Gluon Saturation and Proton - Anti-Proton Cross Sections

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We study proton - anti-proton cross sections in the framework of an updated minijet eikonal model. We propose a different scheme for fixing the parameters, in which we make use of the measured minijet cross section. We compare the results obtained with the GRV98, MRST98, CTEQ6-L and KLN gluon distributions. The latter includes gluon saturation effects. We conclude that in the very high energy regime the use of the KLN distribution improves significantly the behavior of the cross sections. However this improvement is due to the shape of the KLN gluon density and has little to do with saturation effects.

Keywords: Gluon saturation; Minijet eikonal model; Hadronic cross sections

I. INTRODUCTION

The growth of hadronic total cross sections was theoretically predicted many years ago [1] and observed in many experiments at CERN, Fermilab [2, 3] and, more indirectly, in cosmic rays .

Computing total cross sections as a function of the collision energy is one of the great unsolved problems of QCD. Unlike processes which are computed in perturbation theory, calculating total hadronic cross sections appears to be an intrinsically non-perturbative procedure.

In the absence of a pure QCD description, phenomenological models have been used to compare experimental data with theoretical schemes. For a long time the main theoretical approaches to total cross sections were the Regge-type models and the QCD-inspired models. In the latter, the (too fast!) energy rise of the total cross sections is driven by the increasing number of the low x gluon-gluon collisions. These models [4–6] need to be embedded in an eikonal formalism to soften the violent energy rise of the mini-jet cross sections. Of course, this is only one among several unitarization methods (for a discussion see, for example [7]). Some other QCD inspired models based on non-perturbative physics, such as the stochastic vacuum model, may lead to no energy independent cross sections [8].

Even after eikonalisation the predicted energy rise is stronger than the gentle one observed experimentally. Attempts to further tame this rise were advanced by the mini-jet supporters in [9], invoking the increasing soft gluon emission by valence quarks in hadron collisions at increasing energies. Here we discuss another possible mechanism to be included in the eikonal mini-jet model (EMM): gluon saturation due to recombination.

The same gluon recombination which was found to be responsible for reducing the growth of the gluon distribution might prevent the total cross section from growing too fastly with energy, violating the Froissat bound. Of course, from the pure idea to its implementation there is a long way. One important result was presented in [10], where it was shown that the cross section of a very small and neutral color dipole colliding against a hadronic target at fixed impact parameter follows the Froissat behavior at aymptotic energies. In this particular case, gluon saturation in the target was enough to unitarize the scattering amplitude. Inspite of this remarkable result, this is a very special case and the extension to larger, colored dipoles summed over all impact parameters still needs some modelling. For real projectiles and targets the unitarization of the total cross section still must be done in an ad-hoc way, as done, for example, in the eikonal formalism.

II. THE EIKONAL MINI-JET MODEL

At high energies there is a significant increase of the number of the gluons inside the hadron and hadronic cross sections are dominated by the mini-jets coming from gluon-gluon interactions.

The perturbative expression of the jet cross section is given by:

$$\sigma_{\rm jet} = \frac{1}{2}\int dx_1\, dx_2 \int_{P_0} dp_T^2\; G(x_1,Q^2)\; G(x_2,Q^2)\; \hat{\sigma}_{gg} \qquad (1)$$

where $G(x,Q^2)$ is the gluon distribution in the proton extracted from deep inelastic scattering (DIS), x and $\hat{\sigma}_{gg}$ are the proton momentum fraction x and the elementary gluon-gluon cross section respectively. There are different parametrizations for these distribution functions given, for example, by the collaborations GRV [11], MRST [12] and CTEQ [13]. $P_0 = p_{Tmin}$ is a parameter which defines the energy scale at which semi-hard interactions start and perturbative QCD is applicable. The increase of the number of gluons in the high energy region ($x << 1$) makes these functions increase very rapidly at small x and hence the cross section (1) violates the

