Effects of Noise on Entanglement Dynamics in Atom-Field Interactions

Hünkar Kayhan*
Department of Physics, Abant Izzet Baysal University, Bolu-14280, Turkey
(Received on 01 April, 2008)

The entanglement dynamics in a system of the interaction of an atom with a single-mode cavity field in the presence of noise is studied by the Jaynes-Cummings model. Random phase telegraph noise is considered as the noise in the interaction and an exact solution to the model under this noise is obtained. The obtained solution is used to investigate the entanglement dynamics of the atom-field interaction. The mutual entropy is adopted for the quantification of the entanglement in the interaction. It is found that the entanglement is a non monotonic function of the intensity of the noise. The degree of the entanglement decreases to a minimum value for an optimal intensity of the noise and then increases for a sufficiently large intensity.

Keywords: Entanglement; Random phase telegraph noise; Mutual entropy

1. INTRODUCTION

Entanglement can display nonlocal correlations between quantum systems that have no classical counterpart. As a physical resource it plays a key role in Quantum Information Processing (QIP) [1–3]. Its preparation is thus a primary goal of this field. Real quantum systems will unavoidably interact with their surrounding environments. The main problem that must be overcome in QIP is decoherence, an effect that results from the coupling of the system to its surroundings or noise described by the stochastic processes associated with the system. The influence of noise (jump-type) on the atom-field interactions was first introduced by Burshtein [4, 5] in quantum optics. The simplest model of such jump-like processes is the two-state random telegraph. Eberly *et al* [6, 7] discussed laser-atom interactions that are subjected to two-state random (phase and frequency) telegraph noise.

The interaction of a two-level atom with a single-mode field which makes a single-photon transitions in an ideal cavity is described by the Jaynes-Cummings model (JCM) [8]. An employment of noise into the JCM was studied by Joshi *et al* [9–12]. In these works, the authors have treated the incorporation of noise into the JCM as the stochastic fluctuations in the atom-field coupling parameter (that is assumed to fluctuate in phase or in amplitude) with following possible physical reasons: The stochastic fluctuations associated with the coupling parameter in the cavity quantum electrodynamics may presumably be inherited from several reasons such as due to the source of the single-mode coherent cavity field or due to any variation in the mechanism of the production of Rydberg atom because of the instability in the atomic vapour production. Or, the motion of an ion in a harmonic trap interacting with a standing wave or a traveling wave may introduce another possibility for introducing the JCM with the stochastic fluctuations. Because, under a particular approximation [13], the equation of the motion for the ion in the trap may reduce to a similar form with the JCM. In this case, the fluctuations in the coupling coefficient can be considered both in the amplitude and the phase of the standing wave. Another possibility for introducing the stochastic fluctuations in the JCM is that the fluctuations of vacuum Rabi frequency or the atom-field coupling coefficient are known to wash out the trapping states in the micromaser system. Such fluctuations are possible in the case of an electric field generated by rubidium deposits at the cavity coupling holes or the electric field between the adjacent crystal domains in the cavity walls made of niobium.

In this paper, the interaction of a two-level atom with a single-mode field is studied in the environment of the random phase telegraph noise (RPTN) by the JCM [12]. By the method introduced in this reference, an exact solution to the JCM under this noise is obtained. The obtained solution is used to investigate the entanglement dynamics of the atom-field interaction. The mutual entropy is adopted for the quantification of the entanglement in the interaction.

The organization of the paper is as follows; In section 2, the formulation of the problem is obtained. The JCM with the RPTN is introduced and an exact solution to the model under this noise is presented. In section 3, the results and discussions are given. The solution obtained in the previous section is applied to a system in which the atom is initially taken in a pure state and the field initially in a thermal state. The entanglement properties of the system under the RPTN are explored. Finally, in section 4, the conclusions are presented.

2. THE FORMULATION OF THE PROBLEM

2.1. The Model and Solution

It is considered the interaction of a two-level atom described by spin-1/2 operators $S_\pm, S_z$ with a single-mode of quantized radiation field described by the annihilation and the creation operators $a, a^\dagger$. For the sake of the simplicity, it is assumed that the field is in resonance with the atomic transition frequency ω_0. In this case, the Hamiltonian of the system under the rotating wave approximation takes the form of ($\hbar = 1$)

$$H = \omega_0 S_z + \omega_0 a^\dagger a + (g^*(t) S_+ a + g(t) S_- a^\dagger) \qquad (1)$$

