Primordial Bubbles Evolution with Beta Equilibrium and Charge Neutrality

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We discuss macroscopic bulk properties of primordial bubbles quark matter which survived the confinement phase transition in the early universe. Electron and quark components are considered at zero temperature maintaining beta equilibrium and charge neutrality. We analyze the possibility that a superconducting phase transition occurs, changing the initially unpaired quark matter phase to the colour-flavor locked (CFL) alternative without electrons. We had considered the gap energy and the Goldstone bosons condensation for the pressure calculation in the CFL phase.

Witten\cite{1} proposed that a first-order cosmic quark-hadron phase transition at a critical temperature $T_c \sim 150\text{MeV}$ in the early universe, could lead to the formation of quark bubbles with $u$, $d$, and $s$ quarks at a density of the order of the normal nuclear matter density.

We begin our work considering a bulk of quark matter, so the surface effects can be ignored. In order to obtain electrically neutral bulk matter, a nonzero density of electrons is required. But at extremely large chemical potential, and consequently neutral bulk matter, a nonzero density of electrons is not maintained by the weak interaction processes.

The bulk of quark matter is considered as a degenerate Fermi gas of $u$, $d$, $s$ and electrons with chemical equilibrium satisfied. Charge neutrality is automatically satisfied.

In Fig.(1a). The detailed change in electron density vs. baryon number density, is shown in Fig.(1b). For low density, a small fraction of electrons is needed to maintain charge neutrality since the $s$ quark is not present. When strange matter appears Eqs.(1,3) impose a decreasing in the electron density.

At low temperature and very high pressure, QCD has a superconducting phase \cite{3}. The appearance of this new state of matter can be understood considering that in the asymptotic freedom regime the distribution of quarks in the energy levels (which depends on $E - N_f \mu$) does not change adding or subtracting a single particle. But including the attractive exchange of gluon between a pair produces a rearrangement of the states near the Fermi surface, favouring energetically the pair condensation with a fermionic gap formation.

We will consider no electrons because their density is always very small and introducing electrons while maintaining charge neutrality with beta equilibrium preserved could cost too much pairing energy\cite{4}.

The gap $\Delta$ (Fig.1c) can be calculated with perturbative QCD with quark-gluon coupling $g$, giving\cite{5}

\begin{equation}
\Delta \simeq 512 \pi^4 \left( \frac{2}{N_f} \right)^{\frac{1}{2}} \mu g^{-5} \exp\left(-\frac{3\pi^2}{\sqrt{2} g}\right),
\end{equation}

where $N_f = 3$ is the number of flavours.

The gauge coupling constant between quarks and gluons varies with energy through ”vacuum polarization” effects as\cite{6}

\begin{equation}
\alpha_s(\mu) = \frac{4\pi}{\ln\left(\frac{\Lambda_{QCD}^2}{\mu^2}\right)}, \quad \alpha_s = \frac{g^2}{4\pi},
\end{equation}

with $\Lambda_{QCD} \simeq 200\text{MeV}$, and $\mu$ the baryonic chemical potential.

In this CFL phase both the gauge $SU(3)_c$ and the global chiral $SU(3)_{L,R}$ symmetry are broken giving respectively massive gluons and mesons as Goldstone particles.

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The bulk of quark matter is considered as a degenerate Fermi gas of $u$, $d$, $s$ and electrons with chemical equilibrium maintained by the weak interaction processes $d(s) \leftrightarrow u + e^− + \bar{\nu}, u + d \leftrightarrow u + s$, which imply the following relation

\begin{equation}
\mu_u + \mu_e = \mu_d = \mu_s = \mu,
\end{equation}

leaving only two independent chemical potentials for the thermodynamic treatment of the system.

We consider that neutrinos escape from the bubble, playing no role on the beta equilibrium conditions.

When $\mu > m$ and the temperature small enough, the anti-quarks are statistically negligible and the density of fermions in absence of interactions may be approximated by

\begin{equation}
N \simeq \frac{1}{\pi} \left( \frac{\mu^2 - m^2}{\mu} \right)^2.
\end{equation}

For the electrons, the same expression is valid with a factor $\frac{1}{3}$ in the right hand side. The light quarks $u$, $d