Disappearance of Squeezed Back-to-Back Correlations - a New Signal of Hadron Freeze-Out from a Supercooled Quark Gluon Plasma

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We briefly discuss four different possible types of transitions from quark to hadronic matter and their characteristic signatures in terms of correlations. We also highlight the effects arising from mass modification of hadrons in hot and dense hadronic matter, as well as their quantum statistical consequences: the appearance of squeezed quantum states and the associated experimental signatures, i.e., the back-to-back correlations of particle-antiparticle pairs. We briefly review the theoretical results of these squeezed quanta, generated by in-medium modified masses, starting from the first indication of the existence of surprising particle-antiparticle correlations, and ending by considering the effects of chiral dynamics on these correlation patterns. Nevertheless, a prerequisite for such a signature is the experimental verification of its observability. Therefore, the experimental observation of back-to-back correlations in high energy heavy ion reactions would be a unique signature, proving the existence of in-medium mass modification of hadronic states. On the other hand, their disappearance at some threshold centrality or collision energy would indicate that the hadron formation mechanism would have qualitatively changed: asymptotic hadrons above such a threshold are not formed from medium modified hadrons anymore, but rather by new degrees of freedom characterizing the medium. Furthermore, the disappearance of the squeezed BBC could also serve as a signature of a sudden, non-equilibrium hadronization scenario from a supercooled quark-gluon plasma phase.

Keywords: Correlations; Femtoscopy; Quantum optics; Squeezed coherent states; Particle-antiparticle pairs

I. INTRODUCTION

The major goal of the programme of high energy heavy ion physics has been to explore the phases of hot and dense, strongly interacting matter. More specifically, it has been to find a new phase where quarks and gluons are no longer confined to their $T = 0$ bound states, the hadrons. This new phase of matter may be referred to as "Quark-Gluon Plasma" (QGP) in analogy to the electromagnetic plasma, the phase in which electrons and ionized atoms appear as new degrees of freedom, as compared to the neutral atomic or molecular forms of matter.

It has been theoretically predicted by a number of authors [1–6] that the emergence of a large number of colored degrees of freedom in an expanding QGP could be signaled by correlation measurements. However, some of the theoretical expectations related to a first order phase transition, such as a long-lived QGP, are not being observed experimentally.

Nevertheless, we could still ask the question: can a phase transition be signaled by correlation measurements *at all*? Fortunately, the answer is positive: in section II, we shall discuss correlation measurements that have indeed signaled experimentally the onset of first order as well as second order phase transitions, in solid-state physics measurements.

II. PHASE TRANSITIONS AND CORRELATIONS

Returning to the question if an experimental measurement of the two-particle correlation function could be utilized to signal a phase transition, a clear yes answer has recently been given in condensed matter physics by Schellekens, Hoppeler, Perrin, Viana Gomes, Boiron, Aspect and Westbrook [7]. In their measurement, a cloud of ultra-cold gas of meta-stable He atoms was released from its magnetic trap, and after 47 cm ballistic fall, the position of each atom was detected. Single particle detection of these neutral atoms was possible as the 20 eV internal energy of each atom was released when the atoms hit the detector.

The initial temperature has been varied from above to below the Bose-Einstein condensation threshold, $T_{BEC} = 0.5\,\mu K$. The observed two-body correlation function for initial conditions with $T > T_{BEC}$ showed a bunching behavior, while the correlation function became flat for the coherent, Bose-Einstein condensed sample, with initial conditions $T < T_{BEC}$. The observed quantum statistical correlations are the atomic analogue of the Hanbury Brown – Twiss effect [8], and in the case of atomic Bose-Einstein condensation, the disappearance of the bunching behavior signaled the phase transition from the usual thermal state to the Bose-Einstein condensate. The intercept parameter of the correlation function served as an order parameter of this phase transition. Thus the correlation function can clearly signal a phase transition.

III. FOUR POSSIBILITIES FOR TRANSITION FROM DECONFINED TO HADRONIC MATTER

What kind of phase transitions may occur in hot and dense, nuclear matter? According to recent lattice QCD calculations, three types of equilibrium transitions are possible in the (T, μ_B) plane: if the temperature is increased at zero, or nearly zero chemical potentials, the transition from confined to deconfined matter is a cross-over [9], i.e. this is not a phase transition in the strict, thermodynamical sense: quark and gluon degrees of freedom are present below T_c, hadrons are present above T_c and various observables yield different estimates for the critical temperature itself. In contrast, at rather large baryochemical potentials, the transition from hadronic matter to deconfined matter is of first order. The line of first order transitions ends at a critical end point (CEP) that separates the first order phase transitions from the cross-over region. At this critical end point, (μ_{CEP}, T_{CEP}), the transition from confined to deconfined matter is a second order phase transition. There are other more exotic states like color superconducting quark matter, and there is a nuclear liquid-gas phase transition, but the region of the phase diagram which is relevant for high energy heavy ion collisions at CERN/SPS, RHIC and LHC, is the region around the CEP [10]. Recent calculations and experimental data indicate that in central Au+Au collisions at RHIC, the transition is likely a cross-over. The corresponding extracted temperatures for the 0-5 % most central $\sqrt{s_{NN}} = 130$ GeV Au+Au data sample are $T_0 = 214 \pm 7$ MeV, and $\mu_B = 77 \pm 38$ MeV, respectively [11]. These values are to be contrasted with the critical end point location of the line of first order QCD phase transition estimated by lattice QCD [10] to be $T_{CEP} = 162 \pm 2$ MeV and $\mu_{CEP} = 360 \pm 40$ MeV.

If the evolution of the strongly interacting matter produced in relativistic heavy ion collisions happens through thermodynamically equilibrated states, first- and second-order phase transitions, as well as an analytic cross-over, are the only three possibilities for the state of the system. However, heavy ion collisions create violently exploding mini-bangs, where the time-scales of the expansion are rather short as compared to the typically ~ 100 fm/c nucleation times of hadronic bubbles in a first order phase transition from QGP to a hadron gas. Hence non-equilibrium transitions are also possible. In ref. [12], another scenario is suggested, where a rapidly expanding QGP state might strongly supercool in Au+Au collisions at RHIC and then suddenly hadronize while emitting a flash of pions. The predicted signals of this scenario are not, even currently, in disagreement with RHIC data on Au+Au collisions. So the possibility of non-equilibrium hadronic formation must be kept in mind and further experimental tools must be found to pin down if this mechanism is in fact present in Au+Au collisions at RHIC.

IV. CORRELATION SIGNATURES FOR THE FOUR REHADRONIZATION SCENARIOS

Let us itemize the possible types of rehadronization transitions, and summarize their correlation signatures:

- A strong first order QCD phase transition has been studied in the predominant fraction of the literature. Such a transition is characterized by long life-times and time spreads, since the latent heat and the initial entropy densities are large. The picture behind this scenario is that of a slowly burning cylinder of QGP [6]. In such a scenario, the extended duration of particle emission adds up to source parameter in the direction of the average momentum of the pion pair, resulting in a substantial increase of the effective source size. This is signaled as $R_{out} \gg R_{side}$, regardless of the exact details of the calculation [1–6]. However, experimentally $R_{out} \approx R_{side}$, both at CERN SPS [13, 14] and at RHIC [15–17]. Hence, a strong first order transition from deconfined to hadronic matter seems to be excluded by current correlation measurements in high energy heavy ion collisions, both at CERN/SPS and at the RHIC energy range.

However, three other alternatives still remain possible.

- A second order QCD phase transition from deconfined to hadronic matter has been considered recently in ref. [18]. The interesting conclusion is that this phase transition is not signaled by the scale parameters of the correlation function, as the spatial correlations develop a power-law tail in these kind of transitions, and power-laws have no characteristic scale. Instead, the second order phase transitions are characterized by critical exponents. One of these, traditionally denoted by η, characterizes the tail of the spatial correlations of the order parameter. In ref. [18], this exponent was shown to be measurable with the help of the two-particle correlation function in momentum space, and it was shown that $\eta = \alpha$, where α stands for the Lévy index of stability of the correlation function itself, i.e., $C(q) = 1 + \lambda \exp[-(qR)^\alpha]$.