in which the interaction part is

$$H_{int} = (g^*(t) S_+ a + g(t) S_- a^\dagger) \qquad (2)$$

*Electronic address: hunkar_k@ibu.edu.tr

where $g(t)$ is the coupling coefficient between the atom and the field and is time-dependent. In order to employ the RPTN into the problem, the nature of $g(t)$ is considered to be completely stochastic which is defined as

$$g(t) = g_0 e^{i\phi(t)} \tag{3}$$

where g_0 is a positive real constant for the amplitude and $\phi(t)$ is a stochastic variable which fluctuates between different arbitrary phases in a manner of jumps. The jumps are separated by the time intervals called the mean dwell time. It is considered that $\phi(t)$'s in the neighbouring intervals are not correlated. Then, the probability for finding ϕ remains at any instant is the same. Therefore, $\phi(t)$ is undergoing random continuous change of Markov type which allows to take the average over the stochastic fluctuations.

An exact solution of the system is obtained by the method introduced by Joshi [12]

$$\begin{aligned}\rho(\tau)\exp\frac{\tau}{\tau_0} &= \int U(\phi;\tau;0)\rho(0)U^{-1}(\phi;\tau;0)dQ(\phi)\\ &+\frac{1}{\tau_0}\int \exp(\frac{t}{\tau_0})\int U(\phi;\tau;t)\rho(t)U^{-1}(\phi;\tau;t)dQ(\phi)dt\end{aligned} \tag{4}$$

where $dQ(\phi) = \frac{d\phi}{2\pi}$ and τ_0 is the mean dwell time. The mean dwell is the only factor which determines the strength/insensity of the noise. The shorter the mean dwell time means the stronger/the more intense noise. In this integral equation, the statistical average is taken over the stochastic variable $\phi(t)$. So, $\rho(t)$ is the noise-averaged density matrix of the system. The most general solution of Eq. (4) for the atom-field interactions with RPTN may be obtained as follows;

One can obtain the unitary transformation in atomic bases $|e\rangle$ and $|g\rangle$ in the interaction picture as

$$U = \begin{pmatrix} \cos(\sqrt{aa^\dagger}g_0 t) & -i\frac{\sin(\sqrt{aa^\dagger}g_0 t)}{\sqrt{aa^\dagger}}a\exp(-i\phi) \\ -i\frac{\sin(\sqrt{a^\dagger a}g_0 t)}{\sqrt{a^\dagger a}}a^\dagger\exp(i\phi) & \cos(\sqrt{a^\dagger a}g_0 t)\end{pmatrix} \tag{5}$$

The most general expression of the atom-field state at $t = 0$ can be written as

$$\rho(0) = \sum_{m,n=0}\{\rho_{mn}^{++}(0)|me\rangle\langle ne| + \rho_{mn}^{--}(0)|mg\rangle\langle ng| + \rho_{mn}^{+-}(0)|me\rangle\langle ng| + \rho_{mn}^{-+}(0)|mg\rangle\langle ne|\} \tag{6}$$

During the interaction, this state will evolve in time into the state

$$\rho(t) = \sum_{m,n=0}\{\rho_{mn}^{++}(t)|me\rangle\langle ne| + \rho_{mn}^{--}(t)|mg\rangle\langle ng| + \rho_{mn}^{+-}(t)|me\rangle\langle ng| + \rho_{mn}^{-+}(t)|mg\rangle\langle ne|\} \tag{7}$$

where the diagonal elements are

$$\begin{aligned}\rho_{mn}^{++}(t) &= \langle me|\rho(t)|ne\rangle\\ \rho_{mn}^{--}(t) &= \langle mg|\rho(t)|ng\rangle\end{aligned} \tag{8}$$

and the off-diagonal elements are

$$\begin{aligned}\rho_{mn}^{+-}(t) &= \langle me|\rho(t)|ng\rangle\\ \rho_{mn}^{-+}(t) &= \langle mg|\rho(t)|ne\rangle\end{aligned} \tag{9}$$

From these terms, one may obtain the integral expressions for these elements as

$$\begin{aligned}\rho_{mn}^{++}(\tau)\exp(\tau/\tau_0) &= \rho_{mn}^{++}(0)\cos\alpha_m\tau\cos\alpha_n\tau + \rho_{m+1n+1}^{--}(0)\sin\alpha_m\tau\sin\alpha_n\tau\\ &+\frac{1}{\tau_0}\int_0^\tau dt\exp(t/\tau_0)\{\rho_{mn}^{++}(t)\cos\alpha_m(\tau-t)\cos\alpha_n(\tau-t)\\ &+\rho_{m+1n+1}^{--}(t)\sin\alpha_m(\tau-t)\sin\alpha_n(\tau-t)\}\end{aligned} \tag{10}$$