Froissart bound:

$$\sigma_{pp}(s \to \infty) \propto log^2(s/s_0) \tag{2}$$

In Fig. 1 we compare the distributions GRV98, MRST98 and CTEQ6-L. We can see that in the small x region they start to be very different from each other. Using these parametrization in (1) we have computed the corresponding mini-jet cross sections and compared them with the experimental data [14], as shown in Fig. 2. In this figure we have adjusted P_0 in order to describe the data. We can see that, although all of them are able to reproduce the data there is a clear trend of growing too fast. The early mini-jet models tried to apply (1) directly to the total hadronic cross sections. This exercise is updated in Fig. 3 where a compilation of data [3] is shown and also the Froissat limit (2). As anticipated in the introduction this simple use of pQCD does not work.

In order to ensure unitarity, the most common procedure is to utilize an eikonal formalism, evaluating the eikonal in the two-dimensional transverse impact parameter space $\vec{b}$. The total cross section is given by [4]:

$$\sigma_{\text{tot}} = 4\pi \int_0^\infty db\, b \left[1 - cos\chi^I(b,s)\, e^{-\chi^R}(b,s)\right] \tag{3}$$

where $\chi^I(b,s)$ and $\chi^R(b,s)$ are the the imaginary and real part of the eikonal function respectively. The function $\chi(b,s)$ is usually split into a soft and a hard piece:

$$\chi(b,s) = \chi_{hard}(b,s) + \chi_{soft}(b,s) \tag{4}$$

and each of these pieces has a real and an imaginary part. The soft part of the eikonal comes from a parametrization valid at lower energies, in the range $20-60$ GeV [4] and the hard one contains the mini-jet cross section and the impact parameter dependence.

$$\chi = \frac{1}{2} A(b)\, \sigma_{jet} \tag{5}$$

where $A(b)$ is given by the Fourier transform of the electromagnetic form factor and is the same for the soft and hard parts of χ.

III. GLUON SATURATION

In high energy experiments we expect to observe the nonlinear behavior of QCD. In this regime, the growth of parton distributions should saturate and we should observe the state called "Color Glass Condensate" (CGC) [15]. In fact, signals of parton saturation have already been observed both in ep deep inelastic scattering at HERA and in deuteron-gold collisions at RHIC [16]. However, the observation of this new regime still needs confirmation. In the saturation regime the gluon distributions are no longer given by the parametrizations used above, which only contain the DGLAP (linear) evolution. Instead, they are the solutions of non-linear evolution equations. However, these solutions are not yet known and one has to use parametrizations. Karzeev, Levin and Nardi (KLN) [17] developed a model for $G(x)$ that simulates the saturation effects and generates a distribution function that has been used to describe the new data from RHIC, in studies of multiplicity and rapidity distribution of charged particles. A common feature of all saturation models is the existence of a scale that separates the dense and dilute regions of the hadrons. It is known as the saturation scale and has been parametrized as [18]:

$$Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x}\right)^\lambda \tag{6}$$

The KLN distribution function is:

$$xG(x,Q^2) = \begin{cases} \frac{1}{\alpha_s(Q^2)} S Q^2 (1-x)^4 & \text{if } Q^2 < Q_s^2 \\ \frac{1}{\alpha_s(Q^2)} S Q_s^2(x) (1-x)^4 & \text{if } Q^2 > Q_s^2 \end{cases} \tag{7}$$

where S is the proton area and x_0, Q_0 and λ are the parameters of the model fitted from RHIC data: $x_0 = 0.3\, 10^{-4}$, $Q_0^2 = 0.3\, GeV^2, \lambda = 0.288$. The distribution above contains α_s (which is small) in the denominator, which can cancel the α_s factor in $\hat{\sigma}$. This is typical of the non-linear regime, where we have to deal with weak couplings but strong fields. The alluded cancellation casts some doubts on the use of the collinear factorization formula. In fact, the collinear factorization is violated in many cases in the context of saturation physics. Fortunately, for many cases of interest, an expression analogous to (1) is valid, in which we have to replace $G(x)$ by the unintegrated (in the gluon transverse momentum) gluon distribution $\phi(x,k_T^2)$. This is called k_T factorization and was proven to hold in many cases. As shown in [19] k_T factorization is valid for gluon production and is violated in quark production, especially in p-A and A-A collisions. Since we are addressing mostly gluon-gluon interactions (with subsequent gluon production) and only $p-\overline{p}$ collisions we shall assume that k_T factorization holds. Moreover, as shown in [20], k_T and collinear factorization are equivalent at the leading twist level. Given the exploratory nature of this study, we shall assume that (1) holds also for distributions like (7).