  A strong decrease of the Lévy index of stability α down to about 0.50 ± 0.02 in the vicinity of the critical end point is a theoretical prediction for the localization of this outstanding landmark of the QCD phase diagram. This value was obtained based on universality class arguments. Rajagopal, Wilczek and others argued that the universality class of QCD at the critical point is that of the 3-dimensional Ising model [19]. The exponent of the correlation function η, however, is extremely small in that model, i.e., $\eta = 0.03 \pm 0.01$ [19]. In a violent heavy ion collisions, random external fields are also present, which change the universality class and, thus, might increase the value of the correlation exponent. In the 3-dimensional random field Ising model, η increases to 0.50 ± 0.05. When interpreted as a Lévy index of stability, it still corresponds to an extremely peaked correlation function, which should be very clearly measurable. Nevertheless, the excitation function of the *shape parameter* α has not yet been determined experimentally. Therefore, it is possible that at certain colliding energies, either at CERN/SPS range or at RHIC, there is a dramatic change in the shape of the correlation function, which has not yet been identified.

- A cross-over from QGP to confined matter is the last remaining possibility of an equilibrium phase transition. This scenario can be signaled by emission of hadrons from a region above the critical temperature, $T > T_c$, or, by finding deconfined quarks or gluons at temperatures $T < T_c$. Recent lattice QCD calculations suggest that, at zero baryochemical potential, the transition from quarks to hadrons is a rapid cross-over, different observables yielding different transition temperatures at $\mu_B = 0$. The peak of the renormalized chiral susceptibility predicts T_c=151(3) MeV, whereas critical temperatures based on the strange quark number susceptibility, and Polyakov loops, result in values higher than this by 24(4) MeV and 25(4) MeV, respectively. Signs of quarks or gluons below these temperatures have not yet been observed experimentally. However, emission of hadrons from a small but very hot region, with $T > T_c \simeq 176 \pm 7$ MeV [10, 20], has been suggested by Buda-Lund hydro model fits to the identified particle spectra and two-pion correlation functions in Au+Au collisions at RHIC [11].

- The fourth possibility corresponds to hadron formation out of thermal equilibrium. A sudden recombination from quarks to hadrons has been considered as the mechanism for hadron formation in the ALCOR model [21]. Other realizations of sudden hadron formation and freeze-out were tools used to explain the observed scaling properties of elliptic flow [22] with the number of constituent quarks. In the context of femtoscopy analyses, it has been suggested that the comparable magnitude of $R_{out} \approx R_{side} \approx R_{long}$ could be a signature of a flash of hadrons (pion-flash) from a deeply supercooled QGP phase [12]. If this mechanism would indeed be responsible for the hadronization in high energy heavy ion collisions, it would also predict that the freeze-out distributions of temperature, flow, and density would not depend on the particle type (or cross-section), since all hadrons would be produced in the same flash. Strangeness would be enhanced, as strangeness production predominantly happens in the pre-hadron state of matter. Finally, no in-medium mass modification of hadrons could be observed, since deeply supercooled QGP is a state of over-stretched matter, has negative pressures internally, which cause the sudden break-up and coalescence of matter. However, in this picture, the hadron gas is produced in a large volume, and re-scattering effects are small. Hence, the interactions which would cause the in-medium mass modification of hadrons, do not take place, as the hadron gas would freeze out as soon as it was produced. Note, however, that the thermodynamic considerations in Ref. [12] would also allow, in about 50 % of the parameter space, for the production of a super-heated hadron gas. Therefore, the indication of a hot spot in the central region, and of particle emission from a region with $T(x) > T_c$ [11] would be compatible with this non-equilibrium hadronization mechanism as well.

In the end of this review, we shall propose a new method to observe the onset of sudden hadronization from a supercooled QGP state as a possible rehadronization mechanism.

V. COHERENT STATES

In this session, we will briefly present the theory of quantum optical coherence and chaos, by introducing the concept of coherent states and an explanation of the HBT effect. Then we discuss squeezed states and review their applications in high energy heavy ion and particle physics. At the end we will focus on how these quantum statistical correlations of squeezed hadronic states could be experimentally used as tools to search for a sudden freeze-out of hadrons from a super-cooled QGP state.

Let us mention an inspiring historical review by Michel Martin Nieto [23], who compared the well known discovery of the coherent states by E. Schrödinger in 1926 [24] with the much less well known discovery of squeezed states by E. H. Kennard in 1927 [25]. According to Nieto [23], Schrödinger's original discovery of coherent states was inspired by a question from Lorentz, in a letter on May 27, 1926, in which he lamented the fact that Schrödinger's wave functions were stationary, and did not display classical motion. On June 6, 1926 Schrödinger replied that he had found a system with classical motion, and sent Lorentz a draft copy of his paper, published in Ref. [24]. According to it, coherent states can be defined with the help of the so-called minimum-uncertainty method. In this, the mean position and mean momentum are required to follow the trajectory of the classical motion corresponding to the same Hamiltonian, and also that the vacuum state is a member of the set of states.

The development of understanding about the dual nature of light, evident in its wave-like properties and quantized detections has been summarized recently in ref. [26].

When the tools to handle quantum electrodynamics were developed, they were applied mainly to high energy processes, and it was still naively assumed, that the conflict between Maxwell's treatment - who focused on the wave-like properties of the electromagnetic fields, and whose formulas were the basis of radio engineering - and Planck's theory, which emphasized the quantum nature of light, would be of no significance in optical observations. This state of blissful indifference [26] was however changed by the landmark experiment of R. Hanbury Brown and R. Q. Twiss [8], who proposed an interferometric method to determine the angular extension of distant stellar objects. They found out that intensity correlation between photocurrents, recorded in two separated detectors, displayed a bump when the difference in optical path lengths between the signals was decreased towards zero. These observations have shown that the photons in the two different detectors were correlated, although they stemmed from two different surface elements of distant stars. This way, individual photons, as well as pairs of such photons, entered the realm of observational optics. Ever since, this phenomenon is known as the HBT effect.

The correlated emission of particle pairs is a fundamental property of quantum fields. The effect has been observed in particle physics and been explained based on the bosonic nature of pions by Goldhaber, Goldhaber, Lee, and Pais in 1960 [27]. Recently, such positive correlations have been observed also in ultracold quantum gases by Schellekens et al [7], as detailed in Section II, which indicates that quantum fluctuations and correlations are important basic properties of matter fields, just as well as that of the electromagnetic (photon) field [28].

After the discovery of the HBT effect and the discovery of lasers, there was a theoretical debate in the literature, arguing whether photons from a laser light would show the HBT effect or not. The correct theory was published first by Glauber who had shown that there is no HBT effect in the coherent fields of lasers. In fact, the lack of second- and higher-order HBT type of intensity correlations *defines* quantum optical coherence. Thus, Glauber related the bunching properties of the HBT observations to the random, chaotic nature of the photon field in thermal radiation. He also pointed out that more information is necessary to characterize the quantum state of the photon field: *"Whereas a stationary Gaussian stochastic process is described completely by its frequency-dependent power spectrum, a great deal more information in the form of amplitude and phase relations between differing quantum states may be required to describe a steady light beam. Beams of identical spectral distributions may exhibit altogether different photon correlations or, alternatively, none at all. There is ultimately no substitute for the quantum theory in describing quanta".* Based on Glauber's theory, the puzzling HBT effects observed in thermal radiation and the lack of HBT effect in lasers were understood in the same framework, and the characterization of quantized electromagnetic radiation has been reduced to counting photons. The new field of *quantum optics* was born. These days, we witness the birth of *quantum atomics* [28] as coherent and incoherent beams of fermionic [29] or bosonic [7] atoms enter the realm of experimental investigations.

