho=-L$. When $-L>0$, ρ represents the density of the cloud. For multiple vortex solutions ρ is a distribution with support on straight lines. From (3), and (7) we have that the Einstein equations reduce to the Laplace equation and can be explicitly integrated. Moreover, one can show for the field equations (6) the existence of solutions for the boundary conditions that define one or several vortices[3,4,9].

Now we shall consider a σ-model with target metric on a Kahler manifold. Let $\phi^{A}(x^{i})$ a map from S into a 2n-dimensional Kahler manifold M with metric $G_{AB}(\phi)$ and complex structure $J^{A}_{B}(\phi)$, A=1,2,...,n,. The Lagrangean for this 2-dimensional model is

$$L=-(1/2)\mu^{2}G_{AB}\ \partial_{i}\phi^{A}\ \partial_{j}\phi^{B}\ \gamma^{ij}. \tag{9}$$

The quantity μ is another coupling constant. When the fields are related b $\partial_{i}\phi^{A} = J^{A}_{C}\ \eta^{k}_{i}\ \partial_{k}\phi^{C}$, the Lagrangean is a topological invariant and in consequence the Euler-Lagrange equations associated to (9) are identically satisfied. Again, one can show by direct substit u tion[8] that $T_{ij}=0$. Thus, when the field are holomorphic the EMT for the σ-model (9) can be cast as (7), i.e., as a cloud of cosmic strings.

Since in the interaction of cosmic strings with plane fronted gravitational waves the spacetime can develop nontrivial curvature singularities[10] we have that in the cosmic string limit the vortex and the holomorfic σ-model solutions will present the same singular behavior. In other words we have proved that the singularities studied in Ref.10 have a physical origin.

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A Constante Cosmológica na Cosmologia de Membranas

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Abstract

O problema da constante cosmológica é examinado em uma cosmologia de volume mínimo. Nesta cosmologia a constante cosmológica é o quadrado da curvatura extrínseca do espaço-tempo.

O aparecimento da constante cosmológica em relatividade geral é uma conseqência da geometria riemanniana adotada para a descrição do espaço-tempo e da estrutura das equações de Einstein. De fato, o tensor mais geral, construido com a métrica e suas derivadas até ordem 2 e satisfazendo a condição $G^{ij}{}_{;j} = 0$ é o tensor de Einstein com a constante Λ (já que $g_{ij;k} = 0$):

$$G_{ij} = R_{ij} + \frac{1}{2}Rg_{ij} + \Lambda g_{ij}. \tag{1}$$

Isto resulta da integral de ação

$$A = \int (R + \Lambda)\sqrt{-g}dv,$$

onde o termo em Λ é dinâmico e assume o papel da densidade de energia do vácuo. As estimativas atuais da astrofísica sugerem o valor $\Lambda \approx 10^{-47}\mathrm{GeV}^4$ e consequentemente o termo em Λ pode ser desprezado em considerações clássicas. Por outro lado, se levarmos em conta a teoria quântica de campos em espaços curvos, este termo de vácuo com uma constante de proporcionalidade Λ_0, sofre uma correção $\delta\Lambda$ resultante das flutuações quânticas do campo, resultando em um valor efetivo para a constante cosmológica

$$\Lambda_{eff} = \Lambda_0 + \delta\Lambda$$

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Tomando o exemplo do campo escalar obtem-se [1] $\delta\Lambda \approx 10^{74}GeV^4$. Assim, deve-se proceder uma regularização do campo de modo que Λ_0 compense o $\delta\Lambda$ para comparar Λ_{eff} com o valor observado. Como isto deverá ser repetido a cada interação e como o universo se expande continuamente, o processo de regularização nunca cessa, persistindo mesmo nos dias atuais. Assim, o problema da constante cosmológica pode ser resumido como sendo um problema de "sintonia fina" em teoria de campo, relativamente ao valor observado de Λ [2]. A mera substituição de Λ por uma função escalar como sugerido em [3] e outros, não resolveria o problema pois de qualquer forma teriamos uma constante Λ em (1), a menos que se altere a geometria de modo que $g_{ij;k} \neq 0$.

Nesta nota, exploramos a possibilidade de que Λ possa ser interpretada como um campo escalar de natureza geométrica (já que a mesma está no lado esquerdo das equações de Einstein), sem contudo modificar a geometria riemanniana. Para implementar isto, considere o espaço-tempo como uma hipersuperficie de um espaço plano D-dimensional M_D. As coordenadas de imersão X^μ satisfazem as equações[1]

$$g_{ij} = X^\mu_{,i} X^\nu_{,j} \eta_{\mu\nu}, \quad N^\mu_A X^\nu_{,i} \eta_{\mu\nu} = 0, \quad g_{AB} = N^\mu_A N^\nu_B \eta_{\mu\nu}. \tag{2}$$

onde N_A são vetores ortogonais ao espaço-tempo e $g_{AB} = \pm 1$. As condições de integrabilidade de (2) são as equações de Gauss-Codazzi-Ricci para subvariedades. Para as nossas considerações é suficiente tomar a equação de Gauss

$$R_{ijkl} = 2g^{AB} K_{i[kA} K_{l]jB} \tag{3}$$

que relaciona a curvatura riemanniana com os coeficientes da segunda forma fundamental K_{ijA}, definidos por

$$K_{ijA} = X^\mu_{,ij} N^\nu_A \eta_{\mu\nu}. \tag{4}$$

Por contração tensorial em (3) obtemos

$$R_{ij} - \frac{1}{2}Rg_{ij} = K_{imA}K_j^{mA} - H_A K_{ij}^A - \frac{1}{2}(K^2 - H^2)g_{ij} \tag{5}$$

onde denotamos as cuvaturas extriseca e média respectivamente por

$$K^2 = g^{AB}K_{miA}K^{mi}_B = K_{miA}K^{miB} \text{ e } H^2 = g^{AB}H_A H_B, \quad H_A = g^{ij}K_{ijA}.$$

Assumindo agora que g_{ij} satisfaz as equações de Einstein para um dado tensor de energia-momento T_{ij}

$$R_{ij} - \frac{1}{2}g_{ij}R = T_{ij} - \Lambda g_{ij} \tag{6}$$

[1] Os índices latinos pequenos variam de 1 à 4 e os índices latínos maiusculos variam de 5 à D. Todos os índice gregos variam de 1 à D.

e comparando esta equação com com (5), resulta

$$K_{imA}K_j^{mA} - H_A K_{ij}^A - \frac{1}{2}(K^2 - H^2)g_{ij} = T_{ij} - \Lambda g_{ij}. \tag{7}$$

Estas equações dizem que a segunda forma fundamental K_{ijA} deve ajustar-se com a fonte dada e a constante cosmológica. Note que (7) é uma equação algébrica em K_{ijA}, cujo traço é

$$(K^2 - H^2) = 4\Lambda - T \tag{8}$$

Para determinar a geometria associada à constante cosmológica, considere o caso do vácuo $T_{ij} = 0$,, obtendo de (8)

$$K_{imA}K_j^{mA} - \frac{1}{2}K^2 g_{ij} = -\Lambda g_{ij} \quad \text{e} \quad K^2 - H^2 = 4\Lambda \tag{9}$$

Vemos que Λ esta associado á curvatura extrinsec K^2 e a curvatura média H^2, as quais devem se ajustar de forma a compensar o pequeno valor observado de Λ.

Um caso particularmente interessante é aquele em que o espaço-tempo possui volume mínimo[4], caracterizado por $H = 0$, de forma que o espaço-tempo comporta-se como uma membrana de 4 dimensões imersa em M_D. Neste caso e obtemos de (9) $\Lambda = K^2/4$, permitindo descrever a constante cosmológica exclusivamente em termos da curvatura extrinseca K^2. Como esta curvaturas extrinseca não é acessivel ao observador riemanniano classico, interpretamos este resultado afirmando que a adoção da geometria riemanniana é correta ao nivel clássico da teoria. Por outro lado em teoria quântica, Λ comporta-se como um campo escalar devidamente inserido na geometria, o qual deve ser sintonizado.

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TOPOLOGICAL EFFECTS DUE TO A COSMIC STRING *

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Topological defects of spacetime can be characterized by a spacetime metric with null Riemann-Christoffel curvature tensor everywhere except on the defects, that is, by conic type of curvature singularities. Recent attempts to marry the grand unified theories of particle physics with general relativistic models of the early evolution of the universe have predicted the existence of such topological defects. One example of these topological defects are the cosmic strings[1] which appear naturally in gauge theories with spontaneous symmetry breaking.

Cosmic strings are expected to be created during the phase transitions. Some may still exist and may even be observable; others may have collapsed long ago, yet have served as the seeds of the galaxies[1,2].

The line element of the spacetime described by an infinite, straight and static cylindrically symmetric cosmic string[2], lying along the z-axis, is given by[3]

$$ds^2 = dt^2 - d\rho^2 - \alpha^2\rho^2 d\varphi^2 - dz^2 \tag{1}$$

in a cylindrical coordinate system (t,ρ,φ,z) with $\rho \geq 0$ and $0 \leq \varphi \leq 2\pi$, the hypersurface $\varphi = 0$ and $\varphi = 2\pi$ being identified. The parameter α is related to the linear mass density μ of the string by $\alpha = 1 - 4\mu$. This metric describes the spacetime which is locally flat (for $\rho \neq 0$) but has conelike singularity at $\rho = 0$ with the angle deficit $8\pi\mu$. Then, the spacetime around an infinite straight and static cosmic string is locally flat but of course not globally flat, it does not differ from Minkowski spacetime locally, it does differ globally. There is no Newtonian gravitational potential around the string, however we have some very interesting gravitational effects associated with the non-trivial topology of the space-like sections around the cosmic string. Among these effects, a cosmic string can acts as a gravitational lens[2] and can induces a repulsive force on an electric charge at rest[4]. Others effects include pair production by a high energy photon when it is placed in the spacetime around a cosmic string[5] and a gravitational analogue[6] of the electromagnetic Aharonov-Bohm effect[7].

In this paper we study some effects of the global features of the spacetime of a straight cosmic string on quantum particles. To do this we use the Klein-Gordon and Dirac equations in covariant forms.

Let us consider a scalar quantum particle imbedded in a classical background gravitational field. Its behavior is described by the covariant Klein-Gordon equation

$$[\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\, g^{\mu\nu}\partial_\nu) + m^2]\psi = 0 \tag{2}$$

where m is the mass of the particle and $\hbar = c = 1$ units are chosen.

The spacetime corresponding to a cosmic string is time independent, so the time dependence of the wave function that solves Eq(2) may be separated as e^{-iEt} and one is led to a stationary problem at fixed energy E. Moreover, rotational invariance and invariance along the z axis of the metric allow us to separate the φ and z dependences. In view of these we choose the solutions of Eq(2), $\psi(t,\rho,\varphi,z)$ in the form

$$\psi(t,\rho,\varphi,z) = exp(-iEt + i\ell\varphi + ikz)R(\rho) \tag{3}$$

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where E, ℓ and k are constants.

In the spacetime corresponding to a cosmic string, the Klein-Gordon equation [Eq (2)] takes the form

$$\left\{\rho\partial_\rho(\rho\partial_\rho)+\left[E^2-(k^2+m^2)\right]\rho^2-\frac{\ell^2}{\alpha^2}\right\}R(\rho)=0 \tag{4}$$

where we have used the ansatz given by Eq(3).

Equation (4) is a Bessel differential equation with the general solution given by

$$R_{\nu k}(\rho)=C^{(1)}_{\nu k}J_{|\nu|}(\lambda\rho)+C^{(2)}_{\nu k}N_{|\nu|}(\lambda\rho) \tag{5}$$

where $\lambda^2=E^2-(k^2+m^2)$, $\nu=\ell/\alpha$, $C^{(1)}_{\nu k}$ and $C^{(2)}_{\nu k}$ are normalization constants, and $J_{|\nu|}(\lambda\rho)$ and $N_{|\nu|}(\lambda\rho)$ are Bessel functions of the first and second kind, respectively.

We assume that the scalar quantum particle is restricted to move in a region bounded by the cylindrical surfaces $\rho=a$ and $\rho=b$, where $b>a$. The boundary conditions

$$R(a)=R(b)=0 \tag{6}$$

determine the energy levels of the particle. This condition yields the following equation for the energy spectrum of the particle

$$J_{|\nu|}(\lambda a)N_{|\nu|}(\lambda b)-J_{|\nu|}(\lambda b)N_{|\nu|}(\lambda a)=0 \tag{7}$$

In order to obtain the spectrum explicity we will consider a situation in which $\lambda a >> 1$ and $\lambda b >> 1$. Then using Hankel's asymptotic expansion when ν is fixed, we get

$$E=\sqrt{m^2+k^2+\frac{\ell^2}{\alpha^2 a^2}} \tag{8}$$

From Eq.(8) we see that the energy spectrum depends on the factor α (as well as the wave function) relative to the Minkowski case. But the spacetime is locally flat; the Riemann curvature tensor vanishes everywhere outside the string. So, the fact that this spacetime is locally flat but not globally (it is conical with deficit angle $2\pi\alpha$) deforms the energy spectrum respect to α.

Now let us consider the Dirac equation in a curved spacetime, which is taken to be

$$\left[\,i\gamma^\mu(x)\frac{\partial}{\partial x^\mu}-i\gamma^\mu(x)\,\Gamma_\mu(x)\,\right]\psi(x)=m\psi(x) \tag{9}$$

where $\gamma^\mu(x)$ are the generalized Dirac matrices and are given in terms of the standard flat spacetime gammas $\gamma^{(a)}$ by the relation

$$\gamma^\mu(x)=e^\mu_{(a)}(x)\,\gamma^{(a)} \tag{10}$$

where $e^\mu_{(a)}(x)$ are vierbeins defined by the relations

$$e^\mu_{(a)}e^\nu_{(b)}\,\eta^{(a)(b)}=g^{\mu\nu} \tag{11}$$

The product $\gamma^\mu\Gamma_\mu$ that appears in the Dirac equation can be written as[9]

$$\gamma^\mu\Gamma_\mu=\gamma^{(a)}\left(A_{(a)}(x^\mu)+i\gamma^5 B_{(a)}(x^\mu)\right) \tag{12}$$

where $\gamma^5=i\gamma^{(0)}\gamma^{(1)}\gamma^{(2)}\gamma^{(3)}$ and $A_{(a)}$ and $B_{(a)}$ are given by

$$A_{(a)} = \frac{1}{2}\left(e^{\mu}_{(a),\mu} + e^{\rho}_{(a)}\Gamma^{\mu}_{\rho\mu}\right) \qquad (13)$$

$$B_{(a)} = \frac{1}{2}\epsilon_{(a)(b)(c)(d)}\, e^{(b)\mu} e^{(c)\nu} e^{(d)}_{\nu,\mu} \qquad (14)$$

where $\epsilon_{(a)(b)(c)(d)}$ is the completely antisymmetric, fourth-order unit tensor, and the comma denotes $\partial/\partial x^{\mu}$.

For the metric corresponding to a cosmic string we shall use the following set of vierbeins:

$$\begin{aligned} e^{\mu}_{(0)} &= \delta^{\mu}_{0},\quad e^{\mu}_{(1)} = \cos\varphi\, \delta^{\mu}_{(1)} - \frac{1}{\alpha\rho}\sin\varphi\, \delta^{\mu}_{2} \\ e^{\mu}_{(2)} &= \sin\varphi\, \delta^{\mu}_{(1)} + \frac{1}{\alpha\rho}\cos\varphi\, \delta^{\mu}_{2},\quad e^{\mu}_{(3)} = \delta^{\mu}_{3} \end{aligned} \qquad (15)$$

which yield the properflat spacetime limit ($\alpha = 1$). Using Eqs.(12)(13) and (14) and the above set of vierbeins we get,

$$\Gamma_0 = \Gamma_1 = \Gamma_3 = 0 \qquad \text{and} \qquad \Gamma_2 = \frac{i}{2}(1-\alpha)\gamma^{(1)}\gamma^{(2)} \qquad (16)$$

Choosing the ansatz

$$\psi = \begin{pmatrix} \sqrt{E+m} & u_1(\rho) \\ i\sqrt{E-m} & u_2(\rho)e^{i\varphi} \end{pmatrix} exp\,(-iEt + i\ell\varphi) \qquad (17)$$

The Dirac equations become

$$\begin{pmatrix} (E-m) & i\left[\left(\partial_\rho + \frac{1}{2\rho}\right) + \frac{1}{\alpha\rho}\left(\ell + \frac{1}{2}\right)\right] \\ i\left[-\left(\partial_\rho + \frac{1}{2\rho}\right) + \frac{1}{\alpha\rho}\left(\ell + \frac{1}{2}\right)\right] & -(E+m) \end{pmatrix} \cdot \begin{pmatrix} \sqrt{E+m} & u_1 \\ i\sqrt{E-m} & u_2 \end{pmatrix} = 0 \qquad (18)$$

The general solutions of the above equations are given by

$$u_i(\rho) = C^{(1)}_{i\ell}\, J_{|\nu+(i-1)|}(\lambda\rho) + C^{(2)}_{i\ell}\, N_{|\nu+(i-1)|}(\lambda\rho) \qquad (19)$$

where $i = 1,2$, $\lambda^2 = E^2 - m^2$ and $\nu = \frac{\ell+\frac{1}{2}}{\alpha} - \frac{1}{2}$, $C^{(1)}_{i\ell}$ and $C^{(2)}_{i\ell}$ are normalization constants and $J_{|\nu+(i-1)|}(\lambda\rho)$ and $N_{|\nu+(i-1)|}(\lambda\rho)$ are Bessel functions of the first and second kind, respectively.

Now, let us compute the current. If ψ is a massive field, j^{μ} can be written as

$$\begin{aligned} j^{\mu} = {} & \frac{1}{2m}\left(\bar\psi\sigma^{\mu\lambda}\psi\right)_{;\lambda} + \frac{i}{4m}\, g^{\mu\lambda}\bar\psi\overleftrightarrow{\partial}_{\lambda}\psi + \frac{i}{4m}\,\bar\psi\left(\left[\gamma^{\lambda}_{;\lambda},\gamma^{\mu}\right] + \right. \\ & \left.\left[\gamma^{\lambda},\gamma^{\mu}_{;\lambda}\right]\right)\psi + \frac{i}{2m}\bar\psi\left[\gamma^{\lambda}\Gamma_{\lambda},\gamma^{\mu}\right]\psi \end{aligned} \qquad (20)$$

or, writing in components, in this case, we have

$$
\begin{aligned}
j^0 &= \nabla \cdot \mathbf{P} + \rho_{\text{convective}} \\
j_{(\rho)} &= -\partial_t P_{(\rho)} + (\nabla \times \mathbf{M})_{(\rho)} + j_{(\rho),\text{convective}} \\
j_{(\varphi)} &= -\partial_t P_{(\varphi)} + (\nabla \times \mathbf{M})_{(\varphi)} + j_{(\varphi),\text{convective}} + \frac{1}{\rho}\left(\frac{1-\alpha}{\alpha}\right) M_z
\end{aligned}
\tag{21}
$$

and

$$
j_{(z)} = -\partial_t P_{(z)} + (\nabla \times \mathbf{M})_{(z)} + j_{(z),\text{convective}}
$$

where the convective parts are derived from $\frac{1}{4m} g^{\mu\lambda} \bar{\psi} \overleftrightarrow{\partial}_\lambda \psi$, the polarization densities are given by

$$
\begin{aligned}
P_{(\rho)} &= \frac{i}{2m} \bar{\psi} \gamma_{(0)} \gamma_{(\rho)} \psi \\
P_{(\varphi)} &= \frac{i}{2m} \bar{\psi} \gamma_{(0)} \gamma_{(\varphi)} \psi \\
\text{and} \\
P_{(z)} &= \frac{i}{2m} \bar{\psi} \gamma_{(0)} \gamma_{(3)} \psi
\end{aligned}
\tag{22}
$$

and the components of $\mathbf{M}$ are given by

$$
\begin{aligned}
M_{(\rho)} &= \frac{i}{4m} \bar{\psi} \left[\gamma_{(\varphi)}, \gamma_{(3)}\right] \psi \\
M_{(\varphi)} &= \frac{i}{4m} \bar{\psi} \left[\gamma_{(3)}, \gamma_{(\rho)}\right] \psi \\
\text{and} \\
M_{(z)} &= \frac{i}{4m} \bar{\psi} \left[\gamma_{(1)}, \gamma_{(2)}\right] \psi
\end{aligned}
\tag{23}
$$

The vector $\mathbf{M}$ has the meaning of a magnetization current density if we regard to an external electromagnetic field.

Note the dependence of j^μ, thought the component $j_{(\varphi)}$, on the parameter α. Then, the current differs from the Minkowski spacetime case by a term containing a dependence on α. So, the fact that the spacetime corresponding to a cosmic string is locally flat but not globally is also coded into the probability current. There is a physical effect on the current relative to Minkowski spacetime which comes out from the topological features of the spacetime surrounding a cosmic string.

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SOME COSMOLOGICAL CONSEQUENCES OF A Λ-TERM VARYING AS $\beta H^2+\alpha R^{-n}$(β, α and n constants)

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ABSTRACT - A phenomenological decay law for the cosmological Λ-term is proposed and its influence on the standard universe model is examined. As a general feature, singular and nonsingular solutions are present and the age universe problem can be solved. It is also shown that kinematic expressions such as the luminosity distance and angular diameter versus red-shift relation are significantly modified.

1. INTRODUCTION

In the framework of the quantum field theories, the cosmological Λ-term present in Einstein's equations can be interpreted as the vacuum energy density. On the other hand, the cosmological estimatives of such a term ($\Lambda/8\pi G \le 10^{-47}GeV^4$) is smaller than the limits derived from gauge theories by at least forty orders of magnitude. Such a puzzle is the essence of the so-called cosmological constant problem[1].

Some physical mechanisms have been proposed to explain the current small value of the cosmological constant. Recently, several authors have argued that the vacuum energy density, coupled with the other fields, is a time dependent quantity[2-6]. In this way the Λ-term is small today because the universe evolves. From a phenomenological point of view, the problem reduces to determine the dependence of Λ on the scale factor R and its first derivatives, taking into account the cosmological data.

In this article we examine some consequences of an effective Λ-term varying as

$$\Lambda = 3\beta H^2 + 3\alpha R^{-n}, \qquad (1)$$

where α, β and n are constants, R is the universal scale function, $H=\dot{R}/R$ is the Hubble parameter with the factor 3 being introduced by mathematical convenience.

2. THE MODELS

We start writing the Einstein's equations for the FRW line element with a comoving perfect fluid plus a Λ-term as source of curvature (a dot means time derivative)

$$8\pi G\rho + \Lambda = 3\,\frac{\dot{R}^2}{R^2} + 3\,\frac{k}{R^2}\,, \tag{2}$$

$$8\pi G p - \Lambda = -\,2\,\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2}\,. \tag{3}$$

By considering the "γ-law" equation of state $p=(\gamma-1)\rho$, and the Λ-term defined in eq.(1), one obtains the following differential equation for the scale factor

$$R\ddot{R} + \Delta_1\dot{R}^2 + \Delta k - \frac{3\alpha\gamma R^{-n}}{2} = 0, \tag{4}$$

the first integral of which is given by

$$\dot{R}^2 = AR^{-2\Delta_1} + \frac{3\alpha\gamma R^{-n}}{2\Delta_1+2-n} - \frac{\Delta k}{\Delta_1}\,; \quad (\Delta_1 \neq 0, \frac{n-2}{2}), \tag{5}$$

where

$$\Delta_1 = \frac{3\gamma(1-\beta)-2}{2}, \quad \Delta = \frac{3\gamma-2}{2} \text{ and } A \text{ is a } \gamma\text{-dependent constant.}$$

From eqs. (1)-(3) one obtains for ρ and ρ_v, the matter and vacuum energy densities, the following expressions:

$$\frac{8\pi G\rho}{3} = (1-\beta)AR^{-2\Delta_1-2} + \frac{3\gamma(1-\beta)+n-2\Delta_1-2}{2\Delta_1+2-n}\,\frac{\alpha}{R^n} + \frac{\Delta_1-(1-\beta)\Delta}{\Delta_1}\,\frac{k}{R^2}\,, \tag{6}$$

$$\frac{8\pi G\rho_v}{3} = \beta AR^{-2\Delta_1-2} + \frac{3\beta\gamma+2\Delta_1+2-n}{2\Delta_1+2-n}\,\frac{\alpha}{R^n} - \frac{\beta\Delta k}{\Delta_1 R^2}\,. \tag{7}$$

For $\alpha=\beta=0$, the dynamic equation and the energy density of the standard FRW models are recovered[7]. Universes with Λ constant can also be described putting $\beta=n=0$. Further, recent models with variable Λ are simple particularizations of eqs. (4)-(7), namely: Ozer and Taha[2]($\beta=0$, $\alpha=k=1$ and $n=2$), Freese et al[3] ($\alpha=k=0$, $\beta=\rho_v/\rho+\rho_v$), Gasperini[4]($\beta=0$, $9/5<n<2$), Chen and Wu[5] ($\beta=0$, $n=2$), Carvalho et al[6] ($n=2$). It is easy to see that singular and nonsingular solutions are present in our equations. If we put $A<0$ the singularity can be avoided for generic choices of the constants α, β and n. This happens, for instance, in the Ozer and Taha model. Such solutions are, in fact, compatible with the weak and dominant energy conditions. Conversely, taking $A>0$ singular solutions are obtained as in the Chen and Wu model. It is worth mentioning that the several phenomenological laws analysed by the mentioned authors are grounded in different arguments which will not be critically discussed here. Formally, the behaviour assumed in eq.(1) is the simplest generalization of the above considered particular cases.

3. SOME PHYSICAL RESULTS

(i) Universe Age

Defining the present time quantities $q_0 = -R\ddot{R}/\dot{R}^2|_{t=t0}$ and $H_0=\dot{R}/R|_{t=t0}$ one obtains from eqs.(4)-(5) the following expression for the universe age

$$t_0 = H_0^{-1}\int_0^1 \frac{dx}{f(x)}\ , \tag{8}$$

where $x=R/R_0$ and the function $f(x)$ is defined by

$$f(x) = 1 - \frac{2q_0}{1-3\beta} + \frac{2q_0}{(1-3\beta)x^{1-3\beta}} + \\ + \left(1 - \frac{2q_0}{1-3\beta}\right)\left[\frac{(n-2)(1-x^{3\beta-1}) - (1-3\beta)(1-x^{2-n})}{3-3\beta-n}\right] \tag{9}$$

In general, the above integral cannot be exactly solved in terms of elementary functions. However, some interesting particular cases emerge from eqs. (8) and (9). If $n=2$, the expression derived in ref. (7) is recovered. Moreover, if $n-2=1-3\beta$ then, $t_0=2H_0^{-1}/(3-3\beta)$ showing that ages greater than H_0^{-1} can be obtained from eq.(8). The same result holds for $n=2$ and $k=0$ (see ref. (7)).

(ii) Matter Creation

For models with variable Λ the energy conservation law ($T^{\mu\nu}{}_{;\nu} = 0$) takes the following form

$$\dot{\rho} + 3\,\frac{\dot{R}}{R}\,(\rho + p) = -\frac{\dot{\Lambda}}{8\pi G}\ . \tag{10}$$

Thus, if $\dot{\Lambda}<0$ energy is transfered from decaying vacuum to the material component. In the present matter dominated phase, the matter creation rate can be written as

$$\frac{1}{R_0^3}\,\frac{d}{dt}(\rho_0 R_0^3) = 3\rho_0 H_0\left[\frac{n}{3}\left(\frac{1-\Omega_0}{\Omega_0}\right) + \beta\frac{(\Omega_0 - \frac{n}{2})}{\Omega_0}\right], \tag{11}$$

where $\Omega_0=\rho_0/\rho_{cr}$ is the present value of the density parameter. For $n=2$, this expression reduces to the case studied by Carvalho et al[6]. The factor $3\rho_0 H_0$ is exactly the creation rate of the steady state universe[8].

(iii) Luminosity Distance (d_L) and Angular Diameter Distance (d_A)

The kinematical relation distances must be confronted with the observational data in order to put limits on the free parameters of the model. Using the canonical procedure to compute the luminosity and diameter angular distances[9], analytical expressions are obtained in the following cases:

a) k=0 and n-2=1-3β

$$d_L = \frac{2H_0^{-1}}{1-3\beta}(1+z)\left(1-(1+z)^{-\frac{1-3\beta}{2}}\right) \quad (12)$$

b) n=2, k=0,±1

$$d_L = \frac{R_0(1+z)}{\sqrt{k}} \sin\left[\frac{2\sqrt{k}}{1-3\beta}\left(\frac{2q_0}{1-3\beta}-2\right)^{-1/2}(\sin^{-1}\alpha_1 - \sin^{-1}\alpha_2)\right], \quad (13)$$

where

$$\alpha_1 = \left[\frac{1}{(1+z)^{1-3\beta}}\left(1-\frac{1-3\beta}{2q_0}\right)\right]^{1/2}, \quad \alpha_2 = \left(1-\frac{1-3\beta}{2q_0}\right)^{1/2}. \quad (14)$$

For both cases $d_A = d_L(1+z)^{-2}$, so that the distance relations are modified by the presence of the β parameter. As one should expect, if $\beta \to 0$ the results of the FRW universes are recovered.

4. CONCLUSION

We investigate some physical consequences of a decaying vacuum energy density. It was implicitly assumed that the vacumm couples only with the dominant component in each phase. Note also from eq. (5) that the recollapse conditions are strongly modified. In fact, models with k>0 may expand forever regardless the value of the parameters β and n. Alternatively, universes with $k \leq 0$ may recollapse in a finite time interval. Finally, we call atention that the Landau-Lifshits fluctuation theory was applied by Pavón[9] to study the physical consistency of the several phenomenological laws for the Λ-term. Such a paper was recently generalized in the spirit of the present article by Salim and Waga[10].

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New Baryonic Force for the Universe

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It has been established, beyond any doubt, that the Universe is undergoing an expansion. Recent data of several investigators show that galaxies form gigantic structures in space. De Lapparent et al.[1](also, other papers by the same authors) have shown that they form bubbles which contain huge voids of many megaparsecs of diameter. Broadhurst et al.[2] probed deeper regions of the universe and showed that there are (bubble) walls up to a distance of about 2.5 billion light-years from our galaxy. Even more disturbing is the apparent regularity of the walls with a period of about $130h^{-1}$Mpc. Recent data[3] show, however, that the bubble walls are not so regularly spaced and, therefore, the medium formed by them is rather a liquid than a solid. We may call this medium the 'galactic liquid'.

At the other end of the distance scale, in the fermi region, it appears now, that the quark is not elementary after all. This can be implied just from their number, which, now, stands at 18. Theorists in particle physics have already begun making models addressing this compositeness[4].

In the past, science has utilized specific classifications of matter which have revealed hidden laws and symmetries. Two of the most known classifications are the Periodic Table of the Elements and Gell-Mann's classification of particles(which paved the way towards the quark model).

Let us attempt to achieve a general classification of matter, including all kinds of matter, and by doing so we may find the links between the elementary particles and the large bodies of the universe. This classification, although empirical, is surprisingly consistent.

It is well known that the different kinds of matter of nature appeared at different epochs of the universal expansion, and that, they are imprints of the different sizes of the universe along the expansion. Taking a closer look at the different kinds of matter we may classify them as belonging to two general states. One state is characterized by a single entity with angular momentum, and we may call it, the 'whirling' state. The angular momentum may be either the intrinsic angular momentum, spin, or the orbital angular momentum. The other state is characterized by collective interactions and may be called the 'soup' state. In the whirling states we find the fundamental matters that make the soups. The different kinds of fundamental matter are the building blocks of everything, *stepwise*. In what follows we will not talk about the weak force since it does not form any stable matter. Later on we will include it in the discussion. The whirling state is formed by only one kind of fundamental force. In the soup state one always finds two types of fundamental forces, i.e., this state is a link between two whirling states. Due to the interactions among the bodies (belonging to a particular whirling state) one expects other kinds of forces in the soup state. In this fashion we *can* form a chain from the quarks to the galactic superstructures.

The kinds of matter belonging to the whirling states are the nucleons, the atom, the galaxies, etc. The 'et cetera' will become clearer later on in this article. In the soup state one finds the quarks, the nuclei, the gases, liquids and solids, and the galactic liquid. Let us, for example, examine the sequence nucleon-nucleus-atom. A nucleon is made out of quarks and held together by means of the strong force. The atom is made out of the nucleus and the electron(we will talk about the electron later), and is held together by means of the electromagnetic force. The nucleus, which is in the middle of the sequence, is held together by the strong force and by the electromagnetic force. In other words, we may say that the nucleus is a link between the strong and the electromagnetic forces. Let us, now, turn to the sequence atom-(gas,liquid,solid) galaxy. The gases, liquids and solids form the link between the electromagnetic force and the gravitational force because they form big clumps of matter, which are all, part of the biggest individual clumps, the galaxies. In the same fashion as with nuclear matter, one expects other kinds of forces in the gases, liquids and solids due to collective interactions. We arrive again at a single fundamental force that holds a galaxy together, which is the gravitational force. There is always the same pattern: one goes from one fundamental force which exists in a single entity(nucleon, atom, galaxy) to two fundamental forces which coexist in a medium. The interactions in the medium form a new entity in which the action of another fundamental force appears.

By placing all kinds of matter together in a table in the order of the *universal expansion* we can construct the two tables below, one for the states and another for the fundamental forces.

In order to make the atom we need the electron besides the nucleus. Therefore, just the clumping of nucleons is not enough in this case. Let us just borrow the electron for now.

In order to keep the same pattern, which should be related to an underlying symmetry, the tables reveal that there should be another force, other than the strong force, holding the quarks together, and that this force alone should hold together the prequarks. Let us name it the superstrong force. Also, for the galactic liquid, there must be another fundamental force at play. From the enormous distances involved(and thus, the very slow transmission of this force at the present epoch) we expect it to be a very weak force. Let us call it the superweak force.

Summing up all fundamental forces forming the single entities we arrive at five forces. The electron, apparently, belongs to a separate class. Adding the weak force to the other five we obtain *six forces*. Placing all five forces at the corners of an hexagon(Fig.1) in the order in which they appeared in Table 2(*the order of the expansion*), and adding the weak force to the missing corner we obtain very interesting relationships among the forces. For example, we find that the electromagnetic and the weak forces *are coupled*(as they should be); the superstrong force is coupled to the gravitational force; and, the strong force is coupled to the superweak force. These relationships indicate that in the Planck era there are three forces, *not one*: 1) The electroweak force; 2) The superstrong-gravitational force; 3) The strong-superweak force. This would explain the unexplained "threenesses" of the standard model(in particle physics) as discussed by Fritzsch[11]. If the "threenesses" are related to the number of forces 'in the beginning of the universe', then the number of quark generations should be $3l$ where l is an integer larger or equal to one. Therefore, three would be the minimum number of generations.

The ultimate superstructure formed out of the galactic liquid is the universe, of course. There should exist only one universe otherwise there would still exist another fundamental force involved in the interaction among universes.

Let us now consider the 'soups' and let us focus our attention in the forces which form neutral ordinary matter(gases, liquids and solids) and in the nuclear force. There is one type of force which is commom to both cases. It is known that the nuclear force can be represented in terms of the Seyler-Blanchard interaction[5,6] which is a type of Van der Waals equation of state. The Van der Waals interaction is also very common in ordinary matter and is described by several kinds of equations depending on the nature of the dipoles.

In order to have the galactic liquid it is also necessary to have a sort of Van der Waals interaction. Therefore, we need another force with a repulsive character(at present).

As we saw above, the superweak force is coupled to the strong force and they must have been unified 'in the beginning of the universe'. Let us now try to find a possible mathematical expression for this force. There have been reports of a fifth force inferred from the reanalysis of the Eötvös experiment and from the mine-gravity data(Fischbach 1987). The discrepancies suggest the existence of a composition dependent intermediate-range force.

The potential energy of such hypothetical force is usually represented by a Yukawa potential which, when added to the standard Newtonian potential energy, becomes[7]

$$V(r) = -\frac{G m_1 m_2}{r}(1 + \alpha \exp(-r/\lambda)), \tag{1}$$

where α is the new coupling in units of gravity and λ is its range. The dependence on composition can be made explicit by writing $\alpha = q_i q_j \varsigma$ with

$$q_i = cos\theta(N + Z)_i/\mu_i + sin\theta(N - Z)_i/\mu_i, \tag{2}$$

where the new effective charge has been written as a linear combination of the baryon number and nuclear isospin per atomic mass unit, and ς is the coupling constant in terms of G.

Until now the results confirming the existence of such a force have been inconclusive[8], although they do not rule it out because its coupling constant(s) may be smaller than previously thought. *It is worth noting that the experiments performed until now did not involve very large masses(i.e., a large number of baryons.)*

The superweak force proposed in this paper, although being a long range force, has the same character as the one of the proposed fifth force does. Since it should be unified with the strong force at short distances, it may be connected with baryon number or isospin. From the above expression for the fifth force potential we may express the potential of the superweak force in terms of the baryon numbers and isospins of two bodies i and j as

$$\begin{aligned} V(r,N,Z) = {} & (A_B(N+Z)_i(N+Z)_j + A_I(N-Z)_i(N-Z)_j + \\ & + A_{IB}((N+Z)_i(N-Z)_j + (N+Z)_j(N-Z)_i))\, g^2 \frac{\exp(-r/\lambda)}{r} \end{aligned} \tag{3}$$

where A_B and A_I are the force coupling constants of the baryon number and isospin terms, respectively, and A_{IB} represents the mixing coupling of isospin and baryon number, and g is the strong force charge. Let us assume that the constants A_B, A_I and A_{IB} are positive. Taking into account the homogeneity of the universe we may disregard the distinction between i and j and the formula becomes simplified somewhat,

$$V(r,N,Z) = \left(A_B(N+Z)^2 + A_I(N-Z)^2 + 2A_{IB}(N+Z)(N-Z)\right) g^2 \frac{\exp(-r/\lambda)}{r}. \tag{4}$$

The superweak force is given by minus the derivative of the above potential with respect to r, which is a function of time(along the expansion). Taking into account conservation of baryon number we have

$$\begin{aligned} F(r,N(r),B) = {} & -4\frac{dN(r)}{dr}(A_I(2N(r) - B) + A_{IB}B)\, g^2 \frac{\exp(-r/\lambda)}{r} + \\ & + \left(\frac{1}{\lambda} + \frac{1}{r}\right) V(r,N(r),B) \end{aligned} \tag{5}$$

where B is the baryon number of any of the two portions. The number of baryons of these two portions has to be extremely large, otherwise we would already have clearly identified this force on Earth.

With the above expression for the superweak force we will be able to explain the expansion of the universe itself and its cyclic behavior.

At some time in the 'beginning' of the universe N was equal to Z. Let us name it $t = 0$. For $t > 0$, N decreases(from $B/2$) with respect to Z via the weak interaction. Therefore, the asymmetry begins and the repulsive part of the superweak force increases. Asymmetry here means the asymmetry in the number of neutrons with respect to the number of protons. Let us call it nucleonic asymmetry. During the next epoch, the lepton era, the nucleonic asymmetry increased and the repulsion outpaced gravity easily, for, during this era N decreased drasticaly. At the end of the lepton era the neutrons made up only 13% of all baryons, the remaining 87% being protons. Therefore, at the end of this era the repulsion attained its maximum value. After this point the repulsion decreased due to the combined effect of the dependence of the superweak force with r and to the halt in the production of protons. As the universe ages the stars become white dwarfs, neutron stars and black holes(not observed yet). During the aging process the core density of a star increases and the high electron Fermi energy drives electron capture onto nuclei and free protons. This last process, called neutronization[9], happens via the weak interaction. The most significant neutronization reactions are electron capture by nuclei and electron capture by free protons.

Of course, neutronization takes place in the stars of all galaxies, and thus, the number of neutrons increases relative to the number of protons as the universe ages. For example, a white dwarf in the slow cooling stage(for $T \leq 10^7$K) reaches a steady proton to neutron density of about 1/8, and takes about 10^9 years to cool off completely. At a later time one expects that the neutrons will decay via the weak interaction and tha number of protons will increase again(with respect to the number of neutrons). Therefore, we expect to have N and Z as a function of time as shown in Fig.2. The end of the lepton era is represented by $t = t_L$, and t_e is the time when N and Z become equal again. According to the arguments above, there is a time which is the inverse of the end of the lepton era, with much more neutrons than protons. Let us name it $t = t_n$. In this way, the attractive terms of the equation of the superweak force above become more dominant than the repulsive term(s) and the force closes the universe. It will be shown that this force drives the expansion and contraction of the universe and behaves overall as a spring-like force. In Fig.3 H_o^{-1} is the present age of the universe(H_o is the Hubble constant). The two turning points, where the force changes sign are $t = 0$ and $t = T_t$, and T_U is the maximum age of the universe including expansion and contraction.

From the arguments presented above, the superweak force should be zero at $t = 0$ and at $t = T_t$. Moreover, around $t = T_t$, this force must be a restoring force. Let us expand the potential around $t = T_t(r = r_T)$ and find the condition for a minimum(in the potential). Up to third order in $r - r_T$ the potential is given by

$$\begin{aligned}
\frac{V(r)}{(Bg)^2} &= \frac{1}{r_T}\left(A_B + 2A_{IB}(2\eta_T - 1) + A_I(2\eta_T - 1)^2\right) + \frac{1}{r_T}(4a_T A_{IB} \\
&\quad + 4a_T A_I(2\eta_T - 1))(r - r_T) + \frac{1}{r_T}\left(4b_T A_{IB} + 4a_T{}^2 A_I + 4b_T A_I(2\eta_T - 1)\right)(r - r_T)^2 \\
&\quad + \frac{1}{r_T}(4c_T A_{IB} + 4c_T A_I(2\eta_T - 1) + 8a_T b_T A_I)(r - r_T)^3 \qquad (6)
\end{aligned}$$

where a_T, b_T and c_T are the first, second and third derivatives of $\eta(r)$ with respect to r. The linear term in $r - r_T$ should be zero so that we have a minimum at $r = r_T$. This leads to the condition $\eta_T = \frac{1}{2}(1 - A_{IB}/A_I)$. Using this condition and the condition $F = 0$, at $t = 0$ and $t = T_t$ we obtain $A_B A_I = A_{IB}{}^2$ and $\frac{d\eta}{dr} = -\frac{1}{4r_o}(A_B/A_I)^{1/2}$ where r_o is the distance between the two bodies in consideration at $t = 0$.

Taking into account that $A_B A_I = A_{IB}{}^2$, η_T becomes $\eta_T = \frac{1}{2}(1 - \sqrt{\frac{A_B}{A_I}})$ from which we obtain $A_B < A_I$.

Considering that $A_B A_I = A_{IB}^2$, the expressions for the potential and for the force become

$$\frac{V(r)}{(Bg)^2} = \frac{(\sqrt{A_B} + \sqrt{A_I}(2\eta(r) - 1))^2}{r} \tag{7}$$

and

$$\frac{F(r)}{(Bg)^2} = \frac{4}{r}\frac{d\eta(r)}{dr}\left(A_I(2\eta(r) - 1) + \sqrt{A_B A_I}\right) + \frac{1}{r}\frac{V(r)}{(Bg)^2}. \tag{8}$$

The potential around $r = r_T$, up to third order in $r - r_T$ becomes

$$\frac{V(r)}{A_I(Bg)^2} = \frac{1}{r_T}\left(a_T^2(r - r_T)^2 + 8a_T b_T(r - r_T)^3\right). \tag{9}$$

One can easily notice that $V = 0$ at $r = r_T$. Let us analyze in some detail the point $r = r_T$(or $t = T_l$).

The expression for the force around $t = T_l$ up to first order in $r - r_T$, is given by

$$\frac{F(r)}{A_I(Bg)^2} = -\frac{2a_T^2}{r_T}(r - r_T). \tag{10}$$

From this expression we obtain

$$\frac{dF}{dt}\Big|_{t=T_l} = -\frac{2A_I(a_T Bg)^2}{r_T v_T}, \tag{11}$$

which shows that v_T can not be zero.

Now, let us show that the contraction begins at $t = t_e(when\ \eta = 1/2)$. At $t = t_e$ we have

$$\frac{d\eta}{dt}\Big|_{t=t_e} = \frac{\dot{\eta}_e}{v_e} \tag{12}$$

and

$$\frac{F_e}{(Bg)^2} = -\frac{4\sqrt{A_B A_I}}{r_e}\frac{\dot{\eta}_e}{v_e} + \frac{A_B}{r_e^2}. \tag{13}$$

Since $F_e < 0$, we can not have $\dot{\eta}_e > 0$, for, in that case v_e would have to be positive, and so we would just leave the question of the contraction to a later time, but the shape of $\eta(t)$ would not allow it to happen. We can have a contraction at $t = t_e$ if we have $\dot{\eta}_e = 0$, because in this case we must also have $v_e = 0$. Let us show that we have a maximum for $\eta(t_n)$ and a minimum for $\eta(t_L)$, and by doing so we justify the shape of the curve shown in Fig.(2). Taking the derivative of the force as expressed by Eq.(5), and considering that it must be zero, and $\frac{d\eta}{dr} = 0$ at $t = t_L$ and at $t = t_n$, we obtain

$$\frac{d^2\eta}{dr^2} = -\frac{Q + 2\eta - 1}{2r} \tag{14}$$

at these two times, where $Q = (A_B/A_I)^{1/2}$. At $t = t_n$, $2\eta - 1 > 0$, and therefore, $\frac{d^2\eta}{dr^2} < 0$, and thus $\eta(t)$ has a maximum at $t = t_n$. At $t = t_L$, we may rearrange the above expression as

$$\frac{d^2\eta}{dr^2} = -\frac{\eta(t_L) - \eta(T_l)}{r_L} \tag{15}$$

where we have used the relation $\eta(T_t) = \frac{1}{2}(1 - \sqrt{\frac{A_B}{A_I}})$. Because $\eta_L < \eta_T$, the second derivative is positive, and so at $t = t_L$, $\eta(t)$ has a minimum. This is consistent with the qualitative shape of η shown in Fig.2.

We can also show that around $t = 0$ the superweak force is given by an expression identical to the strong force. By expanding the potential around $r = r_o$ up to first order in $r - r_o$, and calculating F, we have

$$|F(r \approx r_o)| = \frac{A_B(Bg)^2}{r_o^2}. \tag{16}$$

This is, of course, the expression for the strong force at $t = 0$. *Therefore, the strong force and the superweak force are unified at $t = 0$.* This result is consistent with Fig.1.

We represent in Fig.4 the potential of the superweak force according to our calculations and considerations. According to this figure the universe spends most of its time at the bottom of the potential, where it is more stable.

Expressing the expansion rate as $\frac{1}{R(t)}\frac{dR(t)}{dt} = f(t) = H(t)$ where $H(t)$ is Hubble's constant, and making $L(\eta) = A_B + A_I(2\eta - 1)^2 + 2(A_I A_B)^{1/2}(2\eta - 1)$ we obtain(disregarding gravity)

$$\dot H = \frac{B^2g^2|\dot L(\eta)|}{m_p r_o^3 H R^3} + \frac{B^2g^2 L(\eta)}{m_p r_o^3 R^3} - H^2 \tag{17}$$

where $R(t) = r(t)/r_o$, $r_o = r(0)$ and m_p is the mass of the proton. In the range between $t = t_L$ and $t = t_s$, $\dot\eta > 0, \eta < 1/2$, and therefore $\dot L$ is negative. H^{-1} is the time of expansion between t_L and t. $|\dot L|/H$ and L may have the same order of magnitude for times close to T_t. Let us consider t as being the present epoch of the universe. If the expansion is slowing down we must have $\dot H < 0, L > 0$ in this range. Solving the cubic equation in H for $\dot H < 0$, we obtain

$$\begin{aligned} H(t) \;>\; & \left(\frac{B^2g^2|\dot L|}{2m_p r_o^3 R^3} + \sqrt{-\frac{B^6g^6L^3}{27m_p^3 r_o^9 R^9} + \frac{B^4g^4|\dot L|^2}{4m_p^2 r_o^6 R^6}}\right)^{1/3} \\ & - \left(\frac{B^2g^2|\dot L|}{2m_p r_o^3 R^3} - \sqrt{-\frac{B^6g^6L^3}{27m_p^3 r_o^9 R^9} + \frac{B^4g^4|\dot L|^2}{4m_p^2 r_o^6 R^6}}\right)^{1/3} \end{aligned} \tag{18}$$

which means that H is positive.

If we include the gravitational force, we add an extra negative term to $\dot H$ which makes the expansion to slow down more.

In the range $t_n < t < T_U + t_L$, taking into account special relativity and gravity and considering that the two bodies are identical, we obtain

$$\frac{c^3 r_o}{(c^3 - H^2R^2r_o^2)^{3/2}}(\dot H R + H^2 R) = \frac{B^2g^2\dot L(\eta)}{m_p r_o^2 R^2 H} + \frac{B^2g^2 L(\eta)}{m_p r_o^2 R^2} - \frac{Gm_o}{r_o^2 R^2}. \tag{19}$$

We can calculate how $H(t)$ depends on time around $t = 0$ if we make some assumptions on the relative proportions of neutrons to protons prevailing around $t = 0$. The nuclear reactions which must be considered in determining the proton-neutron ratio are the following:

$$n \rightleftharpoons p + e^- + \bar\nu_e,\; n + \nu_e \rightleftharpoons p + e^-,\; n + e^+ \rightleftharpoons p + \bar\nu_e. \tag{20}$$

By considering that the temperature is *not* very high so that $m_e c^2 \approx kT$, Alpher et al.[10] have shown(in another context) that, among the reactions above, free neutron decay is the dominant reaction. Taking this into account and considering the condition that we found at $t = 0$($F = 0$), $\frac{d\eta}{d\tau} = -\frac{A_B}{4A_I D\tau}$, we have

$$H(t) = 2\sqrt{\frac{A_I}{A_B}\frac{t}{1100}} \tag{21}$$

where t is given in seconds.

We can explain the (flat)rotational curve of galaxies, v versus r in the following way: We expect that, since the time of its formation, a galaxy experiences an overall repulsion, which must be stronger between its bulge and its outskirts. Since the gravitational force is responsible for holding all the galaxy's stars together, the repulsion must cause a small effect, only observable over a long time. This repulsion is consistent, for example, with the outward motion of two large expanding arms of hydrogen gas which have been observed close to the center of the Milky Way[11]. This phenomenom is not particular to our galaxy, and similar outbursts are happening in many other galaxies. Let us take a look at Fig.(3). Galaxy formation happened at a time $t = t_G > t_L$. From t_G up to the present epoch, a galaxy is subjected to the repulsive force shown as the first hump in Fig.(3).

Because of the repulsion the tangential velocities of the stars of a galaxy(out of the bulge) are kept constant due to the positive work performed by the repulsion on a particular star. This work is done against the gravitational potential and the star gains gravitational energy and moves outward. Therefore, the star's tangential velocity *does not change*.

Let us consider a star in an orbit(1) at a distance R_1. The total energy, E, of the system *(inner galactic region)-star*, is

$$E = K_1 + U_1 + V_1 \tag{22}$$

where K_1 is its kinetic energy, U_1 is its gravitational energy, and V_1 is its superweak potential energy. The inner galactic region includes the bulge of the galaxy and the stars up to the radius R_1. In the orbit with radius R_2, the energy E, is given by

$$E = K_2 + U_2 + V_2. \tag{23}$$

Because the gain in gravitational energy, $U_2 - U_1$, was obtained at the expense of the superweak potential energy released, $V_1 - V_2$, the kinetic energy remains the same, i.e., $K_2 = K_1$. Therefore, $v_2 = v_1$. Since the star moves outwards and keeps the same tangential velocity its angular momentum increases. The increase is given by

$$\Delta L = mv(R_2 - R_1) \tag{24}$$

where m is the mass of the star, v is its tangential velocity, R_1 is its original orbital radius and R_2 is its final(at a particular time) orbital radius. Because v remains constant its angular velocity decreases with respect to the central part of the galaxy.

Because of conservation of angular momentum the galactic bulge must decrease its angular momentum by the same amount, ΔL. If we consider that the angular velocity of the bulge does not diminish(which is more plausible than otherwise), then its mass must diminish, i.e., the central hub sheds more matter outwards. This fact has been observed in many galaxies. Thus, matter is shed outwards because of repulsion and because of angular momentum conservation. Therefore, as the galaxy ages, its nucleus diminishes in size. The opposite happens to the arms which become bigger and bigger. This helps us understanding the formation of arms in galaxies.

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?	quark	nucleon
nucleon	nucleus	atom
atom	gas liquid solid	galaxy
galaxy	galactic liquid	?

Table I. The two general states which make everything in the universe, stepwise. The table is arranged in a way to show the links between mediums and single entities. The leptons are not included because they belong to a separate class

?	? strong force	strong force
strong force	strong force electromagnetic force	electromagnetic force
electromagnetic force	electromagnetic force gravitational force	gravitational force
gravitational force	gravitational force ?	?

Table II. Three of the fundamental forces of nature. Each force appears three times and is linked to another force through a medium. Compare with Table I

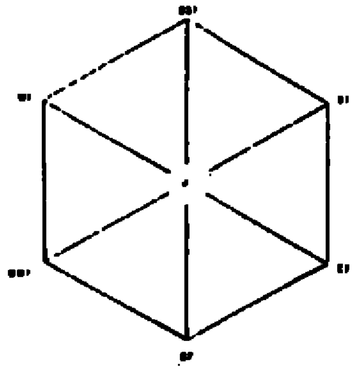

Fig.1. Diagram of the forces of nature. J stands for angular momentum. SSF - Superstrong force; SF - Strong force; EF - Electromagnetic force; GF - Gravitational force; SWF - Superweak force; WF - Weak force.

