# Low-Energy Levels Calculation for ${ }^{193} \mathbf{I r}$ 

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#### Abstract

In this work, a model based on single particle plus pairing residual interaction was used to study the low-lying excited states of the ${ }^{193}$ Ir nucleus. In this model, the deformation parameters in equilibrium were obtained by minimizing the total energy calculated by the Strutinsky prescription; the macroscopic contribution to the potential was taken from the Liquid Droplet Model, with the shell and paring corrections used as as microscopic contributions. The nuclear shape was described using the Cassinian ovoids as base figures; the single particle energy spectra and wave functions for protons and neutrons were calculated in a deformed Woods-Saxon potential, where the parameters for neutrons were obtained from the literature and the parameters for protons were adjusted in order to describe the main sequence of angular momentum and parity of the bandheads, as well as the proton binding energy of ${ }^{193}$ Ir. The residual pairing interaction was calculated using the BCS prescription with Lipkin-Nogami approximation. The results obtained for the first three bandheads (the $3 / 2^{+}$ground state, the $1 / 2^{+}$excited state at $\mathrm{E} \approx 73 \mathrm{keV}$ and the the $11 / 2^{-}$isomeric state at $\mathrm{E} \approx 80 \mathrm{keV}$ ) showed a very good agreement, but the model so far greatly overestimated the energy of the next bandhead, a $7 / 2^{-}$at $\mathrm{E} \approx 299 \mathrm{keV}$.


Keywords: Ir-193; Beta decay; Spectroscopy

## I. INTRODUCTION

This work is a theoretical support for the experimental analysis of the excited states in ${ }^{193} \mathrm{Ir}$ populated by the $\beta^{-}$decay of ${ }^{193} \mathrm{Os}$ [1].

The low-lying states in ${ }^{193}$ Ir have been thoroughly studied and, although this nucleus is usually described as triaxiallydeformed [2-5], it has also been recently described as a prolate rotor [6]. Most of these theoretical studies describe rather well the binding energies of the $3 / 2^{+}$ground state and the $1 / 2^{+}$excited state at 73 keV , but fail to describe the negativeparity bandheads; in this regard, the best effort to this date seems to be the one by Drissi [7], where a variation of the Interaction Boson-Fermion Model (IBFM) describes to some degree the $11 / 2^{-}$isomeric state at 80 keV and has the next bandhead, a $7 / 2^{-}$state, quite shifted from its original 299 keV excitation energy, but still below 1 MeV .

In this work, the low-lying states of ${ }^{193}$ Ir were evaluated using a single-particle model in an axially-deformed potential, with pairing interaction corrections applied. The deformation parameters in equilibrium were obtained by minimizing the total energy calculated by the macroscopic-microscopic method [8]. The single particle energy spectra and wave functions for protons and neutrons were calculated in a deformed Woods-Saxon potential [9], with the potential parameters for neutrons obtained from the literature [10] and the parameters for protons adjusted in order to describe the main sequence of angular momentum and parity of the bandheads, as well as the proton binding energy. The residual pairing interaction was calculated using the BCS prescription with the LipkinNogami approximation [11].

## II. THE NUCLEAR MODEL

## A. Nuclear Deformation

The nuclear deformation was calculated using the macroscopic-microscopic method. Within this method, the total energy is given by

$$
\begin{equation*}
E_{\text {tot }}(\varepsilon, \hat{\alpha})=E_{\text {macr }}(\varepsilon, \hat{\alpha})+E_{\text {micr }}(\varepsilon, \hat{\alpha}) \tag{1}
\end{equation*}
$$

where $\varepsilon$ and $\hat{\alpha}$ are the deformation parameters.
The macroscopic term is given by the liquid drop model. The microscopic portion can be divided into two components: the contribution associated with the shell correction energy and the pairing contribution. In order to obtain the equilibrium deformation the total energy is minimized with respect to the deformation parameters. The Cassini ovaloid shape parametrization was used, taking into consideration the terms associated with the quadrupole $(\varepsilon)$ and hexadecapole $\left(\alpha_{4}\right)$ moments.

## B. Single Particle Energies

The single particle states were calculated using the WoodsSaxon (W-S) potential. To determine the W-S potential twelve constants should be given, six for neutrons and six for protons:
$V_{0}$ depth of the central potential;
$a_{s o}$ diffuseness parameter of the spin-orbit part;
$R_{0}$ radius parameter;


FIG. 1: Dependence of the total energy with the deformation parameters.
$r_{0-s o}$ radius parameter of the spin-orbit potential;
$a$ diffuseness nuclear parameter;
$\lambda$ strength of spin-orbit interaction.
Several parameter sets have been proposed for the WoodsSaxon potential, usually determined by a global fit to various ground state nuclear properties of $\beta$-stable nuclei in a mass number range.
The Woods-Saxon potential consists of the central part $V_{\text {cent }}$, the spin-orbit part $V_{s o}$ and the Coulomb potential $V_{\text {Coul }}$ for protons:

$$
\begin{align*}
& V^{W S}(r, z, \varepsilon, \hat{\alpha})= \\
& \quad V_{\text {cent }}(r, z, \varepsilon, \hat{\alpha})+V_{\text {so }}(r, z, \varepsilon, \hat{\alpha})+V_{\text {Coul }}(r, z, \varepsilon, \hat{\alpha}) \tag{2}
\end{align*}
$$

where $(r, z)$ are cylindrical coordinates.
The central part is defined in order to describe the density distribution function

$$
\begin{equation*}
V_{c e n t}(r, z, \varepsilon, \hat{\alpha})=\frac{V_{0}}{1+e^{\frac{\operatorname{dist(r,z,z,\hat {\alpha })}}{\alpha}}} \tag{3}
\end{equation*}
$$

where dist is equal to the distance of a given point to the nuclear surface.