$$\begin{aligned}\rho^{--}_{mn}(\tau)\exp(\tau/\tau_0) &= \rho^{--}_{mn}(0)\cos\beta_m\tau\cos\beta_n\tau+\rho^{++}_{m-1n-1}(0)\sin\beta_m\tau\sin\beta_n\tau \\ &+\frac{1}{\tau_0}\int_0^\tau dt\exp(t/\tau_0)\{\rho^{--}_{mn}(t)\cos\beta_m(\tau-t)\cos\beta_n(\tau-t) \\ &+\rho^{++}_{m-1n-1}(t)\sin\beta_m\sin\beta_n(\tau-t)\}\end{aligned} \tag{11}$$

$$\begin{aligned}\rho^{+-}_{mn}(\tau)\exp(\tau/\tau_0) &= \rho^{+-}_{mn}(0)\cos\alpha_m\tau\cos\beta_n\tau \\ &+\frac{1}{\tau_0}\int_0^\tau dt\exp(t/\tau_0)\{\rho^{+-}_{mn}(t)\cos\alpha_m(\tau-t)\cos\beta_n(\tau-t)\}\end{aligned} \tag{12}$$

$$\begin{aligned}\rho^{-+}_{mn}(\tau)\exp(\tau/\tau_0) &= \rho^{-+}_{mn}(0)\cos\beta_m\tau\cos\alpha_n\tau \\ &+\frac{1}{\tau_0}\int_0^\tau dt\exp(t/\tau_0)\{\rho^{-+}_{mn}(t)\cos\beta_m(\tau-t)\cos\alpha_n(\tau-t)\}\end{aligned} \tag{13}$$

where, $\alpha_{m-1}=\beta_m=g_0\sqrt{m}$.

By using the Laplace transformation technique, the following expressions from the above equations can be obtained as

$$\rho^{++}_{mn}(s)=\frac{(s+1/T)\rho^{++}_{mn}(0)[(s+1/T)^2+\Omega^+_{mn}]+\rho^{--}_{m+1n+1}(0)\Gamma_{mn}}{s^4+3s^3/T+(2\Omega^+_{mn}+3/T^2)s^2+(1/T^3+(3\Omega^+_{mn}-\Gamma_{mn})/T)s+A_{mn}} \tag{14}$$

$$\rho^{--}_{mn}(s)=\frac{(s+1/T)\{\rho^{--}_{mn}(0)[(s+1/T)^2+\Theta^+_{mn}]+\rho^{++}_{m-1n-1}(0)\Lambda_{mn}\}}{s^4+3s^3/T+(2\Theta^+_{mn}+3/T^2)s^2+(1/T^3+(3\Theta^+_{mn}-\Lambda_{mn})/T)s+B_{mn}} \tag{15}$$

$$\rho^{+-}_{mn}(s)=\frac{(s+1/T)\rho^{+-}_{mn}(0)[(s+1/T)^2+\Upsilon^+_{mn}]}{s^4+3s^3/T+(2\Upsilon^+_{mn}+3/T^2)s^2+(1/T^3+3\Upsilon^+_{mn}/T)s+C_{mn}} \tag{16}$$

$$\rho^{-+}_{mn}(s)=\frac{(s+1/T)\rho^{+-}_{mn}(0)[(s+1/T)^2+\Upsilon^+_{mn}]}{s^4+3s^3/T+(2\Pi^+_{mn}+3/T^2)s^2+(1/T^3+3\Upsilon^+_{mn}/T)s+D_{mn}} \tag{17}$$

where $T=\tau_0$, $\Omega^\pm_{mn}=\alpha^2_m\pm\alpha^2_n$, $\Gamma_{mn}=2\alpha_m\alpha_n$, $\Theta^\pm_{mn}=\beta^2_m\pm\beta^2_n$, $\Lambda_{mn}=2\beta_m\beta_n$, and $\Upsilon^\pm_{mn}=\alpha^2_m\pm\beta^2_n$ and $\Pi^\pm_{mn}=\beta^2_m\pm\alpha^2_n$. And also

$$\begin{aligned}A_{mn} &= (\Omega^+_{mn}/T^2-\Gamma_{mn}/T^2+\Omega^-_{mn}) \\ B_{mn} &= (\Theta^+_{mn}/T^2-\Lambda_{mn}/T^2+\Theta^-_{mn}) \\ C_{mn} &= (\Upsilon^+_{mn}/T-(\Upsilon^-_{mn})^2/T) \\ D_{mn} &= (\Pi^+_{mn}/T-(\Pi^-_{mn})^2/T)\end{aligned}$$

