Ideal Types of Magnetic Materials

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Magnetic materials can be classified according to the degree of similarity with three ideal types of materials: 1) ideal hard magnetic materials, 2) ideal soft magnetic materials, and 3) ideal diamagnetic materials (superconductors). The analysis of the behavior of these classes of materials under an applied magnetic field is illustrative for students of magnetism. This behavior is discussed, and the ranges of values of the initial permeability and susceptibility that characterize these materials are given.

On peut classifier les matériaux magnétiques selon le degré de similitude avec trois types ideales de matériaux: 1) matériaux durs ideales, 2) matériaux doux ideales, et 3) matériaux diamagnétiques ideales (supraconducteurs). L'analyse des comportements de ces classes de matériaux sous un champ magnétique apliqué peut être instructif pour un étudiant de magnétisme. On discute ici ces comportements, et on donne les intervales de variation de la permeabilité initiale et de la susceptibilité que characterisent ces matériaux.

I Introduction

The student of Magnetism is introduced to the usual classification of materials according to their magnetic response as diamagnets, paramagnets and ferromagnets. This response is determined by a simple experiment, where the force exerted on a sample in a magnetic field gradient is measured. Many other classes of magnetic materials are known; for a more complete magnetic taxonomy of materials see Hurd [1].

A more detailed analysis of the properties of magnetic materials usually involves the study of magnetization curves. As the student analyzes the **M** vs. **H** (or **B** vs. **H**) curves, he (or she) deals with much more complex phenomena, related to the multidomain structure of ordinary samples, nucleation, domain wall motion, domain rotation, hysteresis, pinning, the balance between the different sources of anisotropy, and so on. Intrinsic and extrinsic properties of the materials are reflected in these curves.

The shape of the hysteresis curves leads to the more practical grouping of magnetically ordered materials, as magnetically hard and magnetically soft.

Herrmann [2] suggested that the student should be introduced to the magnetic properties of materials, through the $\mathbf{M} \times \mathbf{H}$ curves of some classes of idealized materials, such as: a) ideal non-magnetic materials, b) ideal hard magnetic materials, c) ideal soft magnetic materials, and d) ideal diamagnetic materials (superconductors). The approach of first analyzing magnetic properties of ideal systems parallels the route taken in the study of many other physical problems, for example, in studying the ideal gases before discussing the properties of real gases, and so on.

We have found that the analysis of the magnetic behavior of these ideal materials can help the student in understanding the basic concepts of magnetism.

In the present work we discuss the magnetic properties of these four ideal types of materials. We investigate the values of **B**, **M** and **H** as a function of H_{ext} in these materials, as well as the range of values of their initial magnetic permeability μ_i, and volume susceptibility χ.

II The Quantities B, H and M

In the SI system of units the quantities B (magnetic induction or magnetic flux density), H (magnetic field intensity) and the magnetization M are connected through the relation (e.g. [3]):

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \qquad (1)$$

where μ_0 is a constant, the vacuum magnetic permeability ($=4\pi \times 10^{-7}$ H m^{-1}). Eq. (1) can be seen as the relationship that defines **H**. One way of reading the above equation is that it expresses the contributions to the induction **B**: the field **H** contributes $\mu_0\mathbf{H}$, and inside matter, the magnetization **M** contributes $\mu_0\mathbf{M}$. Assuming **B** and **H** collinear, one may define the permeability of a medium as $\mu = B/H$.

The magnetic response of a medium is measured by its magnetic permeability and magnetic susceptibility; the initial magnetic permeability μ_i is defined through the relation

$$\mu_i = \left.\frac{\partial B}{\partial H}\right|_{H=0} \tag{2}$$

The differential susceptibility is defined as

$$\chi(H) = \frac{\partial M}{\partial H} \tag{3}$$

The internal field **H** in the sample is given by the applied external field H_{ext} plus the demagnetizing field $B_d = -N_d M$:

$$H = H_{ext} - N_d M \tag{4}$$

where N_d is the demagnetizing factor. This factor depends on the shape of the sample, and on the direction of application of the external magnetic field. The demagnetizing field in general varies from point to point, being constant inside samples of ellipsoidal shape.