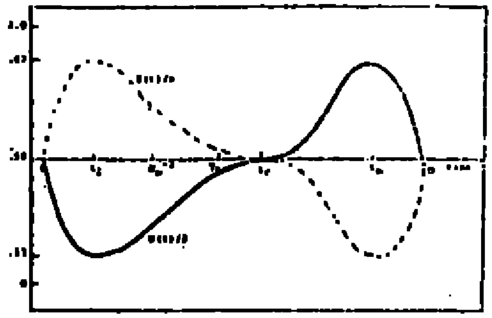

Fig.2. A qualitative shape for H and B as a function of time. H_0^{-1} is the present age of the universe; t_L is the end of the lepton era; t = 0 is the beginning of a cycle; t_E is the time when H and B become equal again. The expansion stops at this time. At t_m B reaches its maximum value, and T_U is the total age of the universe. The time scale is not linear.

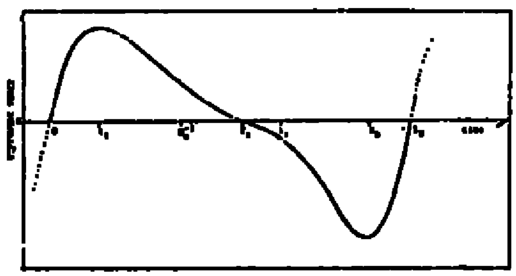

Fig.3. A qualitative shape of the superweak force as a function of time. It is repulsive from t = 0 up to t = T_1 and attractive from t = T_1 up to t = T_U.

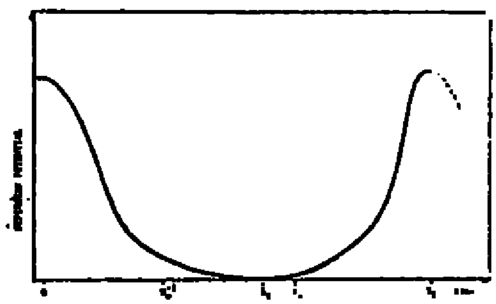

Fig.4. A graph of the potential of the superweak force as a function of time. The time parameters have been defined in other figures and in the text. Time means the age of the universe.

GAUGE E INTEGRABILIDADE EM EQUAÇÕES LINEARES E NÃO LINEARES

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Usamos simetrias permutacionais para introduzir um critério algoritmico de decisão sobre a integrabilidade (solvabilidade) de um sistema de equações. Comparamos as estruturas de gauge das equações de Maxwell e das equações de Einstein, tomadas como exemplos de sistemas lineares e não lineares, respectivamente. Somente para equações lineares podemos fixar o gauge sem perda de completa generalidade (em contradição à literatura corrente), isto porque só para estas equações a condição de gauge coincide com a condição de integrabilidade.

A) SIMETRIAS PERMUTACIONAIS EM SISTEMA DE EQUAÇÕES

1) A ação de operadores de Permutação $\{P_{\alpha\beta},\ \alpha,\beta=1 \text{ a } n\}$, em sistemas de equações, $\mathfrak{F} =\{F^A(x^\alpha,..,Y(x)^\alpha,...]=0,\ A = 1 \text{ a } L\}$,

$$P_{\alpha\beta}F^A(x^\alpha,..,Y(x)^\alpha,...]=F^B(x^\beta,..,Y(x)^\beta,...] \text{com } F^A\ ,\ F^B \in \mathfrak{F}$$

divide o sistema em classes invariantes de equivalência, $[F^A]$.

Obs. Para resolver um sistema de equações no formalismo das simetrias de permutação, trabalhamos com as classes de equivalência, $[F^A]$ e não com as equações, $F^A(\ )=0$.

2) Qualquer equação pode ser escrita como uma equação de autovalor nulo de um polinômio de operadores de permutação.

$$F^A(x^\alpha,..,Y(x)^\alpha,...]=0 \longrightarrow O_I(\alpha)\hat{F}^A(\alpha) = 0, \text{ onde}$$

$O_I(\alpha)$: polinomial de operadores envolvendo α ($P_{\alpha\beta}$, $P_{\alpha\gamma}$, etc)

$\hat{F}^A(\alpha)$: núcleo ou autofunção de $O_I(\alpha)$

Exemplos: $O_I(\alpha)$ $\qquad$ $\hat{F}^A(\alpha)$

$$\nabla^2\phi = \rho \longrightarrow (1 + P_{xy}+ P_{xz}) \quad (\phi,_{xx} -\rho/3) = 0$$

$$\nabla.B = \rho \longrightarrow (1 + P_{xy}+ P_{xz}) \quad (B_{x},_{x}-\rho/3)= 0$$

$$\Box\ \phi = \rho \longrightarrow (1 + P_{xy}+ P_{xz}-P_{xt}) \quad (\phi,_{xx}-\rho/2) = 0$$

BASE DE EXPANSÃO DOS $\hat{F}^{A}(\alpha)$

1) FUNÇÕES DE ARGUMENTOS ORDENADOS, (FAO), são conjuntos de funções, $f|\alpha\beta\gamma\ldots|$, *arbitrárias e genéricas*, que se distinguem entre si pela ordem de seus argumentos e por um fator global.

Ex. Se $f|\alpha\beta\gamma\ldots|$, $= x^{\alpha}+x^{\beta}\cos x^{\gamma}$ então $f|\beta\alpha\gamma\ldots|$, $x^{\beta}+x^{\alpha}\cos x^{\gamma}$

Em um conjunto de FAO só uma é totalmente arbitrária. As outras são determinadas, a partir da primeira, a menos de um fator global.

2) $F^{A}(x^{\alpha}\quad)=0 \Rightarrow O_{I}(\alpha)\hat{F}^{A}(\alpha)=0 \Rightarrow \hat{F}^{A}(\alpha) = O_{II}(\alpha)\ f|\alpha\beta\ldots|$ onde $O_{II}(\alpha)$ é outro polinomial em $P_{\alpha\beta}, P_{\alpha\gamma}$, etc., tal que $O_{I}(\alpha)\ O_{II}(\alpha)\ f|\alpha\beta\ldots| = 0$. Observações:

a) $f|\alpha\beta\ldots|$ sendo genérico e arbitrário $\Rightarrow O_{II}(\alpha)\ f|\alpha\beta\ldots|$ mostra a existência de propriedades algébricas e topológicas comuns a todas as possíveis soluções de uma categoria inteira de equações, à qual pertence $F^{A}(x^{\alpha}\quad)=0$

b) $\hat{F}^{A}(\alpha) = O_{II}(\alpha)\ f|\alpha\beta\ldots|$ é uma equação, em geral, muito mais simples que $F^{A}(x,\quad)=0$ Exemplos:

Se $F^{A}(x^{\alpha}\ldots)=(1+P_{\alpha\beta})\ \hat{F}^{A}(\alpha) \Rightarrow \hat{F}^{A}(\alpha) =(1-P_{\alpha\beta})\ f|\alpha\beta|$ é uma solução.

Se $F^{A}(x^{\alpha}\ldots)=(1+P_{\alpha\beta}+P_{\alpha\gamma})\ \hat{F}^{A}(\alpha)$, então

$$\check{F}^{A}(x^{\alpha},\ldots)_{+} = (2-P_{\alpha\beta}-P_{\alpha\gamma})f|\alpha(\beta\gamma)|$$

é uma de suas infinitas classes de soluções possíveis.

B) CRITÉRIO DE INTEGRABILIDADE (DEFINIÇÕES)

1) Um sistema de equações, $\mathfrak{Z}$, representa um PROBLEMA BEM COLOCADO se o n^{o} de equações independente for igual ao n^{o} de incógnitas.

2) EQUAÇÃO PRESUMIVELMENTE INTEGRAVEL, EPI: a que envolve ou pode ser colocada em uma forma que envolve não mais que uma incógnita.

3) SISTEMA de equações PRESUMIVELMENTE INTEGRÁVEL, SPI: o que tem,ou que pode ser colocado em uma forma que tem pelo menos uma EPI, e que as demais se tornam EPI com a integração das primeiras. Em caso contrário o sistema não é integrável.

4) Um sistema não integrável pode se tornar um SPI com o acréscimo de outras equações, vínculos ou condições adicionais. Estas seriam, então, suas CONDIÇÕES DE INTEGRABILIDADE.

EXEMPLOS (aparentemente em contradição)

a) ∇xB=0. São 3 equações, $(B_{i,j}=B_{j,i},\ i,j = 1,2,3)$, cada uma com duas incógnitas. É SPI, pois pode ser escrito como

$(1-P_{ij})B_{i,j}=0 \Rightarrow B_{i,j}= f|ij|+f|ji|=f|(ij)| \Rightarrow B_{i,j}=\phi,_{ij} \Rightarrow B=\nabla\phi$.

b) ∇.B=0. Uma eq. com 3 incógnitas. Mas é EPI pois $\Rightarrow$ $(1+P_{xy}+P_{xz})B_{x',x}=0$. B = ∇xA é apenas uma de suas (infinitas) classes de possíveis soluções.

Só é um PROBLEMA BEM COLOCADO porcausa de suas simetrias permutacionais $(B_i=P_{ij}B_j)$.

c)Condição de curvatura nula $\Rightarrow$ $A_{\mu,\nu}- A_{\nu,\mu} + [A_\nu,A_\mu] = 0$, ou $(1-P_{\mu\nu})D_\mu A_\nu=0$, com $D_\mu= \partial_\mu + A_\mu$. $\Rightarrow$ $D_\mu A_\nu=f|(\mu\nu)|$ $\Rightarrow$ $A_\mu=- M^{-1}\partial_\mu M$. (M, matriz invertível).

C) EQUAÇÕES DE MAXWELL (em espaço plano)

1) Abordagem usual

$F^{\mu\nu},_\nu = -J^\mu$ com $F_{\mu\nu}=A_{\nu,\mu}- A_{\mu,\nu}$. 4 incógnitas e 3 eqs. já que $J^\mu,_\mu=0$. A 4ª eq. é suprida pela condição de gauge, $A^\mu,_\mu=0 \Rightarrow$ $\Box A^\mu=J^\mu$. As soluções físicas são soluções simultâneas de $A^\mu,_\mu=0$ e de $\Box A^\mu=J^\mu$.

2) Usando as simetrias permutacionais.

$(1+P_{yz}+P_{yt})(A^x;^y_y -A^y;^x_y - J^x/3) = 0$ implica em

$(A^x;^y_y -A^y;^x_y - J^x/3) = 0_{||}(y)f|xyzt|$.

Esta eq. é claramente não integravel, pois envolve 2 incógnitas, A^x e A^y. Precisamos de mais uma eq. relacionando A^x e A^y, dada pela cond. de gauge $(1+P_{yz}+P_{yt})(A^y,_y -A^x,_x/3) = 0$, que resolve o problema.

3) Diferenças (sutís) desta abordagem

a)São 4 eqs. mas consideramos apenas uma $(F^{x\mu},_\mu=-J^\mu)$, representando a classe de equivalência.

b) O fato que as 4 eqs. não são independentes não é utilizado. A não integrabilidade está presente no fato que elas envolvem 2 incógnitas.

c) A condição de gauge é também a condição de integrabilidade já que a adoção da condição de gauge não restringe o universo das possíveis soluções. Não se perde generalidade com a escolha do gauge. Esta afirmativa não pode ser generalizada, como se faz na literatura, para sistemas não lineares.

d) Outra abordagem equivalente seria checar quais soluções da condição de gauge, $(1+P_{xy}+P_{xz}+P_{xt})A^x{}_{,x}=0 \Rightarrow A^x{}_{,x}= O_{||}(x)f|xyzt|$ satisfazem a $\Box A^\mu=J^\mu$.

D) AS EQUAÇÕES DE EINSTEIN ($G_{\mu\nu} = T_{\mu\nu}$)

1) Notação: α, β, γ e δ sempre representam DIFERENTES índices ou componentes. Não vale a convenção de Einstein.

2) Escolhemos , sem perda de generalidade, coordenadas que diagonalizam o tensor métrico em um ponto genérico Q.

$$ds^2|_Q = \varepsilon_\alpha e^\alpha (dx^\alpha)^2+ \varepsilon_\beta e^\beta (dx^\beta)^2+ \varepsilon_\gamma e^\gamma (dx^\gamma)^2+ \varepsilon_\delta e^\delta (dx^\delta)^2,$$

onde α, β, γ e δ são funções genéricas de todas coordenadas, e ε_α, ε_β, ε_γ, ε_δ = ±1. Portanto, estamos considerando as eqs. de Einstein em sua mais completa generalidade, inclusive quanto à assinatura da métrica. Nenhuma hipótese é feita sobre $T_{\mu\nu}$, a não ser da mais completa generalidade, o que corresponde a um tensor totalmente simétrico sob permutações.

3) As eqs. se dividem em duas classes invariantes:

$$|\alpha\alpha>: \quad \theta_\gamma\theta_\beta\{(P_{\alpha\gamma}+P_{\alpha\beta})(1+P_{\alpha\beta})g^{\beta\beta}[(\alpha_{\beta\beta})+2g^{\alpha\alpha}(\alpha_\alpha g_{\alpha\beta',\beta}- g_{\alpha\beta',\alpha\beta}) +$$
$$+4g^{\gamma\gamma}g^{\alpha\alpha}(\Gamma_{\gamma\alpha\alpha}\Gamma_{\gamma\beta\beta}-\Gamma^2_{\gamma\alpha\beta})] + 8/3\ g^{\alpha\alpha}T_{\alpha\alpha}\} = 0$$

$$|\alpha\beta>: \quad (1+P_{\alpha\beta})\theta_\gamma\{(\gamma_{\alpha\beta}+1/2\gamma_\alpha\gamma_\beta -\alpha_\beta\gamma_\alpha -T_{\alpha\beta})+g^{\gamma\gamma}[2\alpha_\beta g_{\alpha\gamma',\gamma}-2\alpha_\gamma\Gamma_{\alpha\beta\gamma}+$$
$$+\gamma_\gamma\ \Gamma_{\gamma\alpha\beta} -2\Gamma_{\gamma\alpha\beta',\gamma} + 2g^{\delta\delta}(\Gamma_{\delta\gamma\gamma}\Gamma_{\delta\alpha\beta} -\Gamma_{\delta\gamma\alpha}\Gamma_{\delta\gamma\beta})]\} = 0$$

onde $\alpha_\beta=\partial\alpha/\partial x^\beta$, $\alpha_\alpha=\partial\alpha/\partial x^\alpha$, etc. , $(\alpha_{\beta\beta}) = 2\alpha_{\beta\beta}+\alpha_\beta(\alpha-\beta)_\beta$.

$2\Gamma_{\alpha\rho\mu}=g_{\alpha\rho',\mu}+g_{\alpha\mu',\rho}-g_{\rho\mu',\alpha}$. $\theta_\gamma=1+P_{\gamma\delta}$, $\theta_\beta=1+P_{\beta\gamma}+P_{\beta\delta}$.

CONDICÕES DE INTEGRABILIDADE

Cada eq. envolve todas as 10 incógnitas de um modo tal que para torna-las integráveis, sem quebrar a simetria, precisaríamos de 10 condições de gauge. Como só dispomos de 4, só nos resta a redução simétrica do n° de incógnitas, o que implica em $g_{\alpha\beta}=0$. A métrica é globalmente diagonalizável. As eqs. se reduzem a

$|\alpha\beta>$: $$(1+P_{\alpha\beta})\theta_{\gamma}\{2\gamma_{\alpha\beta}+\gamma_{\alpha}\gamma_{\beta}-2\alpha_{\beta}\gamma_{\alpha}-2T_{\alpha\beta}\}=0,$$

$$|\alpha\alpha>:\ \theta_{\beta}\theta_{\gamma}\{(P_{\alpha\gamma}+P_{\alpha\beta})(1+P_{\alpha\beta})[g^{\beta\beta}(\alpha_{\beta\beta})+1/2g^{\gamma\gamma}\alpha_{\gamma}\beta_{\gamma}]+8/3\ g^{\alpha\alpha}T_{\alpha\alpha}\}=0$$

Ainda não integráveis.

Para $|\alpha\alpha>$ não há possibilidade de simplificação através de eventual condição de gauge, porque cada um des seus termos envolve mais de uma incognita.

$|\alpha\beta>$ poderia, em princípio, admitir uma condição de gauge do tipo

$$\theta_{\gamma}(1+P_{\alpha\beta})\{2\alpha_{\beta}\gamma_{\alpha}+2T_{\alpha\beta}-F(\gamma)\}=0,$$

onde $F(\gamma)$ é uma dada função de γ e de suas derivadas. Mas para não quebrar a simetria precisaríamos de 6 condições iguais a esta (uma para cada $|\alpha\beta>$). Como só podemos ter 4 temos que reduzir o n^{o} de incógnitas. a única possibilidade é $\alpha=\beta=\gamma=\delta=\phi$, ou seja a métrica tem que ser conforme: "SOLUÇÕES TOTALMENTE SIMÉTRICAS DAS EQUAÇÕES DE EINSTEIN SÃO CONFORMALMENTE PLANAS". E como consequência:"SOLUÇÕES DE VÁCUO TOTALMENTE SIMÉTRICAS SÃO PLANAS".

E) LIBERDADE DE GAUGE E GENERALIDADE

Para as eqs. de Einstein, diferentemente do caso das eqs. de Maxwell, a condição de gauge não é igual à condição de integrabilidade, e isto implica em possível perda de generalidade com a adoção da condição de gauge. Por exemplo, se tívessemos imposto o gauge $g^{\rho\sigma}\Gamma^{\mu}_{\rho\sigma}=0$ (cond. harmônica) só poderíamos ter soluções planas, porque a cond. de integrabilidade não é afetada pelo gauge, e este, para uma métrica conforme implicaria em $\phi,_{\mu}=0$ para todos os pontos da variedade.

Conclusão:

Liberdade de gauge não é garantia de máxima generalidade. Ao se adotar um gauge para sistemas não lineares deve-se atentar para possíveis exclusões no universo de soluções.

FORMALISMO PARA SISTEMAS DE ESTATÍSTICAS GENERALIZADAS

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Sob permutações de índices um sistema de equações se particiona em classes de equivalência. Qualquer equação pode ser escrita como uma equação de autovalor nulo de uma função polinomial de operadores de permutação. Este operador polinomial define uma categoria de equações que tem em comum uma mesma álgebra e uma mesma topologia, definidas sobre uma base apropriada de expansão de suas autofunções. Esta álgebra e esta topologia classificam o sistema de equações de acordo com a estatística de suas soluções, isto é, o comportamento delas sobre permutações. Incluídas nestas classes se encontram as estatísticas de Bose-Einstein, Fermi-Dirac e de sistemas exóticos (anyons).

A) Simetrias Permutacionais em Sistema de Equações

1) A ação de operadores de Permutação $\{P_{\alpha\beta},\ \alpha,\beta=1 \text{ a } n\}$, em sistemas de equações, $\mathfrak{F} = \{F^A(x^\alpha,..,Y(x)^\alpha,...]=0,\ A = 1 \text{ a } L\}$,

$$P_{\alpha\beta}F^A(x^\alpha,..,Y(x)^\alpha,...]=F^B(x^\beta,..,Y(x)^\beta,...] \text{com } F^A\ ,\ F^B \in \mathfrak{F}$$

divide o sistema em classes invariantes de equivalência, $[F^A]$.

Obs. Para resolver um sistema de equações no formalismos das simetrias de permutação, trabalhamos com as classes de equivalência, $[F^A]$ e não com as equações, $F^A(\ )=0$.

2) Qualquer equação é uma equação de autovalor nulo de um polinômio de operadores de permutação.

$$F^A(x^\alpha,..,Y(x)^\alpha,...]=0 \longrightarrow O_I(\alpha)\hat{F}^A(\alpha) = 0, \text{ onde}$$

$O_I(\alpha)$: polinomial de operadores envolvendo α $(P_{\alpha\beta},\ P_{\alpha\gamma},$ etc$)$

$\hat{F}^A(\alpha)$: núcleo ou autofunção de $O_I(\alpha)$

Exemplos: $\qquad O_I(\alpha) \qquad \hat{F}^A(\alpha)$

$$\nabla^2\phi = \rho \longrightarrow (1 + P_{xy} + P_{xz})\quad (\phi,_{xx} - \rho/3) = 0$$
$$\nabla.B = \rho \longrightarrow (1 + P_{xy} + P_{xz})\quad (B_x,_x - \rho/3) = 0$$
$$\Box\,\phi = \rho \longrightarrow (1 + P_{xy} + P_{xz} - P_{xt})\quad (\phi,_{xx} - \rho/2) = 0$$

B) BASE DE EXPANSÃO DOS $\hat{F}^{A}(\alpha)$

1) FUNÇÕES DE ARGUMENTOS ORDENADOS, (FAO), são conjuntos de funções, $f|\alpha\beta\gamma\ldots|$, *arbitrárias e genéricas*, que se distinguem entre si pela ordem de seus argumentos e por um fator global.
Ex. Se $f|\alpha\beta\gamma\ldots|$, $= x^{\alpha}+x^{\beta}\cos x^{\gamma}$ então $f|\beta\alpha\gamma\ldots| \sim x^{\beta}+x^{\alpha}\cos x^{\gamma}$
2) $K^n = \{K_{ij} | \exists\ K_{ij}^{-1};\ K_{ij}K_{ji}=1,\ i,j\in[1,n]\in Z_+\}$, onde K_{ij} é um fator associado ao par de posições dos índices (i,j) no argumento de $f|\alpha_1\alpha_2\ldots\alpha_n|$, quaisquer que sejam os índices. K^n define a estatística (comportamento sob permutações) das FAO (e as divide em classes):

$$P_{\alpha_i\alpha_j} f|\ldots\alpha_i\ldots\alpha_j\ldots| = K_{ij} f|\ldots\alpha_j\ldots\alpha_i\ldots|$$

Observações: i) Isto garante $P^2_{\alpha\beta}=1$, sem nenhuma restrição sobre K^n
ii) Os sistemas usuais de estatísticas de FD e de BE pertencem às classes de FAO com $K_{ij}=\pm1$, $i,\ j \in [1,n]$
iii) Em um conjunto de FAO só uma é totalmente arbitrária. As outras são determinadas, a partir daprimeira, a menos de um fator global.

C) REDES DE FAOs

1) Definimos redes de FAOs associando cada $f|\alpha_1\alpha_2\ldots\alpha_n|$ a um vértice ou ponto, e a cada par de pontos, associamoos uma aresta ou direção, representando um operador $P_{\alpha\beta}$. Cada vértice é ligado a $n(n-1)/2$ novos vértices por $n(n-1)/2$ arestas.
$n = 1$ ⟶ 1 ponto: $f|\alpha|$
$n = 2$ ⟶ 1 dublete: $f|\alpha_1\alpha_2| \overset{\alpha_1\alpha_2}{\longrightarrow} K_{12} f|\alpha_2\alpha_1|$
$n \geq 3$: rede de dimensão $n-1$, de extensão e multiplicidade infinitas, construidas de hexagonos e quadrados Esta rede é uma representação da algebra dos $P_{\alpha\beta}$ sobre as FAOs Para um K abeliano, ela é definida por:

(Geometria algeb.)

$$P_{\alpha_i\alpha_j}P_{\alpha_j\alpha_k}P_{\alpha_i\alpha_j} = P_{\alpha_j\alpha_k}P_{\alpha_i\alpha_j}P_{\alpha_j\alpha_k} \quad \text{para } 0<i<j<k\leq n \qquad \text{HEXAGONO}$$

$$P_{\alpha_i\alpha_j}P_{\alpha_k\alpha_m} = P_{\alpha_k\alpha_m}P_{\alpha_i\alpha_j} \quad \text{para } i,j,k,m, \text{ distintos} \qquad \text{QUADRADO}$$

Obs. Esta álgebra contém o grupo das tranças (braid group) que só admite geradores da forma $P_{\alpha_i\alpha_{i\pm1}}$.

2) Qualquer restrição a K^n gera deformações desta álgebra e desta topologia, porque implica em identificação de vértices. Esta restrição pode ser motivada por argumentos físicos/matemáticos ou pode provir do conjunto de equações que descreve o sistema

D) Expandindo $\hat{F}^A(\alpha)$

$F^A(x^\alpha)=0 \rightarrow O_I(\alpha)\hat{F}^A(\alpha)=0 \rightarrow \hat{F}^A(\alpha) = O_{II}(\alpha)\ f|\alpha\beta\ldots|$ onde $O_{II}(\alpha)$ é outro polinomial em $P_{\alpha\beta}, P_{\alpha\gamma}$, etc., tal que

$$O_I(\alpha)\ O_{II}(\alpha)\ f|\alpha\beta\ldots| = 0.$$

Obs.: sendo $f|\alpha\beta\ldots|$ genérico e arbitrário, $O_{II}(\alpha)\ f|\alpha\beta\ldots|$ denota a existência de propriedades algébricas e topológicas comuns a todas as possíveis soluções de uma categoria inteira de equações, à qual pertence $F^A(x^\alpha)=0$.

Exemplos:

Para n=2, se $F^A(x^\alpha\ldots)=(1+P_{\alpha\beta})\ \hat{F}^A(\alpha) \rightarrow \hat{F}^A(\alpha) = f|\alpha\beta| - K_{12}f|\beta\alpha|$

Cada valor atribuido a K_{12} define uma classe de simetria distinta (estatística). Não há restrição a K_{12}.

Para n=3, se $F^A(x^\alpha\ldots)=(1+P_{\alpha\beta}+P_{\alpha\gamma})\ \hat{F}^A(\alpha)$, suas soluções requerem $K_{12}^2K_{23}^2=K_{13}^2$, e são da forma

$$\check{F}^A(x^\alpha,\ldots)_+ = a\{-K_{12}^{-1}f|\alpha(\beta\gamma)| + f|{}^{(\beta}\alpha^{\gamma)}|\} +$$

$$+ b\ \{-K_{13}^{-1}f|\alpha(\beta\gamma)| + f|(\beta\gamma)\alpha|\};$$

$$\check{F}^A(x^\alpha,\ldots)_- = a\ \{K_{12}^{-1}\quad f|\alpha[\gamma\beta]| + f|{}^{[\beta}\alpha^{\gamma]}|\} +$$

$$+ b\ \{K_{13}^{-1}f|\alpha[\beta\gamma]| + f|[\beta\gamma]\alpha|\},$$

correspondendo a $K_{13}=\pm K_{12}K_{23}$, respectivamente. a e b são constantes arbitrárias, e os parêntesis (colchetes) indicam a simetria (anti-simetria) dos índices envolvidos.

Observe que enquanto não há restrição sobre sistemas bidimensionais, para sistemas tridimensionais a restrição é forte. Sistemas formados por componentes idênticas ($K_{12}=K_{13}=K_{23}$) só podem ter estatísticas de BE ou FD ($K_{ij}=\pm 1$). Topologicamente: a multiplicidade da variedade das FAO se reduz a 1.

ALGUNS EXEMPLOS DE REALIZAÇÕES (para $n=3$, $b=0$, $K_{13}=\pm K_{12}K_{23}$)

1) Classe (111), ou seja $K_{12}=K_{13}=K_{23}=1 \Rightarrow$

$\Rightarrow \quad \check{F}^A(x,\ldots)_{(111)} = (1+P_{yz})(1-P_{xy})f|xyz|$

Para $\nabla^2\phi = 0 \quad \Rightarrow \quad \phi_{xx}=(1+P_{yz})(1-P_{xy})f|xyz|$

$f|xyz|= x^2/r^3$ gera $\phi = 1/r$,

$f|xyz| = [m_x x^3 + x^2(ym_y + zm_z)]/r^7$ gera $\Phi = m.r/r^3$, etc. Todas as soluções de multipolo estão presentes nesta classe.

2) Classe (-1-1-1) ($K_{12}=K_{13}=K_{23}=-1$)

$\Rightarrow \check{F}^A(x,\ldots)_{(-1-1-1)} = (1-P_{yz})(1+P_{xy})f|xyz| = f|(xy)z| - f|(xz)y|$

Para $\nabla.B=0 \quad \Rightarrow \quad B_{x,x}= f|(xy)z| - f|(xz)y| \quad \Rightarrow \quad f|(xy)z| = -A_{z,xy} \Rightarrow$

$B_x=A_{y,z}- A_{z,y}$. Observe que $P_{xy}A_x = - A_y$, pois A pertence à classe (-1-1-1), enquanto que $P_{xy}B_x = B_y$. Não se pode obter $\nabla.B = 0$ em classe (111), e nem $\nabla^2\phi = 0$ em classe (-1-1-1),. o que é um indicativo da relevância da existência de distintas classes.

Sobre a Equação de Dirac em Três Dimensões *

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Em trabalhos anteriores [1], comprovamos a existência de uma ligação bem definida entre a equação de Dirac com o grupo $SU(2^{n/2})$, sendo n um número par de dimensões do espaço-tempo. Mostramos que essa ligação é válida para a formulação usual em termos de matrizes e, também, como uma conseqüência lógica do isomorfismo, provado por Graf [2], entre matrizes de Dirac e formas diferenciais exteriores, estas dotadas ainda de um produto, dito "de Clifford". Mostra-se [3] que estas formas, satisfazendo a álgebra de Kähler-Atiyah, mantém invariantes os ideais mínimos à esquerda do espaço das formas.

Os físicos teóricos estão mais familiarizados, no entanto, com o emprego das matrizes de Dirac. Na última década, por outro lado, houve um interesse crescente pela física num espaço-tempo com dimensão três. Em geral, é costume representar as matrizes de Dirac correspondentes pelas matrizes de Pauli em representação bidimensional. Em alguns casos, usa-se uma representação por matrizes 4×4 em dois blocos idênticos 2×2.

Entretanto, o presente trabalho mostra que as representações habituais das matrizes de Dirac são inconsistentes, não satisfazem as condições que a álgebra correta das matrizes γ deve preencher. A descrição correta exige matrizes de Dirac de dimensão 4, com estrutura de blocos diagonal não idênticos. Isto decorre do fato de que as formas de Kähler-Atiyah podem ser escritas como combinações de geradores do grupo $SU(2) \times SU(2)$, identificáveis unicamente a partir de suas propriedades algébricas. O isomorfismo de Graf [2] ou, simplesmente, a transcrição em termos de matrizes destas propriedades algébricas conduz ao resultado.

Além do mais, a reversão temporal e a de um único eixo espacial são incompatíveis com um formalismo de matrizes 2×2.

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Como é usual, defina-se o produto de Clifford entre duas formas diferenciais exteriores como

$$dx^\mu \vee dx^\nu = dx^\mu \wedge dx^\nu + g^{\mu\nu},$$

em que $\vee$ é o símbolo do produto de Clifford, $\wedge$ denota o produto exterior usual e $g^{\mu\nu}$ é o tensor métrico do espaço-tempo, com $\mu, \nu = 0, 1, 2$.

Podemos construir os seguintes quatro produtos independentes:

$$dx^0 \wedge dx^1,\ dx^0 \wedge dx^2,\ dx^1 \wedge dx^2,\ dx^0 \wedge dx^1 \wedge dx^2.$$

Designando genericamente por dx^H os diferenciais elementares e seus produtos, definimos o comutador "de Clifford" entre qualquer par destes objetos como

$$[dx^K, dx^L]_\vee = dx^K \vee dx^L - dx^L \vee dx^K.$$

Aplicando esta definição, temos os seguintes comutadores não nulos:

$$
\begin{aligned}
&[dx^0, dx^s]_\vee = 2dx^0 \wedge dx^s \quad (s = 1,2)\\
&[dx^0, dx^0 \wedge dx^s]_\vee = 2dx^s \quad (s = 1,2)\\
&[dx^1, dx^2]_\vee = 2dx^1 \wedge dx^2\\
&[dx^s, dx^0 \wedge dx^s]_\vee = 2dx^0 \quad (s = 1,2)\\
&[dx^1, dx^1 \wedge dx^2]_\vee = -2dx^2\\
&[dx^2, dx^0 \wedge dx^2]_\vee = 2dx^0\\
&[dx^2, dx^1 \wedge dx^2]_\vee = 2dx^1\\
&[dx^0 \wedge dx^1, dx^0 \wedge dx^2]_\vee = -2dx^1 \wedge dx^2\\
&[dx^0 \wedge dx^1, dx^1 \wedge dx^2]_\vee = -2dx^0 \wedge dx^2\\
&[dx^0 \wedge dx^2, dx^1 \wedge dx^2]_\vee = 2dx^0 \wedge dx^1.
\end{aligned}
$$

A forma de volume comuta com todos os outros.

Consideremos como exemplo ilustrativo:

$$X_1 = \frac{1}{2}dx^0, \quad X_2 = \frac{1}{2}idx^1, \quad X_3 = \frac{1}{2}dx^0 \wedge dx^1,$$

para os quais temos

$$[X_k, X_\ell]_\vee = i\epsilon_{k\ell m} X_m.$$

Temos também que os duais de Hodge [4] dos X_k acima,

$$Y_1 = \frac{1}{2}idx^1 \wedge dx^2, \quad Y_2 = -\frac{1}{2}dx^0 \wedge dx^2, \quad Y_3 = \frac{1}{2}idx^2,$$

satisfazem

$$[Y_k, Y_\ell]_\vee = i\epsilon_{k\ell m} X_m$$
$$[X_k, Y_\ell]_\vee = i\epsilon_{k\ell m} Y_m.$$

Definimos, então,

$$W_k^+ = \frac{1}{2}(X_k + Y_k), \quad W_k^- = \frac{1}{2}(X_k - Y_k)$$

e temos, portanto,

$$[W_k^\pm, W_\ell^\pm]_\vee = i\epsilon_{k\ell m} W_m^\pm, \quad [W_k^+, W_\ell^-]_\vee = 0,$$

ou seja, estes objetos constituem geradores de uma álgebra $SU(2) \times SU(2)$. Para os X_k, podemos escolher à vontade qualquer par de 1- ou 2-formas e seu produto exterior. É válida a seguinte propriedade:

$$*W_k^\pm = \pm i W_k^\mp, \quad *W_3^\pm = \mp i W^\mp,$$

sendo $*$ o operador de dualidade de Hodge.

Se representados por matrizes, os geradores da álgebra $SU(2) \times SU(2)$ são escritos da seguinte maneira:

$$W_k^+ = \begin{pmatrix} \sigma_k & 0 \\ 0 & 0 \end{pmatrix}, \quad W_k^- = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_k \end{pmatrix}.$$

Com isto e mais o isomorfismo de Graf $\gamma^\mu \leftrightarrow dx^\mu \vee$, podemos reconstruir as matrizes γ de Dirac. Algumas "imagens", com $X_1 \leftrightarrow \gamma^0$, $X_2 \leftrightarrow \gamma^1$, $X_3 \leftrightarrow \gamma^2$, são exemplificadas a seguir:

Dirac-Pauli:

$$\gamma^0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \quad \gamma^1 = -i\begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix}, \quad \gamma^2 = -i\begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}$$

Kramers-Weyl:

$$\gamma^0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \quad \gamma^1 = -i\begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix}, \quad \gamma^2 = -i\begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}.$$

Note-se a diferença de sinais dos blocos. Isto é devido ao fato de que, independentemente da imagem,

$$i\gamma^0\gamma^1\gamma^2 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

onde I é a matriz identidade 2×2. Assim, as representações corretas das matrizes γ são matrizes 4×4, diagonais em blocos 2×2, as quais diferem das utilizadas usualmente para a representação 4×4 de espinores em três dimensões.

Cada bloco de $SU(2)$ identifica uma quiralidade. Assim, os W_k^+ correspondem aos estados em que a rotação do spin é dextrógira e os W_k^- aos de rotação levógira.

Para terminar, temos as expressões das matrizes responsáveis pelas transformações correspondentes à inversão de um eixo coordenado e à reversão temporal [5]:

Dirac–Pauli:

$$P_{(1)}=\begin{pmatrix}0 & \pm\sigma_3\\ \sigma_3 & 0\end{pmatrix},\quad P_{(2)}=\begin{pmatrix}0 & \pm I\\ I & 0\end{pmatrix},\quad T=\begin{pmatrix}0 & \sigma_1\\ \pm\sigma_1 & 0\end{pmatrix}$$

Kramers–Weyl:

$$P_{(1)}=-i\begin{pmatrix}0 & \pm\sigma_2\\ \sigma_2 & 0\end{pmatrix},\quad P_{(2)}=\begin{pmatrix}0 & \pm\sigma_1\\ \sigma_1 & 0\end{pmatrix},\quad T=\begin{pmatrix}0 & I\\ \pm I & 0\end{pmatrix}.$$

Hagen [6] argumentou que a caracterização correta das invariâncias por transformações discretas em três dimensões é crucial para a interpretação em termos de "anyons" das teorias invariantes de gauge tipo Chern–Simons em interação com espinores.

A operação de conjugação de carga não mistura os blocos de $SU(2)$. A operação composta $CP_{(k)}T$ dá, em geral, uma matriz diagonal em blocos, mas cada bloco é não diagonal. Há uma consistência entre o eixo que é invertido e a matriz que aparece no resultado de CPT.

Todas as operações representadas pela ação de uma matriz γ^μ podem ser igualmente descritas por matrizes isomorfas à forma dual de dx^μ.

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A RELAÇÃO ENTRE A EQUAÇÃO DE DIRAC E AS ALGEBRAS DE GRUPOS UNITÁRIOS PARA QUALQUER DIMENSÃO DO ESPAÇO-TEMPO

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Damos a demonstração geral para dimensões pares e ímpares que o anel de Dirac Formado pelas matrizes de Dirac e os seus produtos desenvolve a algebra de comutação do grupo $SU(2^{D/2})$ para as dimensões pares e do grupo $SU(2^{\frac{D-1}{2}}) \times SU(2^{\frac{D-1}{2}})$ para as dimensões ímpares. Discutimos as eventuais conseqüências físicas destes resultados.

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BUBBLES IN THE EARLY UNIVERSE[1]

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ABSTRACT We analyse bubble formation as a result of thermal fluctuations. Bubbles appear whenever there is phase coexistence in the Universe. We have shown how the droplet model of phase transition allows us to determine the radius of the most favorable bubbles and their density contrast. In the case of the $SU(5)$ model we got Zel'dovich spectrum with the proper order of magnitude as well as other interesting consequences to cosmology.

INTRODUCTION

Spontaneous symmetry breakdown seems to play an essential role in formulating theories of the fundamental interactions. Grand Unified theories are based on the idea that at very short distances the fundamental interactions can be described by a theory based on a larger than the standard $SU(3) \times SU(2) \times U(1)$ gauge group G whose symmetry is spontaneously broken at large distances.

If at zero temperature the symmetry is spontaneously broken, then there will be symmetry restoration at high temperatures. The system will then exhibits two phases. Since the Universe started at very high temperatures (in the symmetric phase) one then expects that during the course of its evolution the Universe went through a series of cosmological phase transitions.

The understanding of the dynamics of phase transitions might be relevant in the solution of cosmological problems such as the flatness, horizon, cosmological constant and the large scale structure of the Universe.

In the context of the large scale structure of the Universe cosmological phase transition might play an important role, since the appearance of inhomogeneities (defects) in the system is a common feature of theories whose symmetries are spontaneously broken. In fact, there are suggestions that topological defects such as strings and domain walls generates the required contrast density for giving rise to the observed structures in the Universe.

The formation of bubbles (or droplets) is a feature of systems that exhibits phase coexistence along the phase transition. The approach that we have used (the droplet picture of phase transition (Langer 1967, Gunton, San Miguel and Sahni 1983, Marques and Ramos 1991) has been developed for dealing with bubble formation in phase transitions that are very familiar to physicists. One of our motivations for dealing with this problem, and its role in cosmology, is

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the similarity of the observed geometrical structures and the ones formed along some phase transitions.

FIELD THEORETICAL DESCRIPTION OF CONDENSATION AND COSMOLOGICAL APPLICATIONS

We will show how the evaluation of the partition function for a collection of noninteracting droplets may lead to the thermodynamic properties of a condensing system and the derivation of macroscopic features of a two phase system. This is so called condensation problem (Langer 1967).

The droplet model pictures the system as a *"dilute gas"* of small droplets of radius R. The number of bubbles of size R might be approximated by a simple Boltzman factor, that is

$$N(R) \sim \exp\{-\beta \Delta F^{(1)}(R)\} \tag{1}$$

where $\Delta F^{(1)}(R)$ is the energy cost for introducing a single bubble in the system.

The cost in energy for introducing an interface in the system can be defined as (Marques and Ramos 1991)

$$\Delta F^{(1)} = F(\phi_B) - F(\phi_v) = -\beta^{-1} \ln \left[\frac{Z(\phi_B)}{Z(\phi_v)}\right] \tag{2}$$

For spherical bubbles of radius $R, \Delta F$ is a function for R, and one can write $\Delta F \equiv \Delta F(R)$.

Only bubbles whose size R is above a critical value R_{cr} are stable and they survive in the system. This critical value is given by the condition

$$\left.\frac{d\Delta F^{(1)}(R)}{dR}\right|_{R = R_{cr}} = 0 \ . \tag{3}$$

Bubbles with radius smaller than R_{cr} are unstable and disappear again. These bubbles are assumed to be macroscopic objects.

The value $R = R_{cr}$ determined by (3) corresponds to the limit beyond which large quantities of the new phase begin to be formed. Bubbles beyond the critical range (with $R > R_{cr}$) will inevitably develop into a new phase.

We will assume that the distribution of bubbles is a dilute one. Under these circumstances one can write the partition function Z as

$$\frac{Z}{Z^{(0)}} = \exp\left[\frac{Z^{(1)}}{Z^{(0)}}\right] \simeq e^{-\beta \Delta F^{(1)}} \tag{4}$$

where $Z^{(0)}$ now stands for the partition function at the vacuum field configuration, ϕ_v, and $Z^{(1)}$ at the bubble field configuration ϕ_B.

In the high temperature limit and considering spherical bubbles one can find a general form to F which is given by (Marques and Ramos 1991)

$$F = -T\left[\frac{-4\pi/3\,R^3\,\Delta\Gamma + 4\pi\,R^2\,\sigma(0)}{2\pi T}\right]^{3/2}\left(\frac{1}{\beta}\right)^3 \times$$
$$\times \exp\left[\frac{4\pi/3\,R^3\,\Delta\Gamma(T) - 4\pi\,R^2\,\sigma(T)}{T}\right] \tag{5}$$

$\Delta\Gamma$ in (5) is the energy difference between the two vacua (Carvalho and Marques 1986) (cosmological constant), $\sigma(T)$ is the surface tension and R is the bubble radius.

Let one takes the vacua as a degenerate one, i.e., $\Delta\Gamma$ in (5) is equal zero. In this case, (5) becomes

$$F = -T^4\left[\frac{4\pi\,R^2\,\sigma(0)}{2\pi T}\right]^{3/2}\exp\left[\frac{-4\pi\,R^2\,\sigma(T)}{T}\right] . \tag{6}$$

The critical radius of the bubble can be obtained by minimizing the free energy (6) and one obtains

$$R_{cr}^2(T) = \frac{3T}{8\pi\,\sigma(T)} . \tag{7}$$

From this expression for $R_{cr}(T)$, one can see that for $T = T_c$ the bubble radius becomes infinite.

Within the dilute gas approximation the average number of bubbles is (Gross, Pisarski and Yaffe 1981)

$$N(T) = \frac{Z^{(1)}}{Z^{(0)}} . \tag{8}$$

With (9) and (7) one finds for the bubble density the expression

$$\rho_{\rm bubble} = \frac{(1+\sqrt{3\pi})}{4}\left(\frac{3}{4\pi e}\right)^{3/2}\left(\frac{\sigma(0)}{\sigma(T)}\right)^{5/2} T^4 \tag{9}$$

where one uses the expression (7) for R_{cr} in (6).

The contrast density associated to bubbles is defined as

$$\frac{\delta\rho}{\rho} = \frac{\rho_{\rm bubbles}}{\rho_{\rm elem.part.} + \rho_{\rm bubbles}} \tag{10}$$

where $\rho_{\rm elem.part.}$ is the energy density associated to the elementary particles and it can be written in terms of the number of degrees of freedom fermionic (N_F) and bosonic (N_B) as

$$\rho_{\rm elem.\,part.} = \frac{\pi^2}{30}\left(N_B + \frac{7}{8}N_F\right)T^4 . \tag{11}$$

From the expressions above one can see that all one need to know is the form of $\sigma(T)$, the surface tension, to determine all the quantities of interest.

One can write $\sigma(T)$, in the one loop order and in the high temperature approximation, in the general form given by (Marques and Ramos 1991)

$$\sigma(T) = \sigma(0)\left(1 - \frac{T^2}{T_c^2}\right) \tag{12}$$

where $\sigma(0)$ and T_c depends on the parameters (masses and coupling constants) of the model.

From (9) and (11) one can write the contrast density (10), by taking $\sigma(T)$ given by (12), as

$$\frac{\delta\rho}{\rho} = \frac{1}{1 + \pi^2/30\,(N_B + 7/8\,N_F)\,\frac{4}{(1+\sqrt{3\pi})}\left(\frac{4\pi e}{3}\right)^{3/2}\left(1 - \frac{T^2}{T_c^2}\right)^{5/2}} \ . \tag{13}$$

Furthermore, taking $T < T_c$, one obtains the simple result

$$\frac{\delta\rho}{\rho} \simeq \frac{1}{1 + 25/2\,(N_B + 7/8\,N_F)} \tag{14}$$

which is completely general and leading to a contrast density depending only upon the number of particles in the model.

In the minimal $SU(5)$ model, $N_B + 7/8\,N_F = 160,75$, so that for $T \sim T_c/3$ one gets

$$\frac{\delta\rho}{\rho} \sim 6 \cdot 10^{-4} \ . \tag{15}$$

This result is compatible with the bounds imposed by the anisotropy of the background radiation ($\delta\rho/\rho$ satisfy Zel'dovich's condition) (Zel'dovich 1972; Harrison 1970).

Let us analyze if the length of fluctuations is larger than the Jeans length. The lenght of fluctuations that we propose here is essentially the distance between two bubbles. Unfortunately we are not able to compute this distance, by using thermodynamical arguments, for the range of temperatures covering the critical temperature (10^{15} GeV) until recombination (1 eV). We can do this however, for temperature close to the critical one. For this range of temperatures, one has that if the average number of bubbles is given by (9) with $R = R_{cr}$. This density will be given by $\bar{n} = N(T, R_{cr})/V$ where $N(T)$ is given by (8).

If one assume further that the bubbles are uniformly distributed over the space the (average) distance between two bubbles (theirs centers) will be given by

$$d = \frac{1}{\sqrt[3]{\bar{n}}} \ . \tag{16}$$

For $T \simeq T_c/3$ ($T_c \sim 10^{15}$ GeV) one gets the $SU(5)$ model

$$d^{\rm GUT} \sim 8.2 \times 10^{-5}\,{\rm GeV}^{-1} \simeq 10^{-28}\,{\rm cm} \quad . \tag{17}$$

In order to estimate the lenght of fluctuation in the recombination era, one just makes the hypothesis that the distances between bubbles (λ^B) expands conformally, that is, the ratio between this distance and the horizon distance is constant. Consequently at any time one has

$$\lambda^B(T) = \frac{d^{\rm GUT}}{d_H(0, t_{\rm GUT})}\, d_H(0,t) \quad . \tag{18}$$

So that during the recombination ($t = t_R$) one has, by using (17)

$$\lambda^B(T \simeq 1\,{\rm eV}) = \frac{d_0^{\rm GUT}}{d_H(0.2 \times 10^{-37}\,s)}\, d_H(0,t_R) \sim 1.2 \times 10^{21}\,{\rm cm} \quad . \tag{19}$$

Since the Jeans length at recombination is

$$\lambda_J(t_R) \simeq 2.9 \times 10^{19}\,{\rm cm} \tag{20}$$

it follows from (19) that $\lambda^B > \lambda_J$.

The mass associated to the distance (20) is

$$M^{\rm bubb.} = \frac{4\pi}{3}\, \rho_{\rm rec}(\lambda^B)^3 \sim 10^{10}\, M_\odot \quad . \tag{21}$$

which fits very well in the galactical mass spectrum and is probably consistent with all of them if the dynamics of the bubbles below T_c is considered.

A legitimate conclusion would be that the number of aglutination centers is roughly the number of great structures observed in the Universe today. In fact, one can estimate the number of aglutination centers. This number is roughly given by

$$n^{\rm bubb.}_{\rm agl.cent.} \simeq \left[\frac{d_H(0, t^{\rm GUT})}{d_0^{\rm GUT}}\right]^3 \simeq 1.9 \times 10^6 \quad . \tag{22}$$

The greatest known strucures are the superclusters of galaxies that consist of groups with an average of 10^5 galaxies, that have densites close to critical $\rho_c \sim 10^{-29}\, g\,{\rm cm}^{-3}$ and spread over dimensions from 50 to 100 Mpcs (from 1.5 to 3.0×10^{26} cm). The number of these structures (sub-clusters) may be estimated by the ratio

$$n_{sc} \simeq \left[\frac{d_H(0, t_p)}{d_{sc}}\right]^3 \simeq 7 \cdot 10^5 - 6 \cdot 10^6 \tag{23}$$

because $t_p \sim 10^{10}$ years and $d_H(0,t_p) = 3t_p \simeq 2.7 \times 10^{18}$ cm.

We see that the results from (22) and (23) are quite close to each other.

CONCLUSIONS

Bubbles might appear in cosmological phase transitions for theories with nondegenerate or degenerate vacua. In both cases one can predict phase coexistence in the Universe and the appearance of bubbles as a result of thermal fluctuations. The basic ingredient for making relevant predictons to cosmology is the cost in energy to introduce such an object in the system.

As an application to cosmology we have analysed the GUT phase transition in the minimal $SU(5)$ model. In this application we have assumed that these bubbles survive until the recombination era. This is a dynamical problem that one has to solve in order to be sure that these objects act as seeds for structure formation.

Our simple estimates based only upon the interbubble distance indicates that one might get a surprinsigly good picture for the formation of structures in the Universe from the analysis of bubble formation in the early Universe.

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$SU(3) \otimes U(1)$ Model for Electroweak Interactions and neutrinoless double beta decay

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Abstract

We consider a gauge model based on a $SU(3) \otimes U(1)$ symmetry in which the lepton number is violated explicitly by charged scalar and gauge bosons, including a vector field with double electric charge. This model also produces $(\beta\beta)_{0\nu}$ with massless neutrinos.

Here we are concerned with a gauge model based on a $SU_L(3) \otimes U_N(1)$ symmetry. The model is anomaly free if we have equal number of triplets and antitriplets, counting the color of $SU(3)_c$, and furthermore requiring the sum of all fermion charges to vanish. The anomaly cancellation occurs for the three generations together and not generation by generation. The price we must pay is the introduction of exotic quarks, with electric charge 5/3 and −4/3. [1]

We start by choosing the following triplet representations for the left-handed fields of the first family, $(\mathbf{3},0)$: $(\nu_e, e, e^c)^T_L$ for the leptons, and $(u, d, J_1)^T_L$: $(\mathbf{3}, +\frac{2}{3})$ for the quarks. and the right fields in singlets. Notice that we have not introduced right-handed neutrinos. The numbers $0, 2/3$ and $2/3, -1/3$ and $5/3$ are $U_N(1)$ charges. The other two lepton generations also belong to triplet representations, and the second and third quarks generations belong to antitriplets.

In order to generate fermion masses, we introduce the following Higgs triplets, η : $(\eta^0, \eta_1^-, \eta_2^+)^T, (\mathbf{3}, 0)$, ρ : $(\rho^+, \rho^0, \rho^{++})^T, (\mathbf{3}, 1)$ and χ : $(\chi^-, \chi^{--}, \chi^0)^T, (\mathbf{3}, -1)$ and the sextet $(\mathbf{6}, 0)$ [1]

$$\begin{pmatrix} \sigma_1^0 & h_2^+ & h_1^- \\ h_2^+ & H_1^{++} & \sigma_2^0 \\ h_1^- & \sigma_2^0 & H_2^{--} \end{pmatrix} \tag{1}$$

These Higgs multiplets will produce the following hierarchical symmetry breaking

$$SU_L(3) \otimes U_N(1) \xrightarrow{<\chi>} SU_L(2) \otimes U_Y(1) \xrightarrow{<\rho,\eta>} U_{e.m}(1), \tag{2}$$

The Yukawa interactions with the leptons is

$$2\mathcal{L}_{H} = \sum_l [\bar{\nu}^c_{lL} \nu_L \sigma_1^0 + \bar{l}^c_L l_L H_1^{++} +$$

[1] We thanks R. Foot for calling our attention to this possibility.

$$
\begin{aligned}
&\bar{l}_R l^c_L H^{--}_2 + (\bar{\nu}^c_{lR} l_L + \bar{l}^c_R \nu_L) h^+_2 \\
&+ (\bar{\nu}^c_{lR} l^c_L + \bar{l}_R \nu_{lL}) h^-_1 + (\bar{l}^c_R l^c_L + \bar{l}_R l^c_L) \sigma^0_2.
\end{aligned} \tag{3}
$$

As $\langle\sigma^0_1\rangle = 0$ the neutrinos remains massless. For the first and second quark generations we have Yukawa interactions like $G_u(\bar{u}_L u_R \eta^0 + \bar{d}_L u_R \eta^-_1$, $G_d(\bar{u}_L d_R \rho^+ + \bar{d}_L d_R \rho^0$.