The inverse Laplace transformation gives an exact solution to the atom-field system as

$$\rho^{++}_{mn}(t)=\sum_{j=1}^4\frac{(\lambda_j+1/T)[\rho^{++}_{mn}(0)[(\lambda_j+1/T)^2+\Omega^+_{mn}]+\rho^{--}_{m+1n+1}(0)\Gamma_{mn}}{\prod_{k\neq j}(\lambda_j-\lambda_k)}\exp(\lambda_j t) \tag{18}$$

$$\rho^{--}_{mn}(t)=\sum_{j=1}^4\frac{(\lambda_j+1/T)[\rho^{--}_{mn}(0)[(\lambda_j+1/T)^2+\Theta^+_{mn}]+\rho^{++}_{m-1n-1}(0)\Lambda_{mn}}{\prod_{k\neq j}(\lambda_j-\lambda_k)}\exp(\lambda_j t) \tag{19}$$

where the λ_j s are the roots of the equations

$$\lambda_j^4+3\lambda_j^3/T+(2\Omega_{mn}^{+}+3/T^2)\lambda_j^2+(1/T^3+(3\Omega_{mn}^{+}-\Gamma_{mn})/T)\lambda_j+A_{mn}=0 \tag{20}$$

and

$$\lambda_j^4+3\lambda_j^3/T+(2\Theta_{mn}^{+}+3/T^2)\lambda_j^2+(1/T^3+(3\Theta_{mn}^{+}-\Lambda_{mn})/T)\lambda_j+B_{mn}=0 \tag{21}$$

respectively.

And the other two terms are

$$\rho_{mn}^{+-}(t)=\sum_{j=1}^{4}\frac{(\lambda_j+1/T)\rho_{mn}^{+-}(0)[(\lambda_j+1/T)^2+\Upsilon_{mn}^{+}]}{\prod_{j\neq k}(\lambda_j-\lambda_k)}\exp(\lambda_j t) \tag{22}$$

$$\rho_{mn}^{-+}(t)=\sum_{j=1}^{4}\frac{(\lambda_j+1/T)\rho_{mn}^{-+}(0)[(\lambda_j+1/T)^2+\Pi_{mn}^{+}]}{\prod_{j\neq k}(\lambda_j-\lambda_k)}\exp(\lambda_j t) \tag{23}$$

where the λ_j s are the roots of equations

$$\lambda_j^4+3\lambda_j^3/T+(2\Upsilon_{mn}^{+}+3/T^2)\lambda_j^2+(1/T^3+3\Upsilon_{mn}^{+}/T)\lambda_j+C_{mn}=0 \tag{24}$$

and

$$\lambda_j^4+3\lambda_j^3/T+(2\Pi_{mn}^{+}+3/T^2)\lambda_j^2+(1/T^3+3\Pi_{mn}^{+}/T)\lambda_j+D_{mn}=0 \tag{25}$$

respectively.

Eqs. (18), (19), (22) and (23) describe the dynamics of the atom-field interactions that are subjected to the random phase telegraph noise.

3. RESULTS AND DISCUSSION

The exact solution given by Eqs. (18), (19), (22) and (23) for the RPTN is now used to investigate the entanglement properties of the system. For this, the atom is initially taken in a pure state as

$$|\psi_a(0)\rangle=(\lambda|e\rangle+\sqrt{1-|\lambda|^2}|g\rangle) \tag{26}$$

where λ is the atomic state distribution with $0\leq|\lambda|\leq 1$ and the field is initially taken in a thermal state

$$\rho_f(0)=\sum_{n=0}P_n|n\rangle\langle n| \tag{27}$$

where the thermal field at some temperature of the cavity Te with the probability distribution P_n in the number states $|n\rangle$ being given by,

$$P_n=\frac{1}{(1+\bar{n})}(\frac{\bar{n}}{1+\bar{n}})^n \tag{28}$$

where $\bar{n}=\{e^{\beta\omega}-1\}^{-1}$ is the initial mean photon number in the cavity, $\beta=1/k_BTe$ and k_B is Boltzmann's constant. In this case, the initial atom-field state at $t=0$ becomes

$$\rho^{af}(0)=\sum_{n=0}P_n\{|\lambda|^2|ne\rangle\langle ne|+(1-|\lambda|^2)|ng\rangle\langle ng|\lambda\sqrt{1-|\lambda|^2}|ne\rangle\langle ng|+\lambda^*\sqrt{1-|\lambda|^2}|ng\rangle\langle ne|\} \tag{29}$$