In this section, we highlight some of the basic properties of coherent states, introduced by Glauber in Ref. [30], as eigenstates of the annihilation operator. For simplicity, let us consider a given mode of a free boson field, corresponding to a harmonic oscillator. After quantization and normal ordering, the Hamiltonian operator of the one-mode harmonic oscillator is written as

$$H = \omega a^\dagger a, \tag{1}$$

where $a^\dagger$ and a are the creation and annihilation operators, respectively. Their non-vanishing commutator is

$$[a, a^\dagger] = 1. \tag{2}$$

With the help of these creation and annihilation operators, a Fock space can be built up by considering that

$$a|0\rangle = 0, \tag{3}$$

$$|n\rangle = \frac{1}{\sqrt{n!}} a^{\dagger n}|0\rangle. \tag{4}$$

The creation and annihilation operators step up or down on the infinite ladder of Fock spaces as

$$a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle, \tag{5}$$

$$a|n\rangle = \sqrt{n}\,|n-1\rangle. \tag{6}$$

Here n stands for a non-negative integer number. The number operator is the Hermitian $N = a^\dagger a$, with $N|n\rangle = n|n\rangle$. These Fock states are orthonormal,

$$\langle n|m\rangle = \delta_{n,m}, \tag{7}$$

where $\delta_{n,m}$ stands for a Kronecker-delta. The resolution of the unity operator in terms of Fock states is given by

$$1 = \sum_{n=0}^{\infty} |n\rangle\langle n|. \tag{8}$$

So, these Fock states form a complete, orthonormal basis. The density matrix of the system can be expanded as

$$\rho = \sum_{n=0}^{\infty} p_n |n\rangle\langle n|, \tag{9}$$

where p_n stands for the probability that the quantum mechanical system is in state $|n\rangle$. Its normalization is

$$\mathrm{Tr}\rho = \sum_{n=0}^{\infty} p_n = 1. \tag{10}$$

The above formulas can be generalized with the help of coherent states, but with certain subtleties. Let us start first with definitions.

Coherent states are the eigenstates of the annihilation operator,

$$a|\alpha\rangle = \alpha|\alpha\rangle, \tag{11}$$

where α is a complex (c) number. Their algebraic properties are very interesting. For example, one can express them in terms of Fock states as

$$|\alpha\rangle = \exp(-|\alpha|^2/2) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \tag{12}$$

It is interesting to observe that, removing (detecting) one quantum from a coherent state does not change the probability that yet another quantum could be removed from such a state. This is in sharp contrast to the properties of Fock states where, once a particle is detected, the resulting Fock state is orthogonal to the Fock state before the detection. i.e., $\langle n|n-1\rangle = 0$.

The expansion of $|\alpha\rangle$ in terms of Fock states can be used to prove that these states are properly normalized, $\langle\alpha|\alpha\rangle = 1$. Although different coherent states are not orthogonal, i.e.,

$$|\langle\alpha|\beta\rangle|^2 = \exp(-|\alpha-\beta|^2), \tag{13}$$

they can, nevertheless, also be used to resolve unity as

$$1 = \int \frac{\mathrm{d}^2\alpha}{\pi} |\alpha\rangle\langle\alpha|, \tag{14}$$

where $d^2\alpha = d\mathrm{R}e[\alpha]\, d\mathrm{I}m[\alpha]$. It then follows that any state may be expanded linearly in terms of coherent states. Therefore, the most general light beam (of a given mode $\mathbf{k}$ and of a given polarization) can be described by a density operator of the following form:

$$\hat{\rho} = \int d^2\alpha d^2\alpha' \mathcal{P}_2(\alpha,\alpha')|\alpha\rangle\langle\alpha'|. \quad (15)$$

In many practical important cases, such as the thermal or the coherent radiation, the density matrix turned out to be in a diagonal representation when expanded in terms of the over-complete set of coherent states:

$$\hat{\rho} = \int d^2\alpha \mathcal{P}(\alpha)|\alpha\rangle\langle\alpha|. \quad (16)$$

A similar representation was proposed by Sudarshan [31] shortly after Glauber's work, which emphasized the advantage of this representation when evaluating expectation values for normal ordered products of creation and annihilation operators,

$$\mathrm{Tr}\hat{\rho}\hat{O} = \mathrm{Tr}\left\{\hat{\rho}(a^\dagger)^\lambda a^\mu\right\} = \int d^2\alpha \mathcal{P}(\alpha)(\alpha^*)^\lambda \alpha^\mu. \quad (17)$$

Consequently, the evaluation of expectation values of normal ordered operators in a quantum mechanical system, defined with the help of its density matrix, is reduced to the evaluation of expectation values of the quasi-probability distribution $\mathcal{P}(\alpha)$, defined over the complex plane of α. The Hermiticity of the density matrix implies that $\mathcal{P}(\alpha)$ is a real valued function, although it is not necessarily positive, due to the quantum nature of the field. This representation of the density matrix is frequently called as the Glauber-Sudarshan representation.

Glauber investigated a model of photoionization of a pair of atoms, labelled 1 and 2, lying at $\mathbf{r}_1$ and $\mathbf{r}_2$ within a light beam of sufficiently narrow spectral bandwidth and given polarization. Summing up the transition probabilities over final electron energies, there is no quantum mechanical restriction in defining the time at which each electron emission takes place. The probability density for ionization of atom 1 at time t_1 and for atom 2 at time t_2 can be written as

$$P_2(t_1,t_2) = P_1(t_1)P_1(t_2)C_2(t_1,t_2), \quad (18)$$

where $P_1(t)$ is the transition probability of each atom placed individually in the beam, and $C_2(t_1,t_2)$ is the two-particle correlation function.

Glauber pointed out, that for a coherent state, corresponding to $\mathcal{P}(\alpha) = \delta(\alpha-\beta)$, $C_2(t_1,t_2) = 1$, i.e., there is no HBT type of correlation in a coherent beam, which would be a reasonable model for the laser field. The density matrix of a thermal radiation with mean number of photons $\langle n\rangle$ can be written as

$$\hat{\rho} = \sum_{n=0}^{\infty} p_n|n\rangle\langle n|, \quad (19)$$

$$p_n = \frac{\langle n\rangle^n}{(1+\langle n\rangle)^{1+n}}. \quad (20)$$

This corresponds to a Gaussian distribution in terms of the $\mathcal{P}(\alpha)$ representation,

$$\mathcal{P}(\alpha) = \frac{1}{\pi\langle n\rangle}\exp(-|\alpha|^2/\langle n\rangle). \quad (21)$$

It is easy to show that for such a filtered, single mode, thermal radiation, the correlation function is

$$C_2 = 1 + |\exp(i\omega[t_2-t_1-(x_2-x_1)/c])|^2. \quad (22)$$

This corresponds to the classical limit, when modes are treated as forming a continuum with a stochastic noise. Note that for such a thermal light, the maximum value of the correlator is $C_2 = 2$, in a sharp contrast to the $C_2(t_1,t_2) \equiv 1$ value, that is characteristic of a coherent light.

VI. SQUEEZED CORRELATIONS IN PARTICLE FEMTOSCOPY

Squeezed states are generalized coherent states with very interesting quantum statistical properties. They can be obtained by means of the displacement operator method, as discussed below, by using a Bogoliubov-Valatin transformation, or even as generalized minimum uncertainty states [23]. Let us consider first the displacement operator method, and discuss their properties in terms of Bogoliubov transformation in the next subsection.

A. Squeezed coherent states

The displacement operator has the following properties

$$D^{-1}(\beta)aD(\beta) = a+\beta, \quad (23)$$

$$D^{-1}(\beta)a^\dagger D(\beta) = a^\dagger+\beta^*. \quad (24)$$

It is easy to show that acting with the first equality above on a vacuum state, the displacement operator $D(\alpha)$ creates a coherent state, i.e.,

$$D(\alpha)|0\rangle = |\alpha\rangle \quad (25)$$

On imposing the requirement that $D(\alpha)$ should be a unitary operator, this leads to

$$\begin{aligned} D(\alpha) &= \exp(\alpha a^\dagger - \alpha^* a) \\ &= \exp(-|\alpha|^2/2)\exp(\alpha a^\dagger)\exp(-\alpha^* a), \end{aligned} \quad (26)$$

where the second part expresses the displacement operator in a normal order, i.e., where all the creation operators stand to the left and all the annihilation operators stand to the right. Further discussion of the properties of these displacement operators, can be found in Refs. [32, 33].

