Confinement by Design ?

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The configuration space of SU(N) gauge theory is restricted to orbits with vanishing Polyakov loops of non-trivial N-ality. A practical method of constraining to this orbit space $\mathcal{C}_0$ is found by implementing a certain axial-type gauge. It is shown that the representative of an orbit in $\mathcal{C}_0$ is unique in this gauge up to time independent *Abelian* gauge transformations. The restricted orbit space does not admit non-Abelian monopoles. As long as $\mathcal{C}_0$ is thermodynamically stable, the free energy of the constrained SU(N) gauge model is of order N^0 (even in the presence of dynamical quarks) and confinement is manifest for sufficiently large N. With a free energy of order N^0 and Polyakov loops that vanish by design, there is no transition that deconfines color charge in such an SU(N) model. However, a proliferation of massless hadronic states of arbitrary spin could lead to a Hagedorn transition[1] if the string tension vanishes at a finite temperature T_H. Constraining the orbit space to $\mathcal{C}_0$ can be viewed as a particular boundary condition, and T_H in general is above the first order deconfinement transition of the full theory at T_d. Between T_d and T_H a superheated confining phase may exist for $SU(N>2)$. Perturbation theory in $\mathcal{C}_0$ is sketched. It does not suffer from the severe IR-divergences observed by Linde[2]for the ordinary high temperature expansion. Correlations of the lowest transverse Abelian Matsubara modes develop a renormalization group invariant pole of second order at vanishing spatial momentum transfer when $T=T_H$. The latter could be associated with linear confinement.

Keywords: Finite temperature perturbation theory; QCD; Confinement; Hagedorn transition

I. INTRODUCTION

The Wilson action of SU(N) Lattice Gauge Theory (LGT) is invariant under multiplication of all temporal links on any particular time slice by an element of the center of the structure group. In a phase where this (global) center symmetry (CS) is not spontaneously broken, static color charges in representations with non-trivial N-ality have infinite free energy and are not part of the physical spectrum. Deconfinement above a temperature T_d implies that the CS is broken spontaneously for $T > T_d$. However, the CS by itself does not imply confinement of static color charges in representations to trivial N-nality. CS apparently does not explain confinement when this symmetry is explicitly broken by dynamical fields in representations that have non-vanishing N-ality. One possibility is to include dynamical quarks in the fundamental representation of SU(N), while another[3] is to consider enlarged structure groups for the gluons that include generators in irreducible representations of $SU(N)$ with non-trivial N-ality[16]. In both types of extensions of an SU(N) Yang-Mills theory CS is broken explicitly.

However, the CS of SU(N)-LGT divides gauge orbits into those for which any (global) center transformation is equivalent to a gauge transformation and the complementary orbit space. Within perturbation theory, the set of CS-configurations can be constructed from a CS-symmetric ground state. It has previously been shown[4] that the free energy of $SU(N)$ in this case is of order N^0 to arbitrary perturbative order. This holds even in the presence of fields that break the center symmetry explicitly. For sufficiently large N a free energy of order N^0 implies the absence of non-trivial SU(N)-multiplets from the physical spectrum. One would have *perturbative* confinement at large N if the CS-symmetric vacuum were thermodynamically stable at weak coupling. Unfortunately the spontaneously broken phase generally is thermodynamically preferred at weak coupling. The following investigation explores the possibility of accessing the metastable "superheated" confining phase[5] by a constrained model that may be perturbatively analyzed near a second order Hagedorn transition[1] where the effective string tension vanishes smoothly.

II. THE CONFINING ORBIT SPACE $\mathcal{C}_0$

Consider a Euclidean SU(N)-LGT that is periodic in temporal ($\mu=4$) lattice direction with links $U_\mu(\mathbf{x},\tau)=U_\mu(\mathbf{x},\tau+n_T)$ in the fundamental representation of SU(N). Polyakov loops $L_n(\mathbf{x})$,

$$
\begin{aligned}
&L_n(\mathbf{x}) = \mathrm{Tr}U^n(\mathbf{x},\tau) \quad \text{with} \\
&U(\mathbf{x},\tau) = U_4(\mathbf{x},\tau)U_4(\mathbf{x},\tau{+}1)\ldots U_4(\mathbf{x},\tau{+}n_T{-}1),
\end{aligned} \tag{1}
$$