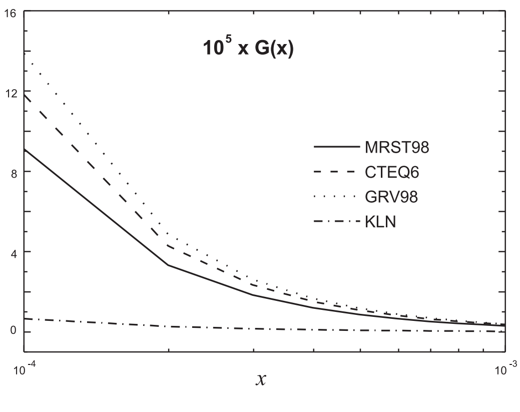

FIG. 1: The gluon distribution function given by GRV98, MRST98, CTEQ6-L and KLN at $Q^2 = 10^4\, GeV$

The KLN distribution is designed to be valid at very low x and its most interesting feature is the comparatively mild behavior in the low x region. It has been used to study gluon

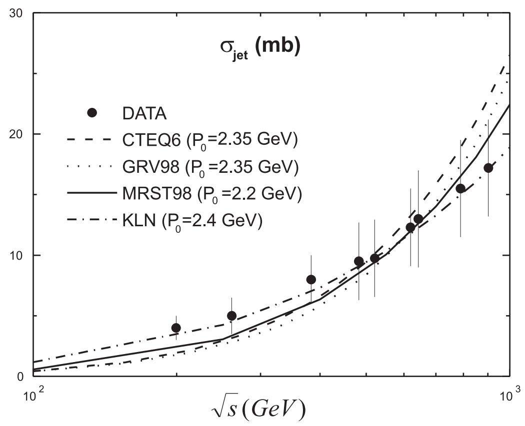

FIG. 2: The mini- jet cross section calculated by Eq. (1) compared to UA1 experimental data [14]

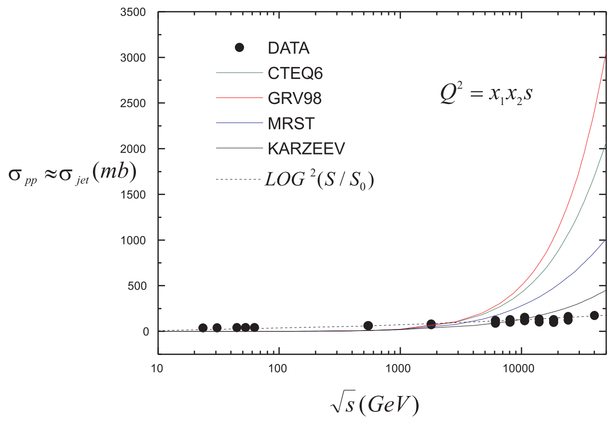

FIG. 3: The cross section calculated by Eq. (1) compared to pp total cross section data [14]

production through the fusion $g+g \to g$ at lower scales $Q^2 \simeq 10$ GeV^2. We shall use the KLN distribution at much higher scales, $Q^2 = x_1 x_2 s$, GeV^2 and therefore we should perform the DGLAP evolution of (7). However we are going to postpone it for a future work. The results without evolution are nevertheless meaningful because the "effective dominant scale", i.e., the one which contributes the most to the integrals in (1) and (3) is not so large, since the gluon distributions are peaked at small values of x, which lead to relatively small values of Q^2 in most of the cases. In Fig. 1 we compare Eq. (7) with the other gluon densities. The difference seems to be very large, of one order of magnitude already at $x = 10^{-4}$. Of course part of it is due to the lack of DGLAP evolution, which is known to enhance the small x region of $G(x)$. However part of this low x behavior is really due to the physical input of (7).