III The Ideal Types of Magnetic Materials

The magnetic properties of real materials may be approximated, following Herrmann [2], by the behavior of one of the following idealized materials: 1) ideal non-magnetic materials, 2) ideal hard magnetic materials, 3) ideal soft magnetic materials, and 4) ideal diamagnetic materials.

III.1 Non-magnetic Materials

The ideal non-magnetic material is an idealization for the non-ferromagnetic materials; i.e., the paramagnetic, diamagnetic, or weakly magnetic materials. This represents the practical approximation usually made in the magnetic analysis of phase mixtures, when the magnetic contributions of paramagnetic and diamagnetic phases are neglected in the presence of ferromagnetic phases. For practical purposes, the non-ferromagnetic phases may be identified to the ideal non-magnetic material. The only medium that behaves strictly in the ideal non-magnetic way is the vacuum.

Since $M = 0$, from Eqs. (1) and (2) one obtains for the initial permeability

$$\mu_i = \mu_0 \tag{5}$$

That is, the initial permeability (and also the permeability μ, in this case) is the same as that of free space. And, from Eq. (3), the magnetic susceptibility is

$$\chi(H) = 0 \tag{6}$$

which means that there is no magnetic response to a field **H**.

The magnetization versus **H** curve, the **B** versus **H** curve, and the **H** versus $\mathbf{H}_{ext}$ curves are given in Fig. 1; $M = 0$ for any value of H, and B varies linearly through the dependence on H in Eq. (1); **B** versus **H** is a straight line forming an angle of artan(μ_0), and finally, $H = H_{ext}$.

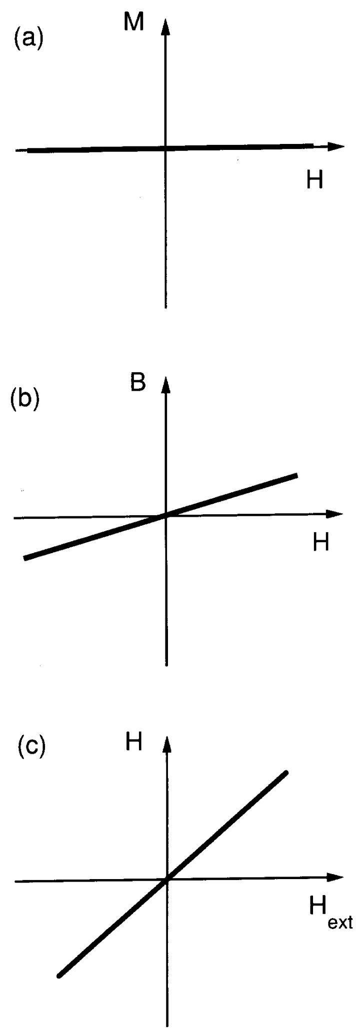

Figure 1. Curves of (a) $\mathbf{M}\times\mathbf{H}$, (b) $\mathbf{B}\times\mathbf{H}$, and (c) $\mathbf{H}\times\mathbf{H}_{ext}$ for an ideal non-magnetic material.

III.2 Hard Magnetic Materials

To define the properties of the ideal hard magnetic material one considers an extrapolation of the behavior of media that have larger and larger coercivities. This means wider and wider hysteresis loops; in the limit, the magnetization M is described by two straight lines parallel to the H axis. Therefore, the ideal hard magnetic material is a medium that retains a constant magnetization M_0, independently of the intensity of the applied magnetic field $\mathbf{H}_{ext}$. The field inside the medium is given by $H_{ext} - N_d M_0$.