are non-contractible gauge invariant Wilson lines that wind around the temporal direction. Under a global center transformation with element $z\in Z_N\subset SU(N)$, Polyakov loops acquire a phase $L_n(\mathbf{x})\to z^n L_n(\mathbf{x})$. All the Polyakov loops $L_{0<n<N}(\mathbf{x})$ therefore must vanish for a center-symmetric orbit. The converse generally will not hold, since *all*, not just straight, Wilson loops of non-trivial N-ality that wind about the temporal direction vanish for a center-symmetric orbit. The Polyakov loops of Eq.(1) are just the "shortest" of these and correspond to the free energy of static color charges only.

However, it is relatively straightforward to constrain the configuration space of an SU(N)-LGT to the set of orbits $\mathcal{C}_0$ with vanishing Polyakov loops,

$$
\mathcal{C}_0 = \{\{U\}; L_n(\mathbf{x})=0,\ \forall\mathbf{x}\wedge N>n>0\} \supset \mathcal{C}_{CS}\,. \tag{2}
$$

The subset $\mathcal{C}_0$ of the full orbit space is gauge invariant because Polyakov loops are. This set in principle may be selected by inserting in the lattice measure (gauge-invariant) δ-functions that enforce the constraints. But it is much easier to solve them explicitly in an axial-type gauge.

Consider a representative configuration of an orbit whose temporal links are all unity except for those on the $\tau = 0$ slice. Such a representative can always be found by an appropriate gauge transformation[6]. Using time-independent gauge transformations, one can exploit the temporal periodicity of the lattice and bring all temporal links of the $\tau = 0$ slice to diagonal form. Since the group of permutations of N elements is a subgroup of SU(N), the diagonal elements of the temporal links can furthermore be conveniently *ordered*. The result of this gauge fixing procedure is that any orbit may be represented by a configuration $\{U\}$ of the form,

$$\begin{aligned} \{U\} &= \{U_\mu(\mathbf{x},\tau);\ U_4(\mathbf{x},\tau\neq 0) = 1\!\!1, \\ &\quad U_4(\mathbf{x},0) = \mathrm{diag}(e^{i\phi_1(\mathbf{x})},\dots,e^{i\phi_N(\mathbf{x})}) \\ &\quad \text{with } -\pi \leq \phi_1(\mathbf{x}) \leq \cdots \leq \phi_N(\mathbf{x}) < \pi\}\ . \end{aligned} \tag{3}$$

The gauge fixing is ambiguous since *time independent Abelian* gauge transformations do not change the form of the representative in Eq.(3). If at some point $\mathbf{x}$ any two of the N phases $\phi_k(\mathbf{x})$ coincide, the ambiguity is enlarged to a *non-Abelian* group. The non-Abelian gauge-fixing singularity is physical and identifies monopoles[7].

Since *all* Polyakov loops of a $\mathcal{C}_0$-orbit vanish, we have for its representative of the form in Eq.(3),

$$0 = \mathrm{Tr}U^n(\mathbf{x},\tau) = \mathrm{Tr}U_4^n(\mathbf{x},0) = \sum_{k=1}^{N} e^{in\phi_k(\mathbf{x})}, \tag{4}$$

for all $\mathbf{x}$ and $0 < n < N$. Including the requirement that $\sum_{k=1}^N \phi_k(\mathbf{x}) = 0$ for $U \in$ SU(N), the N-1 equations of Eq.(4) determine the N phases at each site $\mathbf{x}$. Taking into account the ordering in Eq.(3), the phases $\phi_i(\mathbf{x})$ of a representative of an $\mathcal{C}_0$-orbit are uniquely given by the N roots of unity. The representative of an orbit in $\mathcal{C}_0$ in this gauge thus has the special form,

$$\begin{aligned} \{U\}_{\mathcal{C}_0} &= \{U_\mu(\mathbf{x},\tau);\ U_4(\mathbf{x},\tau\neq 0) = 1\!\!1, \\ U_4(\mathbf{x},0) &= \mathrm{diag}(e^{i\pi\frac{1-N}{N}}, e^{i\pi\frac{3-N}{N}},\dots,e^{i\pi\frac{N-1}{N}})\}\ . \end{aligned} \tag{5}$$