The depth of the central potential is parameterized by:

$$
\begin{equation*}
V_{0}=V_{0}[1 \pm 0.063(N-Z) /(N+Z)] \tag{4}
\end{equation*}
$$

with positive signal for protons and negative for neutrons.
The spin-orbit term is defined by

$$
\begin{equation*}
V_{s o}(r, z, \varepsilon, \hat{\alpha})=\lambda\left(\frac{h}{2 M c}\right)^{2} \nabla V(r, z, \varepsilon, \hat{\alpha}) \cdot(\vec{\sigma} \times \vec{p}) \tag{5}
\end{equation*}
$$



FIG. 2: Comparison of the experimental bandheads of ${ }^{193} \mathrm{Ir}$ to the calculations performed using the present set of parameters and to the calculations using the Universal Parameters from Cwiok [12].
where $M$ is the nucleonic mass, the vector operator $\vec{\sigma}$ stands for the Pauli matrices and $\vec{p}$ is the linear operator.

The Coulomb potential is assumed to be that corresponding to the nuclear charge $(Z-1) e$, uniformly distributed inside the nucleus. For the Hamiltonian diagonalization, the eigenfunctions of an harmonic oscillator with axial symmetry in the cylindrical coordinates were used as a base.

The pairing energy was evaluated as in the commonly used prescription of the BCS approach. The hamiltonian operator in the BCS model contains two parts: the first corresponding to the single particle states and the second corresponding to the pairing interaction. If the particle term is diagonal, the BCS operator can be written in the formalism of the second quantization as

$$
\begin{align*}
& \hat{H}=\hat{H}_{s p}+\hat{H}_{p a i r}= \\
& \quad \sum \varepsilon_{\Omega i}\left(\alpha_{i}^{\dagger} \alpha_{i}+\alpha_{\bar{i}}^{\dagger} \alpha_{\bar{i}}\right)-\sum G_{j i} \alpha_{j}^{\dagger} \alpha_{\bar{j}}^{\dagger} \alpha_{i} \alpha_{i} \tag{6}
\end{align*}
$$

Finally, the system of equations of this model is

$$
\begin{equation*}
E_{\Omega}^{v}=\sum_{i}^{n u} \varepsilon_{\Omega i}+\sum_{i}^{n u}\left(1-\frac{\varepsilon_{\Omega i}-\lambda_{v}}{\sqrt{\left(\varepsilon_{\Omega i}-\lambda_{v}\right)^{2}}+\Delta_{v}^{2}}\right)-\frac{\Delta_{v}^{2}}{G} \tag{7}
\end{equation*}
$$

where:
$\varepsilon$ Energy of single particle states;
$v$ Number of quasi-particles;
$\lambda$ Chemical potential; and
$G$ Pairing constant.

|  | $V_{0}$ <br> $(\mathrm{MeV})$ | $r_{0}$ <br> $(\mathrm{fm})$ | $a$ <br> $(\mathrm{fm})$ | $r_{0-s o}$ <br> $(\mathrm{fm})$ | $a_{\text {so }}$ <br> $(\mathrm{fm})$ | $\lambda$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Universal |  |  |  |  |  |  |
| Parameters <br> $[12]$ | 49.6 | 1.275 | 0.70 | 1.32 | 0.70 | 36.0 |
| Present <br> Work | 54.612 | 1.18992 | 0.70 | 1.27498 | 0.932 | 36.899 |

TABLE I: Parameters of the Woods-Saxon potential

## III. RESULTS

The values of the deformation parameters that minimize the total energy are $\varepsilon=0.1102$ and $\alpha_{4}=-0.0404$; the representation of the total energy dependence with these parameters is shown in Fig. 1.

The parameters of the W-S potential for neutrons were obtained from ref. [10], and the parameters for protons were adjusted in order to describe the main sequence of angular momentum and parity of the three lowest-lying single particle states, as well as their binding energy; from these states it was then possible to calculate the quasi-particle states using
the LINDEN code [11]. The values obtained were compared to the parameters suggested by Cwiok et al. [12], called Universal Parameters (see Table I. In Figure 2, the present single particle states calculation is compared to the experimental results from [1] and to the results obtained using the Universal Parameters.

## IV. CONCLUSIONS

Our calculations reproduced very well the ground state and the two lowest-lying states in ${ }^{193}$ Ir, while the Universal Parameters couldn't even reproduce the ground state. The $11 / 2^{-}$ state at 80 keV , which has presented some problems in other theoretical explanations of this nucleus, in particular, was perfectly reproduced, showing that it can indeed be explained in terms of an axially-symmetric potential.

The next step in this work should be to try to reproduce the next two bandheads in ${ }^{193} \mathrm{Ir}$, a $7 / 2^{-}$state at 299 keV and a $3 / 2^{+}$state at 460 keV , both of which were not included in the present analysis and presented serious trouble in other calculations found in the literature [2-7].
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