The displacement operator is generalized to the squeeze operator $S(z)$ as

$$S(z) = \exp\{\frac{1}{2}(za^\dagger a^\dagger - z^* aa)\}, \quad (27)$$

$$z = r\exp(i\phi) \quad (28)$$

Squeezed states are defined in terms of displacement and squeeze operators as follows:

$$|\alpha,z\rangle = D(\alpha)S(z)|0\rangle, \quad (29)$$

where the ordering $D(\alpha)S(z)$ differs from $S(z)D(\alpha)$ by a change of parameters, being however equivalent.

B. Surprises on π^+-π^- Bose-Einstein correlations

In 1991, Andreev, Plümer and Weiner[34] pointed out the surprising existence of a new boson-antiboson quantum statistical correlation. The surprise was related to the fact that this type of correlation involved particle-antiparticle pairs, differently than the better known Bose-Einstein Correlations (BEC), which occurred between two identical particles. We can understand the origin of this effect in a simple way in terms of creation and annihilation operators, taking the π^0 case for illustration. For instance, the single-inclusive distribution is written as

$$N_1(\mathbf{k}_i) = \omega_{\mathbf{k}_i} \frac{d^3N}{d\mathbf{k}_i} = \omega_{\mathbf{k}_i} \langle \hat{a}^\dagger_{\mathbf{k}_i} \hat{a}_{\mathbf{k}_i} \rangle \,, \qquad (30)$$

and the two-particle distribution, after the decomposition that follows from Wick's theorem, is written as

$$\begin{aligned} N_2(\mathbf{k}_1, \mathbf{k}_2) &= \omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2} \langle \hat{a}^\dagger_{\mathbf{k}_1} \hat{a}^\dagger_{\mathbf{k}_2} \hat{a}_{\mathbf{k}_2} \hat{a}_{\mathbf{k}_1} \rangle \\ &= \omega_{\mathbf{k}_1} \omega_{\mathbf{k}_1} \{ \langle \hat{a}^\dagger_{\mathbf{k}_1} \hat{a}_{\mathbf{k}_1} \rangle \langle \hat{a}^\dagger_{\mathbf{k}_2} \hat{a}_{\mathbf{k}_2} \rangle + \langle \hat{a}^\dagger_{\mathbf{k}_1} \hat{a}_{\mathbf{k}_2} \rangle \langle \hat{a}^\dagger_{\mathbf{k}_2} \hat{a}_{\mathbf{k}_1} \rangle \\ &+ \langle \hat{a}^\dagger_{\mathbf{k}_1} \hat{a}^\dagger_{\mathbf{k}_2} \rangle \langle \hat{a}_{\mathbf{k}_1} \hat{a}_{\mathbf{k}_2} \rangle \} \,. \end{aligned} \qquad (31)$$

In Eq. (31), the first term corresponds to the product of the two single-inclusive distributions, the second one gives rise to the Bose-Einstein identical particle correlation, reflecting their last position just before being emitted. The third term, absent in the $\pi^\pm\pi^\pm$ case, is the responsible for the particle-antiparticle correlation (either $\pi^\pm\pi^\mp$ or $\pi^0\pi^0$, since π^0 is its own antiparticle). They are related to the expectation value of the annihilation (creation) operator, i.e., to $\langle \hat{a}^{(\dagger)}_{\mathbf{k}_1} \hat{a}^{(\dagger)}_{\mathbf{k}_2} \rangle \neq 0$, analogous to what is observed in two-particle squeezing in optics, where the averages are estimated using a density matrix that contains squeezed states, as briefly discussed in the previous section. Under the conditions as those usually considered in femtoscopic analyses, the last term in Eq. (31) vanishes. However, it is non zero if the Hamiltonian of the system is of the type $H = H_0 + H_1$, where H_0 is the free part in the vacuum, corresponding to final particles, and H_1 represents the interaction of quasi-particles, resulting in an effective shift of their masses. Alternatively, as in the pioneer work in Ref. [34], this could be similar to having a *chaotic superposition of coherent states* and a density matrix containing *squeezed states*.

Although that initial discussion by Andreev, Plümer and Weiner was not entirely correct, they clearly pointed out that the particle-antiparticle correlations would manifest themselves as an enhancement above unity of the correlation function, i.e., $C(\pi^+\pi^-) > 1$ and $C(\pi^0\pi^0) > 1$, reflecting particle-antiparticle quantum statistical effects.

In 1994, Sinyukov[35], also discussed a similar effect for $\pi^+\pi^-$ and $\pi^0\pi^0$ pairs, claiming that it would be due to inhomogeneities in the system, as opposed to homogeneity regions in HBT, which comes from a hydrodynamical description of the system evolution. He used Wick's theorem for expanding the two-particle inclusive distribution in terms of bilinear forms.

In 1996, Andreev and Weiner[36] elaborated further their original idea. They considered that in high energy collisions, a blob of strongly interacting pions, which was seen as a liquid, was formed and later suffered a sudden breakup, so that the pionic system having ground state and pioninc excitations was rapidly converted into free pions. In the moment of transition, they postulated that the in-medium creation and annihilation operators $(\hat{b}^\dagger, \hat{b})$ could be related to the corresponding free ones $(\hat{a}^\dagger, \hat{a})$, by means of a squeezing transformation

$$\begin{aligned} \hat{a} &= \hat{b}\cosh(r) + \hat{b}^\dagger \sinh(r) \\ \hat{a}^\dagger &= \hat{b}\sinh(r) + \hat{b}^\dagger \cosh(r) \,, \end{aligned} \qquad (32)$$

where $r = \frac{1}{2}\ln(E_{fr}/E_{in})$ is a squeezing parameter [37], E_{fr} and E_{in} are the asymptotic (free) energy and the in-medium energy, respectively.

In the same year, Asakawa and Csörgő[38] proposed a similar structure to this previous approach, but relating in-medium operators to free ones by means of a two-mode Bogoliubov transformation. They also proposed to observe hadron mass modification in hot medium by means of *Back-to-Back Correlations*, relating particle-antiparticle pair correlations to two-mode squeezed states.

There were a few more tentative works by the two groups but the correct approach was finally written in 1999, by Asakawa, Csörgő, and Gyulassy[39]. The squeezing parameter they proposed was somewhat similar to the one described by Eq. (32).

C. Back-to-back boson-antiboson correlations

The formalism developed by Asakawa, Csörgő and Gyulassy for squeezed bosons in an infinite medium can be summarized as follows. The in-medium Hamiltonian, H, is written as $H = H_0 - \frac{1}{2}\int d\mathbf{x}\, d\mathbf{y} \phi(\mathbf{x})\delta M^2(\mathbf{x}-\mathbf{y})\phi(\mathbf{y})$, where $H_0 = \frac{1}{2}\int d\mathbf{x} \left(\dot{\phi}^2 + |\nabla\phi|^2 + m^2\phi^2\right)$, is the free Hamiltonian, in the matter rest frame. The scalar field, $\phi(\mathbf{x})$, represents quasi-particles whose mass is modified by the medium, and propagate in a momentum-dependent way. The in-medium mass, m_*, is related to the vacuum mass, m, via

$$m_*^2(|\mathbf{k}|) = m^2 - \delta M^2(|\mathbf{k}|).$$

The mass-shift is assumed to be limited to long wavelength collective modes: $\delta M^2(|\mathbf{k}|) \ll m^2$ if $|\mathbf{k}| > \Lambda_s$. As a consequence, the dispersion relation is modified to $\Omega^2_{\mathbf{k}} = \omega^2_{\mathbf{k}} - \delta M^2(|\mathbf{k}|)$, where $\Omega_{\mathbf{k}}$ is the frequency of the in-medium mode with momentum $\mathbf{k}$.