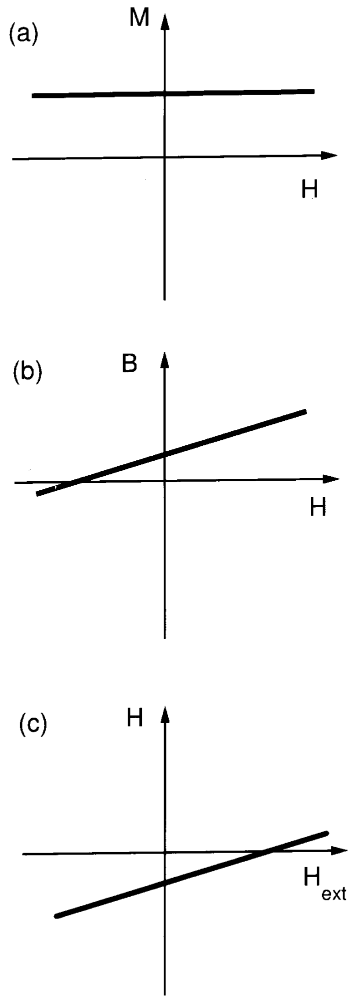

Figure 2. Curves of (a) $\mathbf{M}\times\mathbf{H}$, (b) $\mathbf{B}\times\mathbf{H}$, and (c) $\mathbf{H}\times\mathbf{H}_{ext}$ for an ideal hard magnetic material.

The $\mathbf{M} \times \mathbf{H}$, $\mathbf{B} \times \mathbf{H}$ and $\mathbf{H} \times \mathbf{H}_{ext}$ curves are shown in Fig. 2 (only one branch of the magnetization curve is shown); it is assumed that $\mathbf{M}$ and $\mathbf{B}$ are measured with a parallel field H_{ext}. In Fig. 2, the $\mathbf{M} \times \mathbf{H}$ graph is a straight line parallel to the Maxis; the $\mathbf{B} \times \mathbf{H}$ and $\mathbf{H} \times \mathbf{H}_{ext}$ graphs are straight lines forming, in the first case, an angle artan(μ_0), and in the second case, an angle of 45 degrees with the horizontal axis.

The susceptibility is

$$\chi = \frac{\partial M}{\partial H} = 0 \tag{7}$$

since $M = M_0$=constant. Therefore

$$\mu = \mu_0 \tag{8}$$

which means that the permeability of the ideal hard magnetic material is indistinguishable from that of the vacuum.

III.3 Soft Magnetic Materials

To define the ideal soft magnetic material, one extrapolates from materials that have larger and larger permeabilities, that is, that produce steeper and steeper curves of M versus H. Therefore, the ideal soft magnetic material is a medium that can be magnetized with an arbitrarily small magnetic field intensity.

The magnetization is therefore

$$M = \begin{cases} -M_s & , H < 0 \\ 0 & , H = 0 \\ M_s & , H > 0 \end{cases} \tag{9}$$

where M_s is the value of the saturation magnetization. This behavior can be represented mathematically using the Heaviside 'step function' $\theta(H)$: $M(H) = M_s\theta(H) - M_s\theta(-H)$ for $H \neq 0$, and $M(0) = 0$. The susceptibility is infinite for $H = 0$, and is zero for non-zero values of H. This is represented by a Dirac δ Function, which is the derivative of $\theta(H)$: $\chi(H) = \pm M_s\delta(H)$. For $H = 0$, from Eq. (3),

$$\chi(0) = \infty \tag{10}$$

and for $H \neq 0$, $\chi(H) = 0$.

From Eq. (2) it follows that the initial permeability is

$$\mu_i = \infty \tag{11}$$

and the permeability is $\mu = \mu_0$ for $H \neq 0$.

The curves of $\mathbf{M} \times \mathbf{H}$, $\mathbf{B} \times \mathbf{H}$ and $\mathbf{H} \times \mathbf{H}_{ext}$ neglecting demagnetizing effects are given in Fig. 3. The dependence of M on magnetic field is represented by two straight lines parallel to the H axis; one in the first quadrant, one in the third quadrant. The $\mathbf{B} \times \mathbf{H}$ graph contains two sloping straight lines; the curve of $\mathbf{H} \times \mathbf{H}_{ext}$ is a straight line forming an angle of 45 degrees.