Restricting the configuration space to orbits in $\mathcal{C}_0$ thus amounts to a (unique) determination of the temporal links in a particular axial gauge. Since none of the N phases coincide, the residual gauge group is that of time-independent *Abelian* gauge transformations only. Orbits in $\mathcal{C}_0$ therefore do not have gauge-fixing singularities associated with monopoles. Constraining the configuration space to $\mathcal{C}_0$ in this sense gives a "classical" confinement picture: Infinite free energy of static color-electric charges and a (complete) condensation of non-Abelian monopoles.

It perhaps is remarkable that orbits in $\mathcal{C}_0$ with minimal Wilson action are center symmetric[4]. The Wilson action is minimal (vanishes), if the 1×1 Wilson loop around every plaquette $P_{\mu\nu}(\mathbf{x},\tau)$ is unity. For a representative of an orbit in $\mathcal{C}_0$ of the form given by Eq.(5), the $P_{0i}(\mathbf{x},\tau\neq 0)$-plaquettes in this case imply that spatial links do not depend on τ. Minimal action of the $P_{0i}(\mathbf{x},\tau = 0)$-plaquette, then requires that,

$$\begin{aligned} U_i(\mathbf{x},0)U_4(\mathbf{x},0)U_i^{-1}(\mathbf{x},0)U_4^{-1}(\mathbf{x},0) &= 1\!\!1 \\ \Leftrightarrow\ [U_i(\mathbf{x},0),U_4(\mathbf{x},0)] &= 0\ . \end{aligned} \tag{6}$$

Since the temporal links are of the form given in Eq.(5), Eq.(6) implies that all spatial links are Abelian. Together with our previous conclusion that they do not depend on τ, we have that configurations $\{U\}_{\min} \subset \mathcal{C}_0$ with vanishing Wilson action may be represented by,

$$\begin{aligned} \{U\}_{\min} &= \{U_\mu(\mathbf{x},\tau);\ U_4(\mathbf{x},\tau\neq 0) = 1\!\!1, \\ &\quad U_4(\mathbf{x},0) = \mathrm{diag}(e^{i\pi\frac{1-N}{N}}, e^{i\pi\frac{3-N}{N}},\dots,e^{i\pi\frac{N-1}{N}})\ , \\ &\quad U_i(\mathbf{x},\tau) = \mathrm{diag}(e^{i\phi_{i1}(\mathbf{x})},\dots,e^{i\phi_{iN}(\mathbf{x})})\}\ . \end{aligned} \tag{7}$$

Since temporal and spatial links commute and the spatial links do not depend on time, the minimal action orbits represented by Eq.(7) are invariant under the global center symmetry. Indeed, any Wilson line (not just straight ones) with non-trivial N-ality vanishes for minimal action configurations in $\mathcal{C}_0$.

III. SU(N)-LGT ON $\mathcal{C}_0$

We have seen that restricting the configuration space of an SU(N)-LGT with periodic boundary conditions to orbits in $\mathcal{C}_0$ amounts to specifying the temporal links on a particular time slice in a certain gauge. The constraint thus is akin to imposing particular boundary conditions in a spin model. Configurations of minimal Wilson action in $\mathcal{C}_0$ furthermore are center-symmetric. Since the deconfinement transition is of first order[8] for $SU(N>2)$, it thus may be possible to perturbatively access the superheated confining phase of $SU(N)$ simply by constraining the set of orbits to $\mathcal{C}_0$. Although only static color charges have infinite free energy and are excluded from the physical spectrum in $\mathcal{C}_0$, the free energy of any moving color charge should be even greater. It then is difficult to see how non-trivial color multiplets could arise for configurations in $\mathcal{C}_0$ without violating positivity. We already know[4] that for sufficiently large N only colorless states contribute to the pressure to all orders in perturbation theory about a CS-configuration of minimal action. In the constrained configuration space $\mathcal{C}_0$, the center-symmetric ground state is *perturbatively* stable by design (whereas perturbation theory in the full LGT prefers a ground state that spontaneously breaks the CS). Within[9] $\mathcal{C}_0$, the perturbative regime could be smoothly connected to the confining one. For temperatures above the deconfinement temperature T_d of the full LGT, the constrained model should describe the superheated confining phase up to a second order phase transition at $T = T_H \geq T_d$ at which the effective string tension vanishes smoothly. Non-perturbatively, T_H would correspond to a Hagedorn transition at which the entropy of colorless high-energy (bound) states overwhelms the suppression due to the Boltzmann factor. For $T > T_H$, the constrained model possibly has only a perturbative interpretation. Nevertheless, if it describes a confining phase below T_H, the transition at $T_H \geq T_d$ is of second order[5] and should be visible in the perturbative behavior of 1PI-vertices. The existence of such a (hidden) Hagedorn transition at large N has also been inferred[10] by considering perturbative Yang-Mills on $S_3 \times S_1$ with an infrared cutoff due to the (small) radius of the spatial three-sphere S_3.