The in-medium, thermalized annihilation (creation) operator is denoted by $\hat{b}_{\mathbf{k}}$ ($\hat{b}^\dagger_{\mathbf{k}}$), whereas the corresponding asymptotic operator for the observed quantum with four-momentum $k^\mu = (\omega_{\mathbf{k}}, \mathbf{k})$, $\omega^2_{\mathbf{k}} = m^2 + \mathbf{k}^2$ ($\omega_{\mathbf{k}} > 0$) is denoted by $\hat{a}_{\mathbf{k}}$ ($\hat{a}^\dagger_{\mathbf{k}}$). These operators are related by the Bogoliubov transformation, i.e., $\hat{a}_{\mathbf{k}_1} = c_{\mathbf{k}_1}\hat{b}_{\mathbf{k}_1} + s^*_{-\mathbf{k}_1}\hat{b}^\dagger_{-\mathbf{k}_1}$, which is equivalent to a squeezing operation. For this reason, $r_{\mathbf{k}}$ is called mode-dependent squeezing parameter. The relative and the average pair momentum coordinates are written as $q^0_{1,2} = \omega_1 - \omega_2$, $\mathbf{q}_{1,2} = \mathbf{k}_1 - \mathbf{k}_2$, $E_{i,j} = \frac{1}{2}(\omega_i + \omega_j)$, and $\mathbf{K}_{1,2} = \frac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2)$. For shortening the notation, the squeezed functions are denoted by $c_{i,j} = \cosh[r(i,j,x)]$ and $s_{i,j} = \sinh[r(i,j,x)]$, where

$$r(i,j,x) = \frac{1}{2}\log\left[(K^{\mu}_{i,j}u_{\mu}(x))/(K^{*\nu}_{i,j}(x)u_{\nu}(x))\right] = \frac{1}{2}\log\left[\frac{\omega_{k_i}(x)+\omega_{k_j}(x)}{\Omega_{k_i}(x)+\Omega_{k_j}(x)}\right] \quad (33)$$

is the squeezing parameter. Also, $n_{i,j}$ is the density distribution, which is taken as the Boltzmann limit of the Bose-Einstein distribution, i.e., $n^{(*)}_{i,j}(x) \approx \exp\{-[K^{(*)\mu}_{i,j}u_{\mu}(x)-\mu(x)]/T(x)\}$, where the symbol (*) implies the use of in-medium mass, whereas it is absent if there is no mass-shift.

In the cases of $\pi^0\pi^0$ or $\phi\phi$ correlations, where the boson is its own anti-particle, the full correlation function consists of a HBT part (related to the chaotic amplitude, $G_c(1,2)$) together with a BBC portion (related to the squeezed amplitude, $G_s(1,2)$), as shown below

$$C_2(\mathbf{k}_1,\mathbf{k}_2) = \frac{N_2(\mathbf{k}_1,\mathbf{k}_2)}{N_1(\mathbf{k}_1)N_1(\mathbf{k}_2)} = 1 + \frac{|G_c(1,2)|^2}{G_c(1,1)G_c(2,2)} + \frac{|G_s(1,2)|^2}{G_c(1,1)G_c(2,2)}. \quad (34)$$

The invariant single-particle and two-particle momentum distributions is given by

$$G_c(i,i) = G_c(k_i,k_i) = N_1(\mathbf{k}_i) = \omega_{\mathbf{k}_i}\langle\hat{a}^{\dagger}_{\mathbf{k}_i}\hat{a}_{\mathbf{k}_i}\rangle,$$
$$G_c(1,2) = \sqrt{\omega_{\mathbf{k}_1}\omega_{\mathbf{k}_2}}\langle\hat{a}^{\dagger}_{\mathbf{k}_1}\hat{a}_{\mathbf{k}_2}\rangle,$$
$$G_s(1,2) = \sqrt{\omega_{\mathbf{k}_1}\omega_{\mathbf{k}_2}}\langle\hat{a}_{\mathbf{k}_1}\hat{a}_{\mathbf{k}_2}\rangle. \quad (35)$$

For an infinite, homogeneous and thermalized medium, the part of the full correlation function in Eq. (34) that corresponds to the Back-to-Back Correlation is written as

$$C_{bosons}(\mathbf{k},-\mathbf{k}) = 1 + \frac{|c_{\mathbf{k}}s^*_{\mathbf{k}}n_{\mathbf{k}} + c_{-\mathbf{k}}s^*_{-\mathbf{k}}(n_{-\mathbf{k}}+1)|^2}{n_1(\mathbf{k})\,n_1(-\mathbf{k})}. \quad (36)$$

The effects of finite system size on the BBC are considered afterwards in the text. In Ref.[39], the influence of finite emission times is discussed, observing that the BBC in this case is suppressed when compared to the instant emission. Nevertheless, it was also shown that the maximum of the BBC for each value of $|\vec{k}|$, corresponding to $C_2(\mathbf{k},-\mathbf{k})$, still attained significant magnitude, in spite of considering the finite time suppression. We will see this more explicitly later.

D. Back-to-back correlations for fermions

Previously, the BBC was shown as a different type of correlation between boson-antiboson pairs, occurring if their masses were shifted. In 2001, T. Csörgő, Y. Hama, G. Krein, P. K. Panda and S. S. Padula demonstrated that a similar correlation existed between fermion-antifermion pairs[40], if their masses were modified in a thermalized medium. In the femtoscopic type of correlations, identical bosons have an opposite behavior as compared to identical fermions, resulting from the fact that quantum statistics suppresses the probability of observing pairs of identical fermions with nearby momenta, while it enhances such a probability in the case of bosons. However, regarding the Back-to-Back Correlations resulting from squeezed states, a very different situation occurs: fermionic BBC are positive and similar in strength to bosonic BBC. Besides, contrary to the the femtoscopic correlations, the BBC are unlimited.

The expressions in the fermion BBC case are similar to Eq.(30) and (31),

$$N_1(\mathbf{k}_i) = \omega_{\mathbf{k}_i}\langle\hat{a}^{\dagger}_{\mathbf{k}_i}\hat{a}_{\mathbf{k}_i}\rangle \;;\; \tilde{N}_1(\mathbf{k}_i) = \omega_{\mathbf{k}_i}\langle\hat{\tilde{a}}^{\dagger}_{\mathbf{k}_i}\hat{\tilde{a}}_{\mathbf{k}_i}\rangle\,, \quad (37)$$

$$N_2(\mathbf{k}_1,\mathbf{k}_2) = \omega_{\mathbf{k}_1}\omega_{\mathbf{k}_2}\langle\hat{a}^{\dagger}_{\mathbf{k}_1}\hat{\tilde{a}}^{\dagger}_{\mathbf{k}_2}\hat{\tilde{a}}_{\mathbf{k}_2}\hat{a}_{\mathbf{k}_1}\rangle\,. \quad (38)$$

In the above expressions, $\langle\hat{O}\rangle$ denotes the expectation value of the operator $\hat{O}$ in the thermalized medium and $\hat{a}^{\dagger},\hat{a},\hat{\tilde{a}}^{\dagger},\hat{\tilde{a}}$ are, respectively, creation and annihilation operators of the free baryons and antibaryons of mass M and energy $\omega_k = \sqrt{M^2+|\vec{k}^2|}$, which are defined through the expansion of the baryon field operator as $\Psi(\vec{x}) = (1/V)\sum_{\lambda,\lambda',\vec{k}}(u_{\lambda,\vec{k}}\hat{a}_{\lambda,\vec{k}} + v_{\lambda',-\vec{k}}\hat{\tilde{a}}^{\dagger}_{\lambda',-\vec{k}})e^{i\vec{k}.\vec{x}}$; V is the volume of the system, $u_{\lambda,\vec{k}}$ and $v_{\lambda',-\vec{k}}$ are the Dirac spinors, where the spin projections are $\lambda,\lambda' = 1/2,-1/2$. The in-medium creation and annihilation operators are denoted by $\hat{b}^{\dagger},\hat{b},\hat{\tilde{b}}^{\dagger},\hat{\tilde{b}}$. While the $\hat{a}$-quanta are observed as asymptotic states, the $\hat{b}$-quanta are the ones thermalized in the medium. They are related by a fermionic Bogoliubov-Valatin transformation,

