Chiral Transition and the Quark-Gluon Plasma

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We discuss a few recent results for the equation of state of strongly interacting matter and the dynamics of chiral phase transition. First, we consider the cases of very high densities, where weak-coupling approaches may in principle give reasonable results, and very low densities, where we use the framework of heavy-baryon chiral perturbation theory. We also speculate on the nature of the chiral transition and present possible astrophysical implications. Next, we discuss the kinetics of phase conversion, through the nucleation of bubbles and spinodal decomposition, after a chiral transition within an effective field theory approach to low-energy QCD. We study possible effects resulting from the finite size of the expanding system for both the initial and the late-stage growth of domains, as well as those effects due to inhomogeneities in the chiral field which act as a background for the fermionic motion. We also argue how dissipation effects might dramatically modify the picture of explosive hadronization.

Keywords: Quarks; Gluons; Quark-gluon plasma

I. INTRODUCTION

During the last decade, the investigation of strongly interacting matter under extreme conditions of temperature and density has attracted an increasing interest. In particular, the new data that started to emerge from the high-energy heavy ion collisions at RHIC-BNL, together with an impressive progress achieved by finite-temperature lattice simulations of Quantum Chromodynamics (QCD), provide some guidance and several challenges for theorists.

In fact, lattice QCD results [1] suggest that strongly interacting matter at sufficiently high temperature undergoes a phase transition (or a crossover) to a deconfined quark-gluon plasma (QGP). Despite the difficulties in identifying clear signatures of a phase transition in ultrarelativistic heavy ion collisions, recent data from the experiments clearly point to the observation of a new state of matter [2].

All this drama takes place in the region of nonzero temperature and very small densities of the phase diagram of QCD. On the other hand, precise astrophysical data appear as a new channel to probe strongly interacting matter at very large densities. Compact objects, such as neutron stars, whose interior might be dense enough to accomodate deconfined quark matter, may impose strong constraints on the equation of state for QCD at high densities and low temperatures [3]. Moreover, several preliminary results from the lattice at nonzero values for the quark chemical potential, μ, start to appear [1].

In this paper we briefly review a few recent results for the equation of state of strongly interacting matter and for the dynamics of the chiral phase transition. First, we consider the case of cold and dense QCD. In the extreme cases of very high densities, where weak-coupling approaches may in principle give reasonable results, and very low densities, where we use the framework of heavy-baryon chiral perturbation theory, we can draw a reasonably clear picture of the equation of state. At the end, we discuss results that point to important corrections brought about by a nonzero strange quark mass. We also speculate on the nature of the chiral transition and present possible astrophysical implications. Next, we discuss the kinetics of phase conversion, through the nucleation of bubbles and spinodal decomposition, after a first-order chiral transition within an effective field theory approach to low-energy QCD. We study possible effects resulting from the finite size of the expanding system for both the initial and the late-stage growth of domains, as well as those effects due to inhomogeneities in the chiral field which act as a background for the fermionic motion. We also consider dissipation effects which might dramatically modify the picture of explosive hadronization.

II. EQUATION OF STATE FOR COLD AND DENSE QCD

A. High-density limit

Let us first consider the case of cold and very dense strongly interacting matter. For high enough values of the quark chemical potential, there should be a quark phase due to asymptotic freedom. In this regime of densities, one is in principle allowed to use perturbative QCD techniques [4–8], which may be enriched by resummation methods and quasiparticle model descriptions [8–11], to evaluate the thermodynamic potential of a plasma of massless quarks and gluons (see Figure 1, for the perturbative result). Different approaches seem to agree reasonably well for $\mu >> 1$ GeV, and point in the same direction even for $\mu \sim 1$ GeV and smaller, where we are clearly pushing perturbative QCD far below its region of applicability. However, at some point between $\mu \approx 313$ MeV, and $\mu \approx 1$ GeV, one has to match the equation of state for quark matter onto that for hadrons.