Soft magnetic materials are notable for their ability to shield magnetic fields. We can consider that in an ideal soft magnetic material the external magnetic field is completely shielded, so that the internal field $\mathbf{H}$ is zero. This arises through the appearance, on the surface of the sample, of free poles that compensate the field H_{ext}. In other words, it is expected that the demagnetizing fields $-N_d M$ cancel any finite external field H_{ext} (the geometry has to be such that

$N_d \neq 0$). However, since in a real medium M can only vary in a finite range ($-M_s \leq M \leq M_s$, where M_s is the saturation magnetization), complete shielding will only occur for external magnetic fields in the range $H_{ext} \leq |N_d M_s|$. We will assume H_{ext} to be in this range.

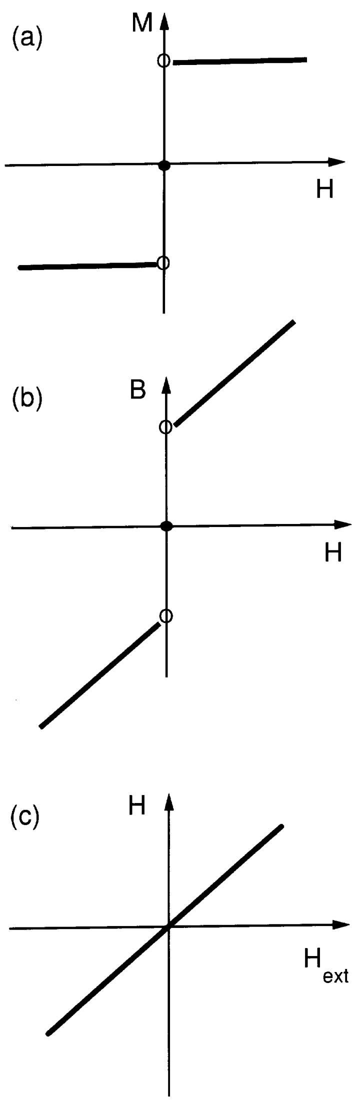

Figure 3. Curves of (a) $\mathbf{M}\times\mathbf{H}$, (b) $\mathbf{B}\times\mathbf{H}$, and (c) $\mathbf{H}\times\mathbf{H}_{ext}$, for an ideal soft magnetic material, neglecting demagnetizing effects; the end-points of the straight lines in the first two graphs are not included. Note that for $H = 0$, $M = B = 0$.

The susceptibility and initial permeability remain unchanged, and are given by:

$$\chi(0) = \infty \tag{12}$$

and

$$\mu_i = \infty \tag{13}$$

The values of $\mathbf{B}$, for $H = 0$ (as H_{ext} varies), vary from $-\mu_0 M_s$ to $+\mu_0 M_s$.

The $\mathbf{M}\times\mathbf{H}$, $\mathbf{B}\times\mathbf{H}$ and $\mathbf{H}\times\mathbf{H}_{ext}$ curves for the case of perfect shielding of $\mathbf{H}$ are shown in Fig. 4; note that the field H is always zero.

III.4 Diamagnetic Materials

In a diamagnet $\chi < 0$, and consequently, under an external magnetic field, the magnetization is negative. The value of B (from Eq. (1)) is reduced below $\mu_0 H$. Most diamagnets show a very weak diamagnetic response, and therefore their magnetic behavior is close to that of the ideal non-magnetic material discussed in Section 3.1.

There is a class of diamagnets, however, that present an important magnetic response – the superconductors. The idealization in this case regards a reduction of B inside the sample such that B is completely canceled. The ideal diamagnet is therefore a material that does not allow the penetration of an external induction $\mathbf{B}$; a susceptibility equal to -1 follows from this hypothesis (using Eq. (1)).

Thus, the corresponding susceptibility and permeability are given by:

$$\chi(H) = \frac{\partial M}{\partial H} = -1 \tag{14}$$

and, from Eq. (2),

$$\mu_i = 0 \tag{15}$$

A type I superconductor behaves, within some limits of intensity of the field H, as an ideal diamagnet. Here the B-field is canceled through the action of surface currents that generate an opposite field. This field does not arise from the alignment of dipole moments [3].