$$\begin{pmatrix}\hat{a}_{\lambda,\mathbf{k}}\\ \hat{\tilde{a}}^{\dagger}_{\lambda',-\mathbf{k}}\end{pmatrix} = \begin{pmatrix}c_{\mathbf{k}} & \frac{f_{\mathbf{k}}}{|f_{\mathbf{k}}|}s_{\mathbf{k}}A\\ -\frac{f^*_{\mathbf{k}}}{|f_{\mathbf{k}}|}s^*_{\mathbf{k}}A^{\dagger} & c^*_{\mathbf{k}}\end{pmatrix}\begin{pmatrix}\hat{b}_{\lambda,\mathbf{k}}\\ \hat{\tilde{b}}^{\dagger}_{\lambda',-\mathbf{k}}\end{pmatrix}, \quad (39)$$

here $c_1 = \cos r_1$, $s_1 = \sin r_1$, and

$$\tan(2r_1) = -\frac{|\mathbf{k}_1|\Delta M(\mathbf{k}_1)}{\omega(\mathbf{k}_1)^2 - M\Delta M(\mathbf{k}_1)} \quad (40)$$

is the fermionic squeezing parameter. Note that in the fermionic case, the squeezing parameter is the coefficient of the sine and cosine functions, differently than in the bosonic cases, in which their hyperbolic counterparts appeared. In Eq. (39) A is a 2×2 matrix with elements $A_{\lambda_1\lambda_2} = \chi^{\dagger}_{\lambda_1}\sigma\cdot\hat{\mathbf{k}}_1\tilde{\chi}_{\lambda_2}$, where $\hat{\mathbf{k}}_1 = \mathbf{k}_1/|\mathbf{k}_1|$, χ is a Pauli spinor and $\tilde{\chi} = -i\sigma^2\chi$. Since r is real in the present case, we drop the complex-conjugate notation in the remaining of this section.

In order to evaluate the thermal averages above, the system is modelled as a globally thermalized gas o quasi-particles (quasi-baryons). In this description, the medium effects are taken into account through a self-energy function, which, for a spin-$\frac{1}{2}$ particle (we will focus on proton and anti-proton pairs), under the influence of mean fields in a many-body system, can be written as $\Sigma = \Sigma^s + \gamma^0\Sigma^0 + \gamma^i\Sigma^i$. In this expression, Σ^0 is a weakly momentum dependent function which, for locally thermalized systems being considered, has the role of shifting

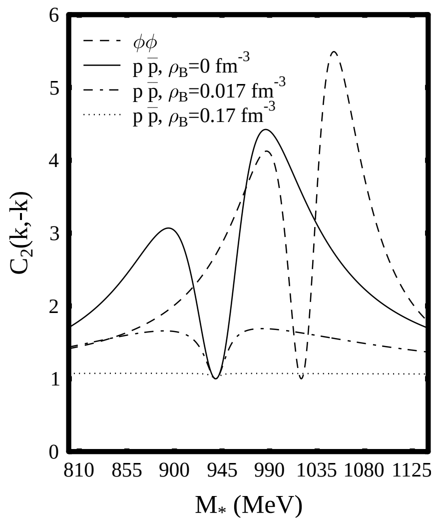

FIG. 1: The plot shows back-to-back correlations of $\bar{p}p$ (fBBC) and of ϕ-meson pairs (bBBC), as a function of the in-medium modified (p or ϕ) mass, m_*, for $|\vec{k}| = 800$ MeV/c. The dependence of the fBBC on the net baryon density is shown for three values of ρ_B. In both cases $T = 140$ MeV and $\Delta t = 2$ fm/c. The plots were extracted from Ref.[40].

the chemical potential, i.e., $\mu_* = \mu - \Sigma^0$. The vector part is very small and is neglected. The scalar part can be written as $\Sigma^s = \Delta M(\mathbf{k})$. Within these approximations the system can be described with a momentum-dependent in-medium mass, $m_*(\mathbf{k}) = m - \Delta M(|\mathbf{k}|)$.

We are mainly interested here in the study of the squeezed correlation function, which corresponds to considering only the joint contribution of the first and third terms of the r.h.s. of Eq. (34). In the fermionic case and for an infinite, homogeneous thermalized medium, the BBC part of the correlation function is written as

$$C^{(+-)}_{fermions}(\mathbf{k}_1, -\mathbf{k}_1) = 1 + \left[1 + (2\Delta t\ \omega_{\mathbf{k}})^2\right]^{-1} \times \left\{\frac{(1-n_{\mathbf{k}}-\tilde{n}_{\mathbf{k}})^2 (c_{\mathbf{k}} s_{\mathbf{k}})^2}{\left[c^2_{\mathbf{k}} n_{\mathbf{k}} + s^2_{\mathbf{k}}(1-\tilde{n}_{\mathbf{k}})\right]\left[c^2_{\mathbf{k}}\tilde{n}_{\mathbf{k}} + s^2_{\mathbf{k}}(1-n_{\mathbf{k}})\right]}\right\}, \quad (41)$$

where $n_{\mathbf{k}} = \frac{1}{\exp[(\Omega_{\mathbf{k}}-\mu_*)/T]+1}$; $\tilde{n}_{\mathbf{k}} = \frac{1}{\exp[(\Omega_{\mathbf{k}}+\mu_*)/T]+1}$, in terms of which the net baryonic density is written as $\rho_B = (g/V)\sum_{\mathbf{k}}\left(n_{\mathbf{k}} - \tilde{n}_{\mathbf{k}}\right)$. In Eq. (41) we have included a more gradual freeze-out by means of a finite emission interval, similarly to what was done in Ref[39], which has the effect of suppressing the BBC signal.

For a numerical study of the fermionic back-to-back correlations, fBBC, for simplicity we considered momentum independent in-medium masses, i.e., $m_* = M - \Delta M$. In Fig. (1) we show fBBC for $\bar{p}p$ pairs as a function of the in-medium mass m_*, for three values of the net baryonic density ρ_B: for the normal nuclear matter, one tenth of this value, and for the baryon free region, i.e., $\rho_B = 0$. We show in the same plot results for the bosonic case, bBBC, corresponding to ϕ meson pair, whose mass is close to the proton mass and was the example used in Ref.[39].

We see from Fig.(1) that fBBC and bBBC are, indeed, both positive correlations, with similar shape, and of the same order of magnitude. We also observe that fBBC is strongly enhanced for decreasing net baryonic density, being maximal for $\rho_B \approx 0$, i.e., for approximately equal baryon and anti-baryon densities.

E. Flow effects on back-to-back correlations

In the previous discussions, an infinite and homogeneous medium was considered. However, we know that the systems produced in high energy collisions, including the ones at RHIC, have finite sizes. Thus, it would be important to test if the BBC signal would survive when more realistic spatial and dynamical hypotheses were considered. For pursuing this purpose, we studied the effects on the squeezing parameter and on the back-to-back correlation of a finite size medium moving with collective velocity[41]. For this, a hydrodynamical ensemble was adopted, in which the amplitudes G_c and G_s in Eq. (34) and (35) were extended to the special form derived by Makhlin and Sinyukov [35],

$$G_c(1,2) = \frac{1}{(2\pi)^3}\int d^4\sigma_\mu(x) K^\mu_{1,2} e^{iq_{1,2}\cdot x}\{|c_{1,2}|^2 n_{1,2} + |s_{-1,-2}|^2 (n_{-1,-2}+1)\}, \quad (42)$$

$$G_s(1,2) = \frac{1}{(2\pi)^3}\int d^4\sigma_\mu(x) K^\mu_{1,2} e^{2iK_{1,2}\cdot x}\{s^*_{-1,2} c_{2,-1} n_{-1,2} + c_{1,-2} s^*_{-2,1}(n_{1,-2}+1)\}. \quad (43)$$

In Eq.(42) and (43) $d^4\sigma^\mu(x) = d^3\Sigma^\mu(x;\tau_f)F(\tau_f)d\tau_f$ is the product of the normal-oriented volume element depending parametrically on τ_f (the freeze-out hyper-surface parameter) and the invariant distribution of that parameter $F(\tau_f)$. We consider two possibilities: i) an instant freeze-out, corresponding to $F(\tau) = \delta(\tau - \tau_0)$; ii) an extended freeze-out, with a finite emission interval, with $F(\tau) = [\theta(\tau - \tau_0)/\Delta t]e^{-(\tau-\tau_0)/\Delta t}$. These cases lead, after performing the integration in $d\tau$ in Eq. (42) and (43) with weight ($E_{i,j}\ e^{-i2E_{i,j}\tau}$), respectively to: i) $(\omega_i + \omega_j)\ e^{-i(\omega_i+\omega_j)\tau_0}$; ii) $(\omega_i+\omega_j)[1 + [(\omega_i+\omega_j)^2\Delta t^2]^{-1/2}$.