From Eq. (29), it is clear that $\rho_{nn}^{++}(0)=P_n|\lambda|^2,\rho_{nn}^{--}(0)=P_n(1-|\lambda|^2),\rho_{nn}^{+-}(0)=P_n\lambda\sqrt{1-|\lambda|^2},\rho_{nn}^{-+}(0)=P_n\lambda^*\sqrt{1-|\lambda|^2}$. With these initial conditions, one can obtain the solution of the system. In this case, the atom-field system will evolve in time into the state

$$\rho^{af}(t)=\sum_{n=0}\{\rho_{nn}^{++}(t)|ne\rangle\langle ne|+\rho_{nn}^{--}(t)|ng\rangle\langle ng|+\rho_{nn}^{+-}(t)|ne\rangle\langle ng|+\rho_{nn}^{-+}(t)|ng\rangle\langle ne|\} \tag{30}$$

The dimension of the system described by Eq. (30) is $2\otimes\infty$. In fact, there is no known an entanglement measure to quantify such systems. For the quantification of the entanglement in the system, it is adopted the mutual entropy of the system $S(A:F)$ which is defined as $S(A:F) = S(A) + S(F) - S(AF)$. Here, $S(A)$ and $S(F)$ are the entropies of the atom and the field, respectively and $S(AF)$ is the entropy of the atom-field system. In order to calculate the entropies, the dimension of the system n_{dim} is taken in such a way that the probability distribution of the field reaches to unity approximately, that is, $\sum_{n=0}^{n_{dim}} P_n \cong 1$ [14].

The entropy of a system is defined as

$$S = -\sum_i \lambda_i \log \lambda_i \tag{31}$$

where λ_is are the non-zero eigenvalues of the density matrix of the system. The density matrix of the atom (and the field) can be found by tracing out the joint density matrix of the system described by Eq. (30) over the degree of freedom of the field (and the atom). So, the time evolution of the density matrix of the atom becomes

$$\rho^a(t) = \sum_{n=0} \rho_{nn}^{++}(t)|e\rangle\langle e| + \sum_{n=0} \rho_{nn}^{--}(t)|g\rangle\langle g| + \sum_{n=0} \rho_{nn}^{+-}(t)|e\rangle\langle g| + \sum_{n=0} \rho_{nn}^{-+}(t)|g\rangle\langle e| \tag{32}$$

And the time-dependent elements of the density matrix of the field becomes

$$\rho_{nn}^f(t) = \rho_{nn}^{++}(t) + \rho_{nn}^{--}(t) \tag{33}$$

The effects of the RPTN on the entanglement dynamics of the atom-field interaction are depicted in Figs. (1)-(4).

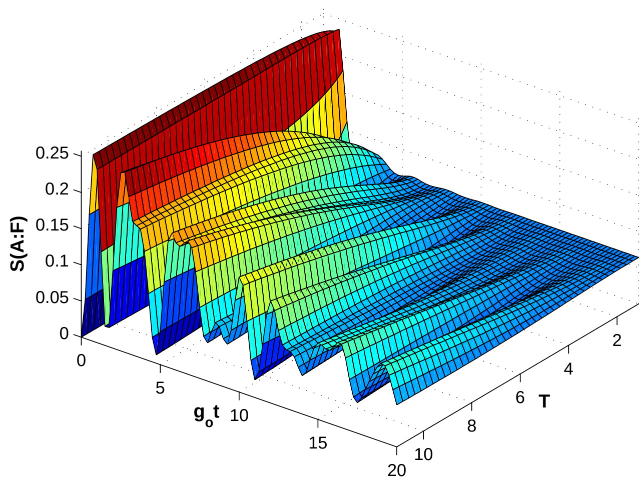


FIG. 1: The mutual entropy as the function of the time $g_0 t$ and the mean dwell time T (in the unit of $1/g_0$) for the large values of the mean dwell time. $\lambda = 1/\sqrt{2}$ and $\bar{n} = 0.1$.

In Figs. (1) and (2), the mutual entropy $S(A:F)$ of the system is plotted as the function of time $g_0 t$ and the mean dwell time T. In Fig. (1), it is clear that the entanglement between the atom and the field reaches its maximum value then decays and eventually will disappear in a finite time interval that depends on the strength/intensity of the noise determined by the mean dwell time. As the value of mean dwell time T increases, the strength and the lifetime of the entanglement increases. Because, as T increases, the effects of the noise on