As we argued in [6, 7], depending on the nature of the chiral transition there might be important consequences for the phenomenology of compact stars. For instance, in the case of a strong first-order chiral transition, a new stable branch may appear in the mass-radius diagram for hybrid neutron stars, representing a new class of compact stars (see Figure 2). On

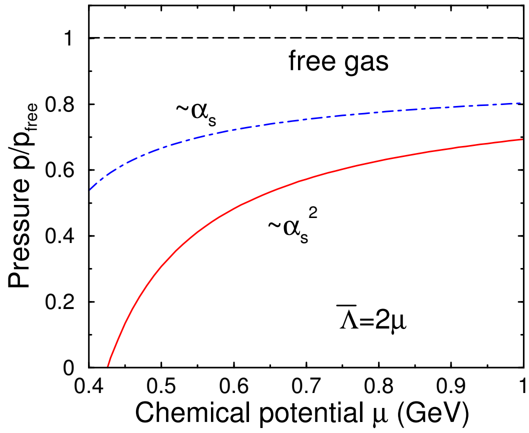

FIG. 1: Pressure in units of the free gas pressure as a function of the quark chemical potential from finite-density perturbative QCD.

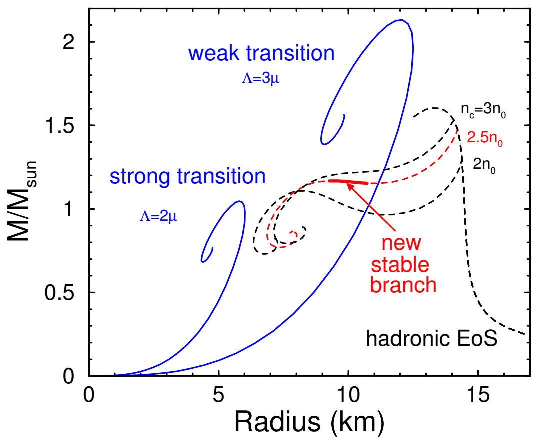

FIG. 2: Mass-radius diagram.

the other hand, for a smooth transition, or a crossover, one finds only the usual branch, generally associated with pulsar data. This is an important issue for the ongoing debate on the radius measurement of the isolated neutron star candidate RX J1856.5-3754, which might be a quark star [14]. For details, see Refs. [6] and [7].

B. Low-density limit

For pure neutron (asymmetric) matter, which will play the role of hadrons at "low" density here, Akmal, Pandharipande and Ravenhall [15] have found that to a very good approximation we have, up to $\sim 2n_0$, the following energy per baryon:

$$\frac{E}{A} - m_N = \frac{\varepsilon}{n} - m_N \approx 15\,\text{MeV}\left(\frac{n}{n_0}\right), \tag{1}$$

which is approximately linear in the baryon density, n. Here, $n_0 \sim 0.16$ baryons/fm^3 is the saturation density for nuclear matter. From this relation, we can extract the pressure:

$$\frac{p_{hadron}}{p_{free}} = \frac{n^2}{p_{free}}\frac{\partial}{\partial n}\left(\frac{\varepsilon}{n}\right) \approx 0.04\left(\frac{n}{n_0}\right)^2. \tag{2}$$

From these results, we see that: (i) even at "low" densities we have a *highly* nonideal Fermi liquid, since free fermions would give $\varepsilon/n - m \sim n^{2/3}$; (ii) energies are very small on hadronic scales (if we take f_π, as a "natural scale"); (iii) energies are small not only for nuclear matter (nonzero binding energy), but even for pure neutron matter (unbound). Then, this might be a generic property of baryons interacting with pions, etc., and not due to any special tuning.

In order to investigate the pion-nucleon interaction at nonzero, but low, density, we considered the following chiral Lagrangian [16] (see also [17] and [18])

$$\begin{aligned}\mathcal{L} &= \overline{\psi}_i\left[i\partial\!\!\!/ - m_N + \mu\gamma^0 - \frac{g_A}{2f_\pi}\gamma_5\gamma^\mu\vec{\tau}\cdot(\partial_\mu\vec{\pi})\right]\psi_i \\ &+ \mathcal{L}_\pi^0 + \left[\begin{matrix}\text{other meson}\\ \text{terms}\end{matrix}\right] + \left[\begin{matrix}\text{higher-order}\\ \text{terms}\end{matrix}\right], \end{aligned}\tag{3}$$

where $\mathcal{L}_\pi^0$ is the free Lagrangian for the pions, $\vec{\pi}$, ψ_i represent nucleons (in n_s species), and μ is the chemical potential for the nucleons. From the Goldberger-Treiman relation we have:

$$g_A = \left(\frac{f_\pi}{m_N}\right) g_{\pi NN} \tag{4}$$

and, from the Particle Data Group, $m_N = 939\,$MeV, $m_\pi = 135\,$MeV, $f_\pi = 130\,$MeV, and $g_{\pi NN} = 13.1\,(g_A = 1.81)$

The goal is to compute the nucleon and the pion one-loop self-energy corrections due to the medium up to lowest order in the nucleon density (only nonzero μ contributions) by using the technique of heavy-baryon chiral perturbation theory. Therefore, we adopt a non-relativistic approximation:

$$\omega_p \approx m_N + \frac{\vec{p}^{\,2}}{2m_N} + \cdots \tag{5}$$

$$\mu \approx m_N + \frac{p_f^2}{2m_N} + \cdots \tag{6}$$

Moreover, we assume that external legs are near the mass-shell $(p_0+\mu)^2 - \omega_p^2 \approx 0$, that pions are dilute, that we have small values of Fermi momentum, and consider only leading order in (p_f/m_N), (p_f/f_π), etc.

One-loop calculations within this framework provide the following result for f_π: as the Fermi momentum, p_f, increases – restoring chiral symmetry – f_π must go down. Indeed, from chiral perturbation theory, we obtain ($a \equiv (g_A^2/48\pi^2)$):

$$\begin{aligned}\frac{f_\pi(p_f)}{f_\pi} &= 1 - a\frac{p_f^3}{m_N f_\pi^2} + \cdots = \\ &= 1 - (15/939)\,(n/n_0) + \cdots \end{aligned}\tag{7}$$

Then, from Brown-Rho scaling [19] one would expect that all quantities should scale in a uniform fashion. However, we obtain the following result for the nucleon mass [16]:

$$\frac{m_N(p_f)}{m_N} = 1 + a\,\frac{p_f^3}{m_N f_\pi^2} - \tag{8}$$
$$- \;\frac{a}{8}\,\frac{m_\pi^2 p_f}{m_N f_\pi^2}\left\{1 + \left(\frac{m_\pi^2 + p_f^2 - p^2}{4p_f^2}\right)\log\left[\frac{m_\pi^2+(p_f+p)^2}{m_\pi^2+(p_f-p)^2}\right]\right\}$$

We can draw some tentative conclusions about the hadronic pressure. To leading order, the non-ideal terms in the pressure are proportional to Σ_0, defined through the nucleon self-energy on mass shell $\Sigma(p^0_{ms}, p) \sim -[(ip^0_{ms} + \mu)\gamma^0 + i\gamma^i p_i + m]\Sigma_0(p)$, which is very small [18]. At higher order, even if corrections to the pion propagator are large, their effect on the nucleon propagator, and the free energy, can still be small, as a large correction to a small number. Thus the possibility of a hadronic phase with a small pressure, required for a new class of quark stars, remains viable.

In order to be able to make clear predictions for the phenomenology of compact stars, and to have a better understanding of this region of the QCD phase diagram, one has to find a way to describe the intermediate regime of densities in the equation of state, where perturbative calculations do not work. The study of effective field theory models might bring some insight to this problem.

C. Role of nonzero quark mass

For almost twenty years the effects of a nonzero strange quark mass on the equation of state of cold and dense QCD were considered to be negligible (less than 5%), thereby yielding only minor corrections to the mass-radius diagram of compact stars. Even the most recent QCD approaches [6, 8–11] generally neglected quark masses and the presence of a color superconducting gap as compared to the typical scale for the chemical potential in the interior of compact stars, ~ 400 MeV and higher. However, it was recently argued that both effects should matter in the lower-density sector of the equation of state [13]. In fact, although quarks are essentially massless in the core of quark stars, the mass of the strange quark runs up, and becomes comparable to the typical scale for the chemical potential, as one approaches the surface of the star.

By computing the thermodynamic potential to first order in α_s, and including the effects of the renormalization group running of the coupling and strange quark mass, we showed that the corrections can be very large (up to 25%), and dramatically affect the structure of compact stars, as can be seen from the modifications of the mass-radius diagram [20].

The effects of the finite strange quark mass on the total pressure and energy density for electrically neutral quark matter (plus electrons) are given in Fig. 3. There we show results for 3 light flavors and running coupling, corresponding to the case considered in [6], and for 2 light flavors and one massive flavor, with both running coupling and strange quark mass (which reaches $m_s \sim 137$ MeV at $\mu = 500$ MeV).

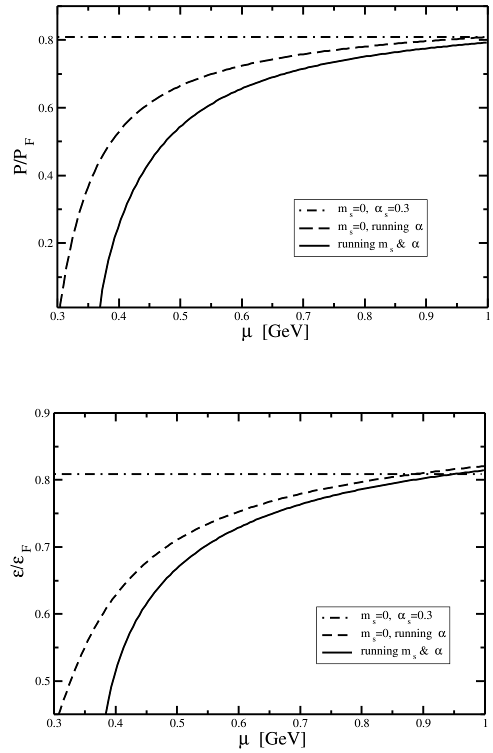

FIG. 3: Pressure and energy density scaled by Fermi values for $\bar{\Lambda} = \frac{2}{3}(\mu_u + \mu_d + \mu_s)$. We show results without renormalization group improvements (dash-dotted lines), running coupling (dashed lines), both for $m_s = 0$, and results with running mass and coupling (full lines).

As can be seen from this Figure, there is a sizable difference between zero and finite strange quark mass pressure and energy density for the values of the chemical potential in the region that is relevant for the physics of compact stars. As has been noticed by several authors [6, 10, 13], the resulting equation of state, $\varepsilon = \varepsilon(P)$, can be approximated by a non-ideal bag model form

$$\varepsilon = 4B_{eff} + aP\ . \tag{9}$$

Here $a \sim 3$ is a dimensionless coefficient while B_{eff} is the effective bag constant of the vacuum. Concentrating on the low-density part of the equation of state, one finds for massless strange quarks the parameters $B_{eff}^{1/4} \simeq 117$ MeV and $a \simeq 2.81$ while the inclusion of the running mass raises these values to $B_{eff}^{1/4} \simeq 137$ MeV and $a \simeq 3.17$ (all values having been obtained by including a running α_s in the equations of state). Therefore, we expect important consequences in the mass-radius relation of quark stars due to the inclusion of a finite mass for the strange quark.

III. FIRST-ORDER CHIRAL TRANSITION AND DYNAMICS OF HADRONIZATION

A. Effective Model

To model the mechanism of chiral symmetry breaking present in QCD, and to study the dynamics of phase conversion after a temperature-driven chiral transition, one can resort to low-energy effective models. In particular, to study the mechanisms of bubble nucleation and spinodal decomposition in a hot expanding plasma, it is common to adopt the linear σ-model coupled to quarks, where the latter comprise the hydrodynamic degrees of freedom of the system [21–26]. The gas of quarks provides a thermal bath in which the long-wavelength modes of the chiral field evolve, and the latter plays the role of an order parameter in a Landau-Ginzburg approach to the description of the chiral phase transition. The gas of quarks and anti-quarks is usually treated as a heat bath for the chiral field, with temperature T. The standard procedure is then integrating over the fermionic degrees of freedom, using a classical approximation for the chiral field, to obtain a formal expression for the thermodynamic potential of an infinite system.

Let us consider a chiral field $\phi = (\sigma, \vec{\pi})$, where σ is a scalar field and π^i are pseudoscalar fields playing the role of the pions, coupled to two flavors of quarks according to the Lagrangian

$$\mathcal{L} = \overline{q}[i\gamma^\mu\partial_\mu + \mu_q\gamma^0 - M(\phi)]q + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\,. \qquad (10)$$

Here q is the constituent-quark field $q = (u, d)$ and $\mu_q = \mu/3$ is the quark chemical potential. The interaction between the quarks and the chiral field is given by $M(\phi) = g\,(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})$, and $V(\phi) = \frac{\lambda^2}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - h_q\sigma$ is the self-interaction potential for ϕ.

The parameters above are chosen such that chiral $SU_L(2)\otimes SU_R(2)$ symmetry is spontaneously broken in the vacuum. The vacuum expectation values of the condensates are $\langle\sigma\rangle = f_\pi$ and $\langle\vec{\pi}\rangle = 0$, where $f_\pi = 93$ MeV is the pion decay constant. The explicit symmetry breaking term is due to the finite current-quark masses and is determined by the PCAC relation, giving $h_q = f_\pi m_\pi^2$, where $m_\pi = 138$ MeV is the pion mass. This yields $v^2 = f_\pi^2 - m_\pi^2/\lambda^2$. The value of $\lambda^2 = 20$ leads to a σ-mass, $m_\sigma^2 = 2\lambda^2 f_\pi^2 + m_\pi^2$, equal to 600 MeV. For $g > 0$, the finite-temperature one-loop effective potential also includes a contribution from the quark fermionic determinant. In what follows, we treat the gas of quarks as a heat bath for the chiral field, with temperature T and baryon-chemical potential μ. Then, one can integrate over the fermionic degrees of freedom, obtaining an effective theory for the chiral field ϕ. Using a classical approximation for the chiral field, one obtains the thermodynamic potential

$$\Omega(T,\mu,\phi) = V(\phi) - \frac{T}{\mathcal{V}}\ln\det\{[G_E^{-1} + W(\phi)]/T\}\,, \qquad (11)$$

where $\mathcal{V}$ is the volume of the system and G_E is the fermionic Euclidean propagator. From the thermodynamic potential one can obtain all the thermodynamic quantities of interest.

B. Inhomogeneities in the Chiral Field

To compute correlation functions and thermodynamic quantities, one has to evaluate the fermionic determinant within some approximation scheme. In $1D$ systems one can usually resort to exact analytical methods[27]. In practice, however, the determinant is usually calculated to one-loop order assuming a homogeneous and static background chiral field. Nevertheless, for a system that is in the process of phase conversion after a chiral transition, one expects inhomogeneities in the chiral field to play a role in driving the system to the true ground state.

We propose an approximation procedure to evaluate the finite-temperature fermionic determinant in the presence of a chiral background field, which systematically incorporates effects from inhomogeneities in the chiral field through a derivative expansion. The method is valid for the case in which the chiral field varies smoothly, and allows one to extract information from its long-wavelength behavior, incorporating corrections order by order in the derivatives of the field.

The Euler-Lagrange equation for static chiral field configurations contains a term which represents the fermionic density $\rho(\vec{x}_0) = (\nu_q/\mathcal{V})\left\langle\vec{x}_0\left|(G_E^{-1} + M(\hat{x}))^{-1}\right|\vec{x}_0\right\rangle$, where $|\vec{x}_0\rangle$ is a position eigenstate with eigenvalue $\vec{x}_0$, and $\nu_q = 12$ is the color-spin-isospin degeneracy factor. In momentum representation:

$$\begin{aligned}\rho(\vec{x}_0) \;=\; \nu_q T\sum_n\int\frac{d^3k}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}\times \\ \frac{1}{\gamma^0(i\omega_n+\mu) - \vec{\gamma}\cdot\vec{k} + M(\hat{x})}e^{i\vec{k}\cdot\vec{x}}\,.\end{aligned} \qquad (12)$$

We can transfer the $\vec{x}_0$ dependence to $M(\hat{x})$ through a unitary transformation, expand $M(\hat{x}+\vec{x}_0)$ around $\vec{x}_0$, and use $\hat{x}^i = -i\nabla_{k_i}$ to write

$$\begin{aligned}\rho(\vec{x}_0) = \nu_q T\sum_n\int\frac{d^3k}{(2\pi)^3}\frac{1}{\gamma^0(i\omega_n+\mu) - \vec{\gamma}\cdot\vec{k} + M(\vec{x}_0)}\times \\ \left[1 + \Delta M(-i\nabla_{k_i},\vec{x}_0)\frac{1}{\gamma^0(i\omega_n+\mu) - \vec{\gamma}\cdot\vec{k} + M(\vec{x}_0)}\right]^{-1}\end{aligned} \qquad (13)$$

where $\vec{x}_0$ is a c-number, not an operator, and $\Delta M(\hat{x},\vec{x}_0) = \nabla_i M(\vec{x}_0)\hat{x}^i + \frac{1}{2}\nabla_i\nabla_j M(\vec{x}_0)\hat{x}^i\hat{x}^j + \cdots$.

If we focus on the long-wavelength properties of the chiral field and assume that the static background, $M(\vec{x})$, varies smoothly and fermions transfer a small ammount of momentum to the chiral field, we can expand the expression above in a power series:

$$\begin{aligned}\rho(\vec{x}) = \nu_q T\sum_n\int\frac{d^3k}{(2\pi)^3}\frac{1}{\gamma^0(i\omega_n+\mu) - \vec{\gamma}\cdot\vec{k} + M(\vec{x})}\times \\ \sum_\ell(-1)^\ell\left[\Delta M(-i\nabla_{k_i},\vec{x})\frac{1}{\gamma^0(i\omega_n+\mu) - \vec{\gamma}\cdot\vec{k} + M(\vec{x})}\right]^\ell,\end{aligned} \qquad (14)$$

$$\Delta M(-i\nabla_{k_i},\vec{x}) = \nabla_i M\left(\frac{1}{i}\right)\nabla_{k_i} + + \frac{1}{2}\nabla_i\nabla_j M\left(\frac{1}{i}\right)^2\nabla_{k_i}\nabla_{k_j} + \cdots , \quad (15)$$

which provides a systematic procedure to incorporate corrections brought about by inhomogeneities in the chiral field to the quark density, so that we can calculate $\rho(\vec{x}) = \rho_0(\vec{x}) + \rho_1(\vec{x}) + \rho_2(\vec{x}) + \cdots$ order by order in powers of the derivative of the background, $M(\vec{x})$. The leading-order term in this gradient expansion for $\rho(\vec{x})$ can be easily calculated and yields the well-known mean field result for ρ. The net effect of this leading term is to correct the potential for the chiral field to $V_{eff} = V(\phi) + V_q(\phi)$, where

$$V_q \equiv -\nu_q T \int \frac{d^3k}{(2\pi)^3} \ln\left(e^{[E_k(\phi)-\mu_q]/T} + 1\right) + (\mu_q \to -\mu_q) , \quad (16)$$

where $E_k(\phi) = \sqrt{\vec{k}^2 + M(\phi)}$. This sort of effective potential is commonly used as the thermodynamic potential in a phenomenological description of the chiral transition for an expanding quark-gluon plasma created in a high-energy heavy-ion collision [22–26]. However, the presence of a nontrivial background field configuration, *e.g.* a bubble, can in principle dramatically modify the Dirac spectrum, hence the determinant[27, 28]. Results for the correction of the Laplacian term will be presented elsewhere[29].

C. Finite-Size Effects on Nucleation of Hadrons

In the process of phase conversion through bubble nucleation in a QGP of finite size, the set of all supercritical bubbles integrated over time will eventually drive the entire system to its true vacuum. The scales that determine the importance of finite-size effects are the typical linear size of the system, the radius of the critical bubble and the correlation length. For definiteness, let us assume our system is described by a coarse-grained Landau-Ginzburg potential, $U(\phi,T)$, whose coefficients depend on the temperature. For the case to be considered, the scalar order parameter, ϕ, is *not* a conserved quantity, and its evolution is given by the time-dependent Landau-Ginzburg equation

$$\frac{\partial\phi}{\partial t} = \gamma\left[\nabla^2\phi - U'(\phi,T)\right] \quad , \quad (17)$$

where γ is the response coefficient which defines a time scale for the system. The equation above is a standard reaction-diffusion equation, and describes the approach to equilibrium.

If $U(\phi,T)$ is such that it allows for the existence of bubble solutions (taken to be spherical for simplicity), then supercritical (subcritical) bubbles expand (shrink), in the thin-wall limit, with the following velocity:

$$\frac{dR}{dt} = \gamma(d-1)\left[\frac{1}{R_c} - \frac{1}{R(t)}\right] \quad , \quad (18)$$

where $R_c = (d-1)\sigma/\Delta F$ and ΔF is the difference in free energy between the two phases. The equation above relates the velocity of a domain wall to the local curvature. The response coefficient, γ, can be related to some characteristic collision time. One can measure the importance of finite-size effects for the case of heavy-ion collisions by comparing, for instance, the asymptotic growth velocity ($R >> R_c$) for nucleated hadronic bubbles to the expansion velocity of the plasma. In the Bjorken picture, the typical length scale of the expanding system is $L(T) \approx (v_z t_c)(T_c/T)^3 = L_0(T_c/T)^3$, where v_z is the collective fluid velocity and $L_0 \equiv L(T_c)$ is the initial linear scale of the system for the nucleation process which starts at $T \leq T_c$.

The relation between time and temperature provided by the cooling law that emerges from the Bjorken picture suggests the comparison between the following "velocities":

$$v_b \equiv \frac{dR}{dT} = -\left(\frac{3b\ell L_0}{2v_z\sigma T_c^2}\right)\left(\frac{T_c}{T}\right)^5\left(1 - \frac{T}{T_c}\right) \quad , \quad (19)$$

the asymptotic bubble growth "velocity", and the plasma expansion "velocity" $v_L \equiv (dL/dT) = -(3L_0/T_c)(T_c/T)^4$. The quantity b is a number of order one to first approximation, and comes about in the estimate of the phenomenological response coefficient $\gamma(T) \approx b/2T$. Using the numerical values adopted previously and $\sigma/T_c^3 \sim 0.1$, we obtain[30]

$$\frac{v_b}{v_L} \approx \frac{20}{v_z}\left(\frac{T_c}{T} - 1\right) \quad . \quad (20)$$

One thus observes that the bubble growth velocity becomes larger than the expansion velocity for a supercooling of order $\theta \approx v_z/20 \leq 5\%$. A simple estimate points to a critical radius larger than 1 fm at such values of supercooling[24]. Therefore, finite-size effects appear to be an important ingredient in the phase conversion process right from the start in the case of high-energy heavy-ion collisions[30].

D. Effects from dissipation

Recently, we have considered the effects of dissipation in the scenario of explosive spinodal decomposition for hadron production [31–34] during the QCD transition after a high-energy heavy ion collision in the simplest fashion [35]. Using a phenomenological Langevin description for the time evolution of the order parameter in a chiral effective model [24], inspired by microscopic nonequilibrium field theory results, we performed real-time lattice simulations for the behavior of the inhomogeneous chiral fields. It was shown that the effects of dissipation could be dramatic in spite of very conservative assumptions: even if the system quickly reaches this unstable region there is still no guarantee that it will explode.

The framework for the dynamics was assumed to be given by the following Langevin equation

$$\Box\phi + \Gamma\frac{\partial\phi}{\partial t} + V'_{eff}(\phi) = \xi(\vec{x},t) \quad (21)$$

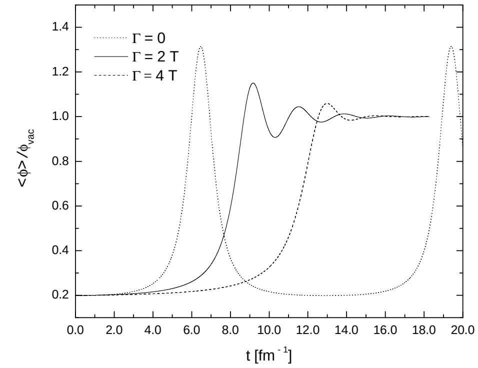

FIG. 4: Average value of the chiral field, ϕ, in units of its vacuum value, ϕ_{vac}, as a function of time for $\Gamma/T = 0, 2, 4$.

where ϕ is a real scalar field, $V'_{eff}(\phi)$ is a Landau-Ginzburg effective potential and Γ, which can be seen as a response coefficient that defines time scales for the system and encodes the intensity of dissipation, is usually taken to be a function of temperature only, $\Gamma = \Gamma(T)$. The function $\xi(\vec{x}, t)$ represents a stochastic (noise) force, assumed Gaussian and white, so that $\langle \xi(\vec{x}, t) \rangle = 0$ and $\langle \xi(\vec{x}, t)\xi(\vec{x}', t') \rangle = 2\Gamma T \delta(\vec{x} - \vec{x}')\delta(t - t')$, according to the fluctuation-dissipation theorem.

Results for the average value of the chiral field, ϕ, in units of its vacuum value, ϕ_{vac}, as a function of time for $\Gamma/T = 0, 2, 4$ are shown in Fig. 4, where one can see the large delay introduced by dissipation effects.

IV. FINAL REMARKS

In the discussion above, we reviewed a few aspects of the physics of the chiral transition and the quark-gluon plasma. Within the issues that have been considered, two points clearly stand out as important challenges for field theorists. The first one is related to the behavior of the equation of state at finite density and zero temperature close to the critical region. There, neither perturbative QCD, improved or not, nor low-density chiral perturbation theory are of great help. Since reliable lattice results are still lacking in this sector of the phase diagram, one could possibly try to incorporate nonperturbative information into a quasiparticle description of the thermodynamics [36] to achieve a more realistic picture. Because this is the most relevant region of the equation of state for the physics of compact stars, astrophysical constraints will certainly be very welcome. The second is a proper understanding of the dynamics of the phase transition and thermalization. Besides the question of the nature of the phase transition as one varies temperature and chemical potential, which still belongs to an equilibrium analysis, one must also provide a satisfactory nonequilibrium description of the phase conversion process. In the case of heavy ion collisions, where relevant scales differ from the cosmological analog by several orders of magnitude, one has to consider in detail a combination of effects coming from dissipation, noise, expansion and finite size of the system, as well as transient non-Markovian corrections [37]. All these challenges, together with so many others not even mentioned here, make the study of the phase diagram of QCD a fascinating subject.

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