According to the hydrodynamical solution, we can express the chemical potential as $\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T} - \frac{\mathbf{r}^2}{2R^2}$, being R the radius of the system, $T = T(x)$ the temperature of the system in each space-time point x, and μ_0 a constant. As an initial study, we assumed that the system expands with four-dimensional flow velocity $u^\mu = \gamma(1,\mathbf{v})$, where $\mathbf{v} = \langle u\rangle \frac{\mathbf{r}}{R}$. In the non-relativistic limit, we can write $\gamma = (1+\mathbf{v})^{-1/2} \approx 1 + \frac{1}{2}\mathbf{v}^2$, thus taking into account all terms up to $O(mv^2)$. We estimate the geometrical and dynamical effects for moderate flow on the BBC in the bosonic case, considering the in-medium changes of ϕ-mesons for illustration.

For small mass shifts, i.e. $\frac{(m-m_*)}{m} \ll 1$, the flow effects on the squeezing parameter are of fourth order, i.e., $O\left(\frac{Kin.\ energy}{m}\right)\left(\frac{\delta m^2}{m^2}\right)$. As a consequence, the flow effects on

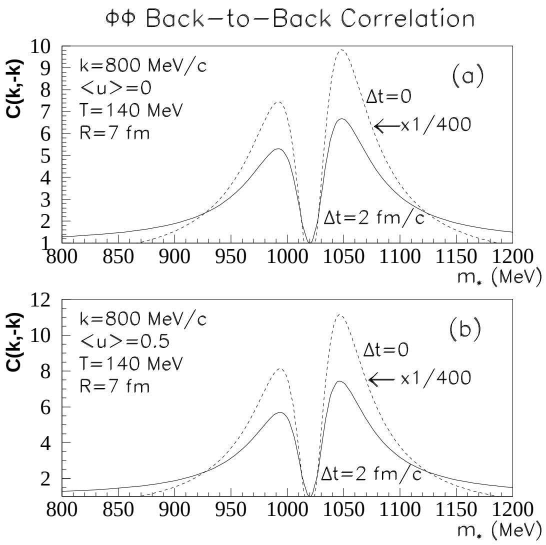

FIG. 2: The effect of a finite emission interval on the back-to-back correlation function, as compared to instant emission, is illustrated by the two plots. The dashed curves have been reduced by a factor of 400, and the solid curves correspond to the suppression by a finite emission duration, of about $\Delta t \simeq 2$ fm/c. The plot in (a) shows this effect in the absence of flow. The plot in (b) shows the corresponding result when flow is included, with $\langle u \rangle = 0.5$; the other parameters adopted to produce the curves are $R = 7$ fm/c, $T = 140$ MeV. The plots were extracted from Ref.[41].

$r_{i,j}$ can be neglected, and the factors $c_{i,j}$ and $s_{i,j}$ become flow independent, although they could still depend on the coordinate r through the shifted mass, m_* (e.g., through $T(x)$, as in hydrodynamics), which is not considered here.

For the sake of simplicity, and trying to keep the results as analytic as possible (for details, see Ref. [41]), we made the hypothesis that the mass-shift was independent on the position within the fireball. We further assumed that this last one had a sharp boundary, i.e., $\delta m = 0$ on the surface, and also the density vanishes outside the system volume. The spatial integration in Eq. (42) and (43) extends over the region where the mass-shift is non-vanishing, which is *not* infinitely large. For instance, in relativistic heavy ion collisions is a finite region $V \approx R^3 \approx (5-10)^3$ fm^3. We should keep in mind that the vacuum term in the integrand vanishes outside the mass-shift region, since it is proportional to $s_{i,j}$, which is identically zero in that region. On the other hand, the terms proportional to $n^{(*)}_{i,j}(x)$ are finite. Being so, we can extend the integration in Eq. (42) and (43) to infinity and, without much loss of generality, we can choose for V a Gaussian profile, $\exp[-\mathbf{r}^2/(2R^2)]$. This study was performed considering two situations: in one of them, the mass-shift was supposed to occur in the entire system region, which was considered as 3-D Gaussian with a circular cross-sectional area of radius $R = 7$ MeV/c at its width. The second case considered this volume split in two regions, the mass-shift occurring only in the internal one, with $R = 5$ MeV/c. Although we adopted several simplifying hypotheses for reducing the problem complexity and treat it analytically, the resulting expressions are still intricate, so that a

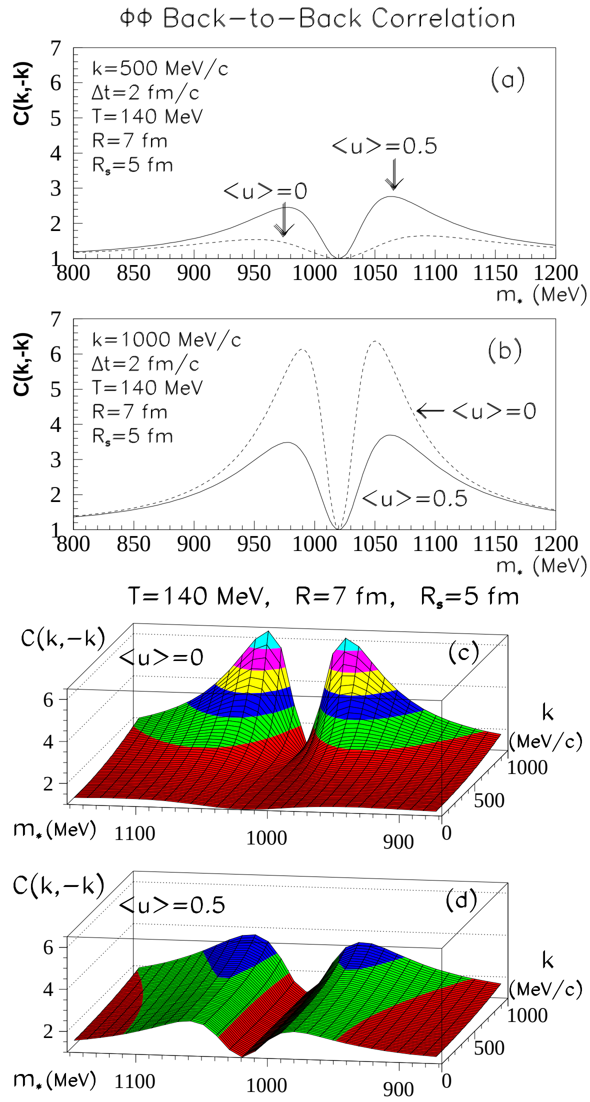

FIG. 3: (Color online) The plots in this panel are similar to the ones in Fig. 2. The main difference is that here we assumed that the mass-shift occurred only in a smaller part of the system volume. The back-to-back correlation is shown as a function of the shifted mass m_* on top, and as function of both m_* and the momentum of each particle ($\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}$), on bottom. The plots in parts (a) and (b) illustrate better the behavior of the BBC signal seen in parts (c) and (d), for $|\mathbf{k}| = 500$ MeV/c and for $|\mathbf{k}| = 1000$ MeV/c, respectively. In both cases, the dashed curve corresponds to $\langle u \rangle = 0$ and the solid curve, to $\langle u \rangle = 0.5$. In (c), the 3-D plot without flow ($\langle u \rangle = 0$) was considered, whereas in (d) a radial flow with $v = \langle u \rangle r/R = 0.5$ was included. The plots were extracted from Ref.[41].

graphical presentation of the results is more beneficial to the reader. A complete discussion with full analytical and numerical results can be found in Ref.[41].