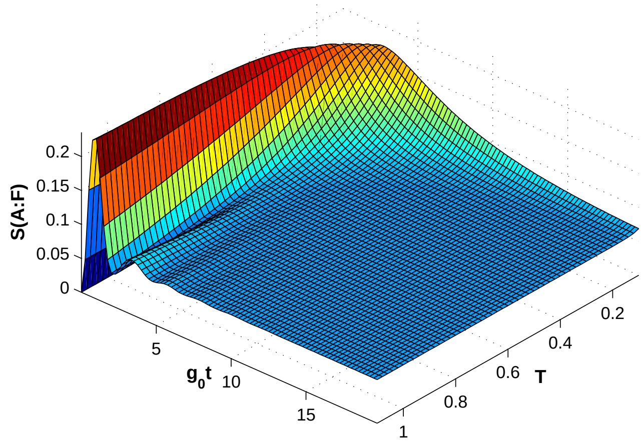


FIG. 2: The mutual entropy as the function of the time $g_0 t$ and the mean dwell time T (in the unit of $1/g_0$) for the small values of the mean dwell time. $\lambda = 1/\sqrt{2}$ and $\bar{n} = 0.1$.

the interaction weakens. Since the intensity/strength of the noise is determined by the mean dwell time, the decoherence mechanism becomes faster as the mean dwell time is shorter. So, it seems that there is a monotonous relation between the strength of the noise (the mean dwell time) and the degree of the entanglement. But, for the sufficiently small values of the mean dwell time, the decrease in the mean dwell time (the increase in the strength of the noise) does not induce a decrease in the degree of the entanglement, instead causes an increase in the degree as shown by Fig. (2). This may be explained as; when the changes in the phase (jump-like) are very fast, the system can not follow all the phase changes and can not respond them completely. So, it only feels the averages of the phase changes. In this case, the entanglement becomes more robust against the noise. Thus, the entanglement is a non-monotonic function of the intensity of the noise. The degree of the entanglement decreases to a minimum value for an optimal intensity of the noise and then increases for a sufficiently large intensity. This situation resembles the stochastic resonance in which the response of a non-linear system to a weak periodic driving can be enhanced when supplemented with a noisy field of certain optimal intensity [15].

In addition, as also shown by Fig. (3), as $T \to \infty$, the effect of the noise disappears and one reaches the entanglement dynamics in the usual JCM [16], in which the entanglement is present at all times and never goes to zero due to the interaction, except at $t = 0$.

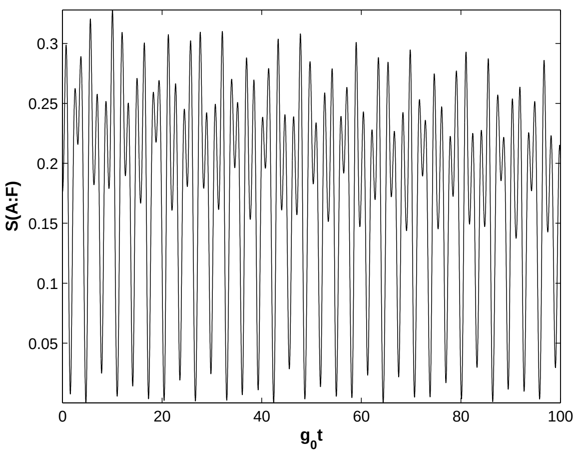
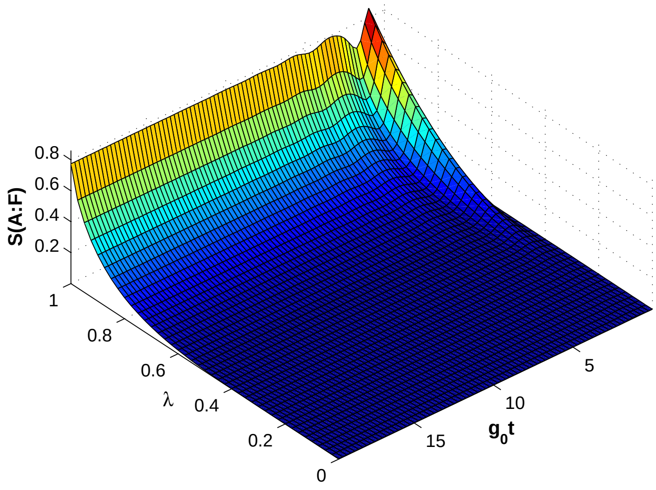
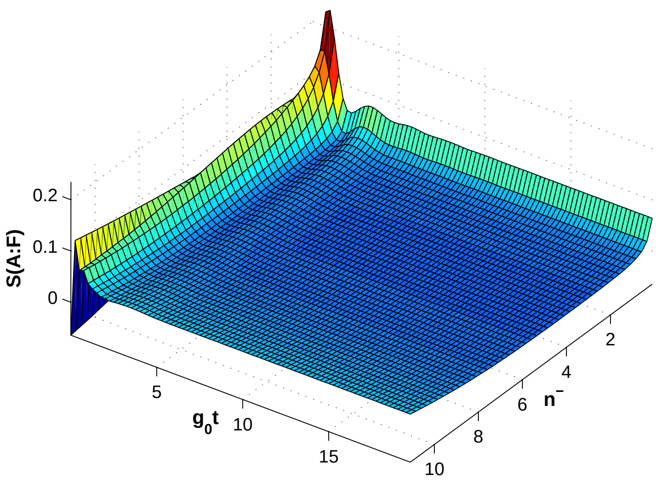