Using the KLN distribution in (1) we obtain a good description of the mini-jet cross section, as shown in Fig.2 and we can improve a lot the description of total cross sections, as it can be seen in Fig. 3.

However a really good fit of data is found only using KLN in (3), as shown in Fig. 4. Notice that here we fit first the mini-jet cross section (in Fig. 2), fixing P_0, and then fit minimum

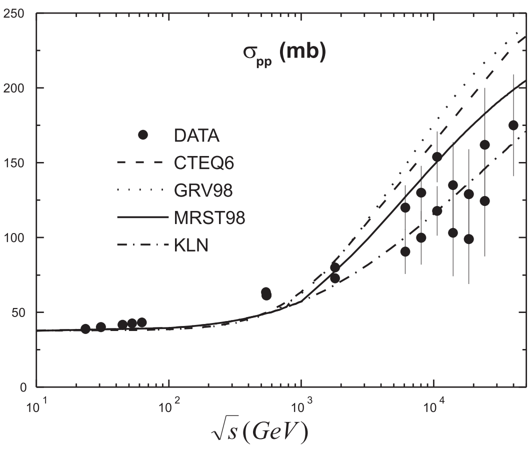

FIG. 4: The total cross section calculated with the EMM (3) compared to the experimental data [3]

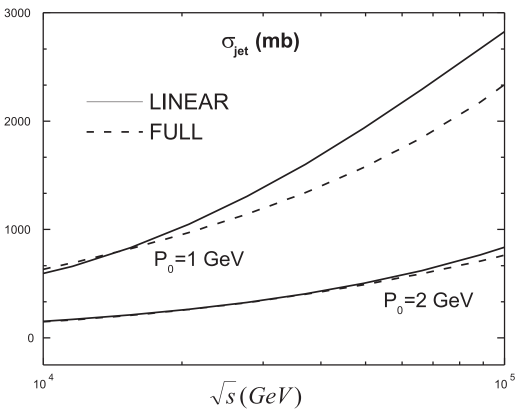

FIG. 5: The mini-jet cross section calculated with the KLN distribution function, with (dashed lines) and without (solid lines) saturation effects.

bias cross section data (in Fig. 4). This is different from what is done in Refs. [4, 5], where the information contained in mini-jet cross sections [14] is not used and all parameters are fixed together fitting the total cross sections. Of course, this conclusion should be better grounded after a least χ^2 fit, which we leave for the future.

Looking at Figs. 3 and 4 we would be tempted to conclude that we are already observing saturation effects in the total $p-\overline{p}$ cross section, since the KLN distribution gives the best fits of the available data. In order to check this conjecture, we have repeated the calculations using only the second line of (7). When $Q^2 > Q_s^2$ we are in the linear regime and no saturation effects are present. The result obtained is shown in Fig. 5 where we compare the cross sections obtained with only the linear part (solid line) and with the two parts (dashed line). The difference between them tells us how important are the saturation effects in this observable. The answer depends on the cut-off P_0. As it can be seen, a significant difference appears only at very high energies and only for a small cut-off. As expected, non-linear effects tend to deplete the cross section, but their magnitude is small. The good agreeement between KLN results and data, shown in the figures, can be attributed to its initial shape rather than to saturation.

To summarize we have used an eikonal mini-jet model to study the behavior of the total hadronic cross section with energy. We have updated previous versions of the EMM in some aspects: we have used more recent versions of the standard gluon parametrizations; we have used the measured mini-jet cross sections [14] to improve the fit and we tested (for the first time in this kind of model) the KLN distribution, which turns out to give the best description of data. This first success suggests that, after the proper incorporation of the DGLAP evolution, which was not included here, the KLN distribtuion may become competitive to the study of total hadronic cross sections in the very high energy limit. Finally, we have observed that, inspite of the phenomenological success of the KLN distribution, saturation effects are very small in the energies considered.

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