Therefore, Fig. (2) and Fig. (3) summarize the main results of that study. The panel in Fig. (2) shows the case where the mass shift extends over the entire system region. It clearly illustrates the dramatic suppression effect of a finite emission duration, since the curves corresponding to instant emission had to be multiplied by a factor 1/400 in order to fit in the same plots. We can also see that the effect of flow, within the approximations used, also reduces the signal but in a moderate way.

For illustrating other interesting features found in this preliminary analysis and shown in Fig.(3), we chose the case with two-regions, as briefly delineated above. It also allows for some comparison with the result on Fig.(2). In parts (a) and (b) of Fig.(3), we can see that, depending on the momenta of the back-to-back pairs suffering the squeezing correlation, the signal is weaker when flow is present (for $|\mathbf{k}| \simeq 1000$ MeV/c), almost unaffected by flow (for $|\mathbf{k}| \simeq 750$ MeV/c), or even slightly stronger in the presence of moderate flow (for $|\mathbf{k}| < 500$ MeV/c), for the set of parameters chosen in those calculations. If flow is absent, however, a monotonically increase can be observed with increasing values of the pair momenta, $|\mathbf{k}|$. A more complete view of the behavior of the maximum of the $\phi\phi$ back-to-back correlation can be seen in the parts (c) and (d) of Fig.(3). It is evident that back-to-back correlation function has a steeper growth with momentum in the no flow case, for the same values of the shifted mass, m_*. The moderate flow picture considered in this study still causes the growth of the BBC with $|\mathbf{k}|$, but in a considerably smaller rate. With this broader panorama in mind, it is easier to understand the plots (a) and (b) of Figs. (2) and (3). Finally, we see that the strength of the signal is directly proportional to the size where the mass-shift occurs. Although the values of $|\mathbf{k}|$ were not the same in those two figures, but already knowing that the signal grows with increasing $|\mathbf{k}|$, we see that the strength in Fig. (2), where the system size is $R = 7$ fm, is bigger than the corresponding ones in Fig. (3), in which the squeezing region has $R_s = 5$ fm, even for $|\mathbf{k}| = 1000$ GeV/c.

Naturally, the above study was mainly a first step towards better understanding the nature and conditions of survival of the squeezing correlation, since it involved several approximations in its derivation. In particular, it was supposed that the squeezing occurred homogeneously throughout the system region, and independently on the particle momentum. Next steps will require a modelling instead of this assumption, which will provide the volume dependence of the squeezing in a more realistic way. Besides, we have presented merely the behavior of the maximum of the back-to-back correlation function. Is is under investigation how the signal behaves for finite systems and small values of the average pair momenta, which hopefully will furnish an optimized form of the BBC signal to be looked for experimentally.

F. Chiral dynamics and back-to-back correlations

This sub-section is based on ref. [42]. The environment generated in the mid-rapidity region of a high-energy nuclear collision might endow the pionic degrees of freedom with a time-dependent effective mass. This implies two-mode squeezing, but with a time-dependent squeezing parameter. Its specific evolution provides a mechanism for the production of back-to-back charge-conjugate pairs of soft pions which may present an observable signal of the non-equilibrium dynamics of the chiral order parameter. The suddenness of the transition to the asymptotic quanta is a condition that is released in this approach. The important point of the numerical investigations along such a model assumptions was that suddenness is not a mandatory requirement. The BBC signal seems to be strong enough to survive even the time-dependence of the effective mass caused by chiral dynamics, resulting in an adiabatic mass variation, modelled by mass oscillations within an exponentially decreasing envelope.

G. Experimental constraints on chiral dynamics as possible explanation for the RHIC HBT puzzle

Recently, Cramer, Miller, and collaborators have been investigating a relativistic quantum mechanical treatment of opacity and refractive effects, which seems able to describe STAR two-pion (HBT) correlation data and pion spectra in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV colliding energies at RHIC. This investigation suggested that an attractive, real part of the optical potential is the critical element needed to reproduce the transverse mass dependence of the sidewards and the outwards HBT radii at RHIC. This optical potential represents the strength of the interactions between the pions and the medium. Chiral dynamics in their model leads to a temperature and density dependent in-medium modification of the pion (pole as well as screening) mass. The results were found to be consistent with a system in which chiral symmetry was partially restored [43].

Cramer, Miller, and collaborators also suggested to experimentally check various new phenomena to see if their explanation was correct. For instance, they argued that a pionic version of the Ramsauer resonances lead to peaks in the HBT radius parameters R_o and R_s in the low momentum region, 15 MeV $< p_T <$ 65 MeV, and also predicted a peaking behavior in the same region in the transverse momentum spectrum of pions [43–45].

Apparently, there are several other tests that could be performed to verify if this connection of pion HBT radii to chiral dynamics is a correct and unique explanation. For example, if in-medium mass modification of pions reflecting their interactions with the medium is the reason for the effect, then similar interactions between kaons and the medium, although of different strength, should also be present. Hence, kaon and pion HBT radii would not be expected to show a similar transverse mass dependence. On the other hand, if the HBT radii are equal due to asymptotic properties of exact solutions of relativistic hydrodynamics, as proposed by the Buda-Lund hydro model [11], kaons, pions, and all other heavier particles should show the same effective, and approximately spherically symmetric, source size at large transverse masses, ($R_{out} \simeq R_{side} \simeq R_{long} \propto 1/\sqrt{m_t}$). Therefore, a calculation of the transverse momentum dependence of the pion HBT radii within this model, above the current upper limit of $p_T \sim 600$ MeV, compared to the PHENIX results on radii up to 1.2 GeV in transverse momentum, could help to clarify this point, when combined with a theoretical and experimental effort to do a similar study for kaons.

Nevertheless, if a chiral phase transition is the key element in the success of the Cramer-Miller approach, there should be a distinct probe of its manifestation. In this regard, and as discussed in this review on hadronic squeezed states, back-to-back correlations of particle-antiparticle pairs were shown to exist in all scenarios involving in-medium mass modification of hadrons, regardless of the details of the calculation (i.e., for bosons or fermions, expanding system, finite duration of particle emission, in-medium mass modification in only a fraction of the total volume, time dependent in-medium mass modification due to chiral dynamics). Therefore, the experimental determination of the in-medium mass modification by means of back-to-back particle-antiparticle correlations could supply this unambiguous answer.

VII. SUMMARY: A NEW CORRELATION SIGNATURE OF A FREEZE-OUT FROM A SUPERCOOLED QUARK-GLUON PLASMA

Currently available correlation data seem to exclude the possibility of a strong first order phase transition in Au+Au collisions at RHIC and in Pb+Pb collisions at top CERN SPS energies. The proposed signature of a possible second order QCD phase transition has not yet been investigated in detail, the excitation function of the Lévy index of stability needs to be determined experimentally. From lattice QCD calculations there is an indication, in case such a critical point is reached at a certain value of the center of mass energy, that above this point a cross-over type of transition should be expected. Presently, a cross-over type of equilibrium hadronization, or a sudden non-equilibrium hadronization mechanism, are both possible scenarios for hadron production in ultra-relativistic heavy ion collisions at RHIC. We have discussed the latter possibility here and focused on quantum optical characteristics of such a scenario, arguing that it leads to vanishing in-medium mass modifications of hadrons. As a consequence, the back-to-back correlated particle-antiparticle pairs should disappear at the onset of such a sudden deconfinement transition, characteristic of quark-gluon plasma hadronization and freeze-out from a deeply supercooled, negative pressure state. Nevertheless, if a chiral phase transition occurs in $\sqrt{s_{NN}} =$ 200 GeV Au +Au collisions at RHIC, these back-to-back correlated particle-antiparticle pairs should be present. In this regard, they could also be employed as a model independent experimental tool to test the validity of the Cramer-Miller approach.

Acknowledgments

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