FIG. 3: The mutual entropy as the function of the time g_0t. $\lambda = 1/\sqrt{2}$, $\bar{n} = 0.1$ and $T \to \infty$.



(a)



(b)

FIG. 4: The mutual entropy as the function of the time g_0t and (a) the atomic state distribution λ. $\bar{n} = 0.1$. (b) the average photon number $\bar{n}$, $\lambda = 1/\sqrt{2}$. T = 1 for both.

As to the influence of the parameters of the system on the entanglement dynamics, Fig. (4) shows the effects of these parameters, atomic state distribution λ and the average photon number $\bar{n}$, on the entanglement dynamics in the presence of the RPTN. The entanglement is obviously the most powerful when the atom is initially in the excited state ($\lambda = 1$) and is the weakest when the atom is initially in the ground state ($\lambda = 0$), as expected. As the value of λ increases, the entanglement becomes stronger. In addition, as the value of $\bar{n}$ increases, equivalently the temperature of the cavity increases, the system acts much like a classical statistical system. So, the strength of the entanglement falls down. But, unlike the non-noisy case, the entanglement of the system will disappear at any temperature of the cavity field no matter what, provided that the monotonous relation between the strength of the noise and the degree of the entanglement. So, the entanglement is very sensitive to the system parameters. The initial conditions of the system play important role on the entanglement dynamics for its strength and for its lifetime to die out.

In the interaction, there is a dephasing mechanism that accounts for the decay of the entanglement. This mechanism arises from the stochastic phase fluctuations of the atom-field coupling parameter. In the mechanism, the phase fluctuations affect only the dipole or the transverse relaxation mechanism of the system. There is not any type of dissipation in the energy of the system. So, this dephasing mechanism is different from the usual dissipation mechanisms (such as cavity field damping and spontaneous spontaneous emission decay or radiative damping of the system) which affects both the energy and the coherence of the system. So, RPTN causes another type of intrinsic decoherence in the JCM, in result the decay of quantum coherences. Milburn [17] proposed a model for intrinsic decoherence. In this model, intrinsic decoherence gives rise the destruction of the quantum coherence in the case that the physical properties of the system approaching a macroscopic level. In this type of dephasing mechanism, the constants of the motion remain constants of the motion and hence stationary states remain stationary states. So, the dephasing mechanisms in these two cases display similar physical features (no energy dissipation and the decay of quantum coherences) but do not have the same origin. Jian *et.al* [18] used Milburn's model to investigate the effects of the intrinsic decoherence on the entanglement properties of the same relevant system. The exact solution in their work reveals that there is an obvious monotonous relation between intrinsic decoherence coefficient γ and the degree of the entanglement. For the small values of the coefficient $\gamma \rightarrow 0$, no entanglement emerges and for its large values $\gamma \rightarrow \infty$, the entanglement in usual JCM is obtained. Therefore, as though the dephasing mechanisms in these both cases display similar physical features (no energy dissipation and the decay of quantum coherences), the entanglement dynamicsses do not. The random phase telegraph noise induces a very special featured entanglement dynamics. Moreover, the noise considered in this study arises in a non-controllable manner. It is completely due to the stochastic behavior of the system itself, not due to an environment effect. There are some works devoted to the environmental noise. For example, one is about preventing or minimizing the influence of environmental noise in quantum information processing [19]. But, instead of attempting to shield the system from the environmental noise, Plenio and Huelge used a white noise to generate a controllable entanglement by incoherent sources [20]. The entanglement dynamics in their work displays a similar character with that of this work. The noise plays a constructive role in quantum information processing but the entanglement arises from a controllable situation. Similar aspects have also been considered elsewhere [21, 22]. In this paper, the revealed properties of the entanglement of the system under the random phase telegraph noise are uncontrollable and unaffected by the surrounding environment. Since the fluctuations in the system are quite random, the entanglement equivalently the information in the system fluctuates randomly.