Here we will not write explicitly the physical gauge bosons, but only to mention that there are consistent with the usual relation between the mass of the lighter neutral gauge boson m_Z and that of the lighter charged one m_W: $m_Z/m_W = 1/c_W$ where c_W is the cosine of the Weinberg angle:

As the sextet does not couple to quarks it is not able to produce $(\beta\beta)_{0\nu}$ by itself. Notwhithstanding, by considering the most general potential involving the η triplet and the H sextet, it is possible to verify that the physical charged scalar are linear combination either of η^-_1-h^-_2 or η^-_2-h^-_1 both degenerates in mass. [2] This degeneration will be broken when we allow the coupling with the other two triplets, ρ and χ, with η. With the sextet they will anly coupled through the term $(1/2)\rho_i\chi_j H^{ij}$.

It is very well known that the observation of neutrinoless double beta decay, $(\beta\beta)_{0\nu}$, will imply a new physics beyond the standard model. Usually, two kinds of mechanisms for this decay were assumed to be independent: massive Majorana neutrinos and right-handed currents [3]. In both cases, the bosons exchanged in diagrams producing the decay are vector ones. Even for those models in which there are contributions of the scalars exchange, they are negligible [4].

Then, the Yukawa interactions with the scalars $h^-_{1,2}$ with leptons and

η_1^- with quarks in, allow all the couplings appearing in Fig. 1, for example, with the physical scalar ϕ_1^-. We can estimate a lower bound on the mass of ϕ_1, by assuming that its contribution to $(\beta\beta)_{0\nu}$ is less than the amplitude due to massive Majorana neutrinos and vector bosons W^- exchange. We obtain $m_\phi > 3\, GeV$.

We can see that $(\beta\beta)_{0\nu}$ proceeds in this model only as a Higgs bosons effect, with massless neutrinos at tree level. There are not contributions to $(\beta\beta)_{0\nu}$ from trilinear Higgs interactions because the charged leptons couple only to the η-like triplets, and those triplets do not contain doubly charged scalars. In models in which these contributions exist, they are negligible [4] unless a neighboring mass scale ($\sim 10^4 GeV$) exist [5].

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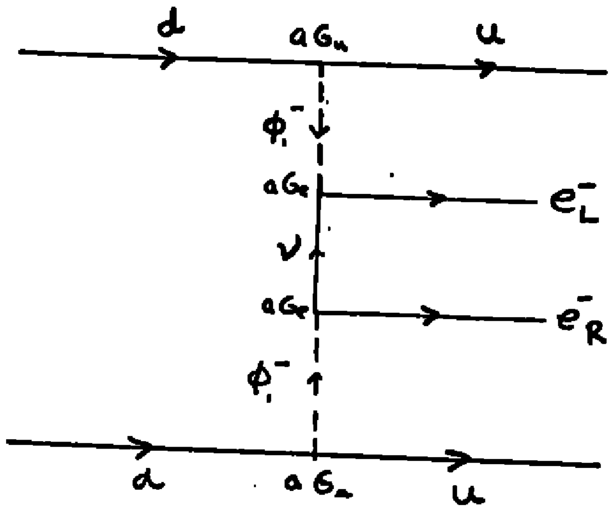
Fig.1 **Scalar contribution to the $(\beta\beta)_{0\nu}$, $G_{u,e}$ are Yukawa couplings and $a_{1,2}$ mixing parameters.**



Form Factors of the Charmed Meson Decays $D^+ \to \overline{K}^{0*} e^+ \nu$

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Abstract. As an attempt to explain discrepancies between experimental results and theoretical calculations on the ratios between longitudinal and transversal polarizations of the $\overline{K}^{0*}$ meson in the semileptonic D^+ decay, we evaluate the simplest hadronic corrections to the form factors. We show that the influence of these corrections is too small to account for the existing discrepancies.

1. Introduction and general method

Semileptonic decays of charmed and beautiful hadrons are a most important source of information on fundamental parameters in the Higgs-sector of the standard model of weak interactions. However, these parameters, which are elements of the Cabibbo-Kobayashi-Maskawa matrix, have to be extracted from the hadron decays taking into account the strong QCD-interaction confining the quarks inside hadrons. The main source of uncertainty in the results that can be thus obtained comes from the treatment given to non-perturbative QCD.

Quark model calculations[1–5] give in general a reasonably adequate description of non-perturbative effects despite the rather crude way of achieving chiral symmetry breaking through non-zero constituent quark masses. However, in the case of the semileptonic decays of charmed mesons into vector mesons $D^+ \to \overline{K}^{*0} e^+ \nu$ there is a rather poor agreement between quark model calculations and experimental results[6–9], concerning both total rates and polarizations. A detailed study of this decay in the framework of QCD-sum rules[10,11] shows better agreement with the data, except for the ratio of the longitudinal to the transversal polarizations of the $\overline{K}^*$-meson. The central experimental value for this ratio is about twice as large as the theoretical result.

It was already pointed out[10,11] that within the sum rule approach a large ratio for the longitudinal over the transversal polarization cannot be obtained and that therefore, in case of confirmation of the present experimental value, one has to look beyond the simple quark levels for sources of the disagreement. The clarification of this question is of great importance, not only for our understanding of the mesons systems consisting of a charmed and a light quark, but also in view of the determination of the weak matrix elements involving bottom quarks.

It was suggested in ref. 10 that hadronic corrections might influence the semileptonic decays and be the source for the discrepancy between theory and experiment. In fig. 1 we show the dominant diagram providing such a correction. In the present work its contribution is evaluated and compared to the simple quark results of ref. 11.

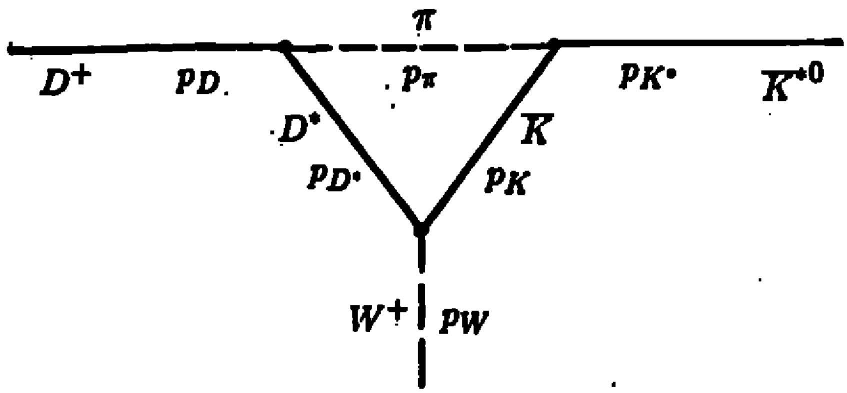

Fig. 1. Dominant hadronic contribution to the semileptonic decay $D^+ \to \overline{K}^{0*} e^+ \nu$, whose contributions to form factors are calculated in the present work.

The calculation of the diagram of fig. 1 must take into account that hadrons are not elementary particles with pointlike couplings, and therefore internal contributions of high virtuality must be supressed. This is done in our calculation in the following way. We represent the hadronic contributions to the different form factors by double dispersion integrals

$$F(t) = \int_{s_1}^{s_m} \frac{ds}{s - m_D^2} \int_{u_1}^{u_m} \frac{du}{u - m_{K^*}^2} \, \rho_{FF}(s, u, t) \tag{1}$$

where the index FF specifies each of the form factors. The quantities s_1 and u_1 are the kinematical thresholds. The double spectral function can be obtained from diagram (1) by using the Cutcosky rules, putting the internal lines on mass shell, so that only on-mass-shell vertex functions occur in the determination of the double spectral function. The internal contributions with high virtuality are then suppressed through cut-offs s_m and u_m in the s and u-integrations.

The cutoffs s_m and u_m are determined by the hadronic excitation energies, since above them the single diagram (1) is no longer representative. We have thus parametrized s_m and u_m as

$$s_m = (m_D + \Delta)^2 \; ; \; u_m = (m_{K^*} + \Delta)^2 \tag{2}$$

where Δ is conservatively varied in the range $0.6 \leq \Delta \leq 2$ GeV.

With a cutoff value s_m the above mentioned non-Landau singularities occur only for $t \gtrsim 4m_{D^*}^2/s_m$.

The point m_D^2 is always outside the integration interval of s, but $m_{K^*}^2$ is not always outside that of u. We therefore take into acocunt the finite width of the K^* meson, through the replacement

$$\frac{1}{u - m_{K^*}^2 + i\epsilon} \to \frac{u - m_{K^*}^2 - im_{K^*}\Gamma_{K^*}}{(u - m_{K^*}^2)^2 + (m_{K^*}\Gamma_{K^*})^2} \tag{3}$$

where Γ_{K^*} is the full width of the K^*-meson. The imaginary part of the hadronic corrections obtained in this way can be viewed as a consequence of a final state interaction.

2. Results

In the table we display the effect on measured quantities due to the hadronic corrections, for cutoff values (in GeV^2 units) given by $(s_m; u_m) = (6.0; 1.7), (8.2; 3.6)$ and $(15.0; 8.35)$. The results of ref. 11 have to be multiplied by the entries of that table in order to include the effects of hadronic corrections. We observe that for the case $(6.0; 1.7)$ the effects are completely negligible, and that even for the extreme large cutoff $s_m = 15$ GeV^2 the effect of the hadronic corrections on the longitudinal to transversal polarization ratio is smaller than 20%. The main influence is on the decay rate, which is increased by 77%. The effect of the imaginary part is always completely negligible.

Table. Multiplicative factors representing the influence of the hadronic corrections on the polarization ratios and on the total decay rate. As in the table, s_m and u_m represent the cutoff values in units of GeV^2.

R_{LT}: Ratio of the longitudinal over transversal ratio including hadronic corrections, divided by the corresponding uncorrected quantity;

$R_{+/-}$: Analogous ratio for the relation between positive and negative helicities;

R_{tot}: Ratio between the total semileptonic decay rates including and not including hadronic corrections.

$(s_m; u_m)$	R_{LT}	$R_{+/-}$	R_{tot}
(6;1.7)	1.01	1.01	1.02
(8.2;3.6)	1.04	1.10	1.15
(15;8.35)	1.19	1.49	1.77

Summarizing we remark that hadronic corrections cannot explain the existing discrepancy between the central value of the experimental results for the $D^+ \to \overline{K}^{0*} e^+ \nu$ decay and the theoretical result obtained from sum rules for the ratio of the longitudinal to the transversal polarization. However, it can be remarked that the corrections obtained slightly move the polarization ratios towards a better agreement with experiment.

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ESTUDO DA DISTRIBUIÇÃO LATERAL DA CASCATA NUCLEÔNICA INDUZIDA POR UM ÚNICO NUCLEON NA ATMOSFERA

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Em recente artigo [1], estudamos o comportamento da cascata nucleônica induzida por um único nucleon, que interage com a atmosfera numa profundidade $t_o (g/cm^2)$ com energia E_o (arbitrariamente escolhida como sendo $E_o = 10^4 TeV$), usando as soluções da equação de difusão tridimensional, obtidas pelo método de ordenação de operadores exponenciais de Feynman [2,3]. Apresentamos aqui somente a discussão de alguns aspectos importantes do comportamento lateral da cascata: dependência com a energia; com a altura de interação; com a dispersão lateral e com a quantidade média de matéria atravessada pela cascata.

A análise da solução é feita com base nas seguintes variáveis: a) E e t, energia e profundidade de detecção; b) a distância r no plano perpendicular à direção da cascata com relação ao seu centro; c) $T = (t - t_o)/\lambda_N$, altura em que ocorre a interação com relação ao detetor, em unidades de livre caminho médio de interação nucleônico; d) $T_f = t/\lambda_N$, nível observacional em unidades de λ_N - nos cálculos usamos $T_f = 6,75$, nível de observação de Chacaltaya, Bolívia ($540 g/cm^2$); e) parâmetro de dispersão lateral $\alpha = Er/p_T H_o$, sendo p_T o momento transversal transferido na colisão, suposto aqui constante e igual ao seu valor médio e H_o um fator de escala, definido pelo modelo isotérmico para a densidade atmosférica.

A componente lateral F_N^L, em função de α, está representada na Figura 1 (multiplicada pela quantidade $p_T^2 H_o^2$), indicando que para cada valor de T existe um alcance máximo ao desenvolvimento lateral, determinado por α_{max}, calculado como sendo

$$\alpha_{max} = -\ln(1 - T/T_f).$$

O comportamento de F_N^L em função da energia E, para diferentes alturas de interação, indica que para eventos iniciados nas proximidades do ponto de medida (T pequeno), o fluxo apresenta um espectro mais rico em energias "altas" ($> 10^3 TeV$) e mais pobre em energias "baixas" ($< 10^2 TeV$) do que para eventos iniciados muito acima do ponto de observação, quando então o desenvolvimento da cascata já está bastante avançado. Com isto, as curvas para diferentes valores de T se cruzam, como mostra a Figura 2, na qual assumimos $\alpha = 0,1$.

O fluxo lateral em função de T demonstra, como era de se esperar, uma intensidade tanto maior quanto mais próximo do centro é realizada a medida (Figura 3, com $E = 100 TeV$). Também é de se notar a presença de um ponto de máximo no fluxo a uma determinada profundidade. Este resultado é analogo ao que ocorre no desenvolvimento da cascata eletromagnética, para a qual existe um ponto crítico na produção de pares e fótons [4].

Uma característica da componente lateral é a dependência com a profundidade do nível de observação T_f. Como exemplo, mostramos na Figura 4 as modificações introduzidas nas curvas de F_N^L em função de T, para diferentes valores de T_f: 6,75 (Chacaltaya, Bolívia); 8,125 (Mt. Fuji, Japão); 10,625 (Gran Sasso, Itália) e 12,875 (nível do mar). Esta dependência é introduzida nos cálculos como consequência da variação da densidade atmosférica ao longo do percurso desenvolvido pela cascata.

Outro fator importante na análise é a dispersão lateral quadrática média, $< \alpha^2 >$. A sua raiz quadrada está relacionada com a largura média da distribuição lateral da cascata e apresenta o mesmo comportamento que o valor de α_{max}, na medida em que se varia T (Figura 5).

A dependência de $< \alpha^2 >$ com a energia é muito fraca, mesmo para diversas ordens de grandeza em E. Este resultado pode ser comparado com aquele obtido por A. Ohsawa e S. Yamashita [5], que resolveram a equação de difusão nucleónica seguindo um método de cálculo bem diverso. A Figura 6 apresenta a variação de $< \alpha^2 >$ com a razão E_o/E, tomando por base o nível de observação $T_f = 6,00$, reunindo resultados da Ref.[5] e os deste trabalho, para 3 diferentes altitudes de interação T. A coincidência é completa e as curvas se superpõe.

Apesar de não existirem dados experimentais só para a cascata nucleónica, esta análise é importante para o estudo do comportamento da cascata hadrónica (nucleons + pions). O conhecimento da solução da equação de difusão para a parte nucleónica da cascata permite resolver de forma completa o sistema de equações diferenciais acopladas que descreve a cascata hadrónica [6], a partir da qual pode-se proceder à análise dos dados experimentais em radiação cósmica.

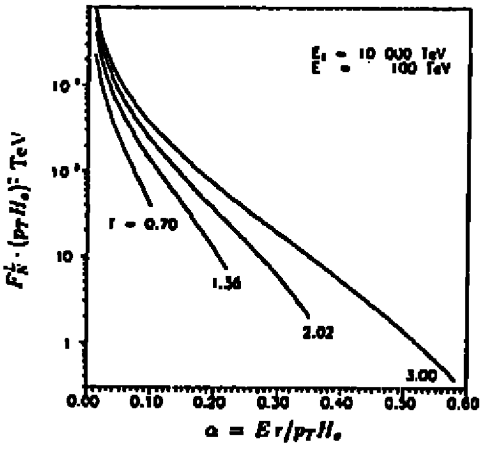

Figura 1. Comportamento de $F_N^L \cdot (p_T H_o)^2$ em função da dispersão lateral α, para diversos valores de T (com $E = 100 TeV$).

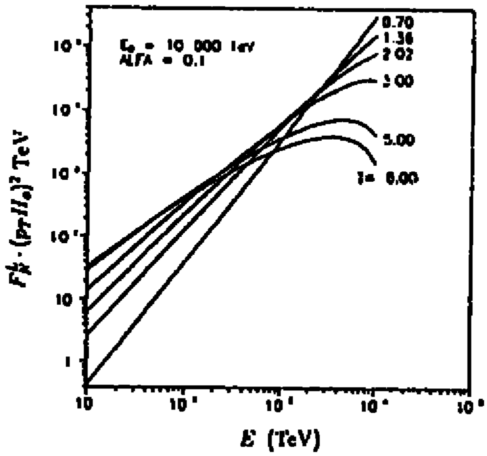

Figura 2. Comportamento de $F_N^L (p_T H_o)^2$ em função da energia E, para diversos valores de T (com $\alpha = 0.1$).

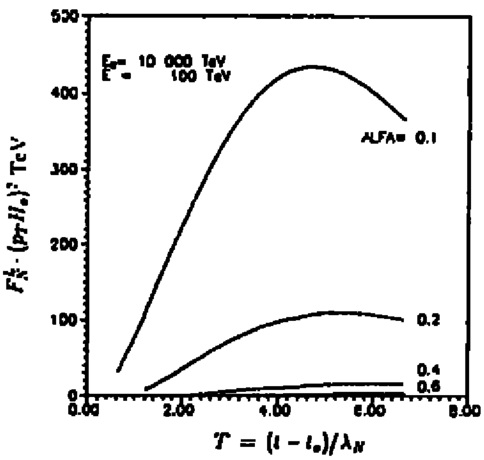

Figura 3. Comportamento de $F_N^L \cdot (p_T H_o)^2$ em função da altura de interação T, para diversos valores de α (com $E = 100 TeV$).

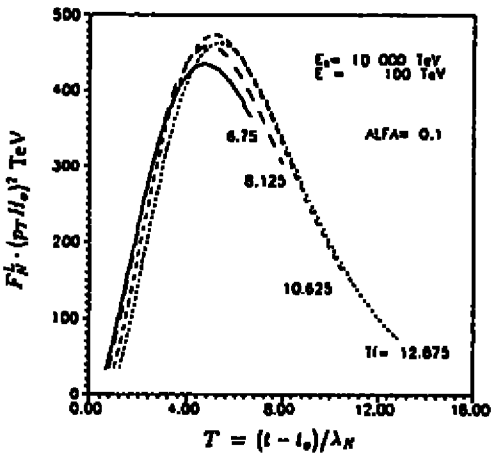

Figura 4. Comportamento de $F_N^L \cdot (p_T H_o)^2$ em função de T, para diversos valores do nível de observação T_f (com $E = 100 TeV, \alpha = 0.1$).

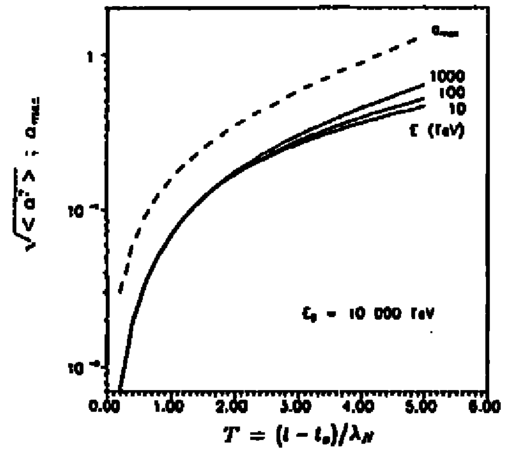

Figura 5. Variação de $\sqrt{<\alpha^2>}$ e de α_{max} em função da altitude de interação T, para diversos valores da energia de limiar E.

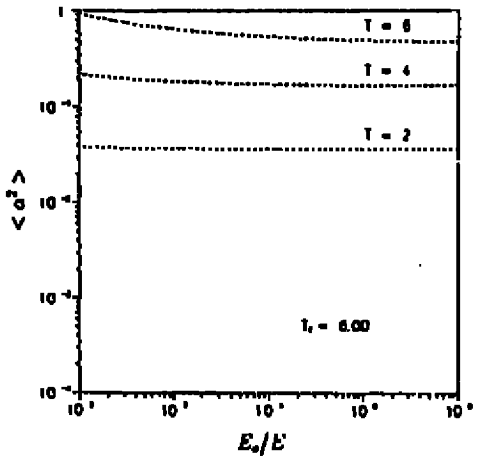

Figura 6. Variação de $<\alpha^2>$ em função da razão E_0/E para 3 valores de T (usando T_f = 6.00). Os cálculos da Ref.[5] e os deste trabalho fornecem os mesmos resultados.

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SKYRMIONS NÃO VIBRANTES E VIBRANTES *

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Abstract

A instabilidade da solução estática do modelo σ não linear levou Skyrme a adicionar uma interação quártica ao lagrangeano. A obtenção de soluções solitônicas no modelo σ não linear continua sob análise na literatura, agora ao nivel quântico, sobretudo com propostas de quantização de grau de liberdade de vibração. Comparamos no presente trabalho os hamiltonianos de algumas destas formulações considerando, também, a interação quártica simétrica incluída em uma generalização do modelo de Skyrme.

1 INTRODUÇÃO

Ao sugerir que os núcleons podem ser descritos por sólitons topológicos de uma teoria de campos mesônicos, Skyrme[1] recorre a uma configuração de campo estática, da forma "ouriço"

$$U_0 = e^{i\vec{\tau}\cdot\hat{r}F(r)} \ , \ U_0 \in SU(2) \ , \ U_0(r=\infty) = 1 \ , \tag{1}$$

com $F(\infty) = 0$ e $F(0) = \pi$ para número topológico $n = 1$.

Considerando que a solução clássica do modelo σ não linear puro não é estável ao colapso, Skyrme adiciona ao lagrangeano uma interação quártica que assegura a estabilidade do sóliton. Omitindo temporariamente o termo de massa:

$$\mathcal{L}_{sk} = Tr\left\{\frac{F_\pi^2}{16}(\partial_\mu U)(\partial^\mu U^\dagger) + \frac{1}{32e^2}[(\partial_\mu U)U^\dagger,(\partial_\nu U)U^\dagger]^2\right\} \tag{2}$$

sendo $F_\pi = 186$ MeV a constante de decaimento do píon e e^2, um parâmetro adimensional a ajustar.

A solução de Skyrme é um estado de "simetria máxima" para o qual uma rotação no espaço de isospin equivale a uma rotação espacial, e em que o ângulo quiral ou função perfil $F(r)$ é solução da equação (de Euler-Lagrange) que minimiza a massa estacionária, definindo um ponto estacionário da ação.

Adkins, Nappi e Witten[2], trataram as flutuações em torno da solução de Skyrme hierarquicamente em uma teoria de campos fracamente acoplados, quantizando os graus de liberdade coletivos rotacionais, e Hajduk e Schwesinger[3] incluíram o modo vibracional do skyrmion para estudar a ressonância Roper e

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isóbaros Δ. Aborda-se, também, na literatura a situação em que o lagrangeano do modelo de Skyrme é, desde o início, quântico[4].

Ademais, como predições sobre a interação NN pelo modelo de Skyrme apresentadas[5,6] evidenciam que o mesmo não descreve a interação atrativa a distâncias médias, é proposta a adição da interação quártica simétrica[7,8]

$$\mathcal{L}_{sim} = \frac{\gamma}{8e^2}\left\{Tr(\partial_\mu U \partial^\mu U^\dagger)\right\}^2 \tag{3}$$

ao lagrangeano de Skyrme, o que resolve em parte[8] tal problema. Como este termo contribui com sinal negativo à massa estática, os possíveis valores de γ são limitados superiormente. Os autores desprezam as derivadas temporais quárticas, por serem de ordem $(v^2/c^2)^2$ em relação à massa estática M.

2 SOLUÇÕES DO MODELO σ NÃO LINEAR

A existência de soluções estáveis para o modelo σ não linear é ainda alvo de discussão na literatura, face à possibilidade de estabilização quântica do sóliton. Carlson[9] incorpora, além dos de rotação, efeitos de vibração em uma teoria vinculada, onde o vínculo preserva a simetria quiral no limite de massa zero para o píon. Dividindo o conjunto de configurações de campo em classes de equivalência de campos que diferem por escalamento do sistema de coordenadas, a integral funcional

$$Z = \int dU \frac{d\lambda}{\lambda} \prod_t \delta\left(1 - \int G[U(\vec{r},t)]d^3r\right) e^{iS[U(\lambda^{-2/3}\vec{r},t)]} \tag{4}$$

inclue um campo representativo de cada classe (em δ), sendo G uma função local positiva da configuração de campo e $\lambda(t)$ a variável que promove a dilatação

$$U(\vec{r},t) \rightarrow U(\lambda^{-2/3}\vec{r},t). \tag{5}$$

Recorrendo ao lagrangeano do modelo σ não linear, ao campo U na forma "ouriço " e adotando como vínculo o termo de Skyrme, Carlson determina o ângulo quiral $F(r)$ que satisfaz $\delta[L - \rho G] = 0$.

A proposição de estabilização das soluções do modelo σ não linear por flutuações quânticas[10], com a introdução de uma variável coletiva vibracional adicional $R(t)$ à função perfil do campo U, é contestada na literatura[11,12], onde se ressalta a importância da presença de uma interação adicional estabilizadora, como o termo de Skyrme, mesmo ao nível quântico. Contudo, para a análise destas situações, são adotadas formas específicas para o ângulo quiral, que não são, obviamente, soluções clássicas das equações de movimento e que dependem de parâmetros clássicos.

Já em uma variante alternativa de estabilização das soluções do modelo pela introdução de um parâmetro de corte ϵ a pequenas distâncias[13] para o funcional de energia, e sua posterior quantização, o perfil $F(r)$ é determinado.

3 HAMILTONIANOS PARA SÓLITONS VIBRANTES E NÃO VIBRANTES

Considerem-se as duas situações seguintes:

I. Ao lagrangeano que inclui os termos do modelo σ não linear, de Skyrme e quártico simétrico, com quantização das variáveis coletivas rotacionais e vibracional para o campo

$$U = A(t)e^{i\vec{\tau}\cdot\hat{r}F(\rho)}A(t)^{\dagger} \quad , \rho = \frac{r}{R(t)} \tag{6}$$

e $A(t) = a_0 + \vec{a}\cdot\vec{\tau}$, incorpora-se a restrição nas variáveis a_μ de SU(2) via multiplicador de Lagrange.

II. Ao lagrangeano do modelo σ não linear, incorpora-se como vínculo a interação composta dos termos de Skyrme e quártico simétrico, via multiplicador de Lagrange.

Desprezando contribuições das derivadas temporais quárticas do último termo e promovendo a quantização covariante[14], após o estabelecimento dos vínculos das duas teorias (todos de 2ª classe) e a determinação dos parênteses de Dirac[15], obtém-se os Hamiltonianos quânticos gerais

$$H = -\frac{1}{2\sqrt{g}}\frac{\partial}{\partial q^{\mu}}g^{\mu\nu}\sqrt{g}\frac{\partial}{\partial q^{\nu}} + V(q) \tag{7}$$

sendo $g_{\mu\nu}(q)$ o tensor métrico, $g = det g_{\mu\nu}$ e q^μ as coordenadas generalizadas.

Como casos específicos, para as situações I e II acima, citam-se:
I.1 Termo quártico simétrico e modo vibracional ausentes: Hamiltoniano de Skyrme[1]; I.2 Modo vibracional ausente[7,8]; I.3 Termo quártico simétrico ausente[12]; I.4 Nenhum termo ausente.
II.1 Termo quártico simétrico ausente[9]; II.2 Nenhum termo ausente.

Aos hamiltonianos I.1 e I.2 correspondem sólitons não vibrantes e aos demais, sólitons vibrantes. De I.4 obtêm-se diretamente I.1, I.2 e I.3 eliminando o parâmetro γ e,ou o grau de liberdade vibracional. A forma funcional dos coeficientes, que são funções do ângulo quiral, nao se altera de um caso a outro, mas o ângulo quiral se modifica.

Os hamiltonianos II.1 e II.2, bem como os das situações III[10] e IV[13] são idênticos na forma diferindo, contudo, os coeficientes, à semelhança da situação I.

Na situação III não existe solução clássica (para o ângulo quiral)[16], sendo esta dificuldade evitada em IV pela introdução do parâmetro de corte ϵ.

A comparação entre I.3 e II.1 (I.4 e II.2) é particularmente interessante porque os coeficientes dependentes do termo de Skyrme (Skyrme mais quártico simétrico) não contribuem a II.1 (II.2) embora os termos contribuam à função perfil correspondente.

Se, para m_π=0, ajustarmos[2] a massa do núcleon a 938.9 MeV na ausência de modo vibracional (F_π e e compatíveis), obteremos para a situação I.3 o valor

m_N=1036 MeV e para I.4, 1045 MeV (γ = 0.11). Já para F_π=186 MeV, II.1 conduz[9] a m_N=1101 MeV e II.2 a 1152 MeV, enquanto o caso I gera valores mais elevados.

Considerações mais detalhadas, serão apresentadas em breve.

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OS SOLITONS DO MODELO DE SKYRME COM O TERMO DE MASSA DO PION

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Damos os desenvolvimentos analíticos na origem e no infinito para a solução da equação de Euler-Lagrange do modelo em questão, e estudamos em função dos parâmetros relevantes as soluções correspondentes a número bariônico inteiro. Mostramos que a inclusão do termo de massa muda a topologia, e o espaço de configuração não é mais uma tri-esfera.

Sobre o Conteudo Físico do Modelo de Skyrme SU(2)

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Este trabalho é um resumo muito comprimido de um outro trabalho onde analisamos, entre outros pontos, as soluções do Modelo de Skyrme na representação ouriço do $SU(2)$ como uma função dos parâmetros do modelo [1]. As regras de soma que obtemos a partir da equação diferencial para a função angulo quiral apresentam-se como ferramentas úteis nesta análise, e também usamos o teorema de Derrick [2] na parte que se refere a estabilidade das soluções sólitons (soluções regulares). Nós enfatizamos que a evolução das soluções é controlada por um parâmetro específico ϕ, que tem valores distintos $\phi_1, \phi_2, \phi_3 \cdots$ para soluções correspondentes a sólitons com diferentes números bariônicos $B = 1, 2, 3 \cdots$, respectivamente. É mostrado que um parâmetro dimensional aparece nas soluções regulares, na origem, e que pode ser tomado como a inclinação das curvas correspondentes às ditas soluçõesAlém disso, mostramos também que o parâmetro adimensional de Skyrme (ϵ) tem um papel peculiar pois ele explicita a instabilidade da solução sóliton clássico visto que, a este nível, não se tem como fixá lo, mesmo sabendo que ele é finito e está definido no intervalo $0 < \epsilon < \infty$. Por outro lado, a quantização do modelo através das coordenadas coletivas nos leva a uma expressão para a energia que é função do parâmetro de Skyrme ϵ que tem um mínimo estável bem definido para cada número bariônico inteiro. Desta maneira fixa-se o valor do parâmetro ϵ.

Neste trabalho obtemos, também, as massas dos sólitons em termos de números específicos dependentes do parâmetro ϕ, do momentum angular e da constante de decaimento do píon (f_π) o qual pode ser tido como um parâmetro livre, para ajustar as previsões do modelo com os dados experimentais ou entrar nos cálculos com o seu valor experimental (0.186 Gev). Mostramos, ainda, um bárion $B = 2, I = J = 1$ com massa quase duas vezes o valor da massa do nucleon $B = 1, I = J = 1/2$, e um estado $B = 3, I = J = 1/2$ que é mais leve que o primeiro acima citado.

No modelo de Skyrme, todos os bárions tornam-se cada vez mais leves em relação às massas conhecidas dos núcleos leves, a medida que B aumenta.

Na Tabela 1 apresentamos os valores do parâmetro ϕ correspondentes a sólitons com número $B = 1, 2, 3$. Nesta mesma tabela damos o valor de e no mínimo quântico (para cada valor de B) e a massa do sóliton nesse mínimo. O valor $f_\pi = 0.129$ Gev é aquele do trabalho de G. S. Adkins, C. R. Nappi e E. Witten [3] e foi colocado para efeito de comparação .

Na Tabela 2 damos as principais características do sóliton bariônico $B = 1$.

Na Figura 1 representamos algumas soluções da equação de movimento, como função do parâmetro ϕ. A solução regular é aquela que vai a zero no infinito.

Na Figura 2 damos alguns resultados obtidos através das regras de soma, como função de ϕ.

Na Figura 3 temos o espectro de massa, de acordo com a Tabela 1.

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ϕ	B	$a_1(\phi)$	$b_1(\phi)$	$i = j$	e	$f_\pi(Gev)$	$F_1(Gev/c)$	$M(Gev/c)$
1.00376...	1	11.6	51.01	1/2	7.67	0.129	0.993	0.817
						0.166	1.421	1.178
	1			3/2	5.13	0.129	0.664	1.222
						0.156	0.950	1.762
1.9630...	2	32.22	219.5	1	11.16	0.129	2.63	1.560
						0.166	4.08	2.250
2.5562...	3	58.1	563.9	1/2	20.02	0.129	7.600	1.500
						0.166	11.2	2.164
				3/2	16.00	0.129	5.21	2.24
						0.156	7.52	3.24

Table 1: Results for B=1, 2 and 3

Quantity	ANW $f_\pi' = 0.129$ Gev	This Work $f_\pi = 0.129$ Gev	This Work $f_\pi = 0.166$ Gev	Experiment
M_N	input	0.817	1.16	0.939 Gev
M_Δ	input	1.222	1.76	1.232 Gev
e	5.45*	7.67	7.67	---
$< r^2 >^{1/2}_{I=0}$	0.59 f_m	0.422 f_m	0.293 f_m	0.72 f_m
$< r^2 >^{1/2}_{M,I=0}$	0.92 f_m	0.65 f_m	0.45 f_m	0.81 f_m
μ_p	1.87	0.61	0.61	2.79
μ_n	−1.31	−0.16	−0.16	−1.91
$\left\lvert\frac{\mu_p}{\mu_n}\right\rvert$	1.43	5.25	5.25	1.46
$g_{I=0}$	1.11	1.36	1.36	1.76
$g_{I=1}$	6.35	2.0	2.0	9.4
g_A	0.61	0.307	0.307	1.23
$g_{\pi NN}$	8.9	3.69	3.69	13.5
$g_{\pi N\Delta}$	13.2	5.51	5.51	20.3
$\mu_{N\Delta}$	2.3	0.71	0.71	3.3
$F_1(Gev/c)$	0.7057	0.993	1.121	---
$K_1(Gev^{-1}/c^2)$	−31.95	−17.65	−6.459	---

Table 2: Results for the Nucleon Physical Parameters

* Obtained by fitting

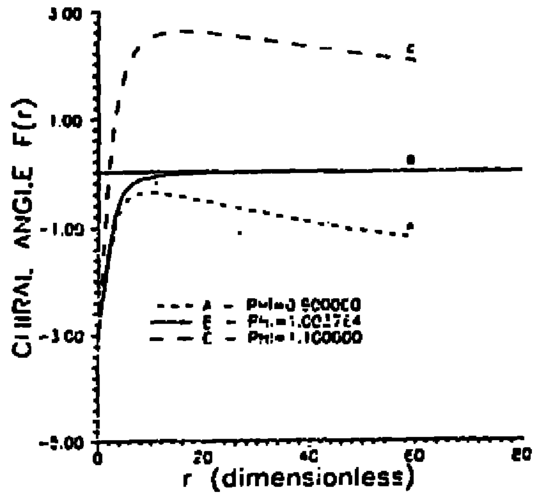

FIG. 1a.

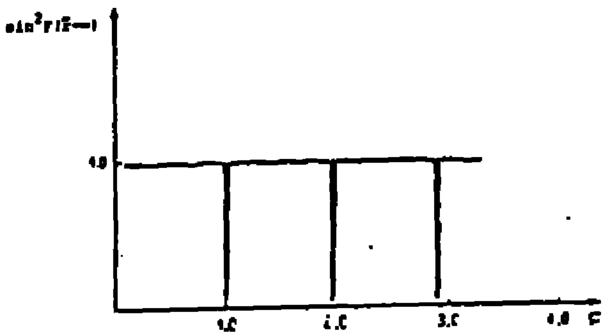

Fig.2

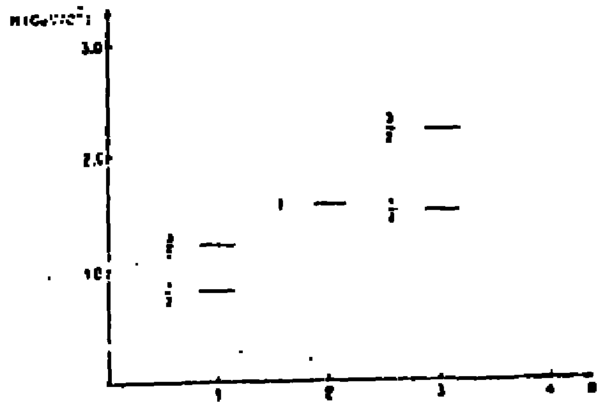

FIG.3

Massas hadrônicas num modelo com confinamento e simetria quiral *

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Resumo

Nesta comunicação apresentamos os resultados de um cálculo de massas de mésons pesados e núcleons e deltas usando um modelo de quarks com confinamento e simetria quiral (P.J.A. Bicudo, G. Krein, J.E.F.T. Ribeiro e J.E. Villate, aceito para publicação em Phys. Rev. D). As massas dos mésons são obtidas resolvendo a equação de Salpeter e as massas dos núcleons e deltas são obtidas usando um método variacional para a equação de Salpeter. Os resultados obtidos são muito bons.

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A cromodinâmica quântica (QCD) tem sido bem sucedida na análise de dados experimentais de sistemas hadrônicos a altas energias e grandes momenta transferidos. Este sucesso é devido principalmente a sua propriedade de liberdade assintótica, a qual permite o emprego da teoria de perturbações para tais processos. Por outro lado, fenômenos de baixos momenta tranferidos e baixas energias, tais como o confinamento da cor e a quebra dinâmica da simetria quiral, não foram ainda derivados da teoria. Estes fenômenos são eminentemente não-perturbativos, o que torna difícil seu estudo de maneira sistemática. Cálculos usando supercomputadores (teorias de calibre na rede) têm se mostrado promissores, mas ainda se encontram num estágio muito preliminar, e os resultados obtidos até o momento ainda não podem ser comparados com a experiência. Portanto, o emprego de modelos fenomenológicos, que incorporam algumas das características básicas da QCD, são a única alternativa disponível no momento para o estudo da QCD a baixas energias. Nesta communicação apresentamos o resultado de um cálculo das massas dos mésons pesados e dos núcleons e das deltas empregando um modelo que incorpora o confinamento da cor e que realiza a simetria quiral no modo de Nambu-Goldstone.

Aqui vamos apenas apresentar os elementos essenciais do cálculo, os detalhes podem ser encontrados na publicação que deverá aparecer em breve na Phys. Rev. D. O modelo está basendo no seguinte Hamiltoniano[1,2]

$$H = \int d^3x \left[H_0(\mathbf{x}) + H_I(\mathbf{x}) \right] \ , \tag{1}$$

onde H_0 é a densidade Hamiltoniana livre e H_I corresponde a uma interação efetiva:

$$\begin{aligned} H_0(\mathbf{x}) &= \psi^\dagger(\mathbf{x}) \left(m\beta - i\vec{\alpha}\cdot\vec{\nabla} \right) \psi(\mathbf{x}) \, , & (2) \\ H_I(\mathbf{x}) &= \frac{1}{2} \int d^3y \, V(\mathbf{x}-\mathbf{y}) \, \psi^\dagger(\mathbf{x}) \frac{\lambda^a}{2} \psi(\mathbf{x}) \, \psi^\dagger(\mathbf{y}) \frac{\lambda^a}{2} \psi(\mathbf{y}) \, . & (3) \end{aligned}$$

As λ^a's são as matrizes de cor de Gell-Mann. A estrutura espinorial desta interação efetiva é do tipo "Coulombiana"; mas o formalismo permite o emprego de outros tipos de interações (possivelmente o retardamento possa ser incluído).

O operador de campo tem a forma

$$\psi_{fc}(\mathbf{x}) = \int \frac{d^3p}{(2\pi)^{3/2}} \left[u_s(\mathbf{p})b_{fsc}(\mathbf{p}) + v_s(\mathbf{p})d^\dagger_{fsc}(-\mathbf{p})\right] e^{i\mathbf{p}\cdot\mathbf{x}} , \quad (4)$$

onde b and d referem-se respectivamente aos operadores de destruição de quarks e antiquarks no espaço de Fock, os quais carregam índices de sabor, spin e cor. Uma soma sobre índices repetidos está implícita. Os espinores u and v, e os operadores do espaço de Fock, não são os corespondentes de uma teoria livre, mas sim combinações lineares destes. u_s e v_s são dados por

$$\begin{aligned}
u_s(\mathbf{p}) &= \frac{1}{\sqrt{2}}\left[f(p) + g(p)\hat{\mathbf{p}}\cdot\vec{\alpha}\right] u^0_s , \\
v_s(\mathbf{p}) &= \frac{1}{\sqrt{2}}\left[f(p) - g(p)\hat{\mathbf{p}}\cdot\vec{\alpha}\right] v^0_s , \\
f(p) &\equiv \sqrt{1 + \sin\varphi(p)} , \\
g(p) &\equiv \sqrt{1 - \sin\varphi(p)} , \qquad (5)
\end{aligned}$$

onde u^0_s e v^0_s são os espinores livres usuais. A função $\varphi(p)$ é chamada de ***ângulo quiral***. Esta função é determinada de modo que a energia do vácuo é mínima. As propriedades de $\varphi(p)$ forum estudadas com detalhe em [1,3] e mais abaixo voltaremos à discussão de algumas destas.

Em termos dos operadores de Fock e do ângulo quiral, o Hamiltoniano fica sendo dado por

$$\begin{aligned}
H &= H_2 + H_4 , \\
H_2 &= \int d^3k\, E(k) \left[b^\dagger_{fsc}(\mathbf{k})b_{fsc}(\mathbf{k}) + d^\dagger_{fsc}(\mathbf{k})d_{fsc}(\mathbf{k})\right] , \\
H_4 &= \frac{1}{2}\int d^3p\, d^3k\, d^3q V(\mathbf{q}) \left(\frac{\lambda^a_{c_1c_2}\lambda^a_{c_3c_4}}{4}\right) :\Theta^j_{c_1c_2}(\mathbf{p},\mathbf{p}+\mathbf{q})\Theta^l_{c_3c_4}(\mathbf{k},\mathbf{k}-\mathbf{q}): .
\end{aligned}$$

Em H_4, os dez diferentes termos (obtidos somando-se sobre os índices j e l) são combinações dos seguintes vértices Θ^j

$$\begin{aligned}
\Theta^1_{c'c}(\mathbf{p},\mathbf{p}') &\equiv u^\dagger_{s'}(\mathbf{p}')u_s(\mathbf{p})\, b^\dagger_{f s' c'}(\mathbf{p}')b_{fsc}(\mathbf{p}) , \\
\Theta^2_{c'c}(\mathbf{p},\mathbf{p}') &\equiv -v^\dagger_{s'}(\mathbf{p}')v_s(\mathbf{p})\, d^\dagger_{fsc}(-\mathbf{p})d_{fs'c'}(-\mathbf{p}') , \\
\Theta^3_{c'c}(\mathbf{p},\mathbf{p}') &\equiv u^\dagger_{s'}(\mathbf{p}')v_s(\mathbf{p})\, b^\dagger_{fs'c'}(\mathbf{p}')d^\dagger_{fsc}(-\mathbf{p}) , \\
\Theta^4_{c'c}(\mathbf{p},\mathbf{p}') &\equiv v^\dagger_{s'}(\mathbf{p}')u_s(\mathbf{p})\, d_{fs'c'}(-\mathbf{p}')b_{fsc}(\mathbf{p}) . \qquad (6)
\end{aligned}$$

Os termos H_2 e H_4 foram ordenados na ordem normal. O ordenamento normal do operador energia potencial introduz termos de auto-energia, os quais estão incluídos em H_2, dando origem ao termo $E(k)$,

$$E(k) = A(k)\sin\varphi(k) + B(k)\cos\varphi(k)\,, \tag{7}$$

$$A(k) \equiv m + \frac{2}{3}\int d^3p V(\mathbf{k}-\mathbf{p})\sin\varphi(p)\,, \tag{8}$$

$$B(k) \equiv k + \frac{2}{3}\int d^3p(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})V(\mathbf{k}-\mathbf{p})\cos\varphi(p)\,. \tag{9}$$

O ordenamento normal dos termos de energia cinética e potencial dá origem, além de $E(k)$, a um termo constante (independente de operadores de criação e destruição), o qual representa a energia do vácuo. A minimização da energia do vácuo com relação a $\varphi(p)$ nos fornece a ***equação de gap***:

$$A(k)\cos\varphi(k) - B(k)\sin\varphi(k) = 0\,, \tag{10}$$

a qual determina $\varphi(p)$.

A física do ângulo quiral é que se $\varphi(p) \neq 0$, temos a quebra dinâmica da simetria quiral: no limite de massas de corrente iguais a zero, o termo de energia dos quarks ($E(k)$) apresenta um termo de massa, o gerado dinamicamente.

Neste trabalho, o potencial confinante empregado foi o seguinte

$$V(\mathbf{x}) = -K_0^3 x^2 + U. \tag{11}$$

O termo constante U, independente das corrdenadas do espaço, é necessário para definir estados assintóticos "in" e "out" . Ambos K_0 e U são positivos e têm dimensão de energia. O potencial $q - \overline{q}$ total pode ser visto como o limite, quando U$\rightarrow +\infty$, de uma sucessão de potenciais cada vez mais profundos com $V(\pm\infty) = 0$ eventualmente. É importante notar que U não corresponde a um deslocamento universal, como num formalismo de primeira quantização, das massas hadrônicas. Este U entra no Hamiltoniano (3) multiplicado por um produto de quatro operadores de campo fermiônicos ψ e duas matrizes de cor. Portanto, U é um operador e não um número-c. Ainda mais, como será visto abaixo, a interação efetiva entre os quarks no interior dos hádrons é atrativa, apesar de V ser positivo.

É possível provar os seguintes resultados:

a) Quando U→ +∞, a autoenergia dos quarks (antiquarks) tende a mais infinito. Isto significa que não existem quarks livres!
b) A adição de um termo constante ao potencial não modifica a equação do gap.
c) Um potencial "constante" não contribui para amplitudes de aniquilação (criação) quark–anti-quark.
d) A equação dos estados ligados (equação de Salpeter) é invariante sob o deslocamento da energia potential por U.
e) O Hamiltoniano confina a cor.

Passemos agora à discussão dos estados ligados correspondentes aos mésons charmosos e núcleons e deltas. Neste trabalho, como estamos tratando com potenciais instantâneos (e desprezamos canais de energia negativa) a equação de Salpeter pode ser escrita como

$$H\,|\psi\rangle = M\,|\psi\rangle\,, \tag{12}$$

onde $|\,\psi\rangle$ é um auto-estado do Hamiltoniano, com massa M. O operador que cria um méson é escrito como

$$\Psi_m^\dagger = \int d^6p\,\delta(\mathbf{p}_1+\mathbf{p}_2)\,\psi(\mathbf{p}_1,\mathbf{p}_2)\,\chi_{f_1 f_2 s_1 s_2}\,b^\dagger_{f_1 s_1 c}(\mathbf{p}_1)d^\dagger_{f_2 s_2 c}(\mathbf{p}_2)\,, \tag{13}$$

e o operador de núcleons (e deltas) é escrito como

$$\begin{aligned}\Psi_b^\dagger \;=\; & \int d^9p\,\delta(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3)\,\psi(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)\,\epsilon_{c_1c_2c_3}\,\chi_{f_1f_2f_3s_1s_2s_3}\\ & \times b^\dagger_{f_1s_1c_1}(\mathbf{p}_1)b^\dagger_{f_2s_2c_2}(\mathbf{p}_2)b^\dagger_{f_3s_3c_3}(\mathbf{p}_3)\,,\end{aligned} \tag{14}$$

onde ψ, χ e ϵ são as funções de onda nas variáveis espaciais, de spin-isospin e cor, respectivamente. Para os mésons, fatorando a parte angular da função de onda de acordo com

$$\phi(\mathbf{k}) = \sum_{L,M}\begin{pmatrix} L & S & J\\ M & M_S & M_J\end{pmatrix} Y_{LM}(\hat{\mathbf{k}})\,\frac{\nu_L(k)}{k}. \tag{15}$$

obtemos a equação para $\nu_L(k)$

$$
\begin{aligned}
&\left\{\frac{d^2}{dk^2}+M-E(k)-\overline{E}(k)-\frac{L(L+1)}{k^2}-\frac{\varphi'^2(k)+\overline{\varphi}'^2(k)}{4}\right.\\
&+\frac{\sin\varphi(k)\,\sin\overline{\varphi}(k)-1}{k^2}+\frac{2}{k^2}\left[g^2(k)\,\mathbf{S}_1+\overline{g}^2(k)\,\mathbf{S}_2\right]\cdot\mathbf{L}\\
&\left.-\frac{2\,g^2(k)\,\overline{g}^2(k)}{k^2}\left[\frac{S}{3}(S+1)+(\hat{\mathbf{k}}\cdot\mathbf{S}_1)(\hat{\mathbf{k}}\cdot\mathbf{S}_2)-\frac{1}{3}\,\mathbf{S}_1\cdot\mathbf{S}_2\right]\right\}\nu(k)=0\,.
\end{aligned}
$$

A equação para ψ para os bárions é dada por

$$
\begin{aligned}
&\left\{3E(p_1)-M-\frac{3}{2}\nabla^2_{\mathbf{p}_{12}}+\frac{3}{4}\varphi_1'^2+\frac{3(1-\sin\varphi_1)}{p_1^2}\right.\\
&\left.+\left[\frac{3}{4}-\frac{1}{3}S(S+1)\right](1-\sin\varphi_1)(1-\sin\varphi_2)\frac{\hat{\mathbf{p}}_1\cdot\hat{\mathbf{p}}_2}{p_1p_2}\right\}\psi(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)=0\,.
\end{aligned}
$$

Nas equações acima, os momenta, energias e massas são dadas em unidades de $(4/3)^{1/3}K_0$ e $\sin\varphi_i$ significa $\sin\varphi(p_i)$. $\mathbf{L}$ é o momento angular orbital total, $\mathbf{S}_i$ é o spin do i'ésimo quark, S é o spin total e $\nabla^2_{\mathbf{p}_{12}}$ é o Laplaciano em relação ao momentum relativo $(\mathbf{p}_1-\mathbf{p}_2)/2$.

As equações acima se parecem com equações de Schrödinger com interações spin-spin, spin-órbita e tensorial. Estas interações implicam em diferentes massas para os diferentes hádrons, dependendo dos valores de S, L and J. É importante notar que todas as interações foram derivadas de um mesmo termo do potencial e dependem do ângulo quiral φ, o qual reflete a estrutura do vácuo.

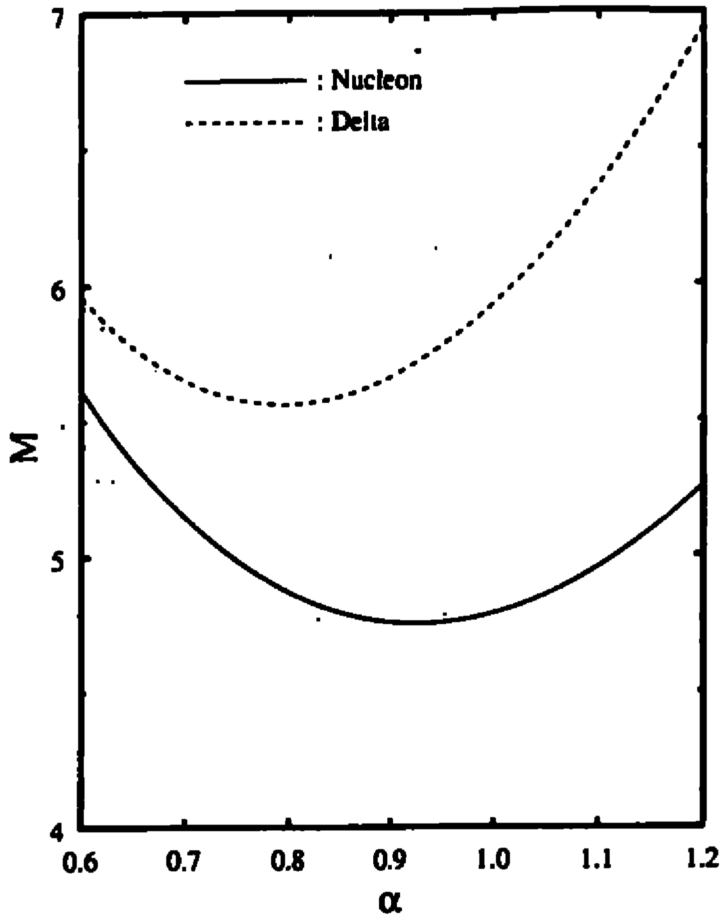
A equação dos mésons é resolvida numericamente usando o método de Runge-Kutta[1]. Os resultados obtidos estão mostrados na Tabela I abaixo.

Tabela 1: Espectro mesônico no setor do charm, com $(4/3)^{1/3}K_0 = 290\ MeV$, $m_c = 1362\ MeV$ e $m_u = m_d = 0$. Os valores experimentais são da Ref. 3

Méson	J^{PC}	${}^{S}L_J$	Calculado (MeV)	Experimental (MeV)
η_c	0^{-+}	${}^1S_0$	3096	2979
J/ψ	1^{--}	${}^3S_1 + {}^3D_1$	3097	3097
χ_{c_0}	0^{++}	${}^3P_0$	3332	3415
χ_{c_1}	1^{++}	${}^3P_1$	3343	3511
χ_{c_2}	2^{++}	${}^3P_2 + {}^3F_2$	3365	3556
ψ'	1^{--}	${}^3S_1 + {}^3D_1$	3579	3686
ψ''	1^{--}	${}^3S_1 + {}^3D_1$	3611	3770
ψ'''	1^{--}	${}^3S_1 + {}^3D_1$	4155	4040
ψ''''	1^{--}	${}^3S_1 + {}^3D_1$	4209	4159
ψ'''''	1^{--}	${}^3S_1 + {}^3D_1$	4935	4415
D	0^-	${}^1S_0$	1998	1869
D_0^*	0^+	${}^3P_0$	2216	---
D^*	1^-	${}^3S_1$	2005	2007
D_1	1^+	${}^3P_1 + {}^1P_1$	2271	---
D_1	1^+	${}^3P_1 + {}^1P_1$	2499	2424
D_2^*	2^+	${}^3P_2 + {}^3F_2$	2552	2459

Tabela 2: Massas dos núcleons e deltas, para diferentes valores para as massas de corrente $m \equiv m_u \equiv m_d$. O parâmetro do potencial é o mesmo que o usado para o charmônio $(4/3)^{1/3}K_0 = 290\ MeV$. α é o parâmetro variacional.

m (MeV)	M_N (MeV)	M_Δ (MeV)	α_N (fm)	α_Δ (fm)
0	1378	1612	0.629	0.540
0.725	1378	1611	0.628	0.539
7.25	1375	1607	0.622	0.537
290	1844	2005	0.479	0.435



Resultados do método variacional para as massas dos núcleons e das deltas. A massa M e o parâmetro variacional α estão apresentados em unidades de $(4/3)^{1/3}K_0$

A equação para núcleons e deltas foi resolvida usando o método variacional. As funções variacionais foram tomadas como gaussianas, com parâmetro α. Como teste do método, empregamos este para o charmônio. Obtemos resultados em excelente acordo com os calculados exatamente (Tabela I). Na Figura abaixo mostramos os resultados para as massas do sistema $N-\Delta$, como função de α, para o caso de massas de corrente iguais a zero. Os valores numéricos para diferentes valores das massas de corrente estão mostrados na Tabela II. Os resultados para as massas $N-\Delta$ são razoavelmente bons, considerando-se que não incluímos canais acoplados (píons, principalmente).

Como conclusão, temos que o presente modelo é capaz de fornecer resultados muito bons para o espectro dos mésons pesados e para o sistema $N-\Delta$. Os resultados aqui obtidos dependem de um *único* parâmetro, K_0. Os desdobramentos dependentes de spin são dependentes do ângulo quiral $\varphi(k)$, o qual é resultado da quebra dinâmica da simetria quiral. O próximo passo, é a inclusão de canais acoplados, bem como o cálculo de outras propriedades dos hádrons (funções de estrutura). Apesar de não-relativístico, o presente modelo certamente é um avanço em relação aos modelos de quarks não-relativísticos do tipo Isgur e Karl[5].

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MINI-JETS SEEN IN COSMIC RAY INTERACTION WITH CARBON TARGET CHAMBER

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In this work we use two differents procedures by mini jets identification in cosmic ray particules interaction with carbon target chamber (C-jets) observed by Brazil-Japan Collaboration. This events concerns the overlapping energy region with CERN and FNAL collider experiments ($\sqrt{s} \sim 500$ GeV). Our results are discuted and interpreted in terms of fire-ball model and we find which those studies are common in many aspects with modern version of multi-particles production models such as quark-string inspired by QCD.

1-INTRODUCTION. The present work covers experimental results and phenomenological studies with use of the emulsion chamber exposed to the cosmic radiation at the top of Mountain Chacaltaya, Bolivia (altitude 5200 above sea level) by Brazil-Japan Collaboration[1]. We carried a systematic analysis on the data of carbon target interaction (C-jets) of cosmic ray particules observed by two storeyed emulsion chamber. The cosmic ray observation is confined in the forward region while the collider experiment works in the central region. However C-jets of Chacaltaya exposure is in good agreement with CERN collider experiment[2]. Both cosmic ray C-jets and CERN collider experiment found frequent emission of "mini-jets" and also rapid increase of its production rate with collision energy. They believe that the association of such mini-jets are the cause of increase of $\langle p_t \rangle$ and multiplicity. According QCD picture mini-jets are the result of "gluon-gluon" collision. The incoming nucleon is a bundle of quark and gluons, where the number of associated gluons (mini-jets) increase with energy.

2-C-JETS CHARACTERISTIC AND MINI-JETS IDENTIFICATION. Lower chamber is designed to be a detector for secondary gamma ray from the cosmic hadrons in carbon target interaction (C-jets). For every C-jet detected in the lower chamber we have energy E and position (r, ϕ) of all detected gamma ray. The detection threshold energy in nuclear emulsion plates for gamma rays in lower chamber is around $E \sim 0.1$ TeV. The energy weighted of C-jet is taken as the origin of the coordinates and after the correction for slanting arrival direction we obtain the zenithal and the azimuthal angles of gamma rays in laboratory system. In this work we use 171 C-jets events with total visible energy greater than 10 TeV and two differents methods by mini-jets identification. a) JETS-ANALYSIS. This method is very similar to use in the study of atmospheric gamma ray families called "de-cascading". The jets analysis procedure use the parameter

Pt_{ij} relative transverse momentum between i and j gamma ray in a C-jet defined as:

$$Pt_{ij} = E_i E_j (R_{ij}/H)/(E_i + E_j) \qquad (1)$$

where E_i and E_j are the respective energies, R_{ij} is their mutual distance and H is the heigth of the interaction ($H \sim 1.7$m for C-jets) and impose the criterion $Pt_{ij} < Pt_{cut}$, putting the pair jointly into one. Repeating the procedure over all possible combination of pairs we arrive at a family composed of jets. The number of mini-jets in every event depending of chosed of Pt_{cut}. The dependence on Pt_{cut} is examined in Fig.1, for the purpose of choosing an appropriate value for Pt_{cut}, where we find a rapid decrease in number of jets as Pt_{cut} increases up to 0.25 GeV/c while the decrease becomes slow as Pt runs in the region beyond and the critical value Pt_{cut} can be taken near 0.3 GeV/c.

b) VETOR-ANALYSIS. According with fire-ball model an intermediate object(fire-ball) is formed in a high energy collision, decays into a number of secondary particles to form a jet in the c.m.s. The momentum of every secondary particles can be resolved in two components p_f and p_t. The p_f is along the direction of the fire-ball momentum and p_t is at right angles to the axis referred to as spin axis, which makes an angle with fire-ball momentum direction[3] as is shown in Fig.2. The momentum conservation impose the condition $\sum_i^N \vec{P}t_i = 0$, where N is the number of secondary particles produced in nuclear collision. If the secondary particle momentum $\vec{p}$ makes angles δ_f and δ_s with the emission direction of the fire-ball and spin axis, respectively, we have:

$$(1-K^2)\cos^2\delta_s + 2K^2\cos\Psi\cos\delta_f\cos\delta_s - K^2\cos^2\Psi = 0 \ , \quad (2)$$

where $K=P_f/P_t$ and $\delta_f = \delta_f(\theta,\phi)$, $\delta_s = \delta_s(\theta,\phi)$. Here θ and ϕ are the zenithal and azimuthal angles of a secondary particle in the c.m.s. When K is constant the relation (2) represents the equation of a scew elliptic curve in θ and ϕ variables. In Fig.3, we can see three difeerent types of distribuition of secondary particles according whit relation (2). The Fig.3C is for all possible values of k and Ψ The Fig.3B is for $k>1$, for both Ψ and k variables, and the Fig.3A is for fixed values of both k and Ψ . Typical c.m.s $\theta - \phi$ plots are shown in Fig.4 A,B,C of secondary gamma rays produced in three differents hadron-nucleus interactions (C-Jets). The elliptic like distribuition is shown by dotted curves.

The number of jets obtained for both procedures, jets-analysis and and vetor-analysis is shown in Fig.5 and we can see a good agreement between both.

3-SECONDARY ENERGY DISTRIBUITION AND "MINI-JETS". In the fire-ball language the scaling characteristical multiple production of hadrons observed in both cosmic ray and accelerator experiment a low energies is a consequence of the existence of a minimum unit fire-ball in multiple-pion production called H-quantum. Those old ideas from cosmic ray studies are common in many respects with modern version of multi-pion production models such as "quark-string" and we may identify H-quantum as an unit piece of the "quark-string". After 1970 Brazil-Japan Collaboration has observed in two storeyed emulsion chamber events with large P_t and large multiplicity and were assumed the existence of a larger fire-ball called "SH-quantum" or "Acu-jets" and is the responsable by scaling break when the energy increases. The "Acu-jets" events are associated with "mini-jets" and "mini-jets" is a consequence of two steps decay of the "SH-quantum" first going into a few "H-quantum" and then "H-quantum" decay in pions like a mini-jet. Analysis of secondary energy distribution for a C-jets with large multiplicity is shown in Fig.6. The full lines is the theorical distribution of gamma-rays from a "SH-quantum" under the assumption of direct decay in gamma-rays through π^0. The broken line is the theorical distribution of gamma-ray from "SH-quantum" under the assumption of two steps decay[4] where a "SH-quantum" first goes into N H-quanta and then emits N gamma-rays through mesons and can be expressed by the convolution of two thermodinamic like distributions.

4-CONCLUSION- We analyse the existence of one or more groups of secondary particles in cosmic ray particles interaction with carbon target(C-jets), each of those groups is represented by points on elliptic curve in c.m.s. Θ-ϕ plot. Each of those groups is identified as a mini-jet. This type of analysis is in good agreement with other type of framework called "jet-analysis". Multiple production is a process which can be considered in two steps: first, a sub-hadronic process(fire-ball production) and second, a subsequent decay or transmutation in jets of hadrons. The sub-hadronic process can be interpreted under the light of the QCD like models.

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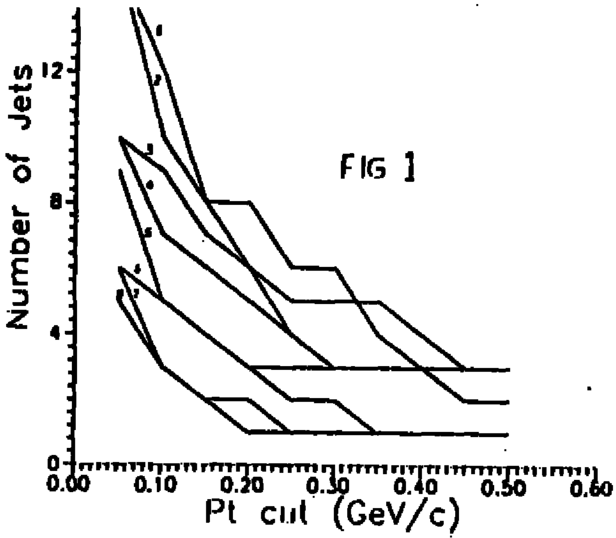

FIG 1

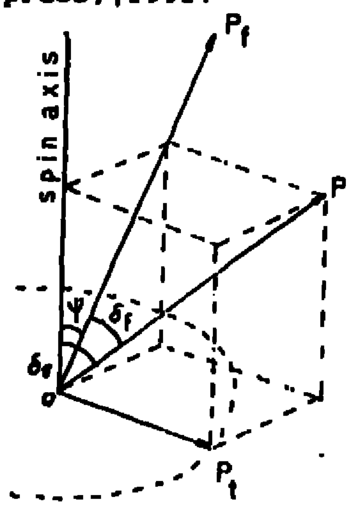

FIG 2

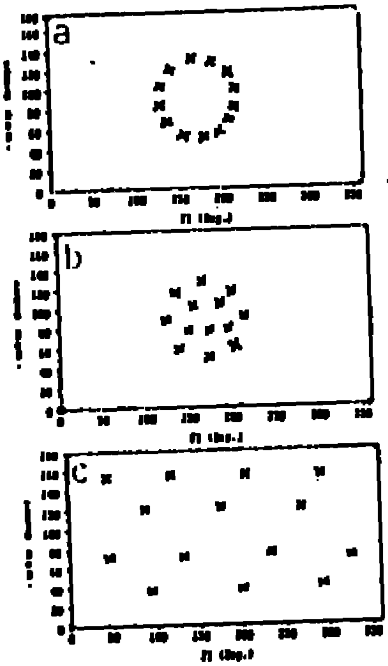

FIG 3

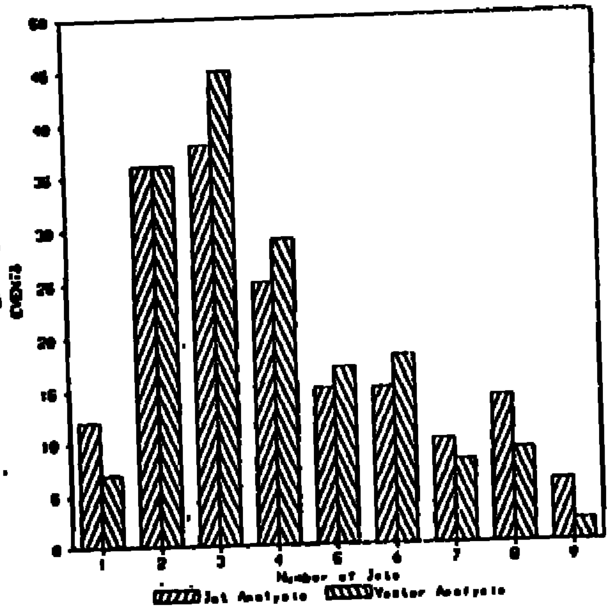

FIG 5

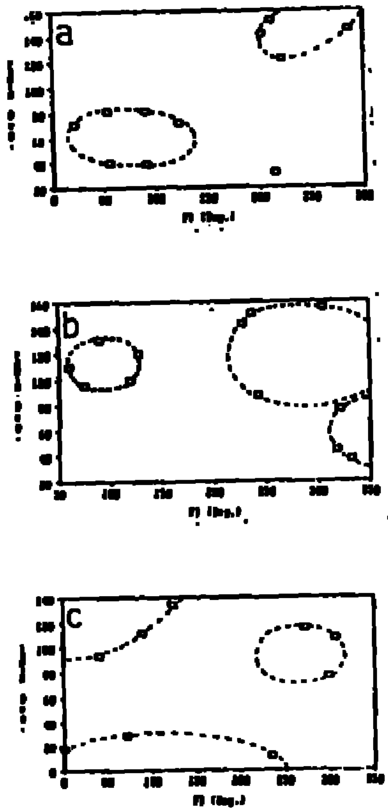

FIG 4

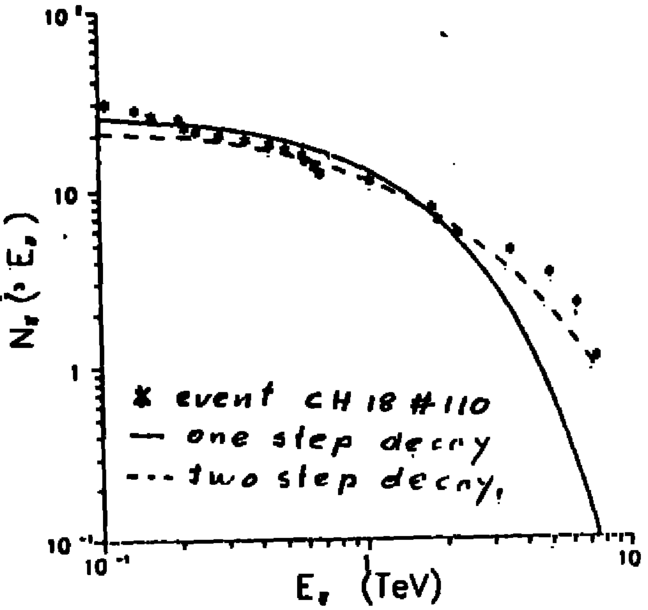

FIG 6

ALGUNS ASPECTOS DA DETECÇÃO DE NEUTRINOS COSMOLÓGICOS E MATÉRIA ESCURA

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RESUMO: Visando a detecção de partículas candidatas à matéria escura, incluindo neutrinos, estudamos o espalhamento coerente inelástico destas partículas por um alvo composto (macroscópico). Através da excitação coletiva do alvo (cristal) investigamos a produção de fônons, fora do equilíbrio térmico. Analisamos o espectro dos fônons produzidos, com especial interesse nos fônons balísticos (GHz-THz), que são mais facilmente detectados. Investigamos ainda a possibilidade de se utilizar estados coerentes como estado inicial do processo.

1 Introdução

Nosso objetivo [1] é investigar processos através dos quais poderiam vir a ser detectados neutrinos cosmológicos, assim como outras partículas, fracamente interagentes, candidatas à matéria escura.

Estamos interessados na magnitude destes processos, isto é, qual a probabilidade com que ocorrem e as respectivas taxas de eventos.

Por se tratar de WIMPS, Weakly Interacting Massive Particles, a seção de choque é proporcional à G_F^2, a constante de Fermi ($\approx 10^{-5}GeV^{-2}$), sendo portanto muito reduzida. Existem outros agravantes, como por exemplo as baixas energias de partículas presas à galáxia (com velocidades da ordem de $300km/s$) ou em equilíbrio térmico a uma temperatura de $1,95K$.

A seção de choque típica destas partículas interagindo com a matéria ordinária (u, d, e) é da ordem de $10^{-36}mb$ para partículas de massa nula, $10^{-27}mb$ para massas da ordem de $10eV$ e $10^{-10}mb$ para massas da ordem de $2GeV$.

A questão é, portanto, como aumentar esta seção de choque. Uma maneira possível é explorar a coerência deste processo. Devido às baixas energia envolvidas (comprimentos de onda longos) ocorre coerência em escala nuclear e atômica em alguns casos.

O espalhamento coerente pode se dar de duas maneiras: elástica e inelasticamente. Vamos tratar aqui apenas do caso inelástico.

2 Espalhamento Coerente Inelástico

Consideramos neste caso, que ao sofrer uma colisão, as partículas transferem energia para o alvo (rede cristalina) de modo a provocar excitações coletivas. Numa rede cristalina o deslocamento dos átomos de suas posições de equilíbrio pode ser descrito através da produção/aniquilação de fônons. A idéia portanto é produzir fônons a partir da interação ν-cristal e detectá-los.

Observou-se que em cristais a baixas temperaturas (com poucos fônons térmicos) pode-se detectar fônons na superfície do cristal que foram produzidos no seu interior mas que não sofreram termalização. São chamados fônons balísticos e possuem frequências da ordem de GHz-THz [2].

Nesta situação teríamos um cristal preparado com número de ocupação de fônons bem definido (auto-estado do operador N) e portanto fora do equilíbrio térmico.

A interação ν-cristal pode ser medida através do monitoramento dos fônons produzidos no estado final. Para tal precisamos conhecer o seu espectro, que é dado por:

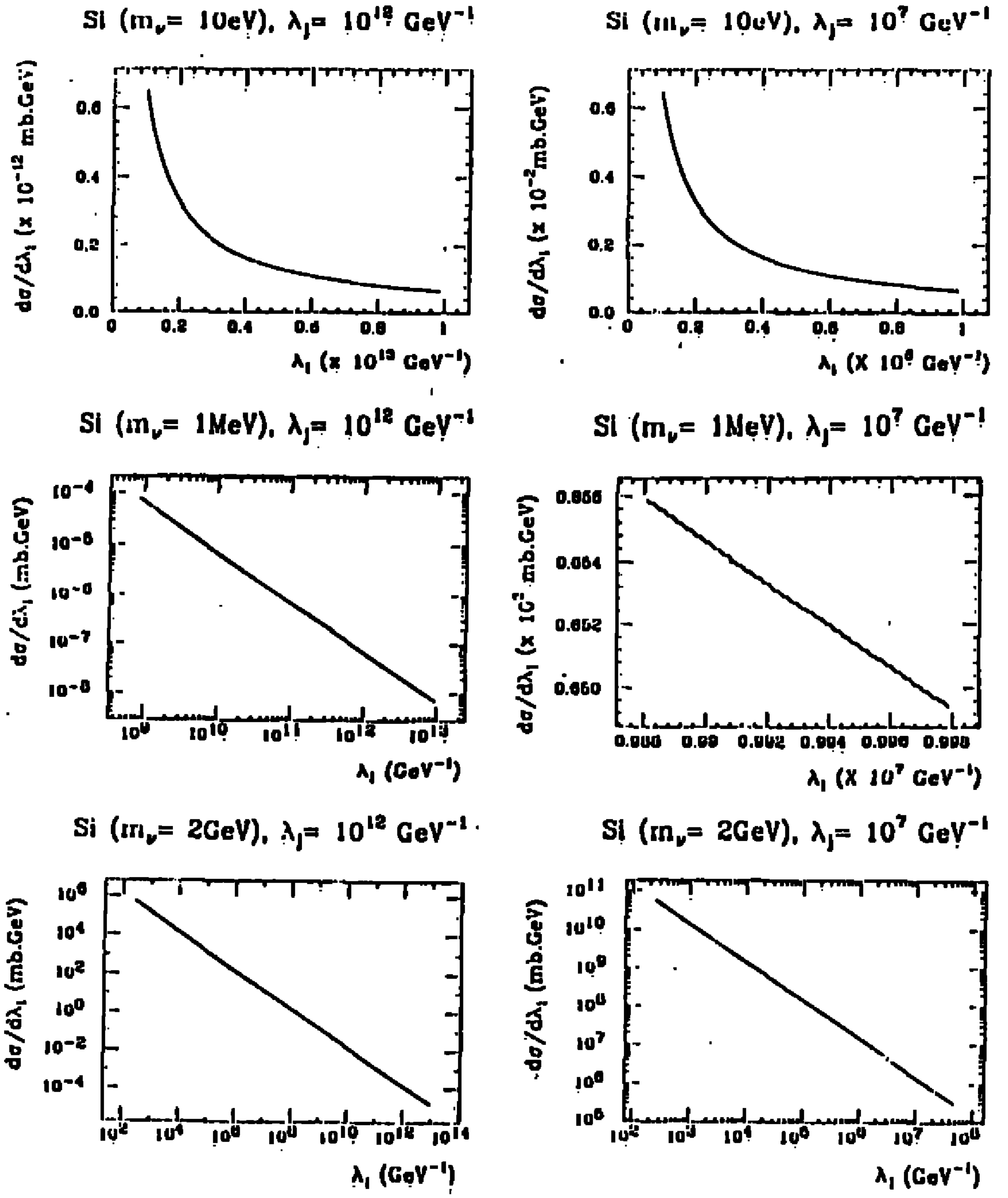
$$\begin{aligned} \frac{d\sigma}{d\lambda_i} &= \frac{\sigma_0}{8k^3}\frac{\lambda_c^2}{\lambda_j}\frac{c_i}{}\, n_j(n_i+1) \\ &\times \left\{\frac{1}{\lambda_i^3}\left[2k^2+\frac{4\pi k c_j}{\lambda_j}\right]-\frac{1}{\lambda_i^1}\left[4\pi k c_i-\frac{8\pi^2 c_i c_j}{\lambda_j}\right]\right\} \end{aligned} \tag{1}$$

para neutrinos sem massa e para neutrinos com massa diferente de zero (ou outras partículas candidatas a ME):

$$\frac{d\sigma}{d\lambda_i}=\frac{\sigma_m}{\pi}\frac{c_i}{m_\nu\beta^2}\frac{\lambda_c^2}{\lambda_j}\frac{1}{\lambda_i^3}\, n_j(n_i+1) \tag{2}$$

Consideramos dois casos, num o cristal é preparado inicialmente com fônons acústicos ($\lambda \approx 10^{12} GeV^{-1}$) e no outro com fônons balísticos ($\lambda \approx 10^{7} GeV^{-1}$).

Os gráficos abaixo apresentam o espectro de fônons produzidos a partir da interação de partículas massivas com um cristal (Si) preparado inicialmente com fônons acústicos (à esquerda) e com balísticos (à direita).



Na tabela abaixo se encontram as taxas de eventos para os processos citados:

m_ν	fônons acústicos	fônons balísticos
0	5.62×10^{-49}	8.42×10^{-36}
$10\ eV$	9.0×10^{-28}	9.0×10^{-18}
$1\ MeV$	5.52	9.12×10^{7}
$2\ GeV$	6.30×10^{13}	6.30×10^{20}

3 Estados Coerentes

Uma outra possibilidade é utilizar um cristal preparado em estados coerentes. O interesse neste caso é investigar se há aumento da seção de choque devido a um fenômeno análogo ao efeito de "super-radiância" em ótica. Em princípio o que se quer observar é a transição entre dois estados coerentes, que pode ser caracterizada pelo surgimento de uma fase relativa. Os resultados numéricos para este processo estão sendo obtidos.

4 Conclusão

Os dados relativos à produção de fônons num cristal inicialmente ocupado com fônons balísticos são bastante animadores, principalmente se considerarmos partículas candidatas à matéria escura com massas superiores a alguns MeV's. Quanto à possibilidade da utilização de estados coerentes é uma questão ainda em aberto, a ser decidida quando estiverem concluídos os cálculos numéricos acima referidos.

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ESPECTRO DE MASSAS DE BÁRIONS NO MODELO QUARK-DIQUARK*

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Calculamos as massas dos estados fundamentais dos bárions de spin 3/2 através da redução do problema de três quarks ao problema equivalente envolvendo um quark e um diquark. O potencial não-relativístico utilizado teve seus parâmetros fixados no setor mesônico, e difere do potencial quark-antiquark apenas por um fator associado com o operador de Casimir quadrático do grupo SU(3) de cor. O espectro de massas obtido por essa aproximação é comparado com o espectro obtido pelo método de Zickendraht e com o espectro experimental. A aproximação quark-diquark e o método de Zickendraht fornecem espectros similares, apesar do diquark ter dimensão de mesma ordem de grandeza do méson.

Acredita-se que a QCD (cromodinâmica quântica) seja a teoria das interações fortes, descrevendo as interações entre quarks e gluons. No modelo de quarks ordinário mésons e bárions são estados ligados $Q\bar{Q}$ e QQQ, respectivamente. Bárions podem ainda ser interpretados como estados ligados de um quark e um diquark. As excitações bariônicas são excitações do diquark, do quark-diquark, ou ambos. O propósito deste trabalho é investigar essa possibilidade calculando as massas dos estados fundamentais dos bárions de spin 3/2 com o modelo de potencial não relativístico.

A equação de Schrödinger com o potencial estático

$$V(r) = F_C \frac{\alpha_s}{r} + Kr^{0,8} + C \tag{1}$$

acrescido de correções relativísticas dependentes de spin, foi usada no setor de quarks leves e pesados, mostrando excelente concordância com os espectros experimentais de mésons [1] e bárions [2]. A extensão para o setor gluônico também já foi considerada [3]. F_C é o valor esperado do produto escalar dos spins F de cor de dois corpos, relacionado com o operador de Casimir quadrático do grupo SU(3) de cor. Para o par $Q\bar{Q}$ no estado singleto de cor $F_C = -4/3$ e a parametrização do potencial (1) para esse sistema é

$$m_b = 4{,}5\ \text{GeV}, \quad m_c = 1{,}5\ \text{GeV} \tag{2a}$$

$$m_s = 0{,}5\ \text{GeV}, \quad m_d = m_u = 0{,}38\ \text{GeV} \tag{2b}$$

$$\alpha_s = 0{,}187, \quad K = 0{,}767\ \text{GeV}^{3/2} \tag{2c}$$

$$C = (0{,}01x^2 + 0{,}146x - 1{,}412)\ \text{GeV} \tag{2d}$$

$$x = \text{Ln}\left[\left(m_q m_{\bar{q}}^2 + m_q^2 m_{\bar{q}}\right)\text{GeV}^{-3}\right] \tag{2e}$$

com as massas em (2e) dadas em GeV. Para o tratamento da espectroscopia de três corpos na Ref. 2 introduziu-se um sistema de coordenadas interno, que descreve a forma do triângulo formado pelos três corpos, e um sistema de coordenadas externo, que descreve a orientação desse triângulo no espaço. A equação de Schrödinger para esse problema reduz-se então a um sistema de equações acopladas nas coordenadas internas. Para os estados de onda S,P e D um método formulado por Zickendraht [4] permite transformar o problema de três corpos num problema unidimensional. O espectro de bárions foi calculado supondo que o potencial de três corpos é uma soma de potenciais entre pares. O par QQ na representação irredutível $\bar{3}$ tem $F_c = -2/3$, de modo que o par QQ tem um peso relativo ao quarkônio igual a 1/2. O potencial de curto alcance para o par QQ é 1/2 vêzes o potencial de curto alcance para o quarkônio. Conjectura-se que essa regra sobrevive para o potencial confinante.

Na aproximação quark-diquark para o cálculo da espectroscopia bariônica supomos, como no caso do método de Zickendraht na Ref. 2, que o potencial de três corpos é uma soma de potenciais entre pares e a regra de multiplicar todos os parâmetros do potencial por um peso relativo. Desse modo não há parâmetros livres. Calculamos as massas dos estados fundamentais dos diquarks de spin 1 ($F_c = -2/3$) e em seguida as massas dos estados fundamentais dos sistemas quark-diquark de spin 3/2 ($F_c = -4/3$), considerando o diquark como uma partícula elementar na combinação com o quark para formar o bárion. Alguns de nossos resultados estão ilustrados na Tabela I.

Na Tabela II constam as massas dos diquarks e as massas dos mésons assim como os respectivos raios quadráticos médios. As massas dos diquarks e mésons não diferem significativamente mas os diquarks são maiores que os mésons por um fator pouco maior que 1,5. Em geral o diquark tem raio quadrático médio maior que o do sistema quark-diquark. Somente diquarks constituidos de dois quarks pesados e combinado com um quark leve reverte essa situação, devido a menor energia cinética dos quarks pesados que tendem a estar mais próximos. Apesar disso nota-se que os resultados obtidos com a aproximação quark-diquark são tão bons (ou tão ruins) quanto a solução (também aproximada) obtida com o método de Zickendraht.

* Trabalho parcialmente financiado pelo CNPq, FAPESP e FINEP

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TABELA I

	Quark-diquark	Zickendraht	Exp.	R_{rqm}
(bb)b	14,495	14,362		0,21
(ss)s	1,707	1,707	Ω^-(1,672)	0,60
(qq)q	1,341	1,339	Δ(1,230-1,234)	0,67
(bb)q	10,422	10,172		0,58
(bq)b	10,185	10,172		0,24
(ss)q	1,611	1,587	Ξ(1,532-1,535)	0,65
(sq)s	1,571	1,587	Ξ(1,532-1,535)	0,61
(qq)s	1,440	1,448	Σ(1,383-1,387)	0,62
(sq)q	1,474	1,448	Σ(1,383-1,387)	0,62
(qq)b	5,747	5,838		0,42
(bq)q	6,055	5,838		0,59
(bc)q	7,200	6,935		0,59
(bq)c	7,083	6,935		0,34
(cq)b	6,924	6,935		0,31

Massas dos estados fundamentais dos bárions de spin 3/2 na aproximação quark-diquark (em GeV). O par entre parênteses é o diquark. As massas obtidas com o método de Zickendraht estão ilustradas para comparação (M.A.B. do Vale et al., Ref.2). Resultados experimentais: Ref.5. O raio quadrático médio também é fornecido (em fm). q = u ou d.

TABELA II

	Diquark	R_{rqm}	Méson	R_{rqm}
bb	9.352	0,35	9.466	0,25
ss	1,159	0,93	1,020	0,69
qq	0,910	1,04	0,770	0,78
bq	5,208	0,80	5,270	0,60
bc	6,285	0,48	6,285	0,48
cq	2,080	0,85	2,088	0,63
sq	1,028	0,99	0,892	0,73

Massas dos estados fundamentais dos diquarks de spin 1 (em GeV) e raios quadráticos médios (em fm). Os resultados para mésons vetoriais também estão ilustrados. q = u ou d.

Simetrias de Spins mais Altos do Modelo de Toda Conforme Afim

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O estudo de modelos integráveis tem levado a observação de estruturas algébricas interessantes. Além disso, alguns desses modelos apresentam invariância conforme, sendo portanto natural procurar uma relação entre essas propriedades. Os modelos de Toda, de certa forma, servem como um laboratório para essas investigações . Estes são classificados em três categorias:

O primeiro, denominado "Toda Molecule", é obtido através da condição de curvatura nula de potenciais de gauge definidos numa álgebra de Lie $\mathcal{G}$ cujos geradores satisfazem $[T^a, T^b] = f^{abc}T^c$. Para o caso sl(2), obtemos a equação de Liouville:

$$\partial_-\partial_+\phi - e^{2\phi} = 0 \tag{1}$$

onde $x_\pm = x \pm t$, $\partial_\pm = \frac{1}{2}(\partial_x \pm \partial_t)$. A invariância conforme é facilmente observada na equação acima a partir das transformações :

$$x^+ \to F(x^+) \tag{2}$$

$$x^- \to G(x^-) \tag{3}$$

$$\phi \to \phi + \frac{1}{2} ln(F'G') \tag{4}$$

Recentemente as cargas desta teoria foram obtidas [1], observando-se serem elas geradoras da álgebra W, que é uma extensão da álgebra de Virasoro.

O segundo modelo de Toda usualmente encontrado na literatura é o "Toda Lattice", cuja equação de movimento é novamente obtida através da condição de curvatura nula de potenciais de gauge descritos por uma "loop algebra", $[T^a_m, T^b_n] = f^{abc}T^c_{m+n}$. Quando $\mathcal{G} = sl(2)$ obtemos:

$$\partial_-\partial_+\phi - e^{2\phi} + e^{-2\phi} = 0 \tag{5}$$

A integrabilidade completa deste modelo foi obtida por Olive e Turok [2], ou seja, foram construídas infinitas cargas conservadas (já que o modelo possui infinitos graus de liberdade) e demonstrada sua involução . No entanto este modelo não apresenta invariância conforme como pode ser observado impondo-se (2), (3) e (4) em (5).

Mais recentemente foi proposto por Babelon e Bonora [3] uma terceira classe de modelos de Toda, descritos por uma álgebra de Kac-Moody:

$$[T^a_m, T^b_n] = f^{abc}T^c_{m+n} + cm\delta^{ab}\delta_{m+n,0} \tag{6}$$

$$[d, T^a_m] = mT^a_m, [c, d] = [c, T^a_m] = 0 \tag{7}$$

Os dois novos geradores d e c implicam a introdução de dois novos campos na teoria, μ e ν respectivamente. As equações de movimento para o caso de sl(2) são:

$$\partial_-\partial_+\phi = e^{2\phi} - e^{-2\phi+2\mu} \tag{8}$$

$$\partial_-\partial_+\mu = 0 \tag{9}$$

$$\partial_-\partial_+\nu = e^{-2\phi+2\mu} \tag{10}$$

A introdução desses novos campos faz com que recobramos a invariância conforme:

$$\phi \rightarrow \phi + \frac{1}{2}ln(F'G') \tag{11}$$

$$\mu \rightarrow \mu + ln(F'G') \tag{12}$$

$$\nu \rightarrow \nu - Bln(F'G') \tag{13}$$

onde B é arbitrário. A questão por nós colocada diz respeito às simetrias do modelo denominado "Toda Conforme Afim".

Propomos uma construção para as cargas deste modelo e explorar as simetrias nele existentes. O fato do modelo ser invariante conforme implica a existência de quantidades construídas a partir das correntes conservadas, $W(x^+)$ e $\bar{W}(x^-)$.

Considerando a quiralidade (x^+) percebemos

$$\partial_- W(x^+) = 0 \rightarrow \partial_x W = \partial_t W \tag{14}$$

Portanto as integrais espaciais de tais densidades são conservadas no tempo:

$$\frac{dQ}{dt} = \int dx\partial_t W = \int dx\partial_x W = 0 \tag{15}$$

Note que qualquer função de W, $F(W)$, também satisfaz $\frac{d}{dt}\int dx F(W) = 0$.

A Lagrangeana do modelo de Toda Conforme Afim, escrita nas coordenadas do cone de luz é dada por:

$$\mathcal{L} = \partial_+\phi\partial_-\phi + \partial_-\mu\partial_+\nu + \partial_+\mu\partial_-\nu + e^{2\phi} + e^{2\mu-2\phi} \tag{16}$$

Definimos os momentos canônicos em relação ao "tempo" x^- como:

$$\Pi_\phi = \partial_+\phi, \Pi_\mu = \partial_+\nu, \Pi_\nu = \frac{1}{2}J^c = \partial_+\mu \tag{17}$$

cuja estrutura algébrica é dada pelos parênteses de Poisson:

$$\begin{aligned} \{\Pi_\phi(x), \Pi_\phi(y)\} &= \frac{1}{2}\delta'(x-y) \\ \{\Pi_\mu(x), J^c(y)\} &= \delta'(x-y) \end{aligned} \tag{18}$$

Escrevendo o tensor de Energia-Momento modificado em termos das quantidades (17)temos:

$$W_2 = \Pi_\phi^2 - \Pi_\phi' + 2\Pi_\mu\Pi_\nu - 2\Pi_\mu' \tag{19}$$

onde os termos com derivadas são introduzidos de tal forma que W_2 tenha traço nulo. Com o auxílio de (18) observamos que o tensor de EM modificado satisfaz:

$$\{W_2(x)\,,\,W_2(y)\} = 2W_2(y)\delta'(x-y) - \partial_y W_2(y)\delta(x-y) - \frac{1}{2}\delta'''(x-y) \tag{20}$$

que é a álgebra de Virasoro. Lembremos que estamos tratando apenas o caso sl(2). A generalização para outras álgebras é encontrada em [4]. Além disso o modelo possui uma corrente conservada J^c, satisfazendo

$$\{W_2(x)\,,\,J^c(y)\} = J^c(y)\delta'(x-y) - \partial_y J^c(y)\delta(x-y) - 2\delta''(x-y) \tag{21}$$

Para construirmos as cargas de spins mais altos introduzimos o seguinte operador

$$\mathcal{D}_s \equiv \partial + \frac{s}{c_J}J(x) \tag{22}$$

onde s e c_J são definidos nas equações que se seguem. Um campo de spin s, V_s deve satisfazer a seguinte relação :

$$\{W_2(x)\,,\,V_s(y)\} = sV_s(y)\delta'(x-y) - V_s'(y)\delta(x-y) + c_V\delta^{(s+1)}(x-y) \quad (23)$$

Assim, uma corrente de spin 1, seguindo a definição acima satisfaz:

$$\{W_2(x)\,,\,J(y)\} = J(y)\delta'(x-y) - J'(y)\delta(x-y) + c_J\delta''(x-y) \quad (24)$$

Portanto, para obtermos um operador de spin $s+1$ a partir de um outro de spin s, aplicamos o operador definido em (22) em V_s:

$$V_{s+1}(x) \equiv \mathcal{D}_s V_s(x) = V_s'(x) + \frac{s}{c_J} J(x)V_s(x) \quad (25)$$

que satisfaz:

$$\begin{aligned}\{W_2(x)\,,\,V_{s+1}(y)\} &= (s+1)V_{s+1}(y)\delta'(x-y) \\ &-V_{s+1}'(y)\delta(x-y) + c_V\mathcal{D}_s(y)\delta^{(s+1)}(x-y)\end{aligned} \quad (26)$$

Observamos na relação (26) que o campo V_{s+1} só será primário se partirmos de um campo primário V_s com $c_V = 0$, pois o último termo descaracteriza a relação (23) (definição de campo primário de spin s).

Voltando ao modelo de Toda Conforme Afim, observamos que a corrente J^c satisfaz a relação (24) com $c_J = -2$, conforme mostrado na equação (21). No entanto o campo primário $W_2(x)$ possui $c_V = -1/2$ (veja (20)), mas a existência de J^c nos permite construir um segundo campo de spin 2, que denominamos $\widetilde{W}_2$:

$$\widetilde{W}_2(x) \equiv \frac{1}{4}\left(J^c(x)\right)^2 - J^{c\prime}(x) \quad (27)$$

que satisfaz a relação de comutação :

$$\{W_2(x)\,,\,\widetilde{W}_2(y)\} = 2\widetilde{W}_2(y)\delta'(x-y) - \widetilde{W}_2'(y)\delta(x-y) - 2\delta'''(x-y) \quad (28)$$

Conseguimos, portanto, construir um campo primário de spin 2 livre de anomalia com o auxílio de (27), dado por:

$$V_2(x) = W_2(x) - \frac{1}{4}\widetilde{W}_2(x) \quad (29)$$

A partir de V_2 construímos uma torre de campos primários, que também serão densidades de cargas conservadas da teoria de Toda Conforme Afim:

$$
\begin{aligned}
W_s^{(1)}(x) \equiv (\partial - (s-1)J^c(x)/2)\,(\partial - (s-2)J^c(x)/2)\ldots \\
\ldots(\partial - 2J^c(x)/2)\,V_2(x)
\end{aligned}
\tag{30}
$$

para $s > 2$.

Notamos, no entanto, que existem outros campos primários livres de anomalia construídos a partir de V_2 e que são as próprias potências dele:

$$
\{W_2(x)\,,\,(V_2(y))^N\} = 2N\,(V_2(y))^N\,\delta'(x-y) - \partial_y\,(V_2(y))^N\,\delta(x-y) \tag{31}
$$

Logicamente estas potências também darão origem a outras torres de campos primários livres de anomalia. Portanto, de maneira geral podemos escrever:

$$
\begin{aligned}
W_s^{(N)}(x) \equiv (\partial - (s-1)J^c(x)/2)\,(\partial - (s-2)J^c(x)/2)\ldots \\
\ldots\;(\partial - (2N+1)J^c(x)/2)\,(\partial - 2N\,J^c(x)/2)\,(V_2)^N(x)
\end{aligned}
\tag{32}
$$

onde $s > 2N$. Resta-nos ainda estudar eventuais degenerescências existentes entre as torres (30) e (32).

Outra questão interessante é a estrutura da álgebra dos campos primários. Mostramos o resultado da relação para $W_3^{(1)}$ obtido a partir de (30):

$$
\begin{aligned}
\{W_3^{(1)}(x)\,,\,W_3^{(1)}(y)\} &= 4W_4^{(1)}(y)\delta'(x-y) - 2W_4^{(1)\prime}(y)\delta(x-y) - B'(y)\delta(x-y) \\
&+ (2B(y) + W_2''(y))\delta'(x-y) - 3W_2'(y)\delta''(x-y) \\
&+ (2W_2(y) + \frac{1}{2}J^c(x)J^c(y))\delta'''(x-y) \\
&+ \frac{1}{2}(J^c(x) - J^c(y))\delta^{(4)}(x-y) - \frac{1}{2}\delta^{(5)}(x-y)
\end{aligned}
\tag{33}
$$

onde $B(y) = J^c(y)W_2'(y) + 2\,I^c(y)W_2(y) - (J^c(y))^2 W_2(y)$. Observamos em (33) uma estrutura de álgebra W um pouco diferente da estrutura para álgebras de Lie ordinárias.

Observamos também que no limite J^c grande:

$$
V_2(x) \approx \pi_\mu(x)J^c(x) - \frac{1}{16}(J^c(x))^2 \tag{34}
$$

$$W_s^{(1)}(x) \approx \pi_\mu(x)(J^c(x))^{s-1} - \frac{1}{16}(J^c(x))^s \qquad (35)$$

obtendo uma relação que se comporta como

$$W_s(x), W_{s'}(y) \approx [(s-1)W_{s+s'-2}(x) + (s'-1)W_{s+s'-2}(y)]\delta(x-y) \qquad (36)$$

que é a relação da álgebra w_∞ [5].A relação acima é válida tanto para geradores obtidos em (30) quanto (32).

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QUANTUM STRING SCATTERING IN SHOCK WAVES BACKGROUNDS

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At particle energies of the order or larger than the Planck mass, the curved space-time geometry created by the particles dominate their collision process. In such situation, the description of fields or strings in flat space-time is no longer valid. The dynamics of the quantum fields or strings is then governed by their equations of motion in the classical background geometry.

This has been the motivation to investigate string propagation in relevant background geometries. In ref. [1] a systematic approach to quantize strings in curved space-times was proposed. It has been applied to cosmological space-times [2], black-hole geometries [3] and more general ones [4]. In addition, the string equations of motion turned out to be exactly soluble, in closed form, for some interesting geometries, like gravitational shock waves [5,6] and conical space-time [7] (the geometry around a straight cosmic string).

The purpose of these papers [8] is to investigate the scattering of particles by a gravitational shock wave in the framework of string theory. As it was stressed in ref. [9], the shock wave described by the Aichelburg-Sexl metric (that is the gravitational field of a neutral spinless ultrarelativistic particle) is relevant to particle scattering at Planck energy. We then choose to investigate specifically the scattering of a string (in one of its stationary states) by a particle with an energy of the order or larger than the Planck mass. The string is considered here as a test string, in other words its energy must be much smaller than the energy carried by the shock wave.

To compute N-point particle amplitudes in a curved, but asymptotically flat, space-time we start from the following generalization of the usual flat space-time formula [6]:

$$A_N(k_1,\ldots,k_N) = \int \prod_{i=1}^{N} [d\sigma_i\, d\tau_i] \, \langle 0_< \mid \prod_{j=1}^{N} : \Psi(k_j, X(\sigma_j,\tau_j)) : \mid 0_< \rangle$$

Here $\Psi(k, X(\sigma,\tau))$ represents the vertex operator for a particle of asymptotic momentum k in curved space-time. It is a solution of the corresponding wave equation in the given geometry, i.e. the Klein-Gordon equation for a scalar particle [6]. Furthermore, the string coordinates $X^\mu(\sigma,\tau)$ fulfil the propagation equations in the choosen curved geometry,

$$\partial_A \left[G_{\lambda\mu}(X)\, \partial^A X^\mu\right] - \frac{1}{2}\, [\partial_\lambda\, G_{\mu\nu}(X)]\; (\partial_A\, X^\mu)(\partial^A\, X^\nu) = 0 \quad ,$$

where $G_{\mu\nu}(X)$ is the space-time metric $(\mu,\nu = 0,1,\ldots,D-1)$ and we use the orthonormal gauge for the world- sheet. Hence, the string interaction with the geometry shows in two different places: the functional form of $\Psi(k,X)$ and the solution for $X^\mu(\sigma,\tau)$.

The vertex operators $\Psi(k_j, X(\sigma_j,\tau_j))$ pinch the world-sheet at N different points. These pinches describe the ingoing and outgoing particles intervening in the process. Of course, the integration in the expression of A_N must cover the whole string world-sheet.

We start by solving exactly the string equations of motion and the constraint equations for a shock wave space-time, in the light-cone gauge [5,6]. We recall that the string obeys the flat equations of motion in one side (<) and the order (>) of the shock wave. There is a non-trivial matching between both flat space-time string solutions, which is reviewed and completed. The ambiguity in the longitudinal coordinate is solved explicitly. We find that the constraints are satisfied if and only if we choose a mean-value prescription. This string solution will be used as the starting point for the computation in of the scattering amplitude in shock wave space-time.

The aim of the present articles is to compute the two-point amplitude, $A_2(k_2,k_1)$, for the scattering of a scalar particle (the tachyon in a bosonic string) by te shock wave,

$$\begin{aligned} A_2(k_2,k_1) \;&=\; \int_0^{2\pi} d\sigma_1\, d\sigma_2 \int_{-\infty}^{+\infty} d\tau_1\, d\tau_2\; \langle 0_< \,|: \Psi^*\,(k_2, X(\sigma_2,\tau_2)) \;: \\ &\quad :\; \Psi\,(k_1, X(\sigma_1,\tau_1)) \;: |\, 0_<\rangle \quad , \end{aligned}$$

where $k_1(k_2)$ is the momentum of the incoming (outcoming) particle. The vertex operator for the scalar particle, $\Psi(k,X)$, is a solution of the Klein-Gordon equation in the shock wave space-time and $X^\mu(\sigma,\tau)$ stands for the string solution in the shock wave metric. The total amplitude A_2 is naturally written as a sum of four terms. They correspond to qualitatively different space-time histories contributing to the scattering process. For simplicity, we choose the light-cone gauge to perform our calculations.

As it is clear, the exact evaluation of the expectation value in the right hand side of the above equation is a difficult task, since it involves the matrix elements of exponentials of non- polynomial functions of oscillator operators. [The operators $X^\mu(\sigma,\tau)$ after the collision contain non- polynomial functions of oscillators]. We then evaluate $A_2(k_2, k_1)$ for large impact parameters q, that is when the scattering angle as well as the momentum transfer are small. In such a regime, we can start by neglecting the oscillator modes since $| q | >> \sqrt{\alpha'}$. This zeroth order approximation can be improved by expanding the string coordinates operators in powers of $\sqrt{\alpha'}$ (i.e. powers of the oscillators modes). Analogous *approximations* have been used in flat space-time [12]. We arrive at an explicit integral representation for the total amplitude in terms of matrix elements $\tilde{S}(\vec{\ell}, \vec{p})$ of the vertex operator. As for $\tilde{S}(\vec{\ell}, \vec{p})$ itself, we show that it admits a series expansion in Gegenbauer polynomials.

In the impact parameter representation, we find that the string contributions for large q appear as corrections of order s/q (s is the usual Madelstam variable) to the Coulombian phase. It must be noticed that flat space-time calculations yield corrections of order s/q^2 and smaller for large q [12]. In oter words, the correction terms we find do not seem to be obtainable through flat space-time computations.

As is well known, the point particle amplitude for te scattering by a gravitational shock wave, as follows from the Klein-Gordon equation, possess an infinite number of purely imaginary poles in s, for $Im\ s < 0$ [9,10]. The $A_2(k_2, k_1)$ amplitudes, here computed in the string framework, exhibit an additional sequence of imaginary poles. Their positions are obtained in the small momentum transfer approximation. On the contrary, *the Coulomb* poles come from the vertex operator as an exact Γ-function factor. In other words, the position of the Coulomb poles are not affected by our approximations.

Up to our knowledge, this is the first time that the amplitude for the quantum scattering of a particle by a curved geometry is computed within the framework of string theory.

We still want to notice that the present calculations can be easily generalized for other string states (that is, for higher spin and higher mass particles) by inserting the appropriate vertex operators. Of course, extensions to superstrings are also possible.

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CORRELATION FUNCTION AND MASS SPECTRUM OF QUANTUM VORTICES†

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ABSTRACT — The method of soliton quantization is used to obtain explicit expressions for the vortex mass spectrum and the asymptotic behaviour of vortex correlation function in the Abelian Higgs Model in 2+1D.

INTRODUCTION

In a recent publication[1], a general method of vortex quantization in continuum QFT was introduced, based on the concept of order–disorder duality of statistical mechanics[2]. It was obtained that the extended topological excitations, which in the Abelian Higgs Model (AHM) in 2+1D is the vortex, could be described by nonlocal fields analogous to the Wilson loop operator[3]. In [1], a general procedure for the obtention of correlation functions involving vortices was established and an explicit operator realization of the vortex field was obtained. In [4] we have take the formulation of [1] and it was applied to the computation of the vortex two point correlation function in the AHM in 2+1D and from the large distance behavior of this function, we obtain an explicit expression for the quantum vortex mass in the tree level and its quantum correction at 1–loop level. The main steps of the procedure are shown below.

CORRELATION FUNCTION OF NONLOCAL VORTEX OPERATORS

Let us consider the AHM in 2+1D, given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* (D^\mu \phi) - m^2 \phi\phi^* - \frac{\lambda}{4} (\phi\phi^*)^2 \tag{1}$$

where $D_\mu = \partial_\mu + ieA_\mu$, A_μ being the electromagnetic field and e, the electronic charge. For $m^2 > 0$ the system is the "unbroken" or disordered phase, where $<\phi> = 0$. For $m^2 < 0$, the Higgs field ϕ develops a nonzero vacuum expectation value and the photon acquires a mass through the Higgs mechanism. We call it the "broken" or

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ordered phase. In this phase Nielsen and Olesen[5] observed that this model possessed classical solutions with the long distance behavior

$$
\begin{aligned}
\phi(\vec{x},t) &\xrightarrow[|\vec{x}|\to\infty]{} \rho_0 \, e^{i \arg(\vec{x})} \\
A_i(\vec{x},t) &\xrightarrow[|\vec{x}|\to\infty]{} -\frac{1}{e}\,\partial_i \arg(\vec{x}) \quad .
\end{aligned}
\tag{2}
$$

These solutions we called vortices and they are associated with the identically conserved topological current $j^\mu = 1/2\, \epsilon^{\mu\alpha\beta} F_{\alpha\beta}$ whose topological charge Q is the magnetic flux along the (x_1,x_2) plane.

In [1] a nonlocal vortex operator μ was introduced through the equal time commutation relations

$$
\mu(\vec{x},t;c)\,\phi(\vec{y},t) = \begin{cases} e^{i \arg(\vec{y}-\vec{x})}\,\phi(\vec{y},t)\,\mu(\vec{x},t;c) & ; \quad \vec{y}-\vec{x} \notin T(c) \\ \phi(\vec{y},t)\,\mu(\vec{x},t;c) & ; \quad \vec{y}-\vec{x} \in T(c) \end{cases}
\tag{3.a}
$$

and

$$
\mu(\vec{x},t;c)\,A_i(\vec{y},t) = \begin{cases} \left[A_i(\vec{y},t) - \frac{i}{e}\,\partial_i^y \arg(\vec{y}-\vec{x})\right] \mu(\vec{x},t;c) & ; \quad \vec{y}-\vec{x} \notin T(c) \\ A_i(y,t)\,\mu(\vec{x},t;c) & ; \quad \vec{y}-\vec{x} \in T(c) \end{cases}
\tag{3.b}
$$

where c defines a certain plane curve on which the vortex operator $\mu(c)$ is defined and $T(c)$ is the minimal surface bounded by c.

The euclidean correlation function of the vortex operator satisfying the algebra (3) is given by[1]

$$
\begin{aligned}
\langle \mu(x;c_1)\,\mu^*(y;c_2)\rangle &= Z^{-1}(0) \int D\phi\, D\phi^*\, D A_\mu \\
&\exp\left\{ -\int d^3z \left[\frac{1}{4}\left(F_{\mu\nu} + \tilde{F}_{\mu\nu}(S)\right)^2 + \left(D_\mu \phi\right)^* \left(D^\mu \phi\right) + V(\phi)\right] \right\}
\end{aligned}
\tag{4}
$$

where $\tilde{F}_{\mu\nu}(S) = \partial_\mu \tilde{A}_\nu(S) - \partial_\nu \tilde{A}_\mu(S)$ and $\tilde{A}_\mu(S)$ is an external field introduced in (4) such that it guarantee that $\langle\mu\mu^*\rangle$ is simultaneously surface and path independence. S is an arbitrary surface such that its boundary is $\partial S = c_1 U c_2$.

MASS SPECTRUM

From (4) one can see that $<\mu\mu^*>$ reduces to

$$<\mu\mu^*> \; = \; e^{\Lambda[x;c_1,\, y;c_2]} \tag{5}$$

where Λ is the sum of all Feynman graphs with the external field $\tilde{A}_\mu(S)$ in the external legs. From the asymptotic behavior of (5) one can predict the following behavior[4]

$$<\mu(x;c_1)\,\mu^*(y;c_2)> \;\xrightarrow[|x-y|\to\infty]{}\; e^{-M_V|x-y|} \tag{6}$$

where M_V will be the mass of the excitations produced by the field $\mu(c)$ and that in this case will be the mass of the vortex excitation.

Choosing $\tilde{A}_\mu(S)$ as defined by the surface $S:\; S_x \cup S_y = (\mathbb{R}^2_x - T_1) \cup (\mathbb{R}^2_y - T_2)$ where T_1 and T_2 are plane surfaces bounded by c_1 and c_2 respectively (with radius R), then $\tilde{A}_\mu(S)$ can be written in the form

$$\tilde{A}_\mu(S) \;=\; -\frac{1}{e}\arg(\hat{z}-\hat{x}) \int_{S_x} \delta^3(z-\xi)\, d^2\xi_\mu + \frac{1}{e}\arg(\hat{z}-\hat{y}) \int_{S_y} \delta^3(z-\xi)\, d^2\xi_\mu \tag{7}$$

and choosing in both phases the Lorentz gauge as the fixing gauge term:

$$\mathcal{L}_{GF} \;=\; \frac{\xi}{2}\left(\partial_\mu A^\mu\right)^2 \tag{8}$$

one can make the shift $A_\mu \longrightarrow A_\mu - \tilde{A}_\mu(S)$ in (4) and define a new D_μ as $\tilde{D}_\mu = \partial_\mu + ie(A_\mu - \tilde{A}_\mu(S))$.

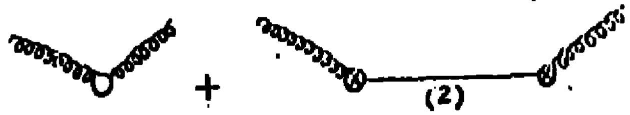
In the symmetric phase ($m^2 > 0$), since $<\phi> = 0$, we need not make any shift in the fields in (1). In the "broken" phase ($m^2 < 0$), we have $<\phi> \neq 0$. Taking $\phi = 1/\sqrt{2}\,(\phi_1 + i\phi_2)$ and choosing $<\phi_1> = b$ and $<\phi_2> = 0$ one will obtain, after the shift in ϕ_1, the mass terms $M^2 = e^2b^2$ for the A_μ field and $m_1^2 = 2|m|^2$ for the Higgs component ϕ_1.

In both phases one extracts from $\mathcal{L}$ the terms that depend on $\tilde{A}_\mu(S)$ and then one has the respectively Feynman rules.

After an explicit calculation one obtains the following expression to Λ in (5) [4]

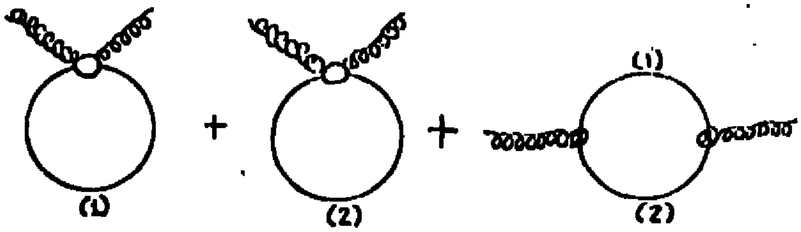
$$\Lambda \;=\; \Lambda_{tree} + \Lambda_{1\text{-loop}} \tag{9}$$

where

$$\Lambda_{\rm tree}\left(\frac{M^2}{e^2\hbar}\right) = $$ 

(10)

with $\underset{(2)}{\overline{\qquad\qquad}} = \Delta_E(z)$, given by $\dfrac{1}{4\pi|z|}$, and

$$\Lambda_{1\text{-loop}}\left(\frac{M^0}{e^0\hbar^0}\right) = $$ 

(11)

where

$$\underset{(1)}{\overline{\qquad\qquad}} = \Delta_E^{(1)}(z) = \int \frac{d^3k}{(2\pi)^3}\,\frac{e^{ikz}}{k^2+m_1^2} \tag{12}$$

in (10) and (11)  represents the external field $\tilde{A}_\mu(S)$.

Taking the limit $|x-y| \to \infty$ in (9) we get[4]

$$\Lambda \xrightarrow[|x-y|\to\infty]{} -M_v\,|x-y| \tag{13}$$

where, in the symmetric phase, $M_v = 0$, as was expected, and in the "broken" phase

$$M_v = \pi\,\frac{M^2}{e^2} - \frac{m_1}{6} \ . \tag{14}$$

In (14), $\pi\,\dfrac{M^2}{e^2}$ is the result at the tree level and is just the semiclassical result for the vortex mass[6]. The second term in (14) represents the 1–loop quantum correction to the vortex mass.

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SUPERSIMETRIA, ALGEBRIZAÇÃO PARCIAL E O POTENCIAL $V(x) = x^2 + \lambda\left[x^2/1 + gx^2\right]$

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RESUMO: Exploramos a relação entre a mecânica quântica supersimétrica e a algebrização parcial do problema espectral para resolver o potencial não polinomial: $V(x) = x^2 + \lambda\,[x^2/1 + gx^2]$.

A supersimetria tem sido aplicada em vários contextos relacionados a mecânica quântica ordinária[1]. Particularmente, a resolução da equação de Schrödinger através da super-algebra tem sido tratada em potenciais exatamente solúveis[2,3]. Estes potenciais possuem uma simetria especial em sua forma, que possibilita a solução exata. Outros potenciais, parcialmente solúveis, também têm sido estudados[4,5].

Por outro lado, o método de algebrização parcial[6] fornece elementos para tratar potenciais onde apenas uma parte do espectro é exatamente solúvel. Este método é baseado na presença de uma simetria escondida no Hamiltoniano (SU(2) no caso 1-dimensional).

Neste trabalho usamos a relação entre a Mecânica Quântica Supersimétrica e o Método de algebrização parcial[6,7] para resolver a equação de Schrödinger para o potencial não-polinomial:

$$V(x) = x^2 + \lambda \frac{x^2}{1 + gx^2} \qquad (1)$$

cujos resultados analíticos vem sendo estudados por vários autores[7-10].

Em Mecânica Quântica Supersimétrica o Hamiltoniano é escrito por

$$H_{ss} = -\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}W^2(x) + \frac{1}{2}\sigma_3\, W'(x) \qquad (2)$$

onde o superpotencial $W(x)$ deve satisfazer a equação de Ricatti (para o setor "bosonico"):

$$V(x) - E = \frac{1}{2} W^2(x) - \frac{1}{2} W'(x) \qquad (3)$$

e a auto função do estado fundamental é

$$\Psi_0 = e^{-\int_0^x w(y)dy} \qquad (4)$$

Para o setor "Fermiônico" o sinal da exponencial é trocado, e neste caso a autofunção não é normalizável.

A ligação entre formalismo supersimétrico e o Método de Algebrização Parcial é feito através do superpotencial[5]

$$W(x) = A(x) - \frac{\tilde{\Psi}'}{\tilde{\Psi}} = A(x) - \sum_{i=1}^{2j} \frac{f'(x)}{f(x) - a_i} \qquad (5)$$

onde $\tilde{\Psi}(x)$ é obtida através de uma transformação de gauge "imaginário" na função de onda original: $\Psi(x) \rightarrow \tilde{\Psi}(x)\, e^{-\int^x A(y)dy}$.
O Hamiltoniano original também sofre uma transformação, a derivada simples é substituída por uma derivada covariante $\left(\frac{d}{dx} \rightarrow \frac{d}{dx} - A(x)\right)$. O Hamiltoniano assim transformado possue uma simetria escondida, neste caso SU(2). O índice j (semi inteiro) indica o número de estados que são diagonalizados: $(2j+1)$ estados.

Para estudar o potencial proposto em (1), nós usamos

$$A(x) = x - \frac{b}{x} \qquad (6)$$

$$f(x) = \sum_{n=0}^{N} c_n x^{2n} \qquad (7)$$

onde b e c_n são coeficientes numericos a serem determinados através da equação (3).

Vamos nos restringir a $j = \frac{1}{2}$, o que significa que apenas $2j + 1 = 2$ estados serão diagonalizados. Assim usando $W(x)$ dado em (5) na equação de Ricatti (3) obtemos

$$\frac{b(b-1)}{x^2} + \frac{\lambda}{g}\frac{1}{1+gx^2} + \left[2E - 2b - 1 - \frac{\lambda}{g}\right] +$$

$$+ \frac{\sum_{n=0}^{N} cn\,(-4nx^{2n} + 4bn\,x^{2n-2} + 2n(2n-1)\,x^{2n-2})}{\sum_{n=0}^{N} c_n\,x^{2n}} = 0 \tag{8}$$

Do resíduo em $x = 0$ obtemos a condição

$$b = 0 \quad \text{ou} \quad b = 1 \tag{9}$$

Manipulando algebricamente a equação (8) obtemos para $b = 1$ e $N = 1$:

$$\frac{c_1}{c_o} = g; \quad E_o = \frac{3}{2} - 3g\ ; \ \frac{\lambda}{g} = -6g - 4 \tag{10}$$

o que nos leva a autofunção e autovalor:

$$\Psi_o = (1 + gx^2)\,x^2\,e^{-1/2x} \quad ; \quad E_o = 3\,\frac{1}{2} - 3g \tag{11}$$

fixamos $c_o = 1$, pois não estamos preocupados aqui com a exata normalização.

Tomando $b = 0$ teremos para cada N uma solução que fixa g e λ e que fornece junto com (11) o par de solução. Para $N = 1$ nós obtemos $g < 0$ que não possue interesse físico. Com $N = 2$ obtemos

$$c_1 = 0; \quad c_2 = -\frac{4}{g} = -g^2\ ; \ g = \frac{2}{3}\ ; \ E = \frac{5}{2} - 3g \tag{12}$$

ou

$$\Psi_e = N(1 - g^2x^2)\,e^{-1/2x^2} \qquad E_e = 5\,\frac{1}{2} - 3g \tag{13}$$

Finalmente, para $N = 3$ a autofunção e autovalor para o estado excitado vale

$$\Psi_e = (1 + C_1x^2 + C_2x^4 + C_3x^6)\,e^{-1/2x^2} \qquad E_e = 9.\,\frac{1}{2} - 3g \tag{14}$$

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$$C_1 = 3g-4 \ , \ C_2 = g^2 - \frac{10}{3} g + \frac{4}{3} \ , \ C_3 = - \frac{1}{5} g^3 - \frac{2}{15} g^2 + \frac{4}{5} g ;$$

$$g = \frac{1}{2} + \frac{1}{2} \sqrt{\frac{11}{3}} \simeq 1,457 \qquad (15)$$

Assim, usando um 'ansatz' apropriado, achamos dois pares de soluções para a equação de Schrödinger original, eq.(11) e (13) com $g = 1/2$ e eq. (11) e (14) com $g = 1,457$. Podemos nos perguntar se é possível obter um número de soluções maior que 2, para que isto ocorra é preciso fixar $j > \frac{1}{2}$ na equação (5) e procurar as soluções de (3).

Finalmente, notemos que o parâmetro b está relacionado a paridade das autofunções: $b = 1$ para o estado fundamental indica paridade impar e $b = 0$ para o estado excitado fornece paridade par.

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RESOLUÇÃO DA EQUAÇÃO DE SCHRÖDINGER COM POTENCIAL BI-DIMENSIONAL USANDO SUPERSIMETRIA

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RESUMO: Introduzimos uma realização não usual para algebra supersimetrica em Mecânica Quântica. Esta realização permite desenvolver um método para determinar a solução da equação de Schrödinger para certos tipos de potenciais bi-dimensionais. O potencial de Hartmann é estudado para ilustrar o método.

Em Mecânica Quântica Supersimétrica (N=2) nós temos 2 geradores (Q e Q^+) que obedecem a seguinte relação de anticomutação.

$$\left\{ Q, Q \right\} = \left\{ Q^+, Q^+ \right\} = 0 \qquad e \left\{ Q, Q^+ \right\} = H_{ss} \tag{1}$$

uma não usual realização desta superalgebra e:

$$Q = d|^- \times \sigma_- = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0_- & 0 & 0 & 0 \\ a & 0_- & 0 & 0 \\ 0 & b^- & 0 & 0 \end{bmatrix};\; d|^- = \begin{bmatrix} a^- & 0 \\ 0 & b^- \end{bmatrix};\; \sigma_- = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{2}$$

$$Q^+ = d|^+ \times \sigma_+ = \begin{bmatrix} 0 & 0 & a^+ & 0_+ \\ 0 & 0 & 0 & b^+ \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};\; d|^+ = \begin{bmatrix} a^+ & 0 \\ 0 & b^+ \end{bmatrix};\; \sigma_+ = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{3}$$

onde a e b são operadores bosônicos (a realização usual da superalgebra pode ser encontrada, por exemplo, na ref.[1]). O Hamiltoniano supersimétrico dado pela eq. (1) assume a forma:

$$H_{ss} = \begin{bmatrix} a^+ a^- & {}_+0_- & 0 & 0 \\ 0 & b^+ b^- & {}_-0_+ & 0 \\ 0 & 0 & a^- a^+ & {}_-0_+ \\ 0 & 0 & 0 & b^- b^+ \end{bmatrix} = \begin{bmatrix} H_+' & 0 & 0 & 0 \\ 0 & H_+'' & 0 & 0 \\ 0 & 0 & H_-' & 0 \\ 0 & 0 & 0 & H_-'' \end{bmatrix} = \begin{bmatrix} H_+ & 0 \\ 0 & H_- \end{bmatrix} \tag{4}$$

Os geradores Q e Q^+ aplicados as autofunções de H_{ss} atuam da seguinte maneira

$$Q \begin{pmatrix} \chi \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ d|^- \chi \end{pmatrix} \qquad e \qquad Q^+ \begin{pmatrix} 0 \\ \bar{\chi} \end{pmatrix} = \begin{pmatrix} d|^+ \bar{\chi} \\ 0 \end{pmatrix} \tag{5}$$

onde χ e $\bar{\chi}$ são autofunções de H_+ e H_-, respectivamente.

O Hamiltoniano original e total a 2 dimensões pode ser encontrado como sendo

$$H_{org} = tr\ H_+ = a^+ a^- + b^+ b^- \qquad (6)$$

Em uma dimensão a superalgebra permite em alguns casos construir uma família de Hamiltonianos ligados entre si pela supersimetria[2] como ilustrado na figura 1.

Figura 1: Ilustração da família de Hamiltonianos gerados por supersimetria.

Sabemos que o espectro e as autofunções destes Hamiltonianos estão relacionados entre si

$$E_n^{(1)} = E_o^{(n)} \quad e \quad \psi_n^{(1)} = a_1^+ a_2^+ \ldots a_n^+ \psi_o^{(n)} \qquad (7)$$

Potenciais uni-dimensionais que permitem tal construção possuem uma invariancia na forma quando passamos de um Hamiltoniano para outro dentro da família, o que significa uma mudança apenas nos parametros do potencial e não na sua forma funcional.

Com a realização que demos para a superalgebra estes conceitos usados em uma dimensão são automaticamente estendidos para cada componente de H_+ (H' e H") no caso bi-dimensional. Em particular, podemos usar a relação (7) para encontrar a solução da equação de Schrödinger[3].

Tomemos agora um exemplo, o potencial de Hartmann[4]

$$V(r,\theta) = \eta\sigma^2 \left\{ \frac{2\, a_o}{r} - \frac{\gamma\, a_o^2}{r^2 \operatorname{sen}^2 \theta} \right\} \varepsilon_o \qquad (8)$$

Este potencial pode ser aplicado, por exemplo, a molecula de benzeno. Em nossa notação a_0 é o raio de Bohr e ε_0 é o estado fundamental do atomo de hidrogênio. γ e σ são parâmetros positivos e constantes. Em termos de coordenadas parabolicas "quadradas"[5] a equação de *Schrödinger* original pode ser separada em duas

$$H'_1\ \chi_1 = \left\{ -\frac{d^2}{dx^2} + \frac{M^2 - 1/4}{x_1^2} + \frac{2\mu\, x_1^2}{h^2} \right\} \chi_1(x_1) = -\frac{2\mu\ \alpha_1}{h\sqrt{|E|}}\ \chi_1(x_1) \tag{9}$$

e

$$H''_1\ \chi_2 = \left\{ -\frac{d^2}{dx_2^2} + \frac{M^2 - 1/4}{x_2^2} + \frac{2\mu\, x_2^2}{h^2} \right\} \chi_2(x_2) = -\frac{2\mu\ \alpha_2}{h\sqrt{|E|}}\ \chi_2(x_2) \tag{10}$$

onde $x_1^2 = \sqrt{|E|}\ \xi^2$ e $x_2^2 = \sqrt{|E|}\ \eta^2$. ξ e η estão relacionadas as coordenadas cartezianas pelas relações:

$$x = \xi\eta \cos\psi \quad , \quad y = \xi\eta \text{ sen } \psi \quad \text{e} \quad z = \frac{1}{2}(\eta^2 - \xi^2)$$

A função de ondas em ternos das novas coordenadas sera

$$\psi\,(x_1, x_2, \psi) = (x_1 . x_2)^{-1/2}\ \chi_1(x_1)\ \chi_2(x_2)\ e^{im\psi} \tag{11}$$

e $M^2 = m^2 + \gamma^2\sigma^2$. Temos ainda um vinculo entre os parâmetros α_1 e α_2:

$$\alpha_1 + \alpha_2 = 4\ \gamma^2\sigma^2\ \varepsilon_0\ a_0 \tag{12}$$

Neste caso em particular, H' e H" são idênticos e devido a sua estrutura de oscilador harmônico com barreira de potencial sabemos que ele apresenta a invariância na forma (a ref.(1) tras alguns aspectos do potencial harmônico com barreira de potencial numa versão supersimetrica). Vamos estudar H', onde os resultados para H" são obtidos por analogia.

Lembrando que $a_n^+\, a_n^- = H_n - E_0^{(n)}$, podemos verificar usando (7) que

$$E_n = 2\ \frac{\sqrt{2\mu}}{h}\,(|M| + 1 + 2n) \qquad (n = 0,1,2,\dots) \tag{13}$$

$$a_n^+ = -\frac{d}{dx} + \frac{\sqrt{2\mu}}{h}\ x - \frac{|M| + 1/2 + n}{x} \tag{14}$$

$$\chi_{1,n} = a_1^+\ a_2^+ \dots a_n^+\ \chi_0 \tag{15}$$

onde χ_o é obtido pela equação diferencial:

$$a_1^- \chi_o = 0 \rightarrow \chi_o = x^{|M|+1/2}\, e^{-\frac{\sqrt{2\mu}}{2\hbar} x^2} \tag{16}$$

No problema original obtemos portanto (da igualdade (15)):

$$E_{org} = E_{n1} + E_{n2} = -2\,\frac{\sqrt{2\mu}}{\hbar}\,(2|M|+2+(2n_1+2n_2)) \qquad n_1 \text{ e } n_2 = 0,1,2, \tag{17}$$

Por outro lado (de (9) e (10))

$$E_{n1} + E_{n2} = -\frac{2\mu}{\hbar\sqrt{|E|}}\,(\alpha_1 + \alpha_2) \tag{18}$$

Usando (17) e (18) obtemos:

$$|E| = \frac{\mu}{8}\left[\frac{4\,\gamma^2\sigma^2\varepsilon_o a_o}{|M|+1+n_1+n_2}\right]^2 \tag{19}$$

que são os autovalores da energia para o potencial original, a função de onda é obtida usando a eq. (11) onde χ_1 e χ_2 tem a forma descrita por (15). Portanto, nosso resultado fornece a solução analitica exata para o potencial de Hartmann (8).

Concluimos frizando que atraves da realização de superalgebra introduzida neste trabalho podemos usar a supersimetria para resolver a equação de Schrödinger para alguns potenciais bi-dimensionais. Entretanto, o metodo não é geral, pois nem todos os potenciais bi-dimensionais, permitem que o Hamiltoniano original seja separado em dois Hamiltonianos uni-dimensionais.

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CÁLCULO DAS FUNÇÕES DE GREEN DO MODELO DE SCHWINGER GENERALIZADO PELO MÉTODO DE INTEGRAÇÃO FUNCIONAL

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Calculam-se as funções de Green do modelo de Schwinger generalizado não-anômalo, usando variadas fixações de calibre, através de uma seqüência de transformações nos campos do modelo. Em particular tratamos do caso em que o parâmetro de regularização assume o valor $a = g_1$ ($a=1$, no modelo de Schwinger quiral). Verifica-se tambem que, como observado por Girotti e Rothe [1], as funções de correlação invariantes de gauge são iguais nos dois modelos. Isto implica que o termo de fonte na integral funcional deve ser tal que gere apenas tais soluções [2].

A Lagrangeana do Modelo de Schwinger generalizado não-anômalo, após serem feitas as transformações que desacoplam os férmions, é dada por

$$\mathcal{L} = \bar{\psi}'(i\gamma_\mu \partial^\mu)\psi' + (1/2e^2)\chi\,\Box^2\chi + (1/2\pi e^2)(g_1^2 - M^2)(\eta + \theta)\Box(\eta + \theta)$$

$$+ (1/2\pi e^2)(g_2^2 + M^2)\chi\Box\chi - (g_1 g_2/\pi e^2)(\eta + \theta)\Box\chi \qquad (1)$$

onde $M^2 = ae^2$ e os campos η e χ são, respectivamente, as componentes longitudinal e transversal do campo de gauge

$$eA_\mu = \partial_\mu\eta + \epsilon_{\mu\nu}\partial^\nu\chi, \qquad (2)$$

e o campo θ é o campo de Wess-Zumino, aquele que restaura a simetria de gauge do modelo. Sendo esta uma versão invariante do modelo, devemos fixar o calibre a fim de obter as funções de Green. Vamos usar os seguintes termos de fixação de calibre:

$$i) \quad \mathcal{L}_{GF} = -(1/2\alpha)(\partial_\mu A^\mu + \beta\epsilon^{\mu\nu}\partial_\mu A_\nu)^2 = -(1/2\alpha)(\Box\eta - \beta\Box\chi)^2; \qquad (3a)$$

$$ii) \quad \mathcal{L}_{GF} = -(1/2\alpha)\partial_\mu\theta\partial^\mu\theta, \qquad (3b)$$

onde α e β são parâmetros de gauge.

CASO $M^2 \neq g_1^2$:

Neste caso, para o primeiro gauge fazemos a transformação:

$$\theta = \theta' + \left[\frac{g_1 g_2}{(g_1^2 - M^2)} - \beta\right]\chi' - \eta' \tag{4a}$$

$$\eta = \eta' + \beta\chi' \quad e \quad \chi = \chi' \tag{4b}$$

de modo que o modelo desacopla:

$$\mathcal{L} = \mathcal{L}_{MSQ} + \mathcal{L}_{GF}^{(i)} = (1/2e^2)\chi'\Box(\Box + m^2)\chi' + (1/2\pi e^2)(g_1^2 - M^2)\theta'\Box\theta' +$$

$$- (1/2\alpha e^2)\eta'\Box\eta', \tag{5}$$

com

$$m^2 = \frac{\left[(g_1 g_2)^2 - (g_1^2 - M^2)(g_2^2 + M^2)\right]}{\pi(M^2 - g_1^2)}. \tag{6}$$

Fazemos as mesmas transformações nos termos de fonte do funcional gerador, por exemplo

$$J_\mu A^\mu = (1/e)J^\mu\left[\partial_\mu(\eta' + \beta\chi') + \epsilon_{\mu\rho}\partial^\rho\chi'\right], \tag{7}$$

de modo que o propagador de fóton, que se obtém a partir de :

$$\langle A_\mu A_\nu\rangle = - \delta^2 Z/\delta J_\mu \delta J_\nu\Big|_{J=0} \tag{8}$$

vai ser dado na representação de momentum por:

$$D_{\mu\nu}(k) = -(1/e^2)\left[k_\mu k_\nu D_{\eta'} + (\beta k_\mu + \bar{k}_\mu)(\beta k_\nu + \bar{k}_\nu)D_{\chi'}\right], \tag{9}$$

onde $\bar{k}_\mu \equiv \epsilon_{\mu\rho}k^\rho$. Usando os propagadores livres dos campos η' e χ' obtem-se:

$$D_{\mu\nu}(k) = -(i/(k^2 - m^2))\left[g_{\mu\nu} - \left[1 + \beta^2 - \alpha + \alpha m^2/k^2\right]k_\mu k_\nu/k^2 + \right.$$

$$\left. - \beta(k_\mu\bar{k}_\nu + \bar{k}_\mu k_\nu)/k^2\right]. \tag{10}$$

No caso da segunda fixação de calibre, teremos analogamente:

1) Trantormações:

$$\eta = \eta' + \left[\frac{g_1 g_2}{(g_1^2 - M^2)}\right]\chi' - \theta' \tag{11a}$$

$$\chi = \chi' \text{ e } \theta = \theta' \tag{11b}$$

2) Lagrangeana:

$$\mathcal{L} = \mathcal{L}_{MSG} + \mathcal{L}_{GF}^{(ii)} = (1/2e^2)\chi'\Box(\Box + m^2)\chi' + (1/2\pi e^2)(g_1^2 - M^2)\eta'\Box\eta' +$$

$$+ (1/2\alpha)\theta'\Box\theta' \tag{12}$$

3) Termo de fonte:

$$J_\mu A^\mu = (1/e)J^\mu\left[\partial_\mu\left(\eta' + \left[\frac{g_1 g_2}{(g_1^2 - M^2)}\right]\chi' - \theta'\right) + \epsilon_{\mu\rho}\partial^\rho\chi'\right], \tag{13}$$

4) Propagador:

$$D_{\mu\nu}(k) = (i/(k^2 - m^2))\left\{-g_{\mu\nu} + \left[\frac{1}{(M^2 - g_1^2)}\right]\left[k_\mu k_\nu\left[(1/e^2)(\pi e^2 +\right.\right.\right.$$

$$- \alpha(M^2 - g_1^2)) + (1/k^2)\left[(m^2\alpha/e^2)(M^2 - g_1^2) - (g_1^2 + g_2^2)\right]\Big] +$$

$$\left.\left.(g_1 g_2/k^2)(k_\mu \bar{k}_\nu + \bar{k}_\mu k_\nu)\right]\right\}. \tag{14}$$

Caso $M^2 = g_1^2$:

Neste caso vamos usar um outro método para calcular os propagadores do fóton. Para isso reescrevemos a Lagrangeana (1), onde omitimos o termo de férmions por simples conveniência, na forma:

$$\mathcal{L} = (1/2)\rho^T \mathbb{M}\, \rho, \tag{15}$$

onde $\rho = \begin{pmatrix} \chi \\ \eta \\ \theta \end{pmatrix}$, de modo que temos por exemplo no caso do segundo gauge ($\partial_\mu\theta = 0$):

$$\mathbb{M} = (\Box/e^2)\begin{pmatrix} \Box + (g_1^2 + g_2^2) & -(g_1 g_2/2\pi e^2) & -(g_1 g_2/2\pi e^2) \\ -(g_1 g_2/2\pi e^2) & 0 & 0 \\ -(g_1 g_2/2\pi e^2) & 0 & (1/2\alpha e^2) \end{pmatrix} \tag{16}$$

Invertendo a matriz acima obtemos a matriz

$$M^{-1} = \begin{bmatrix} \langle\chi\ \chi\rangle & \langle\chi\ \eta\rangle & \langle\chi\ \theta\rangle \\ \langle\eta\ \chi\rangle & \langle\eta\ \eta\rangle & \langle\eta\ \theta\rangle \\ \langle\theta\ \chi\rangle & \langle\theta\ \eta\rangle & \langle\theta\ \theta\rangle \end{bmatrix}, \tag{17}$$

onde os elementos $\langle\chi\ \chi\rangle$, $\langle\chi\ \eta\rangle$, etc., são as funções de correlação de dois pontos entre os respectivos campos, a partir das quais podemos escrever por exemplo o propagador através da relação:

$$\langle A_\mu A_\nu\rangle = (1/e^2)\left[k_\mu k_\nu\langle\eta\ \eta\rangle + k_\mu\tilde{k}_\nu\langle\eta\ \chi\rangle + \tilde{k}_\mu k_\nu\langle\chi\ \eta\rangle + \tilde{k}_\mu\tilde{k}_\nu\langle\chi\ \chi\rangle\right], \tag{18}$$

e usando que $\tilde{k}_\mu\tilde{k}_\nu = -g_{\mu\nu} + k_\mu k_\nu$, obtemos finalmente que:

$$\langle A_\mu A_\nu\rangle = \pi(k_\mu k_\nu/k^2)\left[k^2 - (g_2^2 + g_2^2) + \alpha(g_1 g_2/\sqrt{\pi}\ e^2)\right] +$$

$$- (\pi/g_1 g_2)\left[(k_\mu\tilde{k}_\nu + \tilde{k}_\mu k_\nu)/k^2\right]. \tag{19}$$

Finalmente pode-se verificar que as funções de Green invariantes de gauge são as mesmas que no modelo anômalo, para isso podemos definir a corrente como:

$$J_\mu\left[A^\mu + (1/e)\partial^\mu\theta\right] \equiv J_\mu A^\mu_I\ , \tag{20}$$

onde A^μ_I é definido como o campo invariante de gauge. Observa-se ainda que os resultados acima podem ser comparados com aqueles encontrados na literatura, por exemplo no caso particular do modelo de Schwinger quiral [3,4] para o "gauge θ" no caso em que $\alpha = 0$, e para o "gauge de Lorentz generalizado" quando $\beta = (a - 1)^{-1}$ ou $\beta = 0$. Além disso vê-se que o "mal comportamento" do propagador de fóton quando $e \to 0$ [4] em (14), mostra-se ser um artefato do gauge, pois este termo pode ser eliminado através de uma adequada escolha do parâmetro de gauge α no "gauge θ" ($\alpha = \pi e^2/(M^2 - g_1^2)$).

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An alternative prescription for gauging Floreanini–Jackiw chiral bosons

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We seek new couplings of chiral bosons to $U(1)$ gauge fields. Lorentz covariance of the resulting constrained Lagrangian is checked with the help of a procedure based in the first-order formalism of Faddeev and Jackiw. We find Harada's constraint and another local one not previously considered. We analyze the constraint structure and part of the spectrum of this second solution and show that it is equivalent to an explicitly covariant coupling of Siegel's chiral boson to gauge fields, which preserves chirality under gauge transformations.

In the course of the analysis of chiral bosons properties, one natural step is to couple them to abelian and non–abelian gauge fields[1,2,3] in order to study the corresponding anomalies, or to provide an alternative approach to chiral models in two dimensions[4]. These couplings have been proposed both in Siegel's [5] *explicitly covariant formalism*[2,6] and in the approach of Floreanini and Jackiw (FJ)[7,8].

In the context of chiral theories in two dimensions, Harada has shown recently how to obtain a consistent coupling of FJ chiral bosons with a $U(1)$ gauge field, starting from the chiral Schwinger model (CSM) and discarding the right handed degrees of freedom by means of a projection in phase space implemented by the *chiral constraint* $\pi_\phi = \phi'$[8]. The resulting theory had the same spectrum of the CSM with the additional characteristic that the massless mode was self-dual. There was no trace, at the end, of the right-handed fermion originally present (which, however, was necessary for the eigenvalue problem of computing the fermion determinant to be well defined [9]). It has been shown later by Bazeia[10] that Harada's approach was equivalent to the one of Bellucci, Golterman and Petcher[2] under Faddeev and Jackiw's first–order formalism[11].

We investigate, in this letter, the possibility of obtaining different couplings for the FJ chiral boson, starting from the generalized Schwinger model, where both chiralities interact with the gauge field. We obtain the Lagrangian of the coupled system under a generalized chiral constraint and propose a check test which can straightforwardly decide whether the resulting coupling is Lorentz covariant or not. We observe that starting with the left handed chiral Schwinger model it is possible to couple chiral bosons to $U(1)$ gauge fields in two Lorentz covariant ways, using different *chiral constraints for one chirality* ($\pi_\phi = \phi'$) and for the other ($\pi_\phi = -\phi' + e(A_0 - A_1)$). The constraints $\pi_\phi = -\phi'$ and $\pi_\phi = \phi' + e(A_0 + A_1)$ are the ones allowed for the right handed CSM. The theory obtained using $\pi_\phi = -\phi' + e(A_0 - A_1)$ is shown to be equivalent to a specific coupling of Siegel's chiral bosons with $U(1)$ gauge fields which is symmetric under chirality preserving gauge transformations.

Our starting point is the Lagrangian of the generalized Schwinger model (GSM),

$$\mathcal{L} = \overline{\Psi} \gamma^{\mu} \left(i\partial_{\mu} + e_R A_{\mu} \frac{(1 + \gamma_5)}{2} + e_L A_{\mu} \frac{(1 - \gamma_5)}{2} \right) \Psi \qquad (1)$$

which is equivalent to its bosonized version[12,14]

$$\mathcal{L}_B = \frac{1}{2} (\partial_{\mu} \phi)^2 + \frac{1}{\sqrt{\pi}} \left(\bar{g}_1 \partial^{\mu} - \bar{g}_2 \tilde{\partial}^{\mu} \right) \phi A_{\mu} + \frac{\bar{M}^2}{2\pi} A_{\mu}^2 \qquad (2)$$

where

$$\bar{g}_1 = \frac{\bar{e}_L + \bar{e}_R}{2}, \quad \bar{g}_2 = \frac{\bar{e}_L - \bar{e}_R}{2}, \quad \bar{M}^2 = \frac{\hat{e}_L \hat{e}_R + \bar{e}_L \bar{e}_R}{2}. \qquad (3)$$

In (3) $\hat{e}_L$ and $\hat{e}_R$ are arbitrary couplings introduced by the regularization procedure[13] and $\bar{e}_L$ and $\bar{e}_R$ are defined as[12]

$$\bar{e}_L = \left(\hat{e}_L^2 + (\hat{e}_L - e_L)^2\right)^{1/2}, \quad \bar{e}_R = \left(\hat{e}_R^2 + (\hat{e}_R - e_R)^2\right)^{1/2}. \qquad (4)$$

The Hamiltonian obtained from $\mathcal{L}_B$ is

$$\begin{aligned} \mathcal{H}_B &= \frac{1}{2} (\pi_{\phi} - g_1 A_0 - g_2 A_1)^2 \\ &+ \frac{1}{2} \phi'^2 + \phi'(g_1 A_1 + g_2 A_0) - \frac{M^2}{2} A_{\mu}^2 \end{aligned} \qquad (5)$$

with $\sqrt{\pi}\, g_i = \bar{g}_i$ and $\pi M^2 = \bar{M}^2$.

We project one chirality with the aid of a generalized chiral constraint

$$\Omega = \pi_{\phi} - \alpha \phi' \qquad (6)$$

In (6), α can be a function of ϕ, ϕ' and A_{μ} but not of $\dot{\phi}$, in order that Ω remains constraint. We further impose the requirement on α that $\{\Omega, \Omega\}$ is not to be field dependent, so that it can be absorbed in the normalization of the functional integral

$$Z_{ch}[A] = \int \mathcal{D}\phi\, \mathcal{D}\pi_{\phi}\, \delta(\Omega)\, |det\, \{\Omega, \Omega\}|^{\frac{1}{2}} \exp(i \int d^2x\, (\pi_{\phi} \dot{\phi} - \mathcal{H}_B)). \qquad (7)$$

Under these assumptions the analysis can proceed along classical lines. Functionally integrating over the π_{ϕ} field in (7) we obtain our effective Lagrangian

$$\mathcal{L}_{\alpha} = \alpha \dot{\phi}\phi' - \frac{(\alpha^2 + 1)}{2} \phi'^2$$

$$+\phi' \left((\alpha g_1 - g_2) A_0 + (\alpha g_2 - g_1) A_1 \right)$$
$$- \frac{1}{2} (g_1 A_0 + g_2 A_1)^2 + \frac{M^2}{2} A_\mu^2. \tag{8}$$

Now, we ask which values of α are allowed in order to produce a Lorentz covariant theory. We exemplify our strategy with the non-gauged original FJ Lagrangian,

$$\mathcal{L}_{FJ} = \dot{\phi}\phi' - \phi'^2. \tag{9}$$

Performing a Lorentz rotation,

$$\begin{pmatrix} \dot{\phi} \\ \phi' \end{pmatrix} \longrightarrow \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \phi' \end{pmatrix} \tag{10}$$

this Lagrangian changes to

$$\mathcal{L}_{FJ}^R = a(x)\,\dot{\phi}^2 + b(x)\,\dot{\phi}\phi' + c(x)\,\phi'^2 \tag{11}$$

with

$$a(x) = \frac{(x^2 - 1)}{2x^2}, \quad b(x) = \frac{1}{x^2}$$
$$c(x) = -\frac{(x^2 + 1)}{2x^2}, \quad x = e^\theta. \tag{12}$$

Using the first-order formalism of Faddeev and Jackiw we construct a first-order Lagrangian to (11)

$$\mathcal{L}_{FJ}^{R,1} = \pi_\phi \dot{\phi} - \frac{x^2 \pi_\phi^2}{2(x^2 - 1)} - \frac{x^2 \phi'^2}{2(x^2 - 1)} + \frac{\pi_\phi \phi'}{x^2 - 1}. \tag{13}$$

Now we notice that although $\mathcal{L}_{FJ}$ describes a constrained system, this is not what happens to $\mathcal{L}_{FJ}^R$. It is thus legitimate to ask whether the resulting theory is equivalent to the previous one in the new reference frame, if the constraint is taken into account. Imposing that $\pi_\phi = \phi'$ we obtain simply

$$\mathcal{L}_{FJ}^{R,1}\big|_{\pi_\phi = \phi'} = \mathcal{L}_{FJ}, \tag{14}$$

thus showing that under the chiral constraint assumption, both Lagrangians ((11) and (9)) are equivalent.

Let us make the same analysis for $\mathcal{L}_\alpha$, eq. (8). The Lorentz rotation (10) produces (rotating also, obviously, A_0 and A_1),

$$\mathcal{L}_\alpha^R = a(x)\,\dot{\phi}^2 + b(x)\,\dot{\phi}\phi' + c(x)\,\phi'^2$$
$$+ \left\{ (x^2 - 1)\,\dot{\phi} + (x^2 + 1)\,\phi' \right\} \{ d_+(x)\,A_0 + d_-(x)\,A_1 \}$$

$$- \frac{1}{2} \left(e_+ A_0 + e_- A_1 \right)^2 + \frac{1}{2} M^2 A_\mu^2 \tag{15}$$

with

$$\begin{aligned}
a(x) &= -\frac{(x^2-1)}{8x^2} \left(\alpha^2 (x^2-1) - 2\alpha (x^2+1) + x^2 - 1 \right) \\
b(x) &= -\frac{1}{4x^2} \left(\alpha^2 (x^4-1) - 2\alpha (x^4+1) + x^4 - 1 \right) \\
c(x) &= -\frac{(x^2+1)}{8x^2} \left(\alpha^2 (x^2+1) - 2\alpha (x^2-1) + x^2 + 1 \right)
\end{aligned} \tag{16}$$

$$\begin{aligned}
d_\pm &= \frac{1}{4x^2} \left\{ \alpha \left((x^2 \pm 1) g_1 + (x^2 \mp 1) g_2 \right) - \left((x^2 \mp 1) g_1 + (x^2 \pm 1) g_2 \right) \right\} \\
&\equiv \left(\alpha e_\pm - e_\mp \right)
\end{aligned}$$

The first-order Lagrangian after the imposition of the generalized constraint is

$$\begin{aligned}
\mathcal{L}_\alpha^{R,1} &= \alpha \dot{\phi} \phi' + \frac{4ac - \alpha^2 + b(2a-b)}{4a} \phi'^2 \\
&+ \frac{(2a(x^2+1) + (x^2-1)(\alpha - b))}{2a} \phi' \left((\alpha e_+ - e_-) A_0 + (\alpha e_- - e_+) A_1 \right) \\
&- \frac{1}{2} (e_+ A_0 + e_- A_1)^2 - \frac{(x^2-1)^2}{4a} \left((\alpha e_+ - e_-) A_0 + (\alpha e_- - e_+) A_1 \right)^2 \\
&+ M^2 A_\mu^2
\end{aligned} \tag{17}$$

This expression only equals (8) if

$$(\alpha^2 - 1) \phi' - (g_1 \alpha + g_2) A_0 - (g_2 \alpha + g_1) A_1 = 0 \tag{18}$$

Solving this equation for α we find the set of constraints which preserves relativistic covariance,

$$\begin{aligned}
\pi_\phi &= \pm \frac{1}{2} \left(4\phi'^2 + 4\phi' (g_2 A_0 + g_1 A_1) + (g_1 A_0 + g_2 A_1)^2 \right)^{1/2} \\
&+ \frac{1}{2} (g_1 A_0 + g_2 A_1)
\end{aligned} \tag{19}$$

From (19) we see that there are only two cases where we can get $\{\Omega, \Omega\}$ field independent and simultaneously obtain a polinomial Lagrangian, namely
i) $g_1 = g_2 = e$ (right handed chiral Schwinger model), with constraints

$$\pi_\phi = - \phi' \tag{20}$$

and

$$\pi_\phi = \phi' + e(A_0 + A_1); \tag{21}$$

ii) $g_1 = -g_2 = e$ (left handed chiral Schwinger model), with constraints

$$\pi_\phi = \phi' \tag{22}$$

and

$$\pi_\phi = -\phi' + e(A_0 - A_1). \tag{23}$$

Cases (20) and (22) are the cases studied by Harada and found elsewhere in the literature [2,4,8,10]. Cases (21) and (23) have not been previously considered. To be definite we will start from case (ii) and complete the gauging of the chiral boson within the context of the left-handed chiral Schwinger model. Imposing (23) on (2), with $\bar{g}_1 = -\bar{g}_2 = \sqrt{\pi}e$, we obtain in the same way that we did before

$$\begin{aligned}\mathcal{L}_{CH} &= -\dot{\phi}\phi' - \phi'^2 + e\left(\dot{\phi} + \phi'\right)(A_0 - A_1) \\ &\quad + \frac{M^2}{2} A_\mu A^\mu - \frac{1}{4} F_{\mu\nu}F^{\mu\nu}\end{aligned} \tag{24}$$

where we added a kinectical term to the resulting Lagrangian. From (24) we compute the canonical Hamiltonian

$$\mathcal{H}_C = \frac{(\pi^1)^2}{2} + \pi^1 A_0' + \phi'^2 - e\phi'(A_0 - A_1) - \frac{M^2}{2} A_\mu^2. \tag{25}$$

The constraint (23) is second class,

$$\{\Omega_2(x), \Omega_2(x)\} = +2\delta'(x^1 - y^1) \tag{26}$$

with $\Omega_2 = \pi_\phi + \phi' - e(A_0 - A_1)$. There is another primary constraint, $\Omega_1 = \pi^0$, whose consistence under time evolution produces

$$\begin{aligned}\dot{\Omega}_1 &= \left\{\Omega_1, \int dy^1 (\mathcal{H}_c + u_1\Omega_1 + u_2\Omega_2)\right\} \\ &= \partial_1\pi^1 + e\phi' + M^2 A_0 + eu_2 = 0.\end{aligned} \tag{27}$$

This determines u_2, while u_1 is determined through $\dot{\Omega}_2 = 0$. The inverse of the constraint matrix is given by

$$C_{ij}(x,y) = \frac{1}{e^2}\begin{pmatrix} 2\delta'(x^1 - y^1) & e\delta(x^1 - y^1) \\ -e\delta(x^1 - y^1) & 0 \end{pmatrix} \tag{28}$$

and the non-null Dirac brackets are

$$\{\phi(x), \pi_\phi(y)\}^* = \delta(x^1 - y^1) \quad ; \quad \{\phi(x), A_0(y)\}^* = \frac{1}{e}\delta(x^1 - y^1) \quad ;$$

$$\left\{ \pi_\phi (x) , A_0 (y) \right\}^* = \frac{1}{e} \delta' \left(x^1 - y^1 \right) \quad ;$$

$$\left\{ A_0 (x) , A_0 (y) \right\}^* = -\frac{2}{e^2} \delta' \left(x^1 - y^1 \right) \; ; \; \left\{ A_0 (x) , \pi^1 (y) \right\}^* = \frac{1}{e} \delta \left(x^1 - y^1 \right) \; ;$$

$$\left\{ A_1 (x) , \pi^1 (y) \right\}^* = \delta \left(x^1 - y^1 \right) . \tag{29}$$

One can choose ϕ to be eliminated from (25), after using the constraints strongly, and then arrive to the final Hamiltonian,

$$\mathcal{H} = \frac{\left(\pi^1\right)^2}{2} + \pi^1 A_0' + \pi_\phi^2 - e\, \pi_\phi (A_0 - A_1) - \frac{M^2}{2} A_\mu^2 \tag{30}$$

Thanks to the non standard commutation relations obeyed by A_0, it is not easy to solve the equations of motion obtained from (30). To see something about the spectrum of this theory we can integrate functionally over the A_μ field to obtain an effective Lagrangian for the ϕ field,

$$\mathcal{L}_{eff} = \frac{1}{2} \phi \left(\frac{\left(e^2 - M^2\right) \Box^2 - M^4 \Box + M^2 \left(\Box + M^2\right) \partial_+ \partial_+}{M^2 \left(\Box + M^2\right)} \right) \phi \tag{31}$$

Using $M^2 = e^2 a$, we see that there are poles in the following regions in the (k_+ , k_-)-plane:
i) $a \neq 1$,

$$k_- = \pm a \left(\frac{\left(a^2 e^4 - 2(a-2) e^2 k_+^2 + k_+^4\right)^{1/2} \pm \left(a e^2 + k_+^2\right)}{2 k_+ (a-1)} \right),$$
$$k_+ = 0 \tag{32}$$

ii) $a = 1$,

$$k_- = \frac{e^2 k_+}{e^2 + k_+^2} \quad ,$$
$$k_+ = 0 \tag{33}$$

Although the expression for the k_- curve is not directly Lorentz covariant, we can see explicitly the presence of a self dual pole in the spectrum of the theory, with the correct chirality

Yet the appearance of only the A_- components of the A_μ field in the Lagrangian suggests that this kind of coupling could be obtained by a kind of "self dual" gauging, in which only the ∂_- derivative would be covariantized. This has led us to consider Siegel's formalism for the right handed chiral boson

$$\mathcal{L}_S = \frac{1}{2} \partial_+ \phi\, \partial_- \phi + \frac{1}{2} \lambda \left(\partial_+ \phi\right)^2 . \tag{34}$$

Performing the substitution

$$\partial_-\phi \longrightarrow \partial_-\phi + 2\,e\,A_- \tag{35}$$

we get

$$\mathcal{L}_S^g = \frac{(\lambda+1)}{2}\dot\phi^2 + \lambda\dot\phi\phi' + \frac{(\lambda-1)}{2}\phi'^2 + e\left(\dot\phi+\phi'\right)(A_0 - A_1). \tag{36}$$

The first-order Lagrangian is

$$\mathcal{L}_S^{g,1} = \pi_\phi\dot\phi - \frac{1}{\lambda+1}\left\{\frac{1}{2}\pi_\phi^2 - \lambda\pi_\phi\phi' - e\left(\pi_\phi+\phi'\right)(A_0 - A_1) + \frac{1}{2}e^2(A_0 - A_1)\right\}. \tag{37}$$

Solving the constraint through the equation of motion for λ, we obtain

$$\pi_\phi \equiv -\phi' + e(A_0 - A_1) \tag{38}$$

and, after substitution in (37), we get $\mathcal{L}'_{CH}$ given by

$$\mathcal{L}'_{CH} = -\dot\phi\phi' - \phi'^2 + e\left(\dot\phi+\phi'\right)(A_0 - A_1) \tag{39}$$

which is the same as $\mathcal{L}_{CH}$, eq.(24), without the last two terms.

Finally, we would like to notice that the gauge symmetry of the model proposed in (36) is a kind of "chiral" gauge symmetry: the symmetry of the model is $\phi \to \phi + \epsilon$ and $A_- \to A_- - \frac{1}{2e}\partial_-\epsilon$, $\epsilon = \epsilon(x^-)$. This symmetry preserves the chirality of the chiral boson under gauge transformations. It is also responsible for more degrees of freedom than those present in the case considered by Harada[8], as we can take A_+ as a gauge invariant quantity under these restricted transformations. If this model is an alternative description for the minimal chiral Schwinger model, is a very interesting question to be adressed in the near future.

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[14] Our conventions are $\gamma^0 = \sigma^1$, $\gamma^1 = -i\,\sigma^2$, $\gamma_5 = \gamma^0\gamma^1 = \sigma^3$,
$\eta_{\mu\nu} = \mathrm{diag}(+,-)$, $\epsilon^{01} = -\epsilon_{01} = +1$, $\tilde{\partial}_\mu = \epsilon_{\mu\nu}\partial^\nu$,
$\gamma_5\,\Psi_{L,R} = \mp\Psi_{L,R}$, $\dot{\phi} = \partial_0\phi$, $\phi' = \partial_1\phi$,
$\partial_\pm = \partial_0 \pm \partial_1$, $A_\pm = A_0 \pm A_1$.

ACOPLAMENTO YANG-MILLS/MODELO SIGMA (2,0) EM VARIEDADES COM TORÇÃO

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RESUMO: No superespaço (2,0) efetua-se o acoplamento do supermultipleto de Yang-Mills ao modelo σ não linear. Isto é realizado através do *gauging* das isometrias do espaço-alvo do modelo σ, neste caso, considerado como uma variedade genérica com torção.

A ação do modelo-σ manifestamente invariante sob a supersimetria-(2,0) é a seguinte *:

$$S_\sigma = i\int d^2x\, d\theta_+ d\bar{\theta}_+ \left[K_i(\Phi,\bar{\Phi})\partial_{--}\Phi^i - K_{\bar{i}}(\Phi,\bar{\Phi})\partial_{--}\Phi^{\bar{i}} \right] \quad , \tag{1}$$

onde o vetor $K_i(\Phi,\bar{\Phi})$, algumas vezes chamado de prepotencial, é definido no espaço alvo, cujas coordenadas são os supercampos escalares $\Phi,\bar{\Phi}$ do superespaço (2,0). Os índices latinos $i(\bar{i})=1,\ldots,n(\bar{n})$ denotam um espaço alvo 2n-dimensional. O prepotencial contém toda a informação sobre a geometria do modelo, uma vez que a partir dele podemos obter a métrica e a torção, ou seja

$$g_{i\bar{j}} = \frac{1}{2}\left(\partial_i K_{\bar{j}} + \partial_{\bar{j}} K_i\right) \quad ; \quad b_{i\bar{j}} = \frac{1}{2}\left(\partial_i K_{\bar{j}} - \partial_{\bar{j}} K_i\right) \tag{2}$$

Vale lembrar que estes supercampos são "quirais", no sentido que obedecem aos vínculos $D_+\bar{\Phi}^i = \bar{D}_+\Phi^i = 0$.

Duas invariâncias de "gauge" estão presentes na ação (1):

$$\delta K_i = i\partial_i \Lambda(\Phi,\bar{\Phi}) \text{ com } \Lambda \text{ real} \tag{3}$$

$$\delta K_i = F_i(\Phi)\,; \quad \delta K_{\bar{i}} = F_{\bar{i}}(\bar{\Phi}) \qquad \text{com } \partial_{\bar{j}} F_i = \partial_j F_{\bar{i}} = 0 \quad . \tag{4}$$

Devido à óbvia semelhança com a transformação de Kähler, a menos do fato de que esta é definida para escalares, enquanto que (4) é definida para vetores, rotulemos esta última de transformação vetorial do tipo Kähler.

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* Notações e convenções sobre o superespaço (2,0) podem ser encontradas no trabalho *Modelo de Schwinger Quiral no Superespaço (2,0)*, apresentado neste mesmo volume.

Consideremos transformações nos supercampos (coordenadas da variedade) tais que:

$$\delta\Phi^i = \lambda^\alpha k^i_\alpha(\Phi) \quad ; \quad \delta\bar{\Phi}^{\bar{i}} = \lambda^\alpha k^{\bar{i}}_\alpha(\bar{\Phi}) \quad , \tag{5}$$

onde k^i é um vetor do espaço alvo e λ^α é um parâmetro global. O vetor $k^i(\Phi)$ $(\bar{k}^{\bar{i}}(\bar{\Phi}))$ é uma função holomórfica (anti-holomórfica), no sentido de que dependem apenas de um supercampo quiral (anti-quiral).

Sob as transformações (5), o prepotencial $K_i(\Phi,\bar{\Phi})$ comporta-se da seguinte forma

$$\delta K_i \equiv K_i(\Phi',\bar{\Phi}') - K_i(\Phi,\bar{\Phi}) = (\partial_j K_i)\delta\Phi^j + (\partial_{\bar{j}} K_i)\delta\bar{\Phi}^{\bar{j}} =$$

$$= (\partial_j K_i)\lambda^\alpha k^j_\alpha(\Phi) + (\partial_{\bar{j}} K_i)\lambda^\alpha k^{\bar{j}}_\alpha(\bar{\Phi}) \tag{6}$$

valendo, também, é claro, o conjugado complexo da equação acima.

As condições para a invariância da ação (1) sob as transformações (5), podem ser resumidas na expressão:

$$\mathcal{L}^\alpha K_i(\Phi,\bar{\Phi}) = F^\alpha_i(\Phi) + i\,\partial_i L^\alpha(\Phi,\bar{\Phi}) \tag{7}$$

onde $\mathcal{L}_\alpha K_i$ é a derivada de Lie do prepotencial na direção do vetor k^j e é definida como

$$\mathcal{L}_\alpha K_i \equiv K_{i,j}\,k^j_\alpha + K_{i,\bar{j}}\,k^{\bar{j}}_\alpha + K_j\,\partial_i k^j_\alpha \quad . \tag{8}$$

Lembramos que $L(\Phi,\bar{\Phi})$ é uma função escalar real. A partir da equação (7) podemos obter as condições para a invariância do modelo bosônico, a saber, que o vetor k^i seja um vetor de Killing da variedade e que a derivada de Lie do potencial de torção se anule, caracterizando que as transformações (5) são isometrias da variedade.

Nosso objetivo agora é introduzir supercampos de gauge através do procedimento de elevar as isometrias à condição de simetrias *locais*. Como sabemos, uma transformação de isometria deixa a métrica invariante, portanto o prepotencial $K_i(\Phi,\bar{\Phi})$, também o será (Ver eq. (2)), a menos de uma transformação vetorial de Kähler. Em outras palavras, se a variação δK_i não for zero, deve ser no máximo igual à transformação de Kahler. Desta forma, podemos identificar

$$\delta K_i \equiv K_i(\Phi',\bar{\Phi}') - K_i(\Phi,\bar{\Phi}) = \lambda^\alpha\left(k^l_\alpha K_{i,l} + k^j_\alpha K_{i,j}\right) = \lambda^\alpha F_{i\alpha}(\Phi) \tag{9}$$

Considerando a dependência dos prepotenciais nos supercampos Φ e $\bar{\Phi}$, e levando em conta ainda a simetria da ação e da métrica, a equação acima devem ser modificadas para

$$\delta K_i = \lambda^\alpha\left(k_\alpha^j K_{i,j} + k_\alpha^{\bar{j}} K_{i,\bar{j}}\right) = \lambda^\alpha F_{i\alpha}(\Phi) + i\lambda^\alpha \partial_i M_\alpha(\Phi,\bar{\Phi}) \tag{10}$$

As tranformações *locais* do subgrupo de isometria são escritas na forma

$$\delta\Phi^i = \Lambda^\alpha\, k_\alpha^i(\Phi) \qquad\qquad \delta\Phi^{\bar{i}} = \bar{\Lambda}^\alpha\, k_\alpha^{\bar{i}}(\bar{\Phi}) \quad , \tag{11}$$

onde $\Lambda \equiv \Lambda^\alpha(x;\theta,\bar{\theta})Q_\alpha$ é um supercampo quiral parâmetro de gauge.

Na forma finita estas transformações tornam-se

$$\Phi^i \longmapsto \Phi'^i = \exp(\, L_{\Lambda.k}\,)\Phi^i \quad ; \quad \Phi^{\bar{i}} \longmapsto \Phi'^{\bar{i}} = \exp(\, L_{\bar{\Lambda}.\bar{k}}\,)\Phi^{\bar{i}} \tag{12}$$

onde os operadores $L_{\Lambda.k}$ e $L_{\bar{\Lambda}.\bar{k}}$ são definidos como

$$L_{\Lambda.k}\,\Phi^i \equiv \left[\Lambda^\alpha\, k_\alpha^j \frac{\partial}{\partial\Phi^j}\,,\,\Phi^i\right] \quad ; \quad L_{\bar{\Lambda}.\bar{k}}\,\Phi^{\bar{i}} \equiv \left[\bar{\Lambda}^\alpha\, k_\alpha^{\bar{j}} \frac{\partial}{\partial\Phi^{\bar{j}}}\,,\,\Phi^{\bar{i}}\right] \tag{13}$$

A fim de covariantizar o prepotencial K_i e expressar todas as variações de gauge em termos do supercampo $\Lambda(x;\theta,\bar{\theta})$, de tal forma a imitar o caso das transformações globais, propomos uma redefinição de campos onde esteja embutida a troca $\bar{\Lambda} \to iV$. Definimos um supercampo $\tilde{\Phi}$, que corresponde a uma "covariantização" do supercampo $\bar{\Phi}$, tal que

$$\tilde{\Phi}^i \equiv \exp(L_{iV.\bar{k}})\bar{\Phi}^i \tag{14}$$

onde a transformação de gauge de V é fixada na forma abaixo

$$e^{iL_{V'.\bar{k}}} = e^{L_{\Lambda.\bar{k}}}\; e^{iL_{V.\bar{k}}}\; e^{-L_{\bar{\Lambda}.\bar{k}}} \quad , \tag{15}$$

Portanto , $\tilde{\Phi}$ transforma-se como

$$\tilde{\Phi}^i \longmapsto \tilde{\Phi}'^i = \exp(\, L_{\Lambda.\bar{k}}\,)\, \tilde{\Phi}^i \quad , \tag{16}$$

No entanto, esta prescrição não é suficiente para tornar a ação do modelo-σ simultaneamente invariante sob simetria de gauge e transformações de Kähler *locais*. Para isso, sugere-se a introdução de um par de supercampos auxiliares quirais e anti-quirais, $\xi_i(\Phi)$ e $\bar{\xi}_{\bar{i}}(\bar{\Phi})$, cujas transformações de Yang-Mills são tais que compensam a variação de isometria de K_i. No caso global estes supercampos auxiliares são tais que

$$\delta\xi_i(\Phi) \equiv \lambda^\alpha F_{i\alpha}(\Phi) \qquad ; \qquad \delta\bar{\xi}_{\bar{i}}(\bar{\Phi}) \equiv \lambda^\alpha \bar{F}_{\bar{i}\alpha}(\bar{\Phi}) \qquad . \tag{17}$$

Formulamos a prescrição de *gauging* fazendo as substituições $\bar{\Phi} \longrightarrow \tilde{\Phi}$ e $\bar{\xi} \longrightarrow \tilde{\xi}$, de tal forma que obtemos a seguinte lagrangeana

$$\mathcal{L}_\xi = [K_i(\Phi,\tilde{\Phi}) - \xi_i(\Phi)]\nabla_{-\,-}\Phi^i - [\tilde{K}_i(\Phi,\tilde{\Phi}) - \tilde{\xi}_i(\tilde{\Phi})]\nabla_{-\,-}\tilde{\Phi}^i \tag{18}$$

onde

$$\nabla_{-\,-}\Phi^i \equiv \partial_{-\,-}\Phi^i - g\Gamma^\alpha_{-\,-}\, k^i_\alpha(\Phi) \quad ; \quad \nabla_{-\,-}\tilde{\Phi}^i \equiv \partial_{-\,-}\tilde{\Phi}^i - g\Gamma^\alpha_{-\,-}\, \tilde{k}^i_\alpha(\tilde{\Phi}) \tag{19}$$

Na equação (18), $\tilde{K}_i(\Phi,\tilde{\Phi})$ indica o complexo conjugado de $K_i(\Phi,\tilde{\Phi})$.

Considerando que as derivadas covariantes (19) transformam-se como os supercampos Φ e $\tilde{\Phi}$, e tendo em vista que

$$\delta\xi_i(\Phi) \equiv \Lambda^\alpha F_{i\alpha}(\Phi) \qquad ; \qquad \delta\tilde{\xi}_i(\tilde{\Phi}) \equiv \Lambda^\alpha \tilde{F}_{i\alpha}(\tilde{\Phi}) \qquad , \tag{20}$$

a variação da Lagrangeana (18) é dada por

$$\delta\mathcal{L}_\xi = \Lambda^\alpha\Big[\Big(\mathcal{L}_\alpha K_i - \mathcal{L}_\alpha\xi_i\Big)\nabla_{-\,-}\Phi^i - \Big(\mathcal{L}_\alpha\tilde{K}_i - \mathcal{L}_\alpha\tilde{\xi}_i\Big)\nabla_{-\,-}\tilde{\Phi}^i \Big] \qquad . \tag{21}$$

Portanto a condição para a invariância *local* da *Lagrangeana* (18) é que existam vetores R e $\tilde{R}$, tais que

$$\mathcal{L}_\alpha R_i \equiv \mathcal{L}_\alpha(K_i - \xi_i) = 0 \qquad ; \qquad \mathcal{L}_\alpha\tilde{R}_i \equiv \mathcal{L}_\alpha(\tilde{K}_i - \tilde{\xi}_i) = 0 \tag{22}$$

Acerca dos supercampos auxiliares, assinalamos que no caso da variedade com torção, o *gauging* de um subgrupo de isometria requer a introdução de supercampos auxiliares, os quais são vetores da variedade alvo. No entanto, uma vez que eles são holomórficos ou anti-holomórficos, a métrica definida a partir dos vetores R_i é a mesma obtida a partir dos vetores K_i . Desta forma diferentes escolhas de ξ correspondem à mesma ação do modelo-σ em termos dos campos componentes.

A EXPANSÃO DO HEAT KERNEL NO ESPAÇO-TEMPO CURVO À TEMPERATURA FINITA

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Resumo. Neste trabalho encontramos a expansão do heat kernel no espaço-tempo curvo à temperatura finita. Usamos, então essa expansão para calcular as anomalias quiral e de traço, nessa situação .

O heat kernel no espaço-tempo curvo é bem conhecido [1] e tem sido utilizado em técnicas de regularização em teoria quântica de campos desde a década de 60. A partir dos trabalhos de Fujikawa [2], envolvendo o cáculo de anomalias via método funcional, houve um renovado interesse por essa técnica.

Neste trabalho vamos calcular a expansão assintótica do heat kernel no espaço-tempo curvo à temperatura finita, usando para isto o formalismo de tempo imaginário [3]. Este trabalho é uma generalização de outro anterior, restrito ao espaço-tempo chato [4].

O heat kernel no espaço-tempo curvo, à temperatura zero, é usualmente definido como [1]

$$\begin{aligned} H^{(N)}(x,x';t) &= \langle x'|e^{-tD^2}|x\rangle \\ &= e^{-tD^2} V^{1/2}(x,x') I(x,x') \delta(x,x') \end{aligned} \tag{1}$$

onde $D^2 = D^\mu D_\mu + X$, D_μ é o operador de Dirac, $V(x,x')$ é o determinante de Van Vleck-Morette e $I(x,x')$ o deslocamento geodético paralelo. A expansão assintótica para $t \to 0$ de sua parte diagonal ($x = x'$) é dada por

$$H(x,x;t) \approx \frac{1}{(4\pi)^{N/2}} \sum_m a_m(x)\, t^m \,, \tag{2}$$

onde N é a dimensão do espaço-tempo $e = det\ e^a_\mu$ e os $a_n(x)$ são os coeficientes de Seeley, que são usualmente calculados por fórmulas de recorrência [1]. Entretanto a técnica desenvolvida por Fujikawa para o cálculo de anomalias via integração funcional pode também ser usada para este fim. Esta abordagem é particularmente útil no formalismo de tempo imaginário.

À temperatura finita, o heat kernel pode, então , ser escrito como:

$$H_\beta^{(N)}(x,x';t) = e^{-tD^2}\Delta^{1/2}(x,x')H(x,x')\delta_\beta(x,x') \ , \tag{3}$$

onde a função delta generalizada, à temperatura finita, é dada por

$$\delta_\beta^{(N)}(x,x') = \sum_{n=-\infty}^{+\infty} \int \frac{d^{(N)}k}{(2\pi)^N} \exp\{ik^\mu \sigma(x,x')_{;\mu}\} \left(\frac{2\pi}{\beta}\right) \delta\left(k_0 - \frac{2\pi}{\beta}(n+\frac{1}{2})\right) , \tag{4}$$

análoga à forma correspondente no espaço-tempo chato [3,4]. Cabe ressaltar que esta fórmula é valida para campos fermiônicos, que são antiperiódicos em relação à translações temporais. A fórmula correspondente para campos bosônicos é obtida trocando $n+1/2$ por n na função delta usual. Aplicando a técnica de Fujikawa para o cálculo dos coeficientes à temperatura finita [4], encontramos

$$a_m(x;\beta^2/t) = a_m(x) \left(1 + \sum_n (-1)^n \exp\left\{-\frac{n^2\beta^2}{4t}\right\}\right) , \tag{5}$$

que substituida em (2) nos fornece a expressão do heat kernel no espaço-tempo curvo à temperatura finita. Dessa forma, os kernels às temperaturas finita e zero são relacionados por:

$$H_\beta^{(N)}(x,x';t) = H^{(N)}(x,x';t) \left(1 + \sum_n (-1)^n \exp\left\{-\frac{n^2\beta^2}{4t}\right\}\right) , \tag{6}$$

Os coeficientes à temperatura zero, $a_m(x)$, são os usuais (spin 1/2) [1]:

$$a_0(x) = 1 \tag{7a}$$

$$a_1(x) = \frac{1}{6}R - X \tag{7b}$$

$$\begin{aligned} a_2(x) = {} & \frac{1}{12}\Lambda_{\mu\nu}\Lambda^{\mu\nu} + \frac{1}{180}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu}) \\ & - \frac{1}{6}(\frac{1}{5}R - X)_{;\mu}{}^{\mu} + \frac{1}{2}(\frac{1}{6}R - X)^2 . \end{aligned} \tag{7c}$$

As anomalias, em geral, podem ser escritas como uma soma divergente [2], o mesmo acontecendo à temperatura finita:

$$A_\beta(x) = \sum_n \left(\bar{\phi}_n(x) \gamma \phi_n(x) \right)_\beta , \tag{8}$$

onde $\gamma = \gamma_5$ para a anomalia quiral e $\gamma = 1$ para a anomalia de traço [2]. Sua expressão regularizada é então dada por

$$A_\beta(x) \Big|_{reg} = \lim_{s \to 0} tr \left(\gamma K_\beta(x, x; s) \right) , \tag{9}$$

onde $K_\beta(x, x; s)$ é o kernel do operador potência, definido pela transformada de Mellin [5]

$$K_\beta^{(N)}(x, x; s) = \frac{1}{\Gamma(s)} \int_0^\infty dt\, t^{s-1} H_\beta^{(N)}(x, x; t) . \tag{10}$$

Substituindo (5), (6) e (10) na Eq. (9), fazendo a integração em t e tomando o limite em s, obtemos:

$$A_\beta(x) \Big|_{reg} = A(x) \Big|_{reg} = tr \left(\gamma a_n(x) \right)_{n=N/2} , \tag{11}$$

logo as anomalias quiral e de traço são independentes da temperatura. Esse resultado para a anomalia quiral já é bem conhecido no espaço-tempo chato [3], assim como para a anomalia de traco nos espaço-tempos estáticos [6] ou conformalmente chatos [7]. O resultado apresentado aqui, então , pode ser entendido como uma generalização dos anteriores. Fisicamente, este resultado pode ser explicado com base no fato dessas anomalias serem fenômenos de grandes momentos e consequentemente de pequenos comprimentos de onda. Logo, a pequenas distâncias a estrutura global do espaço tempo não modifica essas quantidades, uma vez que sempre é possível aproximar um espaço-tempo curvo, nas vizinhanças de um ponto, por um espaço-tempo chato.

Um aspecto interessante que convém salientar é a característica topológica da anomalia quiral. Essa anomalia pode ser obtida através do teorema do índice, como é bem sabido [2], logo, era de se esperar que essa anomalia fosse independente da temperatura, já que a temperatura é uma característica global da variedade não modificando sua topologia. Entretanto a anomalalia de traço apesar de não ter tal origem topológica, também exibe um comportamento semelhante. Esse fato sugere um estudo mais profundo das possíveis relações topológicas com essa anomalia.

Cabe ainda lembrar que a análise discutida aqui pode também ser estendida à anomalia gravitacional [8] assim como à de supercorrente [9].

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RADIATIVE CORRECTIONS IN (2+1)-DIMENSIONAL QED

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Abstract

We have calculated the vacuum polarization tensor for (2+1)-dimensional quantum electrodynamics (QED) using the analytic regularization technique by means of a gauge invariant construct. We have thus demonstrated that the gauge boson acquires physical mass at the one-loop level in the Abelian case. A generalization for the non-Abelian case showed up straightforward from this result.

1. Introduction

Gauge theories in (2+1)-dimensions[1,2] are interesting because of their association with high temperature phenomena in four dimensions[3]. They present, however, a challenging theoretical ambiguity in their physical result: gauge field mass may be induced radiatively at the one-loop level, depending on the choice of the method for regularizing ultraviolet (UV) divergent integrals. For instance, the Pauli-Villars method does not generate such a gauge boson mass (also called topological mass), both for Abelian and non-Abelian theories even up to two-loop level, in contrast with other techniques (see, for example, refs. [2,4,5,6]).

Among several regularization techniques available to tackle UV divergent integrals, there is one known as analytic regularization[7], which essentialy consists in considering an analytic extention for the fermion propagator to ensure convergence in the Feynman amplitudes. However, care must be taken, since naive implementation of this technique may violate Ward's identity[8] and so it requires a certain criterion to be implemented such that gauge invariance is preserved.

In this work we shall address this problem through a construct which preserves gauge invariance in the analytic regularization procedure and employ such a method to evaluate the one-loop photon self-energy for QED in three-dimensional space-time.

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2. Analytic Regularized One-Loop Photon Self-Energy

In three-dimensional space time the algebra for Dirac gamma matrices is realized using the Pauli-matrices

$$\gamma^0 = \sigma^3 \ , \ \gamma^1 = i\sigma^1 \ , \ \gamma^2 = i\sigma^2 \ , \tag{1}$$

$$\gamma^\mu \gamma^\nu = g^{\mu\nu} - i\epsilon^{\mu\nu\alpha}\gamma_\alpha \ , \ g_{\mu\nu} = diag(1,-1,-1) \ . \tag{2}$$

Consider now the vacuum polarization diagram. The general structure for the regularized polarization tensor expressed in a gauge-invariant form reads

$$\Pi_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\Pi^{(1)}(k^2) + im\epsilon_{\mu\nu\alpha}k^\alpha \Pi^{(2)}(k^2), \tag{3}$$

where $\epsilon_{\mu\nu\alpha}$ is the usual three-dimensional Levi-Civita tensor. Note that the equivalent for the last term in Eq.(3) is absent in a four-dimensional theory.

Following closely ref. [8], and using the gauge invariant analytic regularization there outlined, the regulated expression for the polarization tensor is given by

$$\Pi^{(\lambda)}_{\mu\nu}(k) = -2im^{2\lambda}f(\lambda)\int_0^1 d\xi \int \frac{d^3p}{(2\pi)^3}\frac{\mathcal{N}_{\mu\nu} + \mathcal{P}_{\mu\nu}}{(M^2 - p^2 - i\epsilon)^{2+\lambda}} \tag{4}$$

where

$$\mathcal{N}_{\mu\nu} = g_{\mu\nu}(M^2 - p^2 - i\epsilon) + \frac{2}{3}(1+\lambda)p^2 g_{\mu\nu} \ , \tag{5}$$

and

$$\mathcal{P}_{\mu\nu} = -(1+\lambda)\left[2\xi(\xi-1)k^2(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) + im\epsilon_{\mu\nu\alpha}k^\alpha\right] , \tag{6}$$

with

$$M^2 = m^2 - \xi(1-\xi)k^2 \ . \tag{7}$$

It is convenient to rewrite Eq.(4) in the form (3), where

$$\Pi^{(1)}(k^2) \equiv 4im^{2\lambda}f(\lambda)(1+\lambda)k^2\int_0^1 d\xi\, \xi(\xi-1)\int\frac{d^3p}{(2\pi)^3}\frac{1}{(M^2-p^2-i\epsilon)^{2+\lambda}} \ , \tag{8}$$

$$\Pi^{(2)}(k^2) \equiv 2im^{2\lambda}f(\lambda)(1+\lambda)\int_0^1 d\xi\int\frac{d^3p}{(2\pi)^3}\frac{1}{(M^2-p^2-i\epsilon)^{2+\lambda}} \ . \tag{9}$$

$$\Pi^{(1)}_{GB} \equiv -2im^{2\lambda}f(\lambda)g_{\mu\nu}\int_0^1 d\xi\int\frac{d^3p}{(2\pi)^3}\frac{1}{(M^2-p^2-i\epsilon)^{1+\lambda}} \ , \tag{10}$$

and

$$\Pi^{(2)}_{GB} \equiv -\frac{4}{3} i m^{2\lambda} f(\lambda)(1+\lambda) g_{\mu\nu} \int_0^1 d\xi \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{(M^2 - p^2 - i\epsilon)^{2+\lambda}} \ . \quad (11)$$

The gauge-breaking terms $\Pi^{(1)}_{GB}$ and $\Pi^{(2)}_{GB}$ add up to zero, since on evaluation the two contributions given by Eqs. (10) and (11) cancel each other out. This means that the regulated polarization tensor $\Pi^{(\lambda)}_{\mu\nu}$ is already gauge-invariant even *before* going to the limit $\lambda \to 0$.

3. Mass Generation for the Photon Field

The polarization tensor leads to the modified gauge boson propagator

$$D_{\mu\nu}(k) = \frac{-i}{k^2 - e^2\Pi(k^2)} \left\{ g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} - i\epsilon_{\mu\nu\alpha} \frac{k^\alpha}{k^2} M(k^2) \right\} \ , \quad (12)$$

where

$$\Pi(k^2) \equiv \Pi^{(1)}(k^2) + m\Pi^{(2)}(k^2) M(k^2) \quad (13)$$

and

$$M(k^2) \equiv \frac{e^2 m \Pi^{(2)}(k^2)}{1 - \frac{e^2\Pi^{(1)}(k^2)}{k^2}} \ . \quad (14)$$

From Eq.(8) one verifies that $\Pi^{(1)}(0) = 0$. As a result, the only contribution to the gauge boson mass comes from the $\Pi^{(2)}(0)$ term. After performing the momentum integral this term results in

$$\Pi^{(2)}(0) = -2\frac{(\pi)^{\frac{3}{2}}}{(2\pi)^3} m^{2\lambda} f(\lambda) \frac{\Gamma(\lambda+\frac{1}{2})}{\Gamma(\lambda+1)} \int_0^1 d\xi \frac{1}{(m^2 - i\epsilon)^{\lambda+\frac{1}{2}}} \ . \quad (15)$$

If we now take the limit $\lambda \to 0$ we finally obtain

$$\Pi^{(2)}(0) = -\frac{1}{4\pi m} \ , \quad (16)$$

so that a topological mass is induced at the one-loop level, in contrast with the Pauli-Villars regularization method where $\Pi^{(2)}(0) = 0$.

4. Conclusion

We have considered the three-dimensional quantum-electrodynamics regularized via analytic extension for the fermion propagator and shown the transversality of the one-loop vacuum polarization tensor. First and originally envisaged for four dimensional gauge theories, this formalism of analytic regularization embedding gauge invariance by construction was shown to produce a one-loop radiatively corrected photon propagator with a dislocated pole in such a way that we can attribute a non-vanishing mass to the real photon. This contrasts with the Pauli-Villars regularization, where the topological mass term in the regularized vacuum polarization tensor does not contribute for such photon mass. We would like to point out here that the odd-parity contribution from fermions is proportional to the sign of their mass, and is therefore naturally cancelled out by the regulator fields in the Pauli-Villars method. On the other hand, since the analytic regularization has no additional fields of that sort, it leaves the original fermions' effect unchanged.

Generalization to the non-Abelian case presents no difficulties since up to the one-loop level, besides the analogous Feynman diagram for QED we would have the additional contributions from diagrams involving gluon self-interaction vertices as well as diagrams with ghost loops, whose corresponding Feynman amplitudes can be regularized by means of the same gauge invariance preserving formalism. However, in this case the only contribution to the topological mass term comes from a closed fermion loop. Thus, the non-Abelian calculation is formally the same, the only difference lying on another coupling constant as well as colour group overall factors.

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FERMIONS AND O(3)-NONLINEAR SIGMA MODEL IN A THREE-DIMENSIONAL SPACE-TIME*

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ABSTRACT

In this paper we study the O(3)-nonlinear σ model soliton coupled with a isospin-1/2 fermion doublet by the Yukawa-type interaction. Describing the bosonic $Q = 1$ sector by collective coordinate, we show that a purely fermionic action can be obtained for this system. We also calculate the induced Hopf *term* for the bosonic sector by integrating out the *fermionic degrees of freedom*, and also the induced fermionic current.

I - INTRODUCTION

It is well known that the O(3)-nonlinear sigma model (NLσM) defined in a 2+1-dimensional space-time presents topologically stable configurations, solitons, that are characterized by a charge Q defined in the next section. These solitons describe continuous maps from the compactified coordinate space S^2 into the group space S^2. Since the second homotopy group $\Pi_2(S^2) = Z$, the NLσM admits infinite class of solitons[1].

In this paper we shall study the interaction of the $Q = 1$ soliton sector with a isospin-1/2 fermionic doublet using a Yukawa-type coupling. Describing the bosonic sector by a collective coordinates, we shall be able to quantize, in a semiclassical way, the total system and also to express it by a *purely fermionic* action. In order to obtain the induced Hopf terms and the topological current we shall develop a perturbation theory in the Yukawa coupling constant.

II-THE SOLITON-FERMION SYSTEM

The 2+1 dimensional O(3)-NLσM is described by the following Lagrangian:

$$L(t) = \frac{1}{2f^2} \int d^2x \, (\partial_\mu \varphi^a)(\partial^\mu \varphi^a) . \qquad (2-1)$$

where $\varphi^a(\vec{x},t)$, for a=1,2,3, are real bosonic field that satisfy the constraint condition $\varphi^a\varphi^a = 1$; so, the field manifold is equivalent to a sphere S^2.

The sector $Q = 1$ can be represented by the isovector $\varphi(\vec{r}) = (\hat{x}\sin g(r), \cos g(r))$, where $\hat{x}$ is a 2-dimensional unit radial vectors, and $g(r)$ being a function that obey the boundary condition $g(0) = 0$ and $g(\infty) = \pi$. The topological charge defined by

$$Q = \int d^2x\, J^0(x), \qquad (2-2)$$

where the identically conserved current J^μ is

$$J^\mu = \frac{1}{8\pi}\epsilon^{\mu\nu\lambda}\epsilon^{abc}\varphi^a\partial_\nu\varphi^b\partial_\lambda\varphi^c. \qquad (2-3)$$

We can study the interaction of an isospinor-1/2 Dirac fermion ψ with the soliton field $\varphi(\vec{x})$, via the Yukawa coupling. So, the fermionic lagrangian density is given by

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - g_y\bar{\psi}\varphi\psi, \qquad (2-4)$$

where ψ has four complex components: $\psi = col(\psi_1^+, \psi_1^-, \psi_2^+, \psi_2^-)$ where the indices 1,2 refer to isospin (charge) and + , - to spin. The gamma matrices are given by $\gamma^\mu = (\sigma^3, i\sigma^1, i\sigma^2)$

Now, let us go back to our original problem. The $Q = 1$ sector configuration is invariant under a combined spatial and isospin $U(1)$ transformation[9]. Let the sub-group of $SU(2)$ genereted by τ^3 be this symmetry group, i.e., $V(x) = e^{i\alpha(x)\frac{\tau^3}{2}} \in U(1)$. So, under this transformation we have

$$\varphi(\vec{x}) \longrightarrow \varphi'(\vec{x}) = V^{-1}\varphi(\vec{x})V = \begin{pmatrix} \cos g(r) & e^{-i(\alpha+\theta)}\sin g(r) \\ e^{i(\alpha+\theta)}\sin g(r) & -\cos g(r) \end{pmatrix} \qquad (2-5)$$

The interaction lagrangian density of this Dirac fermion with this new configuration,

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi - g_y\bar{\psi}\varphi'\psi, \qquad (2-6)$$

can be writen as

$$\mathcal{L} = \bar{\psi}'(i\not{\partial} - m - g_y\varphi)\psi' + \dot{\alpha}(t)\bar{\psi}'\gamma^0\frac{\tau^3}{2}\psi' \qquad (2-7a)$$

if we assume that $\alpha = \alpha(t)$ and $\psi' = V^{-1}\psi$. Dropping the prime indice in the fermionic field, the lagrangian is.

$$L_{F,\alpha} = \int d^2x\, \bar{\psi}(i\,\partial\!\!\!/ - m - g_{\gamma}\varphi(\vec{x}))\psi + \dot{\alpha}(t) \int d^2x\, \bar{\psi}\gamma^0 \frac{\tau^3}{2} \psi \qquad (2-7b)$$

The bosonic lagrangian for the configuration (2 – 5) is:

$$L_{NLsM} = -M + \frac{1}{2} I \dot{\alpha}(t)^2, \qquad (2-8a)$$

where

$$M = \frac{\pi}{f} \int_0^{\infty} dr\, r\, [(g'(r))^2 + \frac{1}{r^2} \sin^2 g(r)], \qquad (2-8b)$$

$$I = \frac{2\pi\lambda}{f}, \qquad (2-8c)$$

We can see that (2-8a) is equivalent to a free rotator lagrangian.

The total lagrangian for our system is written by

$$L_T = L_{NLsM} + L_{F,\alpha} = -M + \frac{1}{2} I \dot{\alpha}(t)^2 + \dot{\alpha}(t) Q + L_F, \qquad (2-9a)$$

with

$$Q = \int d^2x\, \bar{\psi}\gamma^0 \frac{\tau^3}{2} \psi, \qquad (2-9b)$$

and

$$L_F = \int d^2x\, \bar{\psi}(i\,\partial\!\!\!/ - m - g_{\gamma}\varphi(\vec{x}))\psi \qquad (2-9c)$$

One can see that in L_T, $\alpha(t)$ is a cyclic variable, so its conjugate momentum P is conserved

$$P = \frac{\partial L_T}{\partial \dot{\alpha}} = I \dot{\alpha}(t) + Q = \kappa.$$

We can eliminate $\dot{\alpha}(t)$ in favor of P and it lead to purely fermionic hamiltonian

$$H = M + \frac{(\kappa - Q)^2}{2I} + \int d^2x\, \psi^{\dagger} h_0 \psi \qquad (2-10a),$$

$$h_0 = \vec{\alpha}\cdot\vec{p} + \beta\,(m + g_p\varphi(\vec{\pi}))\,. \qquad (2-10b)$$

III - SOME PERTURBATIVE RESULTS

III - 1. The Induced Hopf terms for the NLσM.

The effective lagrangian to the $\dot\alpha(t)$ field can be obtained by the diagram shown in Fig. 1, that is obtained by integrating out the fermionic degrees of freedom.

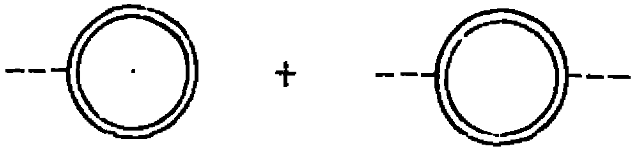

Fig. 1 - Lowest order in $\dot\alpha(t)$ in the bosonic effective lagrangian.

We shall concetrate ourselves only in the first diagrams, that, as we shall see, provides the Hopf term for the bosonic sector.

$$L^{(1)}_{eff}(\dot\alpha) = -\frac{\dot\alpha(t)}{2}\int d^3x\, Tr\left[S(x,x)\gamma^0\frac{\tau^3}{2}\right], \qquad (3-1)$$

where the trace is over the Dirac and isospin matrices.

The full fermion propagator $S(x,y)$ must obey the differential equation:

$$D(x)S(x,y) = \delta^3(x-y),$$

with $D(x)$ being the Dirac operator below

$$D(x) = i\not\partial - m - g_p\varphi(\vec{\pi}).$$

We can obtain $S(x,y)$ by defining

$$S(x,y) = \int_c \frac{dE}{2\pi} e^{-iE(x_0-y_0)} S_E(\vec{x},\vec{y})$$

where C is a causal contour in the complex E plane.

The fermionic propagator can be obtained only by a pertubative expression given by following series in power of g_ν.

$$S_E(\vec{x},\vec{y}) = S_E^{(0)}(\vec{x}-\vec{y}) + g_\nu \int d^2 z_1\, S_E^{(0)}(\vec{x}-\vec{z}_1)\varphi(\vec{z}_1)S_E^{(0)}(\vec{z}_1-\vec{y}) +$$
$$g_\nu^2 \int\int d^2 z_1 d^2 z_2\, S_E^{(0)}(\vec{x}-\vec{z}_1)\varphi(\vec{z}_1)S_E^{(0)}(\vec{z}_1-\vec{z}_2)\varphi(\vec{z}_2)S_E^{(0)}(\vec{z}_2-\vec{y}) + \dots$$

Introducing the Fourier transforms of $S_E(\vec{x},\vec{y})$ and $\varphi(\vec{x})$, we have

$$S_E(\vec{x},\vec{y}) = \int\int \frac{d^2\vec{p}_f}{(2\pi)^2}\frac{d^2\vec{p}_i}{(2\pi)^2} e^{i(\vec{x}\vec{p}_f-\vec{y}\vec{p}_i)} S_E(\vec{p}_f,\vec{p}_i), \qquad (3-2a)$$

where

$$S_E(\vec{p}_f,\vec{p}_i) = S_E^{(0)}(\vec{p}_f)(2\pi)^2\delta^2(\vec{p}_f-\vec{p}_i) +$$

$$g_\nu \int \frac{d^2k_1}{(2\pi)^2} S_E^{(0)}(\vec{p}_f)\varphi(\vec{k}_1)S_E^{(0)}(\vec{p}_i)(2\pi)^2\delta^2(\vec{p}_f-\vec{p}_i-\vec{k}_1) +$$

$$g_\nu^2 \int\int \frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2} S_E^{(0)}(\vec{p}_f)\varphi(\vec{k}_2)S_E^{(0)}(\vec{k}_1+\vec{p}_i)\varphi(\vec{k}_1)S_E^{(0)}(\vec{p}_i)\times$$

$$(2\pi)^2\delta^2(\vec{p}_f-\vec{p}_i-\vec{k}_1-\vec{k}_2) + \dots \qquad (3-2b)$$

with

$$S_E^{(0)}(\vec{p}) = \frac{\gamma^0 E - \not{p} + m}{E^2 - \vec{p}^2 - m^2} \qquad (3-2c)$$

Now, in order to obtain the induced Hopf term for the NLσM we have to use the perturbation expansion (3-2b) in the calculation of the effective action (3-1).

$$L_{eff}^{(1)}(\hat{n}) = -\frac{\hat{n}(t)}{2} \lim_{\epsilon\to 0^+} \int\int \frac{dE}{2\pi} e^{-iE\epsilon} \frac{d^2\vec{p}}{(2\pi)^2} Tr\left[S_E(\vec{p},\vec{p})\gamma_0\tau^3\right]. \qquad (3-3)$$

Our calculation for the induced Hopf term will be developed in the fourth and sixth order in the parameter g_y in the series (3-2b)

a - $O(g_y^4)$.

Calculating the $O(g_y^4)$ in $S_E(\vec{n}_f, \vec{n}_i)$ given in (3-2a) and substuting in (3-3) we get:

$$L_{eff}^{(1)}(\dot{\alpha}(t)) = -g_y^4 \frac{\dot{\alpha}(t)}{2} \int\int\int \frac{d^2 q_1}{(2\pi)^2} \frac{d^2 q_2}{(2\pi)^2} \frac{d^2 q_3}{(2\pi)^2} \varphi^a(\vec{q}_1)\varphi^b(\vec{q}_2)\varphi^c(\vec{q}_3)\times$$

$$\varphi^d(-\vec{q}_1 - \vec{q}_2 - \vec{q}_3)\Pi^{abcd}(\vec{q}_1, \vec{q}_2, \vec{q}_3) \qquad (3-4a)$$

where

$$\Pi^{abcd}(\vec{q}_1, \vec{q}_2, \vec{q}_3) = \lim_{\epsilon \to 0^+} \int\int \frac{dE}{(2\pi)} \frac{d^2 p}{(2\pi)^2} e^{-iE\epsilon} Tr\Big[S_E^{(0)}(\vec{p}) S_E^{(0)}(\vec{p} - \vec{q}_1) S_E^{(0)}(\vec{p} - \vec{q}_1 - \vec{q}_2)\times$$

$$S_E^{(0)}(\vec{p} - \vec{q}_1 - \vec{q}_2 - \vec{q}_3) S_E^{(0)}(\vec{p}) \gamma_0 \tau^a \tau^b \tau^c \tau^d \tau^3\Big]. \qquad (3-4b)$$

So, alfer some steps we get:

$$\Pi^{abcd}(\vec{q}_1, \vec{q}_2, \vec{q}_3) = \frac{3m}{16\pi} \epsilon^{ij}(q_2)_i(q_3)_j \int_0^1 dz_1 \int_0^{1-z_1} dz_2 \times$$

$$\int_0^{1-z_1-z_2} dz_3 \frac{[2(z_1 + z_2) - 1]}{a^{\frac{5}{2}}} A^{abcd}, \qquad (3-5a)$$

where

$$A^{abcd} = Tr\ \tau^a \tau^b \tau^c \tau^d \tau^3, \qquad (3-5b)$$

and

$$a = m^2 + f(q_i, z_i).$$

Unfortunately inserting (3-5) into (3-4a) and making an inverse Wick rotation it is not possible to get a numerical value for the coeficient of the induced Hopf term, and taking the limit for large m, or small momenta q_i, Π^{abcd} is of order $\frac{q^2}{m} \to 0$. So, no Hopf term is induced until this order.

h -$0(g_p^6)$.

The Calculation of the effective action in order (g_p^6) can be done in the same way as the previous calculation, but is much harder. After we have made the Feynman reparimetrization $p \to p + \sum_{i=1}^{6} q_i a_i$, neglected terms with q_i^2, odd power of E, etc, performing the trace oves the gamma matrices, and also using the REDUCE and MAPLE program in some steeps of this calculation, we found that in the limit as m is large the Hopf term is obtained.

$$L_{eff} = -\left(\frac{g_p}{m}\right)^6 \frac{m}{|m|}\vartheta\dot{\alpha}(t), \qquad (3-6)$$

where ϑ is a non-null munerical constant of order 0,1.

III-2 The Induced Topologial current

Although the calculation of the induced topological current has be done by others authors using others models[3] and techniques[4], we also would like to present our calculation for this model using the perturbation method developed previously. So, let us start with the standard expression for the fermionic current.

$$< J^\mu(x) > = < T\bar{\psi}_{(x)}\gamma^\mu\psi_{(x)} >, \qquad (3-7)$$

and obtain this expectation value in presence of a general backgraound bosonic field $\varphi^a_{(x)}$. We shall consider as the interaction action

$$S_I = \int d^3x g_p \bar{\psi}_{(x)}\varphi(x)\psi_{(x)}. \qquad (3-8)$$

Because the topological current for the NLσM, Eq.(2-2b), is a tri-linear expression in the field $\varphi^a(x)$, all that we have to do is to work out (3-7). By three insertion of the perturbation, connected by the unperturbed fermionic propagator, in momentum space, we have

$$< J^\mu(x) > = \int\int\int \frac{d^3q_1}{(2\pi)^3}\frac{d^3q_2}{(2\pi)^3}\frac{d^3q_3}{(2\pi)^3} e^{-i(q_1+q_2+q_3)} \Pi^\mu(q_1+q_2+q_3) \qquad (3-9a)$$

where

$$\Pi^{\rho}(q_1+q_2+q_3)=g_y^3\int\frac{d^3p}{(2\pi)^3}\,Tr\Big\{\gamma^{\rho}S^{(0)}(p)\varphi(q_1)S^{(0)}(p-q_1)\varphi(q_2)\times$$

$$S^{(0)}(p-q_1-q_2)\varphi(q_3)S^{(0)}(p-q_1-q_2-q_3)\Big\}. \qquad (3-9b)$$

The expression above can be obtained by the use of the Feymman reparametrization, collecting all the terms proportional to m^0 and m^2 and performing the trace over the gamma and Pauli matrices; the result, in the limit as m is large, is

$$\Pi^{\rho}(q_1+q_2+q_3)=-\left(\frac{g_y}{|m|}\right)^3\frac{1}{192\pi}\epsilon^{\rho\mu\nu}\epsilon^{abc}\Big[29(q_1)_\mu(q_2)_\nu+$$

$$+14(q_1)_\mu(q_3)_\nu+3(q_2)_\mu(q_3)_\nu\Big]\varphi^a(q_1)\varphi^b(q_2)\varphi^c(q_3). \qquad (3-10)$$

Now, inserting (3-10) into (3-9a) we get, for $g_y=m$,

$$<J^{\rho}(x)>=\frac{m}{|m|}\frac{1}{16\pi}\epsilon^{\rho\mu\nu}\epsilon^{abc}(\partial_\mu\varphi^b)(\partial_\nu\varphi^c), \qquad (3-11)$$

that is in agreement with previous results given in Refs.(3,4).

IV - CONCLUSION AND DISCUSSION

In this paper we have shown that the $Q=1$ soliton sector of the NLσM coupled with isospinor-$\frac{1}{2}$ fermions by a Yukawa coupling can be expressed by a purely fermionic action. For this system it is also possible to develop a perturbative series for the fermion propagator and obtain some perturbative results: i) For the induced Hopf term we have shown that, although it formally can be expressed as a fourth powerof the bosonic field $\varphi^a(x)$, it only appears in the sixth order in g_y in the perturbative series. ii) The topological current agrees, unless a factor $\frac{1}{2}$ with its formal expression.

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O ESPECTRO DO OSCILADOR DE DIRAC VIA ÁLGEBRA DE OSCILADOR GENERALIZADO DE WIGNER-HEISENBERG

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RESUMO

No presente trabalho, incorporamos o oscilador de Dirac dentro da estrutura da álgebra de Wigner-Heisenberg na sua forma super-realizada. Tal conexão nos permite a conversão do problema espectral do oscilador de Dirac para o problema correspondente de uma matriz simples Hermitiana no espaço de número para a partícula de Wigner, proporcionando-nos uma fácil determinação do espectro de energia completo. Do nosso método algébrico, apontamos a assimetria inerente do espectro para energia positiva e negativa e indicamos também a conexão com o oscilador SUSI tridimensional associado ao oscilador de Dirac.

1. INTRODUÇÃO

A equação do oscilador de Dirac[1] ($c = \hbar = 1$)

$$i\frac{\partial}{\partial t}\psi_D = H_D\psi_D, \quad H_D = \underline{\alpha}\cdot(\underline{p} - i\underline{r}\beta) + M\beta \quad (\underline{\alpha} = \Sigma_1\underline{\sigma}, \beta = \Sigma_3), \tag{1}$$
$$\left([\Sigma_i, \sigma_j]_- = 0 \quad (i,j = 1,2,3)\right),$$

tem atraido muita atenção na literatura recente[2] devido os aspectos supersimétricos (SUSI) do seu espectro de energia. No entanto, a interessante conexão de H_D com o oscilador generalizado de Wigner-Heisenberg (WH) parece não ter sido abordada na literatura. No presente trabalho, incorporamos H_D dentro da estrutura da álgebra WH super-realizada (Seção 2) de modo a extrair facilmente as propriedades especiais do seu espectro (Seção 3). A Seção 4 contém os comentários.

2. A ÁLGEBRA WH SUPER-REALIZADA EM TRÊS DIMENSÕES

A Hamiltoniana de Wigner $H(\underline{\sigma}\cdot\underline{L}+1)$ e seus operadores escada $a^{\pm}(\underline{\sigma}\cdot\underline{L}+1)$ nas suas formas super-realizadas dadas por (Jayaraman e Rodrigues[3])

$$a^{\pm}(\underline{\sigma}\cdot\underline{L}+1) = \frac{1}{\sqrt{2M\omega}}\left\{\pm\left(\frac{\partial}{\partial r}+\frac{1}{r}\right) \pm \frac{1}{r}(\underline{\sigma}\cdot\underline{L}+1)\Sigma_3 - M\omega r\right\}\Sigma_1 = \left\{a^{\mp}(\underline{\sigma}\cdot\underline{L}+1)\right\}^{\dagger}, \tag{2}$$

$$H(\underline{\sigma} \cdot \underline{L} + 1) =$$

$$\begin{pmatrix} H_-(\underline{\sigma} \cdot \underline{L}) = \frac{1}{2M} \left\{ -\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} \right) + \frac{1}{r^2} \underline{\sigma} \cdot \underline{L}(\underline{\sigma} \cdot \underline{L} + 1) + M w^2 r^2 \right\} & 0 \\ 0 & H_+(\underline{\sigma} \cdot \underline{L}) \equiv H_-(\underline{\sigma} \cdot \underline{L} + 1) \end{pmatrix}, \quad (3)$$

satisfazem as seguintes relações de (anti-)comutação da álgebra WII em três dimensões (3D):

$$\left[H(\underline{\sigma} \cdot \underline{L} + 1), a^{\pm}(\underline{\sigma} \cdot \underline{L} + 1) \right]_- = \pm \omega a^{\pm}(\underline{\sigma} \cdot \underline{L} + 1), \quad (4)$$

$$\frac{1}{2} \left[a^-(\underline{\sigma} \cdot \underline{L} + 1), a^+(\underline{\sigma} \cdot \underline{L} + 1) \right]_+ = \frac{1}{\omega} H(\underline{\sigma} \cdot \underline{L} + 1). \quad (5)$$

Também os $a^{\pm}(\underline{\sigma} \cdot \underline{L} + 1)$ satisfazem a seguinte relação generalizada da comutação quântica:

$$\left[a^-(\underline{\sigma} \cdot \underline{L} + 1), a^+(\underline{\sigma} \cdot \underline{L} + 1) \right]_- = 1 + 2(\underline{\sigma} \cdot \underline{L} + 1)\Sigma_3. \quad (6)$$

Tal forma super-realizada da álgebra WH, contida nas equações (2)-(6), foi desenvolvida por nós[3] como uma técnica do operador para achar, de modo fácil, a resolução espectral dos potenciais relacionados ao oscilador. (Veja os detalhes em Jayaraman e Rodrigues[3] para obter os espectros de $H(\underline{\sigma} \cdot \underline{L} + 1), H_-(\underline{\sigma} \cdot \underline{L})$ e $H_+(\underline{\sigma} \cdot \underline{L})$ por operações puramente algébricas.)

3. O ESPECTRO DO OSCILADOR DE DIRAC

Após uma transformação unitária feita por U, a equação de autovalor $H_D \psi_D = E_D \psi_D$ se torna

$$\tilde{H}_D \chi_D = E_D \chi_D \quad , \quad \tilde{H}_D = U H_D U^{\dagger}, \chi_D = U \psi_D, U = \begin{pmatrix} 1 & 0 \\ 0 & \sigma_r = \frac{\underline{\sigma} \cdot \underline{r}}{r} \end{pmatrix} \quad (7)$$

onde

$$\tilde{H}_D = \Sigma_1 p_r + \left(\frac{\underline{\sigma} \cdot \underline{L} + 1}{r} - M \omega r \right) \Sigma_2 + M \Sigma_3 = \left\{ i\sqrt{2M\omega}(Q_- - Q_+) + M\Sigma_3 \right\} \quad (8)$$

$$= i\sqrt{2M\omega} \left\{ \frac{1 - \Sigma_3}{2} a^-(\underline{\sigma} \cdot \underline{L} + 1) - \frac{1 + \Sigma_3}{2} a^+(\underline{\sigma} \cdot \underline{L} + 1) \right\} + M\Sigma_3 \quad (9)$$

com $a^{\pm}(\underline{\sigma} \cdot \underline{L} + 1)$ justamente os operadores escada em (2) da Hamiltoniana de Wigner em (3). Tratamos abaixo o caso de $\underline{\sigma} \cdot \underline{L} + 1 \rightarrow \ell + 1$ explicitamente.

Sobre o conjunto completo dos estados $|n; \ell + 1 > \equiv |n >$ $(n = 0, 1, 2, \cdots)$ da partícula de Wigner valem as seguintes propriedades[4]:

$$H(\ell+1)|n> = E^{(n)}(\ell+1)|n> \ , E^{(n)}(\ell+1) = (\ell+\frac{3}{2}+n)\omega, < n|n' > = \delta_{nn'} \, (n, n' = 0,1,\cdots), \quad (10)$$

$$a^-(\ell+1)|2m> = \sqrt{2m}|2m-1> \ , a^-(\ell+1)|2m+1> = \sqrt{2m+1+2(\ell+1)}|2m>,$$

$$a^+(\ell+1)|2m> = \sqrt{2m+1+2(\ell+1)}|2m+1> \ , a^+(\ell+1)|2m+1> = \sqrt{2m+2}|2m+2>,$$

$$\Sigma_3|2m> = |2m>, \ \Sigma_3|2m+1> = -|2m+1> \ (m = 0,1,2,\cdots). \quad (11)$$

Expandindo a parte radial de χ_D em termos da base $|n>$, isto é, $\chi_D = \sum_{m=0}^{\infty} C_{2m}|2m> + \sum_{m=0}^{\infty} C_{2m+1}|2m+1>$ e com uso de (11) em (7) a (9), obtemos após simplificação, que

$$\begin{bmatrix} -M & i\sqrt{4M\omega n} \\ -i\sqrt{4M\omega n} & M \end{bmatrix} \begin{bmatrix} C_{2n-1} \\ C_{2n} \end{bmatrix} = E_D^{(n)}(\ell+1) \begin{bmatrix} C_{2n-1} \\ C_{2n} \end{bmatrix} \quad (n = 0,1,\cdots, C_{-1} \equiv 0) \quad (12)$$

a qual fornece a resolução espectral:

$$E_D^{(0)}(\ell+1) = M, \quad E_{D;\pm}^{(n)}(\ell+1) = \pm\sqrt{M^2+4M\omega n} \quad (n = 1,2,\cdots) \quad (13a)$$

com

$$\chi_D^{(0)}(\ell+1) = |0>, \ \chi_{D;\pm}^{(n)}(\ell+1) \propto \left\{ \left(E_{D;\pm}^{(n)}(\ell+1) + M\right)|2n> + i\sqrt{4M\omega n}|2n-1> \right\} (n = 1,2,\cdots). \quad (13b)$$

A não-dependência de $E_{D;\pm}^{(n)}(\ell+1)$ em ℓ significa que existe um grau de degenerescência infinito desses autovalores.

A repetição da análise acima para $\underline{\sigma}\cdot\underline{L}+1 \to -(\ell+1)$ fornece

$$E_{D;\pm}^{(n)}\{-(\ell+1)\} = \pm\sqrt{M^2+2M\omega(2n+2\ell+3)} \quad (n = 0,1,\cdots; \ell = 0,1,\cdots) \quad (14a)$$

com

$$\chi_{D;\pm}^{(n)}[-(\ell+1)] \propto \Sigma_1 \left\{ \left(E_{D;\pm}^{(n)}[-(\ell+1)] - M\right)|2n> - i\sqrt{2M\omega(2n+2\ell+3)}|2n+1> \right\} (n = 0,1,\cdots). \quad (14b)$$

A ausência do autovalor $-M$ para a energia significa uma assimetria do espectro do oscilador de Dirac entre as energias positivas e negativas.

As autofunções físicas ψ_D na representação de Dirac podem ser facilmente obtidas através da transformação inversa de U sobre χ_D.

A conexão de $\bar{\Pi}_D^2$ com $H(\underline{\sigma} \cdot \underline{L} + 1)$ e $H_{\rm SUSI}$ segue-se das equações (8), (9), (3) e (6):

$$H_{\rm SUSI} = \frac{1}{2M\omega}(\bar{\Pi}_D^2 - M^2) = H(\underline{\sigma} \cdot \underline{L} + 1) - \frac{\omega}{2}\{1 + 2(\underline{\sigma} \cdot \underline{L} + 1)\Sigma_3\} \tag{15a}$$

$$= [Q_-, Q_+]_+ \tag{15b}$$

$$Q_-^2 = Q_+^2 = 0, \quad [H_{\rm SUSY}, Q_\mp]_- = 0 \tag{15c}$$

4. COMENTÁRIOS

Uma questão interessante sobre a existência ou não de uma interação que inverte a assimetria do espectro deduzido acima pode ser respondida afirmativamente. Tal interação não mínima corresponde à $\underline{p} \to \underline{\pi} = \underline{p} + ir\underline{\beta}$ em vez de $\underline{p} \to \underline{\pi} = \underline{p} - ir\underline{\beta}$ como em (1). A não equivalência dos espectros nestes dois casos segue-se da ausência de uma transformação unitária a qual deve transformar $\underline{\alpha} \to \underline{\alpha}, \beta \to \beta$ mas $\underline{\alpha}\beta \to -\underline{\alpha}\beta$.

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ESTADOS COERENTES VIA ÁLGEBRA DE WIGNER-HEISENBERG

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Resumo. Desenvolvemos um formalismo geral para se construir os estados coerentes canônicos de potenciais gerais tipo oscilador com barreira centrífuga. Mostramos que eles são não-ortogonais, super-completos e normalizáveis. Extensões para os estados coerentes generalizados são discutidas.

I. INTRODUÇÃO

Num artigo recente, Jayaraman e Rodrigues (JR) [1] mostraram a utilidade do método algébrico de Wigner-Heisenberg (WH) [2-5] para se resolver os problemas espectrais de sistemas quânticos que possuem conexões com osciladores. Neste trabalho, construiremos os Estados Coerentes Canônicos (ECC) de um oscilador generalizado, embutido no setor bosônico do Hamiltoniano de Wigner (veja eq.(3)), como uma superposição dos seus repectivos autoestados, de modo análogo àqueles do oscilador harmônico simples [6]. Utilizando o sistema de unidades em que $M=1=\omega=\hbar$, a super-realização JR da álgebra WH é alcançada através dos seguintes operadores escada mutuamente adjuntos:

$$a^{\pm}\left(\frac{c}{2}\right) = \frac{\Sigma_1}{\sqrt{2}}\left\{\pm \frac{d}{dx} \mp \frac{c}{2x}\Sigma_3 - x\right\} = \left\{a^{\mp}\left(\frac{c}{2}\right)\right\}^{\dagger} \quad . \tag{1}$$

E, assim, nos proporcionando um Hamiltoniano de Wigner diagonal com dois setores (bosônico e fermiônico),

$$H\left(\frac{c}{2}\right) = \frac{1}{2}\left[a^{-}\left(\frac{c}{2}\right), a^{+}\left(\frac{c}{2}\right)\right]_{+} = \begin{pmatrix} H_{-}\left(\frac{c}{2}-1\right) & 0 \\ 0 & H_{+}\left(\frac{c}{2}-1\right) = H_{-}\left(\frac{c}{2}\right) \end{pmatrix} \quad . \tag{2}$$

cujo setor bosônico é o Hamiltoniano de um oscilador harmônico mais uma barreira centrífuga, a saber,

$$H_{-}\left(\frac{c}{2}-1\right) = \frac{1}{2}\left\{-\frac{d^2}{dx^2} + x^2 + \frac{1}{x^2}\left(\frac{c}{2}-1\right)\frac{c}{2}\right\},\ c \in \mathbb{R} \quad . \tag{3}$$

A partir da relação de comutação escada da álgebra WH,

$$[H(\tfrac{c}{2}), a^{\pm}(\tfrac{c}{2})]_- = \pm a^{\pm}(\tfrac{c}{2}) \quad , \qquad (4)$$

podemos derivar uma relação de comutação generalizada:

$$[a^-(\tfrac{c}{2}), a^+(\tfrac{c}{2})]_- = 1 + c\Sigma_3 \quad . \qquad (5)$$

E das propriedades das matrizes de Pauli, Σ_i (i=1,2,3), obtemos:

$$[\Sigma_3, a^{\pm}(\tfrac{c}{2})]_+ = 0 \Rightarrow [\Sigma_3, H(\tfrac{c}{2})]_- = 0 \quad . \qquad (6)$$

As eqs.(2) e (4) juntamente com as eqs. derivadas (5) e (6), constituem a álgebra WH, a qual é para-bose de um grau de liberdade.

Os autoestados do setor bosônico, $|\psi^{(m)}_-(\frac{c}{2}-1)\rangle$, pertencem ao autoespaço associado aos quanta pares. Os operadores escada destes quanta, $B^{\pm}(\frac{c}{2})$, são realizados por operadores quadráticos, obtidos a partir da relação de comutação escada da álgebra WH:

$$B^{\mp}(\tfrac{c}{2}) = \frac{1}{2}\left\{\frac{d^2}{dx^2} + x^2 - \frac{c}{2x^2}(\tfrac{c}{2}-1) \pm 2x\frac{d}{dx} \pm 1\right\} = \{B^{\pm}(\tfrac{c}{2})\}^{\dagger} \quad . \qquad (7)$$

$$[H(\tfrac{c}{2}-1), B^{\pm}(\tfrac{c}{2})]_- = \pm 2 B^{\pm}(\tfrac{c}{2}) \quad . \qquad (8)$$

Da relação de comutação escada dada por (8), vemos que esses operadores deslocam os quanta pares em duas unidades, i.é, 2m → 2m±2, ou equivalentemente, m → m±1. Neste caso, obtemos:

$$B^-(\tfrac{c}{2})|\psi^{(m)}_-(\tfrac{c}{2}-1)\rangle = \{2m(2m+c-1)\}^{1/2}|\psi^{(m-1)}_-(\tfrac{c}{2}-1)\rangle \quad . \qquad (9)$$

$$B^+(\tfrac{c}{2})|\psi^{(m)}_-(\tfrac{c}{2}-1)\rangle = \{2(m+1)(2m+c+1)\}^{1/2}|\psi^{(m+1)}_-(\tfrac{c}{2}-1)\rangle \quad . \qquad (10)$$

II.1 ESTADOS COERENTES CANÔNICOS DO SETOR BOSÔNICO

Os ECC do setor bosônico do Hamiltoniano de Wigner são definidos como sendo os autoestados do operador de aniquilação dos quanta pares ,

$$B^-(\tfrac{c}{2})|\beta, \tfrac{c}{2}-1\rangle = \beta|\beta, \tfrac{c}{2}-1\rangle \quad . \qquad (11)$$

onde o autovalor β pode assumir valores complexos. Existem outras definições possíveis [8]. Expandindo os ECC na base, $\{|\psi^{(m)}(\frac{c}{2}-1)\rangle\}$, obtemos a seguinte expressão para os coeficientes da expansão, q_m :

$$q_m = \beta\{2m(2m+c-1)\}^{-1/2} q_{m-1} = \left(\frac{\beta}{2}\right)^m \left\{\Gamma\left(\frac{c+1}{2}\right)/m!\,\Gamma\left(m+\frac{c+1}{2}\right)\right\}^{1/2} q_0 \quad . \tag{12}$$

onde Γ é a função Gama ordinária. Agora, usando a condição de normalização, obtemos os seguintes ECC normalizados:

$$|\beta,\tfrac{c}{2}-1\rangle = \left\{\Gamma\left(\frac{c+1}{2}\right) I_{\frac{c-1}{2}}(|\beta|)\right\}^{-1/2} \left(\frac{|\beta|}{2}\right)^{\frac{c-1}{4}} \sum_{m=0}^{\infty} \frac{\left(\frac{\beta}{2}\right)^m}{\left\{m!\,\Gamma\left(m+\frac{c+1}{2}\right)\right\}^{1/2}} |\psi^{(m)}(\tfrac{c}{2}-1)\rangle \ , \tag{13}$$

onde as funções de Bessel modificadas $I_{\mu}(x)$ são dadas por:

$$I_{\mu}(x) = \sum_{m=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{2m+\mu}}{m!\,\Gamma(m+\mu+1)} \quad . \tag{14}$$

Note que o estado de vácuo também é um ECC, o qual está associado ao autovalor zero. Considerando o produto escalar entre dois ECC associados a autovalores diferentes, obtemos:

$$\langle \xi,\tfrac{c}{2}-1|\beta,\tfrac{c}{2}-1\rangle = \frac{\sum_{m=0}^{\infty}\left\{(\xi^*\beta/4)^m/\left[m!\,\Gamma\left(m+\frac{c+1}{2}\right)\right]^{1/2}\right\}}{\Gamma\left(\frac{c+1}{2}\right)\left(\frac{|\beta|}{2}\right)^{\frac{1-c}{4}}\left\{I_{\frac{c-1}{2}}(|\beta|)\, I_{\frac{c-1}{2}}(|\xi|)\right\}^{1/2}\left(\frac{|\xi|}{2}\right)^{\frac{1-c}{4}}} \quad . \tag{15}$$

Isto nos assegura a não-ortogonalidade dos ECC. A importante propriedade de completeza juntamente com a não-ortogonalidade, nos permite fazer a expansão de um estado arbitrário, $|\phi(\frac{c}{2}-1)\rangle$, numa base constituída de ECC. Em particular, podemos expandir um ECC em tal base, o que equivale a dizer que os ECC são super-completos. Isto será mostrado em outra parte.

II. CONCLUSÕES

Construímos os Estados Coerentes Canônicos do setor bosônico do Hamiltoniano de Wigner super-realizado. Eles são os autoestados de um operador de aniquilação quadrático, independente do número de quanta. Os ECC são não-ortogonais, super-completos e normalizáveis.

Ao contrário dos ECC para-bose deduzidos por Sharma, Mehta e Sudarshan [7], estes podem ser identificados com aqueles do oscilador radial tridimensional [8], do oscilador isotrópico 3D de spin 1/2 [9], dos osciladores isotônico 1D e radial D-dimensional [10]. Especificamente, tal correspondência ocorre quando a constante característica da álgebra WH, c/2 ,for substituida por, respectivamente, $(\ell+1)$, ℓ - momento angular orbital; por $(\underline{\sigma}.\underline{L}+1)$, $\underline{\sigma}$-matriz de spin 1/2 de Pauli e $\underline{L}$-operador momento angular orbital em 3 dimensões; por $(\lambda + 1)$, $\lambda \in \mathbb{R}$ e, no caso D-dimensional, por $\left(\ell_D + \frac{D-3}{2}\right)$, ℓ_D-momento angular orbital em D-dimensões. Os operadores $\frac{1}{2}B^{\Sigma}(\xi)$ e $\frac{1}{2}H(\xi-1)$ são geradores do grupo SL(2,R). Logo, podemos construir os estados coerentes generalizados pela ação de um elemento unitário, desse grupo, sobre o vácuo. Um trabalho referente a esta generalização está sendo desenvolvido por nós.

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ESTADOS COERENTES DO OSCILADOR RADIAL 3D.

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Resumo. Encontramos os estados coerentes canônicos e generalizados do oscilador radial 3D, através de operadores derivados da super-realização da álgebra de Wigner-Heisenberg. Usamos os estados coerentes de Perelomov do grupo SL(2,R) para obter o espectro desse oscilador.

I. INTRODUÇÃO

Construiremos os autovalores de energia do Oscilador Radial (OR) via o operador resolvente, na representação de Schwinger [1], sobre uma base de Estados Coerentes Generalizados (ECG). Os EC podem ser definidos de três maneiras, em geral, inequivalentes: (i) EC canônicos, são os autoestados de um operador de aniquilação independente do número de quanta [2]; (ii) EC de incerteza mínima, são as soluções de uma equação diferencial deduzida da relação de incerteza de Heisenberg [3]; (iii) os ECG são aqueles obtidos pela ação de um operador sobre o vácuo (estado fundamental). Tal operador pode ser unitário e pertencer a um certo grupo [4], ou um funcional complicado [5]. No caso do oscilador harmônico simples, estas definições são equivalentes [6]. Aqui utilizaremos as definições (i) e (iii). Os ECG radiais de um operador de aniquilação, dependente do número de quanta foram construídos para o OR 3D [7]. Então, quais são os ECG do OR 3D como autoestado de um operador de aniquilação independente dos quanta? Utilizando a super-realização de Jayaraman e Rodrigues (JR) da álgebra de Wigner-Heisenberg (WH) [8], obtemos a resposta desta questão.

A álgebra WH para o OR 3D, discutidas na seção 3 da ref. [8], é a seguinte:

$$H(\ell+1) = \frac{1}{2}\left[a^{-}(\ell+1), a^{+}(\ell+1)\right]_{+}, \quad \left[a^{\mp}(\ell+1), H(\ell+1)\right]_{-} = \pm a^{\mp}(\ell+1). \qquad (1)$$

$$[\, a^{-}(\ell+1) \,,\, a^{+}(\ell+1) \,] = 1 + 2(\ell+1)\Sigma_3 \quad , \qquad (2)$$

onde os operadores escada do super-oscilador radial de Wigner e o Hamiltoniano do setor bosônico são, respectivamente:

$$a^{\pm}(\ell+1) = \frac{1}{\sqrt{2}}\Sigma_1 \left\{ \pm\left(\frac{d}{dr} + \frac{1}{r} \right) \mp \frac{(\ell+1)}{r}\Sigma_3 - r \right\} \quad , \qquad (3)$$

$$H(\ell) = \frac{1}{2}\left\{ -\frac{d^2}{dr^2} - \frac{2}{r}\frac{d}{dr} + r^2 + \frac{\ell(\ell+1)}{r^2} \right\} \quad . \qquad (4)$$

Seguindo o maquinário para se construir os operadores escada, independentes dos quanta, do setor bosônico [9], obtemos:

$$[\, H(\ell) \,,\, B^{\pm}(\ell+1) \,] = \pm 2 B^{\pm}(\ell+1) \quad , \qquad (5)$$

onde os operadores de criação, B^{+} , e de aniquilação, B^{-} , do oscilador radial são:

$$B^{\pm}(\ell+1) = \frac{1}{2}\left\{ \frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} \mp 2r\frac{d}{dr} + r^2 - \frac{\ell(\ell+1)}{r^2} + 3 \right\} \quad . \qquad (6)$$

Esses operadores quadráticos diminuem ou aumentam os quanta em duas unidades,

$$B^{-}(\ell+1)\,|R^{(m)}(\ell)\rangle = \{2m(2m+2\ell+1)\}^{1/2}\,|R^{(m-1)}(\ell)\rangle \quad , \qquad (7)$$

$$B^{+}(\ell+1)\,|R^{(m)}(\ell)\rangle = \{2(m+1)(2m+2\ell+3)\}^{1/2}\,|R^{(m+1)}(\ell)\rangle \quad . \qquad (8)$$

II. ESTADOS COERENTES CANÔNICOS E GENERALIZADOS

Agora, construiremos os ECC radiais, $|\alpha,\ell\rangle$, $\alpha \in \mathbb{C}$, como sendo uma superposição dos autoestados do OR 3D. Eles são os autoestados do operador de aniquilação, $B^{-}(\ell+1)$, do OR 3D:

$$B^{-}(\ell+1)\,|\alpha,\ell\rangle = \alpha\,|\alpha,\ell\rangle \quad . \qquad (9)$$

Apesar deles serem não-ortogonais,

$$\langle \xi,\ell|\alpha,\ell\rangle = \left(\Gamma(\ell+\tfrac{3}{2}) \left\{ \left(\frac{|\xi|}{2}\right)^{\frac{1}{2}-\ell} I_{\ell-\frac{1}{2}}(|\xi|) \left(\frac{|\alpha|}{2}\right)^{\frac{1}{2}-\ell} I_{\ell-\frac{1}{2}}(|\alpha|) \right\}^{1/2} \right)^{-1} \sum_{m=0}^{\infty} \frac{(\xi^{*}\alpha/4)^{m}}{\{m!\,\Gamma(m+\ell+\frac{3}{2})\}^{1/2}} \, , \qquad (10)$$

eles são normalizáveis, e dados por:

$$|\alpha,\ell\rangle = \left\{\Gamma(\mu+1) I_{\mu}(|\alpha|)\right\}^{-1/2} \left(\frac{|\alpha|}{2}\right)^{\mu-\frac{3}{4}} \sum_{m=0}^{\infty} \frac{(\alpha/2)^m \, |R_{\ell}^{(m)}\rangle}{\{m!\,\Gamma(m+\mu+1)\}^{1/2}} \quad (11)$$

onde $I_{\mu}(x)$ são as funções de Bessel modificadas, Γ é a função Gama e $\mu = \ell + 1/2$. A importante propriedade de completeza será demonstrada num trabalho que vamos submetê-lo a publicação numa revista internacional.

Agora, calcularemos os Estados Coerentes Generalizados(ECG) associados ao grupo de simetria SL(2,R). A partir de (3-5), obtemos a seguinte realização da álgebra de Lie do SL(2,R):

$$[K_0, K_+] = K_+ \quad , \quad [K_0, K_-] = -K_- \quad . \quad (12)$$

$$[K_-, K_+] = 2K_0 \quad , \quad K_{\pm} = \tfrac{1}{2} B^{\pm}(\ell+1) \quad , \quad K_0 = \tfrac{1}{2} H(\ell) \quad . \quad (13)$$

De acordo com a nossa realização acima, $k(k-1)$, $k = \ell + 3/2$ dão os autovalores do operador de Casimir do SL(2,R). Os ECG de Perelomov associados a essa álgebra geradora do espectro do OR 3D, são dados por:

$$|\sigma,k\rangle = (1-|\sigma|^2)^k \sum_{n=0}^{\infty} \left\{\frac{\Gamma(2k+n)}{n!\,\Gamma(2k)}\right\}^{1/2} \sigma^n |k,n\rangle \quad (14)$$

onde a medida de integração é a seguinte:

$$d\mu(|\sigma|) = \left(\frac{2k-1}{\pi}\right)(1-|\sigma|^2)^{-2} d^2\sigma \quad . \quad (15)$$

Estes ECG são análogos aos do potencial Coulombiano na equação de Klein-Gordon [10] e, assim, são não-ortogonais e super-completos.

III. A FUNÇAO DE GREEN E O ESPECTRO

A função de Green definida sobre uma base constituida dos ECG, $\{|\sigma,k\rangle\}$, é uma soma parcial de funções de Green, i.é,

$$G(\sigma;\sigma') = \sum_{\ell=0}^{N-1} G_{\ell}(\sigma,\sigma') = \sum_{k} \langle \sigma,k | G_E | \sigma',k\rangle \quad , \quad (16)$$

onde G_E é o operador resolvente, o qual na representação de Schwinger toma a seguinte forma exponencial:

$$G_E = \left(H(\ell) - E \right)^{-1} = i\int_0^{\omega} \exp\{-i\left(H(\ell)-E\right)s\}\, ds. \qquad (17)$$

De (14) e (17) em (16), a função de Green torna-se:

$$G_E(\sigma';\sigma') = i\sum_{\ell=0}^{N-1}\left(1-\frac{|\sigma'|^2}{2}\right)^{\frac{2\ell+3}{4}}\left(1-\frac{|\sigma'|^2}{2}\right)^{\frac{2\ell+3}{4}}\int_0^{\omega} e^{i\left(E-\frac{2\ell+3}{2}\right)s}\left(1-\frac{\sigma'^*\sigma'}{2}e^{-i2s}\right)^{-\frac{2\ell+3}{2}} ds \qquad (18)$$

Os pólos do traço do operador resolvente, na base dos ECG,

$$\mathrm{Tr}\, G'_E = i\sum_{\ell=0}^{N-1}\sum_{m=0}^{\omega}\left(E-\left(\ell+\frac{3}{2}+2m\right)\right)^{-1}. \qquad (19)$$

são os autovalores de energia do OR 3D.

IV. CONCLUSÕES

Construimos os Estados Coerentes Canônicos e Generalizados (ECC e ECG) para um oscilador harmônico radial 3D, via operadores derivados da álgebra de Wigner-Heisenberg super-realizada. Calculamos a função de Green e o espectro, através do operador resolvente, na representação de Schwinger, definido sobre uma base constituida dos ECG do OR 3D.

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The Feynman - Dyson proof of Maxwell equations and magnetic monopoles

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Abstract. Using a violation of the Jacobi Identity[3,4] we are able to generalize the Feynman's Proof of the Maxwell Equations including magnetic monopoles.

In 1990 Dyson[1] published a proof due to Feynman that the Maxwell equations follow from Newton' s equation

$$m\,\ddot{x}_j = F_j(x,\dot{x},t) \tag{1}$$

and the quantum mechanical canonical rules

$$[x_j, x_k] = 0 \tag{2}$$

$$m[x_j, \dot{x}_k] = i\,\hbar\,\delta_{jk}\,. \tag{3}$$

Soon after, Lee[2] extended the Feynman's proof to non - abelian gauge fields, obtaining the Yang-Mills equations. In his paper, Lee suggested that magnetic monopoles can be introduced, through Feynman's approach using the dual Lorentz force equation

$$F_j = B_j - \varepsilon_{jkl}\dot{x}_k E_l\,. \tag{4}$$

It is possible to obtain the magnetic monopoles without postulating the dual Lorentz force. This is shown below.

In his proof Feynman have used twice the well known Jacobi Identity

$$[A,[B,C]] + [B,[C,A]] + [C,[A,B]] = 0\,. \tag{5}$$

Magnetic monopoles appear when we have a violation of Jacobi Identity for the kinetic momenta $p_k = m\,\dot{x}_k$.

We follow Dyson-Feynman[1] closely and point out the necessary changes to include magnetic monopoles.

From (1) and (3) we have

$$[x_j, F_k] = -m[\dot{x}_j, \dot{x}_k]\,. \tag{6}$$

Now, we use the Jacobi Identity (5) for operators x_j and $\dot{x}_k$ in the form

$$[x_\ell, [\dot{x}_j, \dot{x}_k]] + [\dot{x}_j, [\dot{x}_k, x_\ell]] + [\dot{x}_k, [x_\ell, \dot{x}_j]] = 0\,. \tag{7}$$

From (3) it's easy to see that the two last terms in the left-handed side of above equation, vanish.

So (7) can be writen

$$[x_\ell, [\dot{x}_j, \dot{x}_k]] = 0\,. \tag{8}$$

This equation means that the comutator $[\dot{x}_j, \dot{x}_k]$ is a function of x and t only. So, from (6) and (8) we can define the magnetic field H as

$$[x_j, F_k] = \left(\frac{-i\hbar}{m}\right)\epsilon_{jk\ell}\, H_\ell \tag{9}$$

and the eletric field as

$$E_j = F_j - \epsilon_{jk\ell}\,\dot{x}_k\, H_\ell \tag{10}$$

and, of course, H_ℓ and E_j are also functions of x and t only.

Substituting (6) and (9) in the Jacobi Identity in the form

$$\epsilon_{jk\ell}[\dot{x}_\ell, [\dot{x}_j, \dot{x}_k]] = 0\,. \tag{11}$$

We conclude that

$$[\dot{x}_\ell, H_\ell] = 0 \tag{12}$$

which is equivalent to

$$\text{div}\ \vec{H} = 0\,. \tag{13}$$

Now, as shown by Jackiw[3] and Wu and Zee[4], the existence of magnetic monopoles implies the violation of Jacobi Identity (11) and this is the very definition of magnetic charge, namely

$$\text{div } \vec{H} = \frac{1}{\hbar^2}\,\epsilon_{jk\ell}[p_\ell,[p_j,p_k]] \doteq \rho_{mag} \tag{14}$$

where we have rewritten (11) in terms of kinetic momenta $p_j = m\,\dot{x}_j$.

Using (6) we can rewrite (9) as

$$H_\ell = \frac{-im^2}{\hbar^2}\epsilon_{jk\ell}[\dot{x}_j,\dot{x}_k]\,. \tag{15}$$

The total time derivative of (15) is

$$\frac{\partial H_\ell}{\partial t} + \dot{x}_m\frac{\partial H_\ell}{\partial x_m} = \frac{-im^2}{\hbar^2}\epsilon_{jk\ell}\dot{[\ddot{x}_j,\dot{x}_k]}\,. \tag{16}$$

After some calculations on the right-hand side the above equation we get

$$\frac{\partial H_\ell}{\partial t} - \epsilon_{jk\ell}\frac{\partial E_j}{\partial x_k} = -\dot{x}_\ell\frac{\partial H_k}{\partial x_k}\,. \tag{17}$$

The right-handed side of this equation defines the magnetic current, using (14)

$$-\,\dot{x}_\ell\,\rho_{mag} = j_\ell \tag{18}$$

and so we obtain the second generalized Maxwell equation

$$\frac{\partial H_\ell}{\partial t} - \epsilon_{jk\ell}\frac{\partial E_j}{\partial x_k} = j_\ell\,. \tag{19}$$

The other two non-homogeneous Maxwell equations

$$\text{div } \vec{E} = \rho_{eletric} \tag{20}$$

$$\text{curl } \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j}_{eletric} \tag{21}$$

are interpreted in Feynman-Dyson approach as defining the very eletric charge and current.

This have caused a certain uneasiness[5-10] because apparently there is no physical or mathematical principle to fix the non-homogeneous equations such that the complete set of Maxwell equations results Lorentz invariant.

Nevertheless, we agree with Farina and Vaydia[5], and Hojman and Shepley[10] that it is necessary to introduce a parameter with units of velocity. This arbitrary parameter is shown to be independent of the observer[11] using weaker assumptions on isotropy and homogeneity of space than the original conditions used by Einstein, obtaining in this way the Lorentz transformations. But, unfortunately we can not yet fix the non-homogeneous equations from the postulates (1), (2), (3).

Another shortcoming is related to a Lagrangian formulation of magnetic monopoles theories. Hojman and Shepley[10] have shown that if we don't have a Lagrangian for a physical system we can't quantize it.

However the monopole theory, where the monopole didn't arise from a change of the topology of the world manilfold, is an example of a quantum system for which there doesn't exist a Lagrangian[12] giving simultaneously the field equations and the equations of motion of changes and monopoles. So, it would be interesting to investigate how and why this kind of monopole overrides the Hojman and Shepley's theorem. To end we call the reader's attention that we have shown elsewhere[13] that the equations of motion for both charges and monopoles follows directly from the generalized Maxwell equations without any ad-hoc postulate, a result complementar to the above one.

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The Vacuum Energy of QED with Four-Fermion Interaction

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Abstract

We consider quantum electrodynamics in the quenched approximation including a four-fermion interaction with coupling constant g. The effective potential at stationary points is computed as a function of the coupling constants α and g and an ultraviolet cutoff Λ, showing a minimum of energy in the (α, g) plane for $\alpha = \alpha_c = \pi/3$ and $g = \infty$. When we go to the continuum limit ($\Lambda \to \infty$), keeping finite the dynamical mass, the minimum of energy moves to ($\alpha = 0, g = 1$), which correspond to a point where the theory is trivial.

There are several works devoted to the analysis of a non-trivial phase of quantum electrodynamics (QED) in the strong-coupling regime, where it has been shown that the chiral symmetry is spontaneously broken [1]. For the existence of such symmetry breaking, the gauge-coupling constant α must have a particular relation to an ultraviolet cutoff (Λ), from which it can be inferred that the theory has a non-trivial ultraviolet fixed-point [2]. The existence of a fixed-point changes completely the argument that renormalized QED is a trivial theory [3]. As long as these calculations were made in the quenched approximation, where the coupling constant is not allowed to run, it is far from obvious that we may define a renormalization group β function [4], and from it we are able to determine the presence of the fixed point. Moreover, the results were obtained solving the Schwinger Dyson equations (SDE) in the ladder approximation, and it is not an easy task to determine how accurate these solutions are. However, a strong support for these calculations comes out from lattice simulations, where the same broken symmetry phase was found at strong coupling [5,6].

Another important result is that, in the planar limit the dimension of the four-fermion operators approach dimension four at the critical coupling constant α_c then, to study the fixed points we must include this four-fermion interaction [7] with dimensional coupling constant G. It is interesting to keep in mind that this four-fermion interaction introduced by hand, could be dynamically generated by the theory [6]. The dynamical generation of new interactions at a fixed point occurs also for example in $\eta\phi^6$ theory [8]. It has also been shown that solutions to the gap equation for an arbitrary value of G will break the scale symmetry unless G approaches a fixed-point value [7]. On the other hand, it is well know that weak coupling solutions of the Schwinger-Dyson equations does not produce spontaneous breaking of the chiral symmetry [9,10]. However, when four-fermion interactions are added, spontaneous breaking occurs even for weak gauge coupling, but in this case a critical line in the (α, G) plane appears [11,12].

Even though the triviality of QED does not have any phenomenological consequence, because it will probably be unified to the other interactions before we arrive at the Landau's pole, it is crucial to know if the simplest and (perhaps) the best known gauge theory we have, behaves well at high energies. It is clear that if the theory is not trivial at strong coupling, and chiral symmetry is broken when the coupling constant is larger than a certain critical value, say $\alpha > \alpha_c$, the vacuum energy must be well defined and different from zero. In the case of QED without four-fermion interaction it was verified that the theory has a minimum of energy, exactly at $\alpha = \alpha_c$ [13]. In this work, we will compute an effective potential for composite operators [14] at stationary points in the case of QED with a four-fermion interaction, looking for minima of energy in the (α, g) plane, $(g = G\Lambda^2/4\pi^2)$.

The chiral invariant four-fermion interaction to be added to the QED

Lagrangian is [7]

$$L_4 = (G/2)\left[(\overline{\psi}\psi)^2 - (\overline{\psi}\gamma_5\psi)^2\right]. \quad (1)$$

In the chiral limit, in the quenched (ladder) approximation and in the Landau gauge, the Schwinger-Dyson equation for the fermion self-energy, $\Sigma(p^2)$ takes the form [7],

$$\Sigma(x) = \frac{g}{\Lambda^2}\int_0^{\Lambda^2} dy \frac{y\Sigma(y)}{y+\Sigma^2(y)} + \lambda\int_0^{\Lambda^2} dy \frac{\Sigma(y)}{y+\Sigma^2(y)}\left[\frac{y}{x}\theta(x-y)+\theta(y-x)\right], \quad (2)$$

where, we have made a Wick rotation and integrated over the angular variables, with $x = p^2, \lambda = 3\alpha/4\pi = 3e^2/16\pi^2$ and $g = G\Lambda^2/4\pi^2$. Eq. (2) can be solved by standard methods [7,11,12], and a critical line can be determined from these solutions. This critical line separates the spontaneously broken and unbroken phases of the chiral symmetry. It has also been argued that the whole critical line is the fixed point i.e., we have in this case a "fixed line" [11].

With the non-perturbative solutions of the Schwinger-Dyson equation for the fermionic propagator we can start the calculation of the effective potential of QED. In the Euclidean space and after integrating over the angular variables the effective potential for composite operators [14] is given by :

$$\begin{aligned} V(\Sigma) &= -\frac{1}{8\pi^2}\int_0^{\Lambda^2} dx\, x\left[\ln\left(1+\frac{\Sigma^2(x)}{x}\right) - \frac{2\Sigma^2(x)}{x+\Sigma^2(x)}\right] \\ &\quad + \frac{1}{8\pi^2}\int_0^{\Lambda^2} dx \frac{x\Sigma(x)}{x+\Sigma^2(x)}\int_0^{\Lambda^2} dy \frac{y\Sigma(y)}{y+\Sigma^2(y)} F(x,y,\lambda,\Lambda), \quad (3) \end{aligned}$$

where

$$F(x,y,\lambda,\Lambda) = \frac{\lambda}{x}\theta(x-y) + \frac{\lambda}{y}\theta(y-x) + \frac{g}{\Lambda^2}.$$

By using Eq. (2) as an identity in Eq. (3), we obtain the following expression:

$$\Omega = -\frac{1}{8\pi^2}\int_0^{\Lambda^2} dx\left[x\ln\left(1+\frac{\Sigma^2(x)}{x}\right) - \frac{x\Sigma^2(x)}{x+\Sigma^2(x)}\right]. \quad (4)$$

In virtue of the condition $\delta V/\delta\Sigma = 0$ which implies (2), Ω means the value of the effective potential at the extreme points. Notice that Ω is always negative for any non-trivial solution $\Sigma(x)$.

Without going into the details [15], we now discuss the existence of a minimum for the vacuum energy. In Fig. 1 we show a plot of $(8\pi^2/\Lambda^4)\Omega$ against α for several values of g. Notice that the case $g = 0$ is not reduced directly to the one in Ref. [13], where a simpler approximation to the solution of the Schwinger-Dyson equation was used and where the upper limit of Eq. (4) was approximated to infinity. From Fig. 1 we can see that the minimum of energy tends towards the point $\alpha = \alpha_c$. To illustrate the behavior of Ω as a function of g we show, in Fig. 2, $(8\pi^2/\Lambda^4)\Omega$ against g for values of α around α_c. For large values of g Fig. 2 tells us that the deepest minimum occurs for $\alpha = \alpha_c$. For larger or smaller values of α all curves of Fig. 2 lie above the curve with $\alpha = \alpha_c$. Strictly speaking the minimum will occur at $(\alpha = \alpha_c, g = \infty)$. The position of the minimum in the (α, g) plane is shown in Fig. 3 by the thick solid curve. At the point $(\alpha_0, 0)$ the value of $(8\pi^2/\Lambda^4)\Omega$ is -0.0012, and it becomes deeper and deeper as we increase the value of g and approximate $\alpha = \alpha_c$.

In Fig. 3 we show also another curve (dot-dashed) which can be interpreted as follows. Away from the critical line the fermion self-energy is approximately constant, therefore the solution for the gap equation leads to a consistency condition [16]:

$$1 = \left(\frac{\alpha}{4\alpha_c} + g\right)\left[1 - \frac{\Sigma^2}{\Lambda^2}\ln\frac{\Lambda^2}{\Sigma^2}\right]. \tag{5}$$

Eq. (5) in the limit $\Sigma^2/\Lambda^2 \to 0$, gives a mean-field curve, $g = 1 - \alpha/4\alpha_c$ described by the dot-dashed straight-line in Fig. 3. For larger values of g (above this curve) we approach a trivial Nambu-Jona-Lasinio theory [16]. Therefore, if we allow for large values of g we conclude that the minimum of energy happens for values of the coupling constants where the theory is trivial, and it is clear from Fig. 3 that values of minima are above the mean-field curve described by Eq. (5). However, we have also to keep in mind that the curve $g = 1 - \alpha/4\alpha_c$ was obtained with a crude approximation and it should be regarded more as a qualitative result. The question now is: how arbitrary is g? This point is of fundamental importance because if g is limited to some finite value, we do have a definite minimum of energy in the (α, g) plane (see Fig. 3), otherwise the minimum will be located at g equal to infinity where the theory is certainly trivial.

In our calculations we have a free mass parameter Λ that can be factorized in such a way that Λ enters in Ω only as a multiplicative factor. In fact, the quantity Ω/Λ^4 is independent of Λ. We can ask what happens if we consider the continuum limit $\Lambda \to \infty$. In order to take $\Lambda \to \infty$ seriously we must know the behavior of g and α as a function of Λ and this limit must fulfill the hypothesis of Miransky and others [2,7] about the existence of a fixed-point. In that case we can have a possible limit that results in g, α and m finite and therefore a definite minimum of energy. We notice that there is a possibility of taking this limit even if not in a rigorous way. We can argue that when $\Lambda \to \infty$, g and α are related through the critical line. In this case, Λ goes to infinity but $\overline{m}$ goes to zero keeping m constant and so Ω, i.e., the limits on Λ and $\overline{m}$ are taken in such way that their product is equal to κ over the critical line, where κ has a definite value. In this case all points of minimum in the curve lie under the mean-field line showed in Fig. 3 coinciding with the critical line. However, Ω for $\alpha = \alpha_c$ and $g = 1/4$ is not the deeper minimum. In fact Ω becomes deeper and deeper as we decrease the value of α and approach $\alpha = 0$ and $g = 1$. This result tells us that in this picture i.e., $\Lambda \to \infty$ and g and α related by the critical line, the 4-fermion interaction alone is more efficient to break chiral-symmetry than both interactions together. Notice that the minimum of energy at $(\alpha = 0, g = 1)$ is the only one that also corresponds to a point (according to Eq. (5)) where the theory is trivial. The above procedure is useful to illustrate the possibility that when Λ goes to infinity we can have a well defined minimum of energy.

In conclusion, we computed the vacuum energy of QED with four-fermion interaction. Starting from the solutions of the Schwinger-Dyson equation for the fermion self-energy, we determined the values of minima of energy in the (α, g) plane. The minimum we have found is located at $(\alpha = \alpha_c, g = \infty)$, and we argued that this point corresponds to one where the theory is trivial. The theory has an unique mass parameter which is given by the ultra-violet cutoff Λ. When we go to the continuum limit $(\Lambda \to \infty)$ we only obtain a sensible result imposing the same condition of Miransky and others [2,7] i.e., we must impose a relation between α, g, m and Λ in such a way that when $\Lambda \to \infty$ and m is kept finite α and g go to some specific critical line. However, performing the calculation over the critical line, with $\Lambda/m \to \infty$, we found the global minimum at $(\alpha = 0, g = 1)$ which is again a point that characterize a trivial theory. All these conclusions probably do not hold if the four fermion is generated dynamically, when a well defined minimum of energy could appear as a function of a certain critical value of α.

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FIGURE CAPTIONS

Fig. 1. Ω calculated from Eq.(4) for $\alpha > \alpha_c$ and the following values of $g : g = 0(a), 0.25(b), 0.50(c), 0.75(d), 1.00(e)$.

Fig. 2. Ω calculated from Eq.(4) for the three different regions: Ω_1 for $\alpha = 0.8\alpha_c$ (dot-dashed curve); Ω_2 for $\alpha = \alpha_c$ (solid curve) and Ω_3 for $\alpha = 1.4\alpha_c$ (dashed curve).

Fig. 3. The critical line (solid curve). The line separating the regions with trivial and non-trivial solutions obtained from Eq.(5) (dot-dashed curve). The local minima of Ω (thick solid curve).

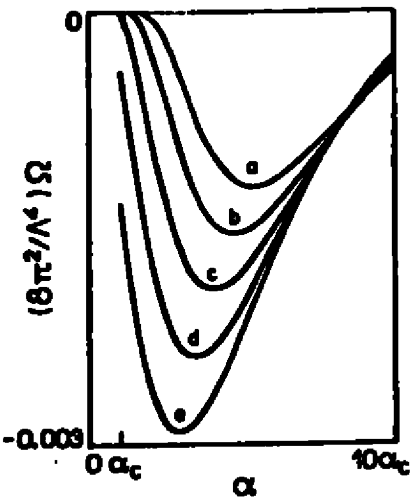

FIG. 1

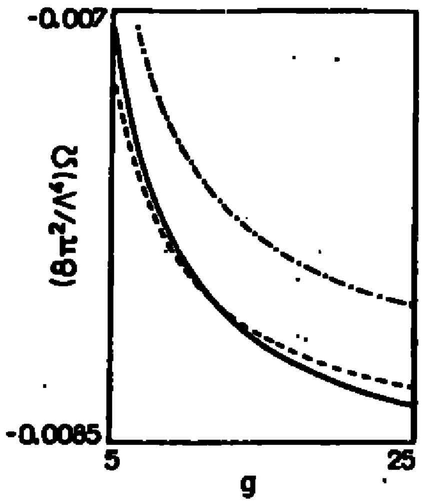

FIG. 2

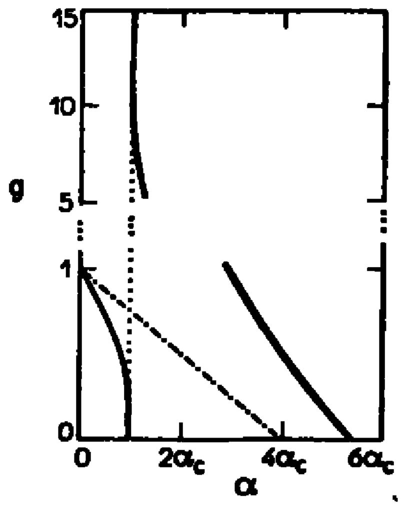

FIG. 3

Unobservability of the Sign Change of Spinors Under a 2π Rotation in Neutron Interferometric Experiments

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Abstract

We show that the neutron interferometric experiments do not imply that *the neutron* wave function must be described by a Pauli c-spinor wave function that changes sign under a 2π rotation. We argue that the papers supporting the opposite view have jumbled up the time evolution of the Pauli c-spinor wave function with its transformation law under rotations. Even more, we show that the experiment can be well described using a Pauli algebraic spinor wave function that does not change sign under a 2π rotation.

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There are essentially three different definitions of spinors in the literature: (i) the *covariant* definition, where a particular kind of covariant spinor (c-spinor) is a set of complex variables defined by their transformations under a particular kind of spin group; (ii) the *ideal* definition, where a particular kind of *algebraic* spinor (e-spinor) is an element of a lateral ideal (defined by the idempotent e) in an appropriate Clifford algebra (when e is a primitive idempotent we call it an a-spinor, instead of e-spinor); and (iii) the *operator definition*, where a particular kind of operator spinor (o-spinor) is a Clifford number in an appropriate Clifford algebra $\mathbb{R}_{p,q}$ determining a set of tensors by bilinear mappings. In [1,2] we have clarified the relations between and the possible equivalence of all these kinds of spinors and in [3,4] we studied the corresponding spinor fields as *sections of appropriate* bundles over a manifold modelling spacetime.

Physicists use almost exclusively c-spinor fields (despite the fact that operator spinor fields have been introduced by Ivanenko and Landau [5] already in 1928 and rediscovered by Kähler[6] in 1961) as the representatives of spin 1/2 fermionic matter. As is well known, a c-spinor wave function has the property of changing its sign under an active 2π rotation, which is not the case for algebraic or operator spinor wave functions interpreted as sections of appropriate Clifford bundles [4]. Which kind of spinor fields, covariant or algebraic/operator gives the best mathematical and physical representation of fermionic matter is a very important problem, since algebraic and operator spinor fields can be written as sums of non-homogeneous differential forms [1,2,4,5,7,8] thus challenging the "majority view" that spinors are objects more fundamental than tensors [9,10,11]. (*We emphasize* here that when a-spinor fields are interpreted as sections of the so called Spin-Clifford bundle they have the usual transformation law [4].)

Bernstein [12], Aharanov and Susskind [13] and Moore [14] proposed experiments for the verification of the *sign change of c-spinors* under an active 2π rotation. Hegerfeldt and Krauss [15] put forth a critical remark on the Aharanov and Susskind argument, showing that it is in flaw (a point on which we agree). Also Jordan [16] invoked the spin statistics theorem for spin 1/2 particles to argue that 2π rotations are unobservable.

After the neutron interferometric experiments [17,18,19] the controversy on the interpretation of the sign change of the neutron c-spinor wave function in a magnetic field went out, as it is well illustrated by the many papers that appeared on this subject [20-30]. It seems to be the "majority view" that the neutron interferometric experiments do indeed prove that the neutron wave function must be described by a Pauli c-spinor wave function (on the nonrelativistic limit appropriate for the experiment) that changes sign under an active 2π rotation.

Here we challenge such a viewpoint. Indeed, we are going to show that the neutron interferometric experiment as described e.g. in [30] can be perfectly explained when the spin 1/2 neutron matter is described by a Pauli a-spinor wave function that does *not* change sign under a 2π active rotation. What happens is simply that the unitary evolution operator for such a wave function is an element of Spin(3) $\simeq$SU(2) ! For what follows nonrelativistic (first quantization) quantum mechanics will suffice. We are going to use *elementary* definitions of the c-spinor and a-spinor wave functions, i.e. we are not going to present these objects as sections of some vector bundle. (The interested reader may consult e.g. [4] on that topic.)

We take as arena of physical phenomena the Newtonian spacetime $N = \mathbb{R}^3 \times \mathbb{R}$ and define a Pauli c-spinor wave function as a mapping

$$\Psi : N \to \mathbb{C}^2 \tag{1}$$

where $\mathbb{C}^2$ is a two-dimensional vector space over the complex field $\mathbb{C}$. The space $\mathbb{C}^2$ is equipped with the spinorial metric

$$\beta_p : \mathbb{C}^2 \times \mathbb{C}^2 \to \mathbb{C};\ \beta_p(\Psi,\Phi) = \Psi^\dagger \Phi \tag{2}$$

where $\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$ and $\dagger$ stands for Hermitian conjugation. The spinorial metric is invariant under the action of SU(2)$\simeq$Spin$_+$(3) (in fact it is invariant under the action of U(2) [2]). As it is well known Pauli c-spinors carry the fundamental representation $D^{1/2}$ of SU(2). Under an active rotation R in the Euclidian space $\mathbb{R}^3$ the Pauli c-spinor wave function transforms as

$$\Psi \overset{R}{\mapsto} U(R)\Psi,\ U(R) \in \mathrm{SU}(2) \tag{3}$$

and if R is a 2π rotation around a given axis, then $\Psi \overset{2\pi}{\mapsto} -\Psi$. In a given magnetic field $B : N \to \mathbb{R}^3$ the neutron wave function Ψ satisfies as it is well known [31] Pauli's equation

$$i\frac{\partial \Psi}{\partial t} = H_i\Psi - \frac{\nabla^2\Psi}{2m} \tag{4}$$

where we use units such that $\hbar = 1$, m is the neutron mass and

$$H_i = -\boldsymbol{\mu} \cdot \boldsymbol{B} = -\mu(\sigma_1 B_1 + \sigma_2 B_2 + \sigma_3 B_3) \tag{5}$$

where σ_j, $j = 1,2,3$ are the Pauli spin matrices, B_j, $j = 1,2,3$ are the components of $\boldsymbol{B}$ in a given reference frame of $\mathbb{R}^3$ and μ is the neutron's magnetic moment. In what follows we are interested only in the spin precession motion and so we consider instead of eq.(4) the equation

$$i\frac{\partial \Psi}{\partial t} = H_i\Psi,\ \Psi : t \mapsto \Psi(t) \in \mathbb{C}^2 \tag{6}$$

We choose $\boldsymbol{B}$ in the z-direction and then write $H_i = -\mu B\sigma_3$. We now write $\Psi = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \sum c_j|j>$ and observe that $\sigma_1\sigma_2\sigma_3|1> = i|1>$ and $\sigma_1\sigma_2\sigma_3|2> = -i|2>$. Then eq.(6) can be written

$$\sigma_1\sigma_2\sigma_3\frac{\partial \Psi}{\partial t} = -\mu B\sigma_3\Psi. \tag{7}$$

We now define the Pauli a-spinor wave function and write the (Pauli) equation satisfied by this object for the situation of the neutron interferometric experiment. We first recall [1,2] that the Pauli algebra $\mathbb{R}_3$ is the Clifford algebra generated by 1 and e_j, $j = 1,2,3$ such that $e_ie_j + e_je_i = 2\delta_{ij}$ where $\{e_j; j = 1,2,3\}$ is a basis of the Euclidian vector space $V \simeq \mathbb{R}^3 \hookrightarrow \mathbb{R}_3$. We take $\{\sigma_i; i = 1,2,3\}$ as a basis of V^*, the dual space of $\mathbb{R}^3$, with $\sigma_i(e_j) = \delta_{ij}$ and call $\mathbb{P}(\simeq \mathbb{R}_3)$ the Clifford algebra generated by 1 and the σ_i, $i = 1,2,3$. A Pauli a-spinor wave function in then defined as a mapping

$$\psi : N \to \{\mathbb{P}e\} \tag{8}$$

where $e = \frac{1}{2}(1 + \sigma_3)$ is a primitive idempotent of $\mathbb{P}$ and $\{\mathbb{P}e\}$ is the class of equivalent minimal left ideal of $\mathbb{P}$ generated by e, i.e. ψ is a sum of non-homogeneous differential forms [3,4,7]. Under an active rotation R in $\mathbb{R}^3$ the Pauli a-spinor wave function transforms as

$$\psi \overset{R}{\mapsto} u(R)\psi u^{-1}(R) \tag{9}$$

where $u \in \mathrm{Spin}_+(3)(\simeq SU(2)) \subset \mathbb{P}$. (More precisely this is the transformation law when $(x, \psi(x))$ is taken as a section of the Clifford bundle. See [3,4] for details.) This has as a consequence that under a 2π rotation $\psi \overset{2\pi}{\mapsto} \psi$. The spinorial metric defined by eq.(2) can also be defined within the Pauli algebra [1,2] but it is not necessary here.

The spinorial basis generated by $e = \frac{1}{2}(1+\sigma_3)$ is $\{e, \sigma_1 e\}$ [1,2] and we can write $\psi = c_1 e + c_2\sigma_1 e$ with $c_1, c_2 \in \mathbb{C}$, generated by $\{1, \mathrm{i}\}$. Also $\mathrm{i} = \sigma_1\sigma_2\sigma_3$ is the volume element of $\mathbb{R}^3$ and $\mathrm{i}\Lambda_p$ is essentially $*\Lambda_p$, where $\Lambda_p \in \bigwedge(T^*\mathbb{R}^3)$ is a p-form and $*$ is the Hodge dual operator. To write the (Pauli) equation satisfied by ψ for the neutron interferometric experiment we need only to take $\psi : t \mapsto \{\mathbb{P}e\}$ and to make in eq.(7) the substitutions $\Psi \mapsto \psi$, $\sigma_i \mapsto \sigma_i$, $(i = 1,2,3)$. We get

$$\frac{\partial\psi}{\partial t} = \mu B(\mathrm{i}\sigma_3)\psi. \tag{10}$$

The solution of this equation is

$$\psi(t) = \exp(\mu B \mathrm{i}\sigma_3 t)\psi(0) \tag{11}$$

where $\mathrm{Spin}_+(3) \ni u(t) = \exp(\mu B\mathrm{i}\sigma_3 t) = \cos(\mu Bt) + \sigma_1\sigma_2 \sin(\mu Bt)$ [33].

Equation (11) shows that the predictions for the neutron interferometric experiment *when one uses a Pauli a-spinor wave function* are the same as when a Pauli c-spinor wave function is used. Since these two kinds of spinor wave functions have different transformation laws under rotations (eq.(3) and eq.(9)), it follows that the experiment *does not prove* that the fermionic matter of the neutron must be described by a Pauli c-spinor wave function.

Before we end we must add that the notion of algebraic spinor fields leads to a new point of view [4] concerning the spinor structure of spacetime and the relation between bosons and fermions (supersymmetry) [34]. Also our translation of the Pauli equation satisfied by Ψ into the (Pauli) equation satisfied by ψ provides a geometrical meaning for the imaginary unit $i = \sqrt{-1}$, a fact that may have nontrivial consequences as already emphasized by Hestenes [35-38] who has been since long using algebraic and operator spinor wave functions for the interpretation of the relativistic quantum mechanics of the electron.

At least, to those who might not be convinced by our arguments, we recall the fact that there are many two-state quantum systems described by equations identical to eq.(6). Indeed as shown in Chap. 11-3 of [31] this is the case of the amonia molecule (a boson) in an eletric field. In a (possible) interferometric two-slit experiment with amonia molecules, with one of the paths passing through an eletric field E, we could see for an appropriate E a phase change $\phi \mapsto -\phi$. Nevertheless we are sure that in such a case nobody would claim that we are observing a 2π rotation of a spinor!

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NON-ANOMALOUS BOSONIZED THEORIES FROM A GAUGE PRINCIPLE

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Abstract: Starting from a gauge principle for a scalar boson we show that one can get, in 1+1 dimensions, the Lagrangian density of the generalized Schwinger model (GSM) in its bosonized version. This occur for a particular value of the regularization parameter. Then we show how to get the model with the arbitrary parameter through the introduction of the Wess-Zumino field.

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As is well known, some two-dimensional models presents the interesting feature of dynamical mass generation for the gauge bosons, and this happen without loss of gauge invariance. In fact this is a mechanism that would be wellcome in higher dimensions, particularly in 3+1, in order to archieve a standard model without Higgs fields.

On the other hand the possibility of transformation of fermionic fields into bosonic ones it is one of the more interesting features of the two dimensional models. The bosonization technique is very important in order to get nonperturbative informations from a given model. In fact this type of approach is nowadays being studied in higher dimensions [1], through the operatorial picture.

In this letter we intend to show that, from the well known gauge principle, we can obtain these bosonized theories in 1+1 dimensions in the abelian case. This is done by observing that the free scalar Lagrangian density

$$\mathcal{L} = (1/2)\partial_\mu\phi\ \partial^\mu\phi\ , \tag{1}$$

is invariant under the global translaction:

$$\phi(x) \to \phi(x) + v\ , \tag{2}$$

where v is a constant. As usual, now we impose that this model be invariant under a local translation

$$\phi(x) \to \phi(x) + v(x). \tag{3}$$

This is made by the introduction of a gauge vector field, that has the usual transformation

$$E_\mu(x) \to E_\mu(x) - (1/g)\partial_\mu v(x)\ , \tag{4}$$

where g is a dimensional coupling constant. With these elements in hands, it is not difficult to see that the invariant Lagrangian density shall be

$$\mathcal{L} = (1/2)\partial_\mu\phi\ \partial^\mu\phi + g\ E_\mu\ \partial^\mu\phi + (g^2/2)E_\mu\ E^\mu. \tag{5}$$

In fact the above Lagrangian density is valid in an arbitrary number of dimensions. Here we will restrict our analysis to 1+1 dimensions. Up to now we do not have identified the gauge field E_μ with the photon field A_μ, but in two dimensions we can relate these fields through the general expression

$$E^\mu = (c_1\ g^{\mu\nu} + c_2\ \epsilon^{\mu\nu})\ A_\nu. \tag{6}$$

Using this relation we get for the Lagrangian density:

$$\mathcal{L} = (1/2)\partial_\mu\phi\ \partial^\mu\phi + g\ (c_1\ g^{\mu\nu} + c_2\ \epsilon^{\mu\nu})\ \partial_\mu\phi\ A_\nu +$$

$$+ (g^2/2)(c_1^2 + c_2^2)A_\mu A^\mu, \qquad (7)$$

choosing the coupling constant g as being the electron charge, that is the dimensional parameter in two dimensions; and identifying c_1 and c_2 with coupling parameters of the GSM [2], g_1 and g_2 respectively, we have that

$$\mathcal{L} = (1/2)\partial_\mu\phi\,\partial^\mu\phi + e(g_1\, g^{\mu\nu} + g_2\, \epsilon^{\mu\nu})\,\partial_\mu\phi\, A_\nu + (e^2/2)A_\mu A^\mu, \qquad (8)$$

where we used imposed that $g_1^2 + g_2^2 = 1$ [2]. The above Lagrangian density, up to the arbitrary regularization parameter, is just the bosonized version of the GSM [3],

$$\mathcal{L} = (1/2)\partial_\mu\phi\,\partial^\mu\phi + e(g_1\, g^{\mu\nu} + g_2\, \epsilon^{\mu\nu})\,\partial_\mu\phi\, A_\nu + (ae^2/2)A_\mu A^\mu \qquad (9)$$

that in the particular cases in which $g_1 = 1 = -g_2$, and that $g_1 = 0$, $g_2 = 1$, recalls the chiral Schwinger model and the Schwinger one respectively [4,5].

However, as can be seen from above, the Lagrangian density of the bosonized GSM, was obtained in a particular value of the regularization parameter. Now we will see how to introduce this arbitrary parameter. This will be made through the use of the Wess-Zumino field.

This is made through the transformations:

$$\phi \to \phi + k_1\, \theta, \qquad (10a)$$

$$A_\mu \to A_\mu + (k_2/e)\, \partial_\mu\theta, \qquad (10b)$$

that after substitution in (9) and rearranging gives:

$$\mathcal{L} = (1/2)\partial_\mu\phi\,\partial^\mu\phi + e(g_1\, g^{\mu\nu} + g_2\, \epsilon^{\mu\nu})\partial_\mu\phi A_\nu + (ae^2/2)A_\mu A^\mu$$

$$+ (k_1 + g_1 k_2)\partial_\mu\phi\,\partial^\mu\theta + (1/2)(k_1^2 + 2g_1 k_1 k_2 + ak_2^2)\partial_\mu\theta\,\partial^\mu\theta$$

$$e\,\theta\left[(q_1 k_1 + a k_2)\partial^{\mu}A_{\mu} + q_2 k_1 \epsilon^{\mu\nu}\partial_{\mu}A_{\nu}\right]. \qquad (11)$$

Now we can eliminate one of the constants k_1 and k_2, by imposing that the crossed term in the fields ϕ and θ vanishes. This condition raises from the fact that such term would corresponds to an interaction between the fermion fields and the Wess-Zumino one, and this would renders the theory anomalous. Using this condition to eliminate the constant k_1, we obtain

$$\mathcal{L} = (1/2)\partial_{\mu}\phi\,\partial^{\mu}\phi + e(q_1 g^{\mu\nu} + q_2 \epsilon^{\mu\nu})\partial_{\mu}\phi A_{\nu} + (ae^2/2)A_{\mu}A^{\mu} + \mathcal{L}_{WZ} \qquad (12a)$$

where

$$\mathcal{L}_{WZ} = (k_2/2)(a - q_1^2)\partial_{\mu}\theta\,\partial^{\mu}\theta + e\,k_2\,\theta\left[(a - q_1^2)g^{\mu\nu} + q_1 q_2 \epsilon^{\mu\nu}\right]\partial_{\mu}A_{\nu}, \qquad (13)$$

and making the finite renormalization $\theta \to -(1/k_2)\theta$, we get finally the correct non-anomalous GSM, with the Wess-Zumino Lagrangian density

$$\mathcal{L}_{WZ} = (1/2)(a - q_1^2)\partial_{\mu}\theta\,\partial^{\mu}\theta - e\,\theta\left[(a - q_1^2)g^{\mu\nu} + q_1 q_2 \epsilon^{\mu\nu}\right]\partial_{\mu}A_{\nu}, \qquad (14)$$

that is the correct Wess-Zumino Lagrangian density, as can be verified from the particular cases of chiral Schwinger model ($q_1 = 1 = - q_2$) and the vectorial Schwinger one ($q_1 = 0$, $q_2 = 1$) [6].

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ESTADOS SUPER-COERENTES DO OSCILADOR RADIAL SUSI 3D

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Resumo. Encontramos os estados super-coerentes canônicos do oscilador radial SUSI 3D. Mostramos que eles são de três tipos: bosônico, fermiônico e super-simétrico (SUSI).

I. INTRODUÇÃO

Os estados coerentes podem ser definidos de várias maneiras, sendo algumas delas equivalentes [1]. Eles têm uma vasta aplicação em física [2]. Uma extensão dos estados coerentes do oscilador harmônico simples são os Estados Super-Coerentes Canônicos (ESCC) do oscilador harmônico supersimétrico (SUSI) 1D, os quais são os autoestados de um operador de aniquilação SUSI de primeira ordem [3]. Estes estados geram espaço de Hilbert bidimensional: um estado é fermiônico puro, e o outro é uma mistura de estados bosônico e fermiônico, i.é, um estado SUSI. Aqui, faremos a extensão dos estados coerentes radiais. Da conexão entre o oscilador radial SUSI 3D e o oscilador radial generalizado de Wigner, obtén-se uma realização das super-cargas, , em termos da super-realização de Jayaraman e Rodrigues (JR) da álgebra de Wigner-Heisenberg (WH) [4]. No sistema de unidades em que $h=1=M=\omega$, tal realização da SUSI em mecânica quântica é a seguinte:

$$H_{SS} = H(\ell+1) - \frac{1}{2}\Sigma_3\left(1 + 2(\ell+1)\Sigma_3\right) = [Q_-, Q_+]_+ \tag{1}$$

$$[H_{SS}, Q_\pm]_- = 0, \quad Q_+^2 = 0 \quad \text{e} \quad Q_-^2 = 0\,. \tag{2}$$

onde $H(\ell+1)$ é o Hamiltoniano do oscilador radial generalizado de Wigner [6], comuta com a coordenada fermiônica, Σ_3 . E as super-cargas são dadas por:

$$Q_- = \frac{1}{2}(1-\Sigma_3)a^-(\ell+1) \quad , \quad Q_+ = \frac{1}{2}(1+\Sigma_3)a^+(\ell+1) \quad . \tag{3}$$

onde $a^{\pm}(\ell+1)$, são os operadores escada da partícula de Wigner.

$$[H(\ell+1), a^{\pm}(\ell+1)]_- = \pm a^{\pm}(\ell+1). \tag{4}$$

Como $H(\ell+1)$ e Σ_3 comutam, então H_{SS} e $H(\ell+1)$ também comutam. Neste caso, podemos diagonalizar este Hamiltoniano SUSI na mesma base dos estados espinoriais de Wigner. Denotando os estados associados aos quanta pares e ímpares, por $|\phi_m\rangle$ e $|\psi_m\rangle$, respectivamente, obtemos :

$$H_{SS}|\phi_m\rangle = 2m|\phi_m\rangle, \quad H_{SS}|\psi_m\rangle = 2(m+1)|\psi_m\rangle. \tag{5}$$

O espectro deste sistema SUSI é degenerado para $m\geq 1$. O vácuo é um estado singleto de energia zero, logo a SUSI é não quebrada.

II. ESTADOS SUPER-COERENTES CANÔNICOS

Os ESCC são os autoestados de um operador de aniquilação SUSI do OR 3D. A partir da álgebra WH obtemos três tipos desse operador, os quais são escritos em termos do operador de aniquilação da partícula de Wigner. O primeiro, é diagonilzável na base dos estados super-coerentes fermiônicos puros [7],

$$A_1(\ell+1) = \frac{1}{2}(1+\Sigma_3)\{a^-(\ell+1)\}^2. \tag{6}$$

Este operador de aniquilação, possuem as seguintes propriedades:

$$A_1(\ell+1)|\phi_m\rangle = 2\{m(m+\ell+\tfrac{1}{2})\}^{1/2}|\phi_{m-1}\rangle, \quad A_1(\ell+1)|\psi_m\rangle = 0. \tag{7}$$

Então, expandindo os ESCC na base dos autoestados ortonormais, pertencentes ao autoespaço associado aos quanta pares, de dimensão um, deduzimos a seguinte forma espinorial:

$$|\xi_F,\ell\rangle = \begin{pmatrix} |\xi,\ell\rangle_- \\ 0 \end{pmatrix} = |\xi,\ell\rangle_- \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{8}$$

onde os kets $|\xi,\ell\rangle_-$ são os estados coerentes do OR 3D da ref. [5]. Estes ESCC são os análogos radiais dos estados super-coerentes fermiônicos puros do oscilador SUSI 1D da ref. [4].

O operador de aniquilação do OR SUSI 3D que, atua sobre os autoestados pertencentes aos quanta impares,

$$A_2(\ell+1) = \frac{1}{2}(1-\Sigma_3)\{a^-(\ell+1)\}^2 \quad . \tag{10}$$

é diagonalizado pelos ESCC bosônicos puros,

$$|\eta_B,\ell\rangle = \begin{pmatrix} 0 \\ |\eta,\ell\rangle_+ \end{pmatrix} = |\eta,\ell\rangle_+ \begin{pmatrix} 1 \\ 0 \end{pmatrix} . \tag{11}$$

onde os kets $|\eta,\ell\rangle_+$, são os ECC não-normalizados do OR 3D com momento angular adicionado de uma unidade , $(\ell+1)$, a saber:

$$|\eta,\ell\rangle_+ = \sum_{m=0}^{\infty}\left\{\frac{\Gamma(\ell+3/2)}{\Gamma(m+\ell+3/2)\,m!}\right\}^{1/2}\left(\frac{\eta}{2}\right)^m |m,\ell\rangle_+ \,. \tag{12}$$

O operador de aniquilação SUSI que atua sobre os autoestados associados aos quanta pares ou impares (estados fermiônicos ou bosônicos) e, consequentemente diagonalizável pelos ESCC SUSI,

$$A_3(\ell+1) = \{a^-(\ell+1)\}^2 \quad . \tag{13}$$

tem as seguintes propriedades:

$$A_3(\ell+1)|\phi_m\rangle = 2\{m(m+\ell+1/2)\}^{1/2}|\phi_{m-1}\rangle \,. \tag{14}$$

$$A_3(\ell+1)|\psi_m\rangle = 2\{m(m+\ell+3/2)\}^{1/2}|\psi_{m-1}\rangle \,. \tag{15}$$

Expandindo os ESCC SUSI na base $\{|\phi_0\rangle, |\phi_m\rangle, |\psi_m\rangle\}$, obtemos:

$$|\theta_S,\ell\rangle = b_0|\theta_B,\ell\rangle + d_0|\theta_F,\ell\rangle = \begin{pmatrix} d_0|\xi,\ell\rangle \\ b_0|\eta,\ell\rangle_+ \end{pmatrix} . \tag{16}$$

onde

$$A_3|\theta_S,\ell\rangle = \theta|\theta_S,\ell\rangle \;,\; A_2|\theta_S,\ell\rangle = \eta\,|\eta_B,\ell\rangle \;,\; A_1|\theta_S,\ell\rangle = \xi|\xi,\ell\rangle . \tag{17}$$

Estes estados Super-Coerentes Canônicos (ESCC) possuem também as duas propriedades importantes dos estados coerentes usuais: não-ortogonalidade e completeza. Estas propriedades serão mostradas por nós num trabalho mais detalhado sobre os estados coerentes do oscilador radial SUSI 3D, o qual está sendo preparado para submetê-lo a publicação numa revista científica internacional.

III. CONCLUSÕES

Construímos os ESCC do OR SUSI 3D. Mostramos que eles são de três tipos: (i) estados canônicos, análogos dos estados super-coerentes fermiônicos puros do oscilador SUSI 1D [3], (ii) ESCC bosônicos puros e (iii) ESCC SUSI. Todos esses estados super-coerentes são super-completos e não-ortogonais. Assim como foi possível esta extensão dos estados coerentes canônicos, podemos encontrar os estados coerentes generalizados associados à álgebra OSP(1/2) da SUSI em mecânica quântica. Um trabalho nesta linha está sendo desenvolvido por nós.

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ESTADOS COERENTES DO OSCILADOR HARMÔNICO ISOTRÓPICO 3D DE SPIN 1/2

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Resumo. Os estados coerentes esféricos são construídos via a técnica algébrica de Wigner-Heisenberg de três graus de liberdade. Eles são os autoestados do operador de aniquilação esférico de um oscilador harmônico isotrópico 3D de spin 1/2.

I. INTRODUÇÃO

Faremos um desenvolvimento análogo ao da nossa construção dos Estados Coerentes Canônicos (ECC) para um oscilador generalizado, o qual emergiu do setor bosônico de um Hamiltoniano de Wigner [1]. Usaremos o sistema de unidades em que $\hbar=1=m=\omega$.

A super-realização da álgebra de Wigner-Heisenberg (WH) proposta por Jayaraman e Rodrigues (JR) possibilitou uma simples resolução espectral do oscilador harmônico isotrópico 3D não relativístico e de spin 1/2 [2]. Este sistema é descrito pela seguinte equação de Schrödinger independente do tempo:

$$H(\underline{\sigma}\cdot\underline{L})\psi_- = \frac{1}{2}\left\{-\frac{\partial^2}{\partial r^2} - \frac{2}{r}\frac{\partial}{\partial r} + r^2 + \frac{\underline{\sigma}\cdot\underline{L}(\underline{\sigma}\cdot\underline{L}+1)}{r^2}\right\}\psi_- = E_-\psi_- \quad , \qquad (1)$$

cujo Hamiltoniano aparece embutido no setor bosônico do Hamiltoniano de Wigner 3D. Na eq.(1), usamos as identidades [2,3] envolvendo as matrizes de spin 1/2 de Pauli, $\sigma_i\,(i=1,2,3)$.

$$(\underline{\sigma}\cdot\underline{L})(\underline{\sigma}\cdot\underline{L}+1) = \mathbb{1}\,\underline{L}^2 , \quad \underline{\sigma}\cdot\underline{p} = \sigma_r p_r + \frac{i}{r}\sigma_r(\underline{\sigma}\cdot\underline{L}+1) \quad , \sigma_r = \frac{\underline{\sigma}\cdot\underline{r}}{r} \quad , \qquad (2)$$

$$\sigma_r^2 = \mathbb{1} \quad , \quad [\underline{\sigma}\cdot\underline{L}+1 \, , \sigma_r]_+ = 0 \quad , \quad \underline{\sigma}\cdot\underline{p} = \sigma_r p_r + \frac{i}{r}\sigma_r(\underline{\sigma}\cdot\underline{L}+1) \, . \qquad (3)$$

A super-realização JR dos operadores escada, mutuamente adjuntos,

$$a^{\pm}(\underline{\sigma}\cdot\underline{L}+1) = \frac{1}{\sqrt{2}}\left\{\pm\left(\frac{\partial}{\partial r} + \frac{1}{r}\right) \pm \frac{1}{r}(\underline{\sigma}\cdot\underline{L}+1)\Sigma_3 - r\right\}\Sigma_1 = \left\{a^{\mp}(\ell+1)\right\}^{\dagger} \quad , \qquad (4)$$

nos proporciona uma álgebra WH em 3D:

$$\mathbb{H}(\underline{\sigma}\cdot\underline{L}+1) = \frac{1}{2}\left[a^{-}(\underline{\sigma}\cdot\underline{L}+1), a^{+}(\ell+1)\right]_{+} \quad . \tag{5}$$

$$\left[\mathbb{H}(\underline{\sigma}\cdot\underline{L}+1), a^{\pm}(\underline{\sigma}\cdot\underline{L}+1)\right]_{-} = \pm a^{\pm}(\underline{\sigma}\cdot\underline{L}+1) \quad . \tag{6}$$

A relação de comutação generalizada derivada desta álgebra é:

$$\left[a^{-}(\underline{\sigma}\cdot\underline{L}+1), a^{+}(\underline{\sigma}\cdot\underline{L}+1)\right]_{-} = 1 + 2(\underline{\sigma}\cdot\underline{L}+1)\Sigma_3 \quad . \tag{7}$$

As coordenadas fermiônicas $\Sigma_i (i=1,2,3)$ são as matrizes de Pauli também, mas não descrevem o spin e, por sua vez, comutam com as matrizes de spin 1/2, σ_i.

As autofunções do operador matricial $(\underline{\sigma}.\underline{L}+1)$ são os bem conhecidos harmônicos esféricos de spin,

$$|+\rangle = |\ell\tfrac{1}{2}, j=\ell+\tfrac{1}{2}, m_j\rangle \; , \quad |-\rangle = |(\ell+1)\tfrac{1}{2}, j=(\ell+1)-\tfrac{1}{2}, m_j\rangle . \tag{8}$$

Pode-se mostrar que $(\underline{\sigma}.\underline{L}+1)$ comuta com todos os elementos da álgebra WH 3D. Então, seus autovalores vão rotular as representações irredutíveis que varrem os autoespaços de $H(\underline{\sigma}.\underline{L}+1)$, para um valor fixo do momento angular total, $j=\ell+1/2=(\ell+1)-1/2$.

Os autovetores da partícula de Wigner no autoespaço de $(\underline{\sigma}.\underline{L}+1) \to (\ell+1)$, formam um conjunto completo associado aos quanta pares ou ímpares, satisfazendo a seguinte equação de autovalor:

$$\mathbb{H}(\underline{\sigma}\cdot\underline{L}+1)|\psi^{(n)}_{\ell\frac{1}{2},jm_j}\rangle = E^{(n)}(\ell+1)|\psi^{(n)}_{\ell\frac{1}{2},jm_j}\rangle \cdot \quad E^{(n)}(\ell+1) = \ell+\frac{3}{2}+n \quad . \tag{9}$$

De (5)-(7), nesse autoespaço, obtemos as seguintes realizações para o operador de aniquilação dos quanta da partícula de Wigner:

$$a^{-}(\ell+1)|\psi^{(2m+1)}_{\ell\frac{1}{2},jm_j}\rangle = \{2(m+j+1)\}^{1/2}|\psi^{(2m)}_{\ell\frac{1}{2},jm_j}\rangle \quad . \tag{10}$$

$$a^{-}(\ell+1)|\psi^{(2m)}_{\ell\frac{1}{2},jm_j}\rangle = \sqrt{2m}\,|\psi^{(2m-1)}_{\ell\frac{1}{2},jm_j}\rangle \, . \tag{11}$$

A projeção do comutador $[H(\sigma\cdot L+1), \{a^{+}(\sigma\cdot L+1)\}^{2}]$, no autoespaço associado aos quanta pares, nos dá os operadores escada esféricos do oscilador isotrópico 3D de spin 1/2, $B^{\pm}(\sigma\cdot L+1)$.

$$B^{+}(\sigma\cdot L+1) = A^{+}(\sigma\cdot L+1)A^{+}[-(\sigma\cdot L+1)] = \{B^{-}(\sigma\cdot L+1)\}^{+}, \qquad (12)$$

$$= \frac{1}{2}\left\{\left(\frac{\partial}{\partial r}+\frac{1}{r}\right)^{2} - 2r\frac{\partial}{\partial r} + r^{2} - \frac{(\sigma\cdot L)(\sigma\cdot L+1)}{r^{2}} - 3\right\} , \qquad (13)$$

onde

$$A^{\pm}[\mp(\sigma\cdot L+1)] = \frac{1}{\sqrt{2}}\left\{\pm\left(\frac{\partial}{\partial r}+\frac{1}{r}\right) \mp \frac{1}{r}(\sigma\cdot L+1) - r\right\} . \qquad (14)$$

A partir de (10)-(14), vemos que os operadores quadráticos, mutuamente adjuntos satisfazem as seguintes propriedades:

$$B^{-}(\sigma\cdot L+1)|m\ell\tfrac{1}{2}, jm_j\rangle = 2\{m(m+j)\}^{1/2}|(m-1)\ell\tfrac{1}{2}, jm_j\rangle . \qquad (15)$$

$$B^{+}(\sigma\cdot L+1)|m\ell\tfrac{1}{2}, jm_j\rangle = 2\{(m+1)(m+j+1)\}^{1/2}|(m+1)\ell\tfrac{1}{2}, jm_j\rangle . \qquad (16)$$

Propriedades semelhantes se verificam também no autoespaço pertencentes aos quanta impares e, por sua vez, os operadores de criação e de aniquilação , deduzidos por nós, independem do número de quanta.

II. ESTADOS COERENTES CANÔNICOS ESFÉRICOS

Os autoestados esféricos do operador de aniquilação, B^{-} , estão associados ao autovalor complexo, γ . Eles são exatamente os ECC do oscilador hamônico isotrópico 3D de spin 1/2. Em plena analogia com a ref.[1], obtemos os ECC normalizados como uma expansão na base ortonormal, $\{|m\ell\tfrac{1}{2}, jm_j\rangle\}$, ou seja:

$$|\gamma, j\rangle = \left\{\left(\frac{2}{|\gamma|}\right)^{j} I_j(|\gamma|)\,\Gamma(j+1)\right\}^{-1/2} \sum_{m=0}^{\infty} \frac{\left(\frac{\gamma}{2}\right)^{m}}{\{m!\,\Gamma(m+j+1)\}^{1/2}} |m\ell\tfrac{1}{2}, jm_j\rangle . \qquad (17)$$

onde $I_j(|\gamma|)$ são as funções de Bessel modificadas, a saber,

$$I_j(|\tau|) = \sum_{m=0}^{\infty} \frac{(|\tau|/2)^{2m+j}}{m!\,\Gamma(m+j+1)} \tag{18}$$

A propriedade de não-ortogonalidade é evidenciada abaixo pelo produto escalar entre dois ECC associados a autovalores distintos,

$$\langle \tau, j | \chi, j \rangle = \left\{ \left(\frac{2}{|\tau|} \frac{2}{|\chi|} \right)^j I_j(|\tau|) I_j(|\chi|) \right\}^{-1/2} \frac{1}{\Gamma(j+1)} \sum_{m=0}^{\infty} \frac{\left(\frac{\tau^* \chi}{2} \right)^m}{m!\,\Gamma(m+j+1)} . \tag{19}$$

Estes estados coerentes canônicos esféricos satisfazem a uma propriedade de completeza e, portanto, são super-completos. Esta propriedade será demonstrada por nós num trabalho que está sendo preparado para ser submetido a publicação internacional.

III. CONCLUSÕES

A partir da super-realização JR da álgebra WH 3D, obtemos o operador de aniquilação de um oscilador harmônico isotrópico 3D de spin 1/2, cujos autoestados são exatamente os estados coerentes canônicos esféricos deste oscilador. Eles possuem as propriedades de não-ortogonalidade e completeza. Os operadores escada obtidos aqui não dependem do número de quanta. Esta construção nos permite várias aplicações em física quântica. Além das possíveis extensões daquelas aplicações usadas no tratamento unidimensional [5], podemos analisar a fase de Berry [6] sobre uma base constituída destes estados coerentes 3D.

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CÁLCULO ALGÉBRICO DE PROPAGADORES EM ESPAÇOS CURVOS

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Resumo. Utilizando a representação de Schwinger, fórmulas BCH, e a álgebra de Lie do grupo SO(2,1) obtivemos as funções de Green do campo escalar em alguns modelos cosmológicos.

O estudo do comportamento de campos quânticos na presença de campos gravitacionais externos é de vital importância no entendimento de fenômenos como a evaporação de buracos negros, o universo primordial, etc (Birrel & Davies 1982 [1]). Neste estudo é necessário obter as funções de Green da teoria, através das quais podemos obter as diversas quantidades de interesse como as ações efetivas e taxas de produção de pares.

O que vamos fazer neste trabalho é obter as funções de Green do campo escalar com acoplamento conforme em alguns modelos cosmológicos, que satisfazem [1]:

$$(\Delta_{LB} + m^2 + \tfrac{1}{6}R)G(x,x') = - \frac{1}{\sqrt{-g}}\, \delta^4(x-x') \qquad (1)$$

Onde $g = \det g^{\mu\nu}$, R é a curvatura escalar e Δ_{LB} é o operador de Laplace-Beltrami,

$$\Delta_{LB} = (-g)^{-1/2}\partial_\mu [g^{\mu\nu}(-g)^{1/2}\partial_\nu] \qquad (2)$$

Na representação de Schwinger temos [1]:

$$G(x,x')=\lim_{\varepsilon\to 0_+} i\int_0^\infty ds\ \exp[-i(\Delta_{LB}+\frac{1}{6}R+m^2-i\varepsilon)s](-g)^{-1/2}\ \delta^4(x-x') \quad (3)$$

O que vamos fazer é obter a atuação da exponencial acima sobre a função delta de Dirac , para tal vamos considerar casos em que o argumento da exponencial pode ser escrito como uma combinação *linear* de geradores da álgebra de Lie SO(2,1):

$$[T_1,T_2] = -i\ T_1 \qquad [T_2,T_3] = -i\ T_3 \qquad [T_1,T_3] = -i\ T_2 \quad (4)$$

Utilizando as relações de comutação acima e *fórmulas* BCH (Baker-Campbell-Hausdorff) poderemos encontrar $G(x,x')$,como foi feito para o problema de Kepler relativístico por Mil'shtein e Strakhovenko 1982 [2] e para diversos potenciais da mecânica quântica por Boschi e Vaidya 1990 [3].

MODELOS ANISOTRÓPICOS DE BIANCHI DO TIPO I

Estes modelos são descritos pela métrica [1]:

$$g^{\mu\nu}= \mathrm{diag}(1,-\ t^2,-\ t^{2p_1},-\ t^{2p_2}) \quad , \quad 0 \le t < \infty \quad (5)$$

Onde p_1 e p_2 são parâmetros constantes que assumem os valores 0 e 1. Para $p_1=p_2=1$ temos um universo isotrópico, espacialmente chato, de Robertson-Walker com expansão linear, enquanto que para outros valores dos p_i's temos universos anisotrópicos.

Como ainda temos simetria por translação podemos escrever:

$$G(x,x') = \frac{1}{(2\pi)^3} \int d^3k \, e^{i\vec{k}\cdot(\vec{x} - \vec{x}')} \, G_k(t,t') \qquad (6)$$

A equ. (1) assume a forma

$$\left[\partial_t^{\,2} + (1+p_1+p_2)\,\frac{1}{t}\,\partial_t + \frac{1}{t^2}k_x^{\,2} + \frac{1}{t^{2p_1}}k_y^{\,2} + \frac{1}{t^{2p_2}}k_z^{\,2} + m^2 + \frac{\gamma}{t^2}\right] G_k(t,t')$$

$$= t^{-(1+p_1+p_2)}\delta(t-t') \qquad (7)$$

onde γ é uma constante determinada por p_1 e p_2 :

$$\gamma = \begin{cases} 1 \;, & p_1=p_2=1; \\ 1/3 \;, & p_1=1 \;,\; p_2=0; \\ 0 \;, & p_1=p_2=0 \;. \end{cases} \qquad (8)$$

podemos identificar na equação (7) o gerador T_1

$$\partial_t^{\,2} + (1+p_1+p_2)\,\frac{1}{t}\,\partial_t + \frac{1}{t^2}(k_x^{\,2} + p_1 k_y^2 + p_2 k_z^2 + \gamma)$$

quando então (9)

$$T_2 = -\frac{1}{2}\, t\partial_t - i\frac{1}{4}(2+p_1+p_2) \qquad , \qquad T_3 = -\frac{1}{8}\, t^2$$

Utilizando o método algébrico ([2],[3]) pode-se mostrar que:

$$G_k(t,t') = -\frac{1}{2}(tt')^{-(p_1+p_2)/2}\int_0^\infty ds\, \frac{e^{\frac{-i}{4s}(t^2+t'^2)}}{s}\, I_\nu\left(\frac{itt'}{2s}\right)\times$$

$$\exp-is[m^2+(1-p_1)k_y^2+(1-p_2)k_z^2-i\epsilon]$$

onde $\nu = \frac{1}{2}[(p_1+p_2)^2 - 4(k_x^2 + p_1k_y^2+p_2k_z^2+\gamma)]^{1/2}$ (10)

Na expressão acima $I_\nu(z)$ é a função de Bessel modificada (Gradshtein e Ryzhik 1965 [4]).

Integrando em s na equ. (10) temos [4]:

$$G_k(t,t') = -\frac{\pi}{2}(tt')^{-(p_1+p_2)/2} H_\nu^{(2)}(\mu t)\, J_\nu(\mu t'),\quad t > t' \quad (11)$$

onde $H_\nu^{(2)}(z)$ é a função de Hankel de segunda espécie e $J_\nu(z)$ é a função de Bessel cilindrica [4].

Os resultados acima conferem com os encontrados por Charach 1982 [5] e Duru e Ünal 1986 [6] ,que utilizaram integrais de caminho.

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GEOMETRIA DOS AUTOESTADOS DE SPIN

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ABSTRACT: usando o fato bem conhecido da geometria spinorial de que à um spinor podemos associar um vetor (por sua vez interpretado como o eixo da rotação associada ao spinor) através do produto direto do spinor pelo seu conjugado hermitiano, mostramos que os operadores de projeção de spin da teoria quântica tem por autoestados os spinores associados, através do produto direto, à direção espacial definida pelo operador.

INTRODUCÃO - Na mecânica quântica os sistemas físicos são descritos por vetores de estado, enquanto que os observaveis físicos são relacionadas a operadores lineares que atuam sobre os estados transformando um estado em outro. [Landau e Lifchitz, pg 19] O principal problema em Mecânica Quântica é obter os autoestados e autovalores dos operadores representando os observaveis físicos relevantes ao problema em questão.

Na teoria Quântica (não relativista) de partículas com spin 1/2, os estados das partículas no que se refere a variável de spin são descritos por espinores [Rodrigues e Zeni; Landau e Lifchitz, pg 232; Santaló, pg 29-33], que são elementos de um espaço vetorial complexo bidimensional, sendo representados por matrizes colunas 2X1. Por outro lado, os operadores de projeção de spin 1/2 (ou simplesmente operadores de spin) são representados por matrizes hermitianas 2X2 complexas, que podem ser escritas em termos das matrizes de Pauli [Landau e Lifchitz, pg 232; Sakurai, pg 163-65].

Assim, um operador de spin $\vec{S}$ é definido por

$$\vec{S} = \hbar/2\, \vec{N} \qquad (1)$$

onde $\hbar$ é a constante de Planck e N é o operador de spin 1/2 adimensional definido como sendo um vetor (real) expandido nas matrizes de Pauli:

$$\vec{N} = \vec{n}.\vec{\sigma} = n_1\sigma_1 + n_2\sigma_2 + n_3\sigma_3 = \begin{bmatrix} n_3 & n_1 - in_2 \\ n_1 + in_2 & -n_3 \end{bmatrix} \qquad (2)$$

A expressão acima é conveniente pois associamos a cada direção do espaço,

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definida pelo vetor $\vec{n}$, um operador de projeção de spin ao longo desta direção.

AUTOESTADOS E PRODUTO DIRETO - os autoestados do operador de spin $\vec{N}$, serão indicados pelo spinor ψ_n. Considerando apenas vetores unitários ($|\vec{n}|^2 = n_1^2 + n_2^2 + n_3^2 = 1$), os autovalores do operador de spin $\vec{N}$ estão restritos aos valores ± 1.

Passamos a expor nosso método para obter o autoestado de spin up. Recordamos inicialmente que o produto direto de uma matriz coluna 2X1, digo o spinor ψ, por uma matriz linha 1X2, como o spinor conjugado hermitiano $\phi^\dagger$, resulta numa matriz quadrada 2X2 cujas linhas (colunas) são caracterizadas por ψ ($\phi^\dagger$), de acordo com a seguinte definição: [Wigner, pg 17; Santalo, pg.99]

$$\psi \times \phi^\dagger = \begin{bmatrix} \psi_1\phi_1^* & \psi_1\phi_2^* \\ \psi_2\phi_1^* & \psi_2\phi_2^* \end{bmatrix} \tag{3}$$

onde ϕ_i^* é o conjugado complexo de ϕ_i. Como qualquer matriz 2X2 complexa, a matriz resultante do produto direto $\psi \times \phi^\dagger$ pode ser escrita como combinação linear das matrizes $(\mathbb{1}, \vec{\sigma})$, onde $\mathbb{1}$ é a matriz identidade 2X2.

Agora, considerando o produto direto de um spinor pelo seu conjugado hermitiano, i.e., $\psi \times \psi^\dagger$, notamos dois fatos importantes que podem ser verificados diretamente da eq.(3) [Rodrigues e Zeni]:

(i) as componentes do produto direto $\psi \times \psi^\dagger$ na base $\{\mathbb{1}, \vec{\sigma}\}$ são reais (a matriz é hermitiana). A componente da identidade é igual a 1/2 $\psi^\dagger\psi$;

(ii) o spinor ψ é um autoestado da matriz resultante do produto direto $\psi \times \psi^\dagger$, com autovalor $\psi^\dagger\psi$. Simbolicamente este fato é expresso como:

$$(\psi \times \psi^\dagger)\, \psi = (\psi^\dagger\psi)\, \psi = (|\psi_1|^2 + |\psi_2|^2)\, \psi \tag{4}$$

O fato (i) nos diz que sempre podemos expressar um operador de spin $\vec{N}$ através do produto direto de um particular spinor e seu conjugado hermitiano como mostrado abaixo:

$$\vec{N} = \vec{n}.\vec{\sigma} = 2\, \psi_n \times \psi_n^\dagger - \psi_n^\dagger \psi_n\, \mathbb{1} \tag{5}$$

O fator 2 foi introduzido na eq.(5) pois deste modo o vetor $\vec{n}$ é unitário se e somente se o spinor ψ_n também o for. As fórmulas relacionando as componentes do operador de spin $\hat{N}$ no spinor ψ_n podem ser deduzidas da eq.(2) para $\hat{N}$ e da formula explicita para o último membro da eq.(5).

Ressaltamos que quando expressamos um operador de spin através do produto direto de um spinor pelo seu conjugado hermitiano, o spinor assim obtido é definido a menos de uma fase global, isto é, se trocarmos ψ_n por $e^{i\theta}\psi_n$, onde θ é um número real, então o mesmo operador $\hat{N}$ é obtido através da eq.(5) [Rodrigues e Zeni]. Esta liberdade na escolha da fase global é também inerente na descrição da mecânica quântica [Landau e Lifshitz, pg 8; Barut, pg 14], de modo que a eq.(5) atribue um único estado físico para o sistema.

Por outro lado, o fato (11), eq.(4), nos garante que o spinor ψ'_n é um autoestado do operador $\hat{N}$, eq.(5), correspondente ao autovalor +1.

Discutimos agora uma interpretação geométrica para a eq.(5), que se mostrará significativa na análise física da variável de spin. Inicialmente, observamos que podemos relacionar um spinor a uma rotação através do seguinte raciocínio [Rodrigues e Zeni; Santalo, pg 35; Penrose e Rindler, pg.10-14]: o eixo de rotação está na direção do vetor definido pelo spinor através da eq.(5); o angulo de rotação é dado pela fase do spinor em relação a um particular spinor, que representa a rotação por 2π radianos ao redor do eixo em questão.

A relação acima entre um spinor e uma rotação leva a seguinte interpretação para a Mecânica Quântica de partículas com spin 1/2: o autoestado de spin up de um dado operador (de projeção) de spin é dado pelo spinor associado a uma rotação ao redor do eixo definido pelo operador de spin em questão.

Para completar a discussão dos autoestados de spin, ressaltamos que o autoestado de spin correspondente ao autovalor −1, denominado spin 'down', pode ser obtido do autoestado de spin up através da inversão temporal [Sakurai, pg 277-8].

CONCLUSÃO: Os métodos usuais de se obter os autoestados de spin (seja resolvendo o problema algébrico de autovalores [Kessler, pg.9-10], seja usando o recurso de que o operador de spin $\hat{N}$ ao longo de uma direção qualquer pode ser obtido do operador σ_3 por uma rotação e portanto os autoestados de $\hat{N}$ estão relacionados aos autoestados do σ_3 pela mesma rotação [Sakurai, pg 167-8]) não fornecem uma interpretação geométrica nem acrescentam outra relação além da definição entre o operador de spin e os spinores que são seus autoestados.

Por outro lado, neste artigo nos servimos de uma relação bem conhecida da teoria de grupos e geometria spinorial [Rodrigues e Zeni; Santaló, pg.35; Penrose e Rindler; pg.32-7] entre spinores e rotações, que resulta no seguinte: o autoestado de spin up para um dado operador de spin é o spinor associado auma rotação ao redor do eixo espacial definido pelo operador de spin.

Além disso, nossa metodologia nos permite concluir que se um sistema físico é descrito por um dado spinor, então o spin do sistema (ou a projeção do spin) está ao longo da direção do vetor definido pela eq.(5).

Por fim, ressaltamos que os resultados aqui discutidos foram originalmente elaborados usando a algebra de Clifford gerada pelos vetores euclidianos [Zeni e Rodrigues, 1990 e 1991] e a teoria de spinores algebricos [Figueiredo, Oliveira e Rodrigues; Rodrigues e Zeni].

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QUANTUM CORRECTIONS TO CLASSICAL SOLUTIONS

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Abstract – In a real scalar field model in $1+1$ dimensions with quartic and sixtic selfcoupling there appears a classically unstable nontopological soliton which, in a certain parameter range of the model, is stabilized by quantum corrections.

1. INTRODUCTION

The real scalar field theory in $1+1$ dimensions with quartic and sixtic self interactions corresponding to a deepest central well and two lateral ones has a static classical solution which takes values in one of these for all the space except for a finite region where it approaches the absolute minimum[1]. In a lattice quantum version of the model it has been shown[2] that the condensation of these bubble type states, together with the kinks, determines the phase diagram, which exhibits a tricritical point that may be related to the $He^3 - He^4$ mixture[3].

The bubble type solution is unstable. Equivalent static classical unstable bubbles appear in the non linear Schroedinger equation[4], but, due to the non relativistic nature of the theory, they may achieve stability when they move exceeding a critical velocity with respect to the medium.

The purpose of this work is to indicate that in the relativistic theory bubbles may be stabilized by quantum corrections. The problem has two aspects. One is that the bubble is classically unstable against small perturbations. The other is that the classical bubble lives mostly in a false vacuum which may tunnel into the true one. Regarding the former problem, it will be seen in section 2 that if higher order terms around the classical contributions are considered, by means of a quadratic approximation, all the energies of the excitations turn out to be real, and therefore no decay is possible. Regarding the later problem, in will be seen in section 3 that quantum corrections at one and two loops give rise to dynamical symmetry breaking, turning the false vacuum into a stable one.

It must be stressed that these indications for the quantum stability of the bubble are different from those corresponding to other non topological solitons which are always related to a Noether charge.

2. BUBBLE STABILIZATION

Given a Lagrangian in 1 + 1 dimensions for a real field $\tilde{\phi}$

$$\tilde{\mathcal{L}}(\tilde{x},\tilde{t}) = \frac{1}{2}(\tilde{\partial}_m\tilde{\phi})^2 - \tilde{V}(\tilde{\phi}) \tag{1}$$

where

$$\tilde{V}(\tilde{\phi}) = \frac{K^2}{2}(\tilde{\phi}^2 - \rho)^2(\tilde{\phi}^2 - A\rho) \tag{2}$$

the change of variables $\phi = \frac{1}{\sqrt{\rho}}\,\tilde{\phi}$, $x_\mu = \rho\tilde{x}_\mu$ allows us to write

$$\mathcal{L}(x,t) = \frac{1}{\lambda}\left[\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)\right] \tag{3}$$

where

$$V(\phi) = \frac{K^2}{2}(\phi^2 - 1)^2(\phi^2 - A) \qquad \text{and} \qquad \lambda = \frac{1}{\rho^3} \tag{4}$$

If $A < 0$ there is spontaneous symmetry breaking and topological solitons of kink type appear. If $0 < A < 1$ the central minimum is the absolute one, the true vacuum corresponds to $\phi = 0$ and there is a static solution of the bubble type

$$\phi_c^2 = A\left\{1 - (1 - A)\mathrm{tgh}^2[K\sqrt{1 - A}(x - x_0)]\right\}^{-1} \tag{5}$$

which satisfies $\frac{1}{2}(\phi')^2 = V(\phi)$.

The bubble Eq.(5) is classically unstable since a small perturbation $\psi(x)\,e^{i\omega t}$ satisfies

$$\left[-\frac{d^2}{dx^2} + V''[\phi_c(x)]\right]\psi(x) = \omega^2\,\psi(x) \tag{6}$$

and being the zero mode $\phi_c'(x)$ a one node function, the ground state of Eq.(6) corresponds to imaginary ω.

Let us see how Eq.(6) is modified when corrections higher than the quadratic ones are included

$$\phi(x,t) = \phi_c(x) + \hat{\phi}(x,t) \tag{7}$$

whith

$$\hat{\phi}(x,t) = \sum_n \frac{1}{\sqrt{2\omega_n}}\left[a_n\psi_n(x)\,e^{-i\omega_n t} + a_n^\dagger\psi_n^*(x)\,e^{i\omega t}\right] \tag{8}$$

where $[a_n, a_n^\dagger] = \delta_{nn}$ and $\{\psi_n(x)\}$ is a complete set of functions.

Keeping only the second order terms in $\hat{\phi}(x,t)$, the Hamiltonian turns out to be

$$H^{(2)} = E_c + \sum_n \omega_n\left(a_n^\dagger a_n + \frac{1}{2}\right) \tag{9}$$

if $\psi_n(x)$ satisfies Eq.(6). The existence of an imaginary frequency formally produces the instability, though the strict treatment of Eq.(8) requires ω_n to be real. Inspired by ref. 5 we keep the cubic and quartic contribution and approximate it as a quadratic expansion around its minimum, which will be valid if $\hat{\phi}$ is small. Now we have

$$H^{(4)} = E_c + \int dx \left[\frac{45}{8}\,\frac{(V''')^4}{(V^{IV})^3} + \frac{1}{2}\,\dot{\hat{\phi}}^2 + \frac{1}{2}\,\hat{\phi}'^2 + f(x)\,\hat{\phi} + \frac{1}{2}\,g(x)\,\hat{\phi}^2\right] \tag{10}$$

with $f(x) = \frac{9}{2}\,\frac{(V''')^3}{(V^{IV})^2}$, $g(x) = V''(\phi_c) + \frac{3}{2}\,\frac{(V''')^2}{V^{IV}}$

Separating from $\hat{\phi}$ a time independent part

$$\hat{\phi}(x,t) = \chi(x,t) + \eta(x) \tag{11}$$

such that $-\eta'' + g\eta + f = 0$, we have for the operator term of Eq.(10)

$$E(\hat{\phi}) = \int dx \left[\frac{1}{2}\dot{\chi}^2 + \frac{1}{2}\,\chi'^2 + \frac{1}{2}\,g\chi^2 + \frac{1}{2}\,\eta'^2 + \frac{1}{2}\,g\eta^2 + f\eta\right] \tag{12}$$

The η dependent contribution to Eq. (12) adds a real constant to the energy whereas the expansion of the operator χ into a complete set produces a Schroedinger equation analogous to Eq.(6) but with $V''(\phi_c)$ replaced by $g(x)$. To see whether this equation has a negative eigenvalue we use the semiquantum method[6], which provides a lower bound to the ground state.

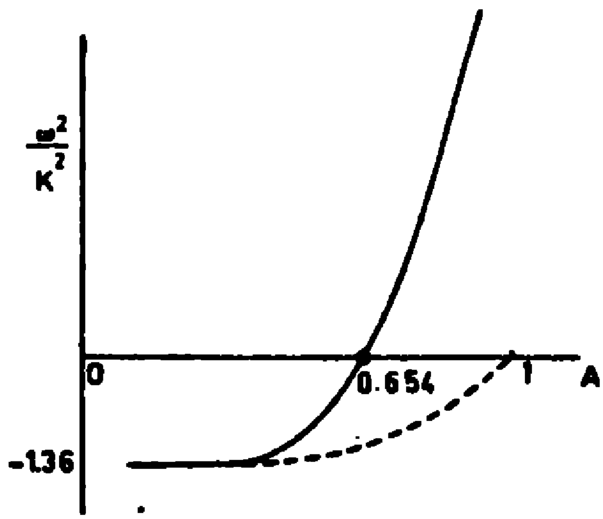

Figure 1.

As shown in Fig. 1, for $A > 0.654$ the eigenvalue lower bound is positive indicating a stabilization of the bubble.

3. DYNAMICAL SYMMETRY BREAKING

The effective potential correspondings to Eq.(4) has been calculated, up to one loop, in ref. 7

$$V_{eff} = \frac{1}{\lambda} V(\phi) + \frac{V''(\phi)}{8\pi}\Big\{1 - \ln\Big[\frac{V''(\phi)}{\mu^2}\Big]\Big\} + \frac{V''(\phi)}{8\pi} \ln\Big(\frac{\Lambda^2}{\mu^2}\Big) + a_1 + b_1\phi^2 + c_1\phi^4 \quad (13)$$

where the contribution divergent for $\Lambda \to \infty$ has been separated and the infinite part of a_1, b_1 and c_1 must be chosen so as to cancel it. The finite part of a_1 merely adds a constant. Two of the other three parameters are independent and, replaced by $\ln \mu^2$, and $\ln \mu_2^2$ allow us to write

$$V_{eff} = \frac{1}{\lambda} V(\phi) + \frac{V''(\phi)}{8\pi}\Big\{1 - \ln\Big[\frac{V''(\phi)}{\mu_1^2}\Big]\Big\} + \frac{15}{8\pi}\phi^4 \ln\Big[\frac{\mu_2^2}{\mu_1^2}\Big] \quad (14)$$

This approximation is not defineu when $V''(\phi)$ is negative but it is valid close to its minima. Therefore, we may obtain the shift ε for the position of the lateral minima from $V''_{eff}(\phi = 1 + \varepsilon) = 0$. Moreover we may define a critical parameter λ_c equating the values of the two minima at $\phi = 0$ and $\phi = 1 + \varepsilon$

$$\frac{A}{2\lambda_c} = \frac{1+2A}{8\pi}\Big\{1 - \ln\Big[\frac{1+2A}{U_1^2}\Big]\Big\} - \frac{4(1-A)}{8\pi}\Big\{1 - \ln\Big[\frac{4(1-A)}{U_1^2}\Big]\Big\} + \frac{15}{8\pi}\ln\Big(\frac{U_1^2}{U_2^2}\Big) \quad (15)$$

in terms of dimensionless parameters $U_i^2 = \mu_i^2/K^2$. These may be chosen for fixed A so that $\varepsilon = 0$ and λ_c is sufficiently small to ensure the validity of the loop approximation[8]. Once this is done, λ and K, the last two parameters of V_{eff}, are determined e.g. by the renormalized mass and quartic coupling at the symmetry breaking vacuum

$$m_R^2 = V''_{eff}|_{\phi=1} \quad , \quad \alpha_R = V_{eff}^{IV}|_{\phi=1} \quad (16)$$

For the two loop correction we have[7]

$$V_2(\phi) = \frac{1}{8\pi}\Big(b_1 + \frac{c_1}{2}\phi^2\Big)(\ln\Lambda^2 - \ln V''(\phi)\Big) - \frac{K(V'''(\phi))^2}{192\pi^2 V''(\phi)} + \frac{V^{IV}}{128\pi^2}(\ln\Lambda^2 - \ln V''(\phi))^2 + a_2 + \frac{b_2}{2}\phi^2 + \frac{c_2}{4!}\phi^4 \quad (17)$$

We will not establish the same renormalization conditions at all orders, except the one that $\varepsilon = 0$ (the lateral minimum occurs always at $\phi = 1$). We may establish e.g. that the difference between the two minima reduces at each order to a half of that of the previous one as a tendency towards symmetry breaking. Once the counterterms are determined in agreement with these conditions, the renormalized mass and quartic coupling will be obtained from Eq.(16). For an indication of the convergence of the loop expansion it will be important that each correction is smaller than the one for the previous order.

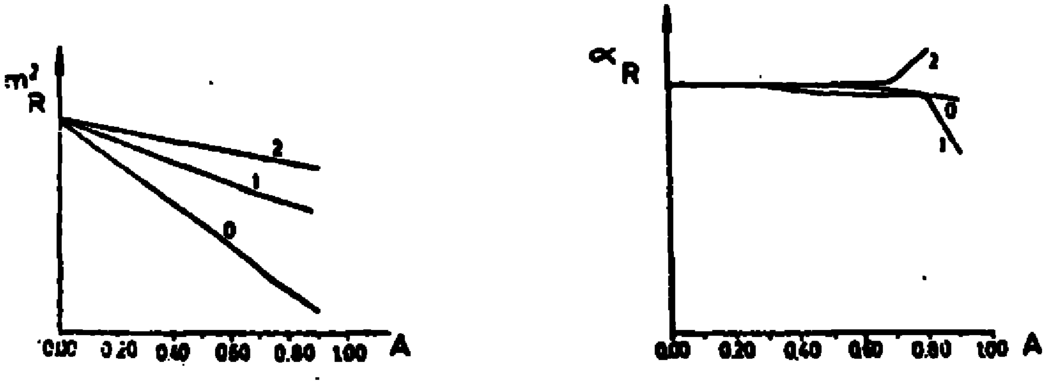

Figure 2.

In fig.(2) the corresponding values for mass and quartic coupling are shown indicating that for small enough values of λ, and A not too close to 1, the expansion seems to converge.

To compare, in the framework of the loop expansion, the previous renormalization with the renormalization done at the origin, we refer the reader to Ref. 8.

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XII ENCONTRO NACIONAL DE FÍSICA DE PARTÍCULAS E CAMPOS

P R O G R A M A

QUARTA FEIRA, 18/09/91

14:00 - Saída dos ônibus para Caxambu
São Paulo - Instituto de Física - USP
Rio de Janeiro - Centro Brasileiro de Pesquisas Físicas

QUINTA FEIRA, 19/09/91

09:00 - "TESTES DO MODELO PADRÃO NO LEP"
Prof. R. Shellard (PUC/RJ)
10:15 - Café
10:30 - Sessões de comunicações
Física de Hadrons
Física das Interações Eletrofracas
Física Experimental de Altas Energias e Raios Cósmicos
Teoria de Campos
Gravitação e Cosmologia
12:30 - Almoço
15:15 - "EQUAÇÕES DE YANG-BAXTER, GRUPOS QUÂNTICOS, INVARIÂNCIA CONFORME ETC "
Prof. R. Köberle (IFQSC-USP)
16:15 - Café
16:30 - Abertura da Sessão de Painéis
17:30 - Grupos de Trabalho
Física de Hadrons
Física das Interações Eletrofracas
Física Experimental de Altas Energias e Raios Cósmicos
Teoria de Campos
Gravitação e Cosmologia
Computação Algébrica
19:00 - Jantar

SEXTA FEIRA, 20/09/91

09:00 - "QUANTIZAÇÃO CANÔNICA DA GRAVITAÇÃO"
Prof. N. Pinto Neto (CBPF)
10:15 - Café
10:30 - Sessões de comunicações
Física de Hadrons

Física das Interações Eletrofracas
Física Experimental de Altas Energias e Raios Cósmicos
Teoria de Campos
Gravitação e Cosmologia

12:30 - Almoço
15:30 - "FIXING THE GAUGE AT FUTURE NULL INFINITY"
Prof. O.M. Moreschi (Univ. de Córdoba)
16:15 - Café
16:30 - "POTENCIAL EFETIVO NÃO RELATIVÍSTICO NA TEORIA DE MAXWELL-CHERN-SIMMONS"
Prof. H. Girotti (UFRGS)
17:30 - "MEAN FIELD APPROACH TO QUANTUM GRAVITY"
Prof. Roberto Percacci (SISSA)
19:00 - Jantar
21:00 - Assembléia

SÁBADO, 21/09/91

09:00 - "TEORIA DE CAMPOS, EFEITO HALL QUÂNTICO, SUPERCONDUTIVIDADE E ANYONS"
Prof. E. Marino (PUC/RJ)
10:15 - Café
10:30 - Sessões de comunicações
Física de Hadrons
Física das Interações Eletrofracas
Física Experimental de Altas Energias e Raios Cósmicos
Teoria de Campos
Gravitação e Cosmologia
12:30 - Almoço
15:00 - "DETECTABILIDADE DA MATÉRIA ESCURA"
Prof. C.O. Escobar (IFUSP)
16:15 - "RECENT DEVELOPMENTS IN CONFORMAL FIELD THEORIES AND INTEGRABLE MODELS"
Prof. F. Toppan (Univ. Pierre et Marie Curie, Paris)
17:30 - "ALGEBRAIC PROPERTIES OF LANDAU GAUGE"
Prof. S.P. Sorella (LAPP- Annecy e UCP-Petrobrás)
19:00 - Jantar

DOMINGO, 22/09/91

09:00 - Saída dos ônibus para São Paulo e Rio de Janeiro