The string tension of SU(2)-LGT vanishes smoothly at the deconfinement transition and $T_H \simeq T_d$ for $SU(2)$ but $T_H > T_d$ for SU(N$\geq$3) where the deconfinement transition is of first order[8]. Lattice simulations in the metastable phase strongly indicate[5] the existence of a finite Hagedorn temperature at large N. Independent support for a Hagedorn transition in restricted SU(2)-LGT is derived from the fact that the specific entropy rises sharply at $T_H \sim T_d$ and appears to be bounded only by the infra-red cutoff of the finite lattice[11].

Since all temporal links can be specified in a particular gauge, LGT on $\mathcal{C}_0$ is best described by its transfer matrix. A spatially constant but time-dependent Abelian gauge transformation of the configurations of Eq.(3), shows that one may equivalently consider representatives in a slightly unconventional axial gauge in which the temporal links are constant matrices given by,

$$\begin{aligned} U_4(\mathbf{x},\tau) &= G = e^{2\pi i\theta/n_T}, \text{ with} \\ \theta &= \mathrm{diag}(\tfrac{N-1}{2N}, \tfrac{N-3}{2N}, \ldots, \tfrac{3-N}{2N}, \tfrac{1-N}{2N})\ . \end{aligned} \tag{8}$$

Two major differences to ordinary axial gauge $U_4(\mathbf{x},\tau) = \mathbb{1}$ perhaps are worth mentioning:

i) any configuration satisfying Eq.(8) represents an orbit in $\mathcal{C}_0$ and every orbit in $\mathcal{C}_0$ on a periodic lattice has a representative whose temporal links are given by Eq.(8). By contrast, the orbit space of ordinary axial gauge is that with *maximal* Polyakov loops.

ii) The residual gauge group in the present case is that of static gauge transformations in the (Abelian) Cartan subgroup of SU(N) only. The residual gauge group of ordinary axial gauge on the other hand includes static non-Abelian gauge transformations.

Identifying a configuration of spatial links $\{U_\mu(\mathbf{x})\}$ on a time slice with a basis vector $|U\rangle$ of Hilbert space, the hermitian transfer matrix[12] of Yang-Mills LGT constrained to $\mathcal{C}_0$ has elements,

$$\begin{aligned} \langle U'|\mathcal{T}|U\rangle &= e^{\mathrm{ReTr}\Phi(\{U',U\})}, \text{with} \\ \Phi(\{U',U\}) &= \beta_T \sum_{\mathbf{x},i} U_i'^{\dagger}(\mathbf{x}) G^{\dagger} U_i(\mathbf{x}) G + \\ &\quad + \beta_s \sum_{j>i} (P_{ij}(\mathbf{x}) + P'_{ij}(\mathbf{x}))/2 \end{aligned} \tag{9}$$