4. CONCLUSION

In this paper, the interaction of a two-level atom with a single-mode field is studied in the environment of the random phase telegraph noise analytically by the Jaynes-Cummings model. Random phase telegraph noise is considered as the noise which arises from the system itself not from its surrounding environment. It is quite due to the stochastic phase fluctuations in the atom-field coupling of the system. It causes another type of intrinsic decoherence in the JCM in a non-controllable manner. An exact solution to the model under this noise is obtained. The entanglement dynamics between the atom and the cavity field is explored by calculating the mutual entropy of the system. The noise manifests itself as a decay factor in the degree of entanglement in time due to a kind of dephasing mechanism which is different from the radiative damping of the atom. This dephasing mechanism affects only the dipole or the transverse relaxation mechanism of the system. The mean dwell time predominantly determines the strength as well as the lifetime of the entanglement. The entanglement is a non monotonic function of the intensity of the noise determined by the mean dwell time. The degree of the entanglement decreases to a minimum value for an optimal intensity of the noise and then increases for a sufficiently large intensity. In addition, the entanglement dynamics is very sensitive to the system parameters. The initial conditions of the system play important role on the entanglement dynamics for its strength and for its lifetime to die out. Unlike the non-noisy case, the entanglement of the system will disappear at any temperature of the cavity field no matter what provided that the monotonous relation between the intensity of the noise and the degree of the entanglement.

Acknowledgments

I am grateful to Aliekber Aktag for his help and discussion.

[1] D. P. DiVincenzo, Science **270**, 255 (1995).
[2] J. I. Cirac and P. Zoller, Nature (London) **404**, 579 (2000).
[3] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, Cambridge, England 2000.
[4] A. I. Burshtein, Zh. Eksp. Teor. Fiz. **49**, 1362(1966), [Sov.Phys. JETP **22**, 939 (1966)]; A. I. Burshtein and Yu. S. Oseledchik *ibid.* **51**, 1071 (1966) [*ibid.* **24**, 716 (1967)]; L. D. Zusman and A. I. Burshtein *ibid.*, **61**, 976 (1971) [*ibid.* **34**, 520 (1972)].
[5] A. I. Burshtein and Yu. S. Oseledchik *ibid.* **51**, (1966) 1071 [*ibid.* **24**, 716 (1967)]; L. D. Zusman and A. I. Burshtein *ibid.*, **61**, 976 (1971) [*ibid.* **34**, 520 (1972)].
[6] J. H. Eberly, K. Wodkiewicz, and B. W. Shore, Phys. Rev. A **30**, 2381 (1984).
[7] K. Wodkiewicz, B. W. Shore, and J. H. Eberly, Phys. Rev. A **30**, 2390 (1984).
[8] E. T. Jaynes, F. W. Cummings, Proc. IEEE **51**, 89 (1963).
[9] A. Joshi, S. W. Lawande, Phys. Lett. A **184**, 390 (1994).
[10] A. Joshi, S. W. Lawande, Phys. Rev. A **50**, 1692 (1994).
[11] S. V. Lawande, A. Joshi, and Q. V. Lawande, Phys. Rev. A **52**, 619 (1995).
[12] A. Joshi, J. mod. Optics **42**, 2561 (1995).
[13] J. I. Cirac, P. R. Zoller, W. D. Phillips, Phys. Rev. A **46**, 2688 (1992); C. A. Blockey, D. F. Walls, H. Risken, Europhys. Lett. **17**, 509 (1992).
[14] E. Boukobza, D. J. Tannor, Phys. Rev. A. **71**, 063821 (2005).
[15] L. Gammaitoni *et al*, Rev. Mod. Phys., **223**, 70 (1998).
[16] S. Bose, I. Fuentes-Guridi, P. L. Night, and V. Vedral, Phys. Rev. Lett. **87**, 050401 (2001).
[17] G. J. Milburn, Phys. Rev. A. **44**, 5401 (1991).
[18] ZHANG Jian, SHAO Bin, J. At. Mol. Phy. **23**, 417 (2006).
[19] L.-M. Duan and G.-C. Guo, Phys. Rev. Lett. **79**, 1953 (1997); P. Zanardi, and M. Rasetti, *ibid.* **79**, 3306 (1997); D. A. Lidar, I. L. Chuang, and K. B. Whaley, ibid. **81**, 2594 (1998).
[20] M. B. Plenio and S. F. Huelga, Phys. Rev. Lett. **88**, 197901 (2002).
[21] A. Beige *et al.*, J. Mod. Opt. **47**, 2583 (2000);
[22] M. S. Kim *et al.*, Phys. Rev. A **65**, 040101 (2002).