The curves $\mathbf{M}\times\mathbf{H}$, $\mathbf{B}\times\mathbf{H}$, and $\mathbf{H}\times\mathbf{H}_{ext}$ are shown in Fig. 5. The $\mathbf{M}\times\mathbf{H}$ and $\mathbf{H}\times\mathbf{H}_{ext}$ graphs are straight lines that form angles of 135 and 45 degrees with the x-axis, respectively; $\mathbf{B}\times\mathbf{H}$ is described by a straight line coinciding with the H axis.

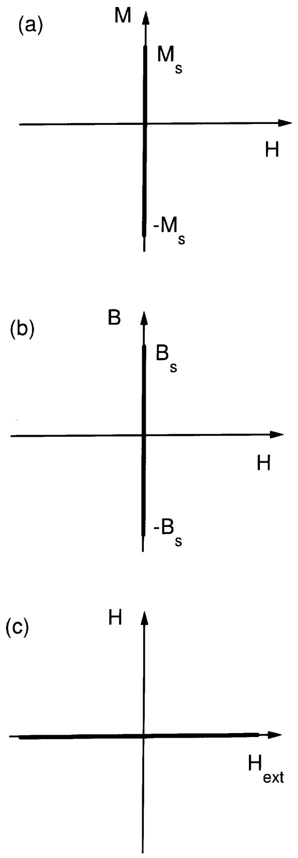

Figure 4. Curves of (a) $\mathbf{M}\times\mathbf{H}$, (b) $\mathbf{B}\times\mathbf{H}$, and (c) $\mathbf{H}\times\mathbf{H}_{ext}$, for an ideal soft magnetic material, including the demagnetizing field $-N_dM$.

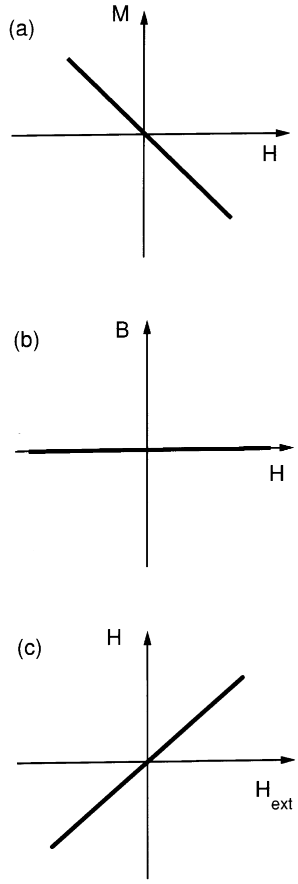

Figure 5. Curves of (a) $\mathbf{M}\times\mathbf{H}$, (b) $\mathbf{B}\times\mathbf{H}$, and (c) $\mathbf{H}\times\mathbf{H}_{ext}$, of an ideal diamagnetic material (the last curve is drawn for $N_d = 0$).

Table I contains a summary of the properties of the ideal magnetic materials under a variable external magnetic field H_{ext}.

Table I - Magnetic Properties of Some Ideal Materials

	M	**B**	**H**	χ	μ_i
Non-magnetic material	0	$\mu_0 H_{ext}$	H_{ext}	0	μ_0
Hard magnetic material	M_0	$\mu_0(H_{ext}+M_0)$	$H_{ext}-N_dM_0$	0	μ_0
Soft magnetic material ($N_d=0$, $H_{ext}\neq 0$)	$\pm M_s$	$\pm\mu_0(H_{ext}+M_s)$	H_{ext}	∞	∞
Soft magnetic material ($N_d\neq 0$, $H_{ext}\neq 0$)	$-M_s\leq M\leq M_s$	$-\mu_0 M_s\leq B\leq \mu_0 M_s$	0	∞	∞
Diamagnetic material (superconductor)	$H_{ext}/(N_d-1)$	0	$H_{ext}/(1-N_d)$	-1	0

IV Conclusions

We have analyzed the magnetic behavior of some ideal materials, and have presented the range of values of some quantities that characterize these materials, as well as the graphical representation of M, B, and H. The discussion of the properties and magnetization curves of these materials is recommended to students of magnetism.

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