where $P_{ij}(\mathbf{x}) = U_i(\mathbf{x})U_j(\mathbf{x}+a\hat{\mathbf{i}})U_i^\dagger(\mathbf{x}+a\hat{\mathbf{j}})U_j^\dagger(\mathbf{x})$ is the plaquette in the spatial ij-plane at $\mathbf{x}$ ($P'_{ij}(\mathbf{x})$ is similarly defined with $U \to U'$). $\beta_T = \frac{a}{g^2 a_T}$ and $\beta_s = \frac{a_T}{g^2 a}$ are inverse coupling constants related to the temporal- (a_T) and spatial- (a) spacing of the lattice by dimensional transmutation. The dependence of the kinetic term on the matrix G of Eq.(8) ensures that only $\mathcal{C}_0$ configurations are generated by the transfer matrix of Eq.(9). The transfer matrix in Eq.(9) differs from that of Creutz in the bilinear kinetic term only. The matrix $G \neq \mathbb{1}$ of Eq.(8) leads to vanishing Polyakov loops, whereas they are maximal for the transfer matrix in ($U_4 = \mathbb{1}$) axial gauge obtained by Creutz[6].

The conjecture that a second order Hagedorn transition occurs at a finite temperature T_H in the constrained model suggests that the transfer matrix of Eq.(9) is positive definite for $T < T_H$ only. For $T < T_H$, the transfer matrix of Eq.(9) in this case defines a hermitian Hamiltonian H, with $\mathcal{T} = e^{-a_T H}$, whose spectrum and eigenstates have a physical interpretation[12]. Above T_H, the transfer matrix perhaps develops negative eigenvalues and the LGT on $\mathcal{C}_0$ ceases to be unitary on sufficiently large physical volumes.

IV. THE CONTINUUM MODEL ON $\mathcal{C}_0$

Although simulating the LGT defined by Eq.(9) may be of interest on its own, the present motivation for restricting to $\mathcal{C}_0$ is that a second order (Hagedorn) transition at T_H could make a *perturbative* investigation of the *confining phase* of this model possible when $T \simeq T_H$. The character of the perturbative series on $\mathcal{C}_0$ in fact changes dramatically at a finite temperature T_H.

In the limit of a divergent correlation length, the continuum model on $\mathcal{C}_0$ can be found by formally expanding the spatial LGT-connection $U_j(\mathbf{x},\tau) = e^{iagV_j(\mathbf{x},\tau)}$ in powers of the spatial lattice spacing a. If we decompose the hermitian connection V_j into diagonal (Abelian) and off-diagonal (coset) parts as,

$$\begin{aligned} V_j &= A_j + W_j, \text{ with} \\ W_j^{ab} &= \begin{cases} V_j^{ab} & \text{,for } a \neq b \\ 0 & \text{,for } a = b\ , \end{cases} \end{aligned} \tag{10}$$

the critical Yang-Mills action corresponding to Eq.(9) becomes,

$$\begin{aligned} S = \int d\mathbf{x} \int_0^{1/T} d\tau\, \mathrm{Tr}\{ &(\partial_4 A_i)^2 + (D_4 W_i)^2 + \\ &+ \tfrac{1}{2} G_{ij} G_{ij} + \eta(\mathbf{x})(\partial_i A_i)\}\ . \end{aligned} \tag{11}$$

Here

$$\begin{aligned} G_{ij} &= \partial_i A_j - \partial_j A_i + \\ &\quad + D_i W_j - D_j W_i + ig[W_i, W_j] \\ D_i W_j &= \partial_i W_j + ig[A_i, W_j] \\ D_4 W_j &= \partial_4 W_j + 2\pi i T[\theta, W_j]\ , \end{aligned} \tag{12}$$

with the constant diagonal connection θ given in Eq.(8). The matrix components of the last line in Eq.(12) thus are,

$$(D_4 W_j)^{ab} = \partial_4 W_j^{ab} + i T_N (a-b) W_j^{ab}\ , \tag{13}$$

with a rescaled temperature $T_N = 2\pi T/N$. The covariant temporal derivative in Eq.(11) thus generates color-dependent

Matsubara-frequencies for the coset fields. Since none of these vanish at finite temperature, Linde's observation[2] that the perturbative series of a non-Abelian gauge theory develops uncontrollable infra-red divergences at finite temperature, does not hold on $\mathcal{C}_0$.

To have a well-defined continuum model, the residual gauge invariance of Eq.(9) under *Abelian* gauge transformations was used to eliminate the τ-independent longitudinal part of the *Abelian* connection $\mathbf{A}$. In Eq.(11) a time-independent Lagrange multiplier $\eta(\mathbf{x})$ enforces the gauge condition,

$$0 = \int_0^{1/T} d\tau\, \partial_i A_i(\mathbf{x},\tau)\ . \tag{14}$$

Note that ghosts decouple in this gauge. The representative in this gauge is unique and the Gribov ambiguity[13] does not arise. It is a "physical" gauge in the sense that (the decoupled) unphysical degrees of freedom are not dynamical. Although distinct from Coulomb-like gauges[13, 14], Eq.(14) also preserves manifest invariance under translations and spatial rotations[17].

A. Perturbative Continuum Propagators at Finite Temperature

Since gluons at finite temperature are periodic fields in temporal direction, it is convenient to consider their Fourier components,

$$V_j^{ab}(\mathbf{x},\tau) = \sum_{n=-\infty}^{\infty}\int \frac{\mathbf{dk}}{(2\pi)^d} V_j^{ab}(\mathbf{k},n) e^{i(2\pi n T\tau + \mathbf{k}\cdot x)},$$
$$\text{with } V_j^{ab}(\mathbf{k},n) = V_j^{ba*}(-\mathbf{k},-n)\ . \tag{15}$$

Note that the gauge condition of Eq.(14) is a constraint on the diagonal gauge field of vanishing Matsubara frequency only. It eliminates longitudinal modes and in Fourier space implies that $\mathbf{k}\cdot\mathbf{A}(\mathbf{k},0) = 0$. The quadratic terms of Eq.(11) then lead to the following non-trivial tree level propagators,

$$\begin{aligned}
\langle A_i(\mathbf{k},0)A_j(-\mathbf{k},0)\rangle &= \frac{\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}}{\mathbf{k}^2} \\
\langle A_i(\mathbf{k},n\neq 0)A_j(-\mathbf{k},-n)\rangle &= \frac{\delta_{ij} + \frac{k_i k_j}{(T_N N n)^2}}{(T_N N n)^2 + \mathbf{k}^2} \\
\langle W_i^{ab}(\mathbf{k},n)W_j^{cd}(-\mathbf{k},-n)\rangle &= \\
= \delta^{bc}\delta^{ad} & \frac{\delta_{ij} + \frac{k_i k_j}{(T_N(Nn+a-b))^2}}{(T_N(Nn+a-b))^2 + \mathbf{k}^2} \\
&,
\end{aligned} \tag{16}$$

where the (trivial) color dependence of the Abelian propagators has been suppressed. A QED-like, Abelian Ward Identity ensures that radiative corrections to the $n = 0$ part of $\langle AA\rangle$ are transverse and that the corresponding two-point vertex function vanishes at $\mathbf{k}^2 = 0$ for any temperature. This Ward Identity also implies that $g^2\langle AA\rangle\big|_{n=0}$ is a renormalization group invariant function. To one loop in a minimal subtraction scheme that analytically continues the number of spatial dimensions one finds[15],

$$g^2\langle A_i(\mathbf{k},0)A_j(-\mathbf{k},0)\rangle = \frac{\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}}{\mathbf{k}^2\Gamma(\mathbf{k}^2)} \text{ with,} \tag{17}$$
$$\Gamma(\mathbf{k}^2) = \beta_0 \ln\frac{4\pi T^2 e^{\frac{1}{\beta_0 g^2}}}{N^2\mu^2 e^{\gamma_E + \frac{1}{11}}} - \frac{N^3\zeta(3)\mathbf{k}^2}{1920\pi^4 T^2} + O(\mathbf{k}^4)\ .$$

μ here is the renormalization scale of the scheme, γ_E is Euler's constant and $\beta_0 = 11N/(48\pi^2)$ is the leading coefficient of the β-function of purely gluonic SU(N), $\beta(g) = -\beta_0 g^3 - \beta_1 g^5 \ldots$. At the (renormalization group invariant) temperature,

$$T_H = \frac{N\mu e^{\frac{\gamma_E}{2} + \frac{1}{22} - \frac{24\pi^2}{11Ng^2}}}{2\sqrt{\pi}}\ , \tag{18}$$

the Abelian correlation function of Eq.(17) develops a second order pole at vanishing spatial momentum transfer $\mathbf{k}^2 = 0$ and the long-range behavior of the model changes qualitatively. In coordinate space, the double pole at vanishing momentum transfer corresponds to a linearly rising potential. Contrary to confinement of static color charges by a linearly rising instantaneous Coulomb-interaction[13, 14], the present correlation is between transverse components of time-independent Abelian color-*currents*.

V. CONCLUSION

The configuration space $\mathcal{C}_0$ of orbits with vanishing Polyakov loops defined in Eq.(2) includes all center-symmetric orbits and may be selected by imposing the particular axial-type gauge of Eq.(8). This is to be contrasted with the more common axial gauge $U_4 = \mathbb{1}$, which in fact constrains the configuration space to orbits with maximal Polyakov loops. The residual gauge freedom in our case is just that of *Abelian* gauge transformations that do not depend on time. The latter were used to eliminate the longitudinal component of static Abelian gauge fields in a linear gauge for which ghosts decouple (see Eq.(14)). The Gribov problem[13] does not arise and each orbit of $\mathcal{C}_0$ is uniquely represented in this gauge. The gauge condition manifestly preserves all the symmetries of the finite temperature theory.

A perturbative treatment of the continuum model at finite temperature T reveals that only gluon propagators in the Abelian subgroup generally are truly massless. All other perturbative 2-point functions are finite at vanishing momentum transfer. The severe infrared divergences of high orders of perturbation theory observed by Linde[2] do not occur in $\mathcal{C}_0$ and there is no principal obstruction for constructing the finite temperature perturbative series. At extreme temperatures, the only low-lying degrees of freedom are the Abelian gluons and the dimensionally reduced model apparently describes a Coulombic phase. We on the other hand know[4] that the free energy for large N in fact is $O(N^0)$ to all orders in perturbation theory. It evidently depends on the order the limits are taken whether a Coulombic phase is realized at high temperatures and large N.

A QED-like Ward Identity implies that the time-independent Abelian propagator is transverse and that the anomalous dimension of $gA_i(\mathbf{k},0)$ vanishes. In one-loop approximation the long-range behavior of the model is characterized by the renormalization group invariant temperature T_H of Eq.(18). For $T > T_H$ transverse Abelian modes are massless in perturbation theory. At $T = T_H$ the time-independent part of the Abelian gluon propagator develops a double pole at vanishing momentum transfer. The perturbative interactions at this point are of particularly long range. The transition between these two perturbative phases is associated with a negative norm state that becomes massless at $T = T_H$. This transition thus *is not* the first order deconfinement transition of $SU(N > 2)$ at T_d. As discussed earlier, $T_H \geq T_d$ probably is the temperature associated with a second order Hagedorn transition from a "superheated" confined phase to one whose string tension vanishes, allowing the formation of low-mass gauge invariant states of arbitrary spin. Note that for temperatures just above T_H, the series in Eq.(17) leads to a (Landau) pole at low spatial momentum transfers – an indication, supported by lattice simulations[11], that the model is thermodynamically unstable in this regime. Below T_H on the other hand, the perturbative analysis does not show an instability due to low-energy modes and hints at the possibility that $T < T_H$ may be accessible by perturbing about the (marginally) confining model at $T = T_H$. A two-loop calculation would determine T_H in terms of an asymptotic scale like $\Lambda_{\overline{MS}}$.

Models that statistically constrain non-Abelian Coulomb gauge to configurations within the Gribov horizon[13, 14], also generate long-range correlations that vanish as $\mathbf{k}^{-4}$. However, such long-range Coulomb interactions are instantaneous and occur in the temporal component of the correlation function. Instantaneous long-range Coulomb interactions tend to confine at all temperatures[14]. We here restricted the orbit space ab initio to $\mathcal{C}_0$ and were able to avoid Gribov's[13] as well as Linde's[2] problem. The analysis of primitive vertices was entirely perturbative and the physical scale T_H was not introduced as a stochastic constraint parameter, but rather represents a transition temperature.

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[15] the detailed calculation will be provided elsewhere.
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[17] At finite temperature all these gauge conditions are manifestly covariant since the $O(d+1)$-invariance of Euclidean space-time is explicitly broken to the group of spatial rotations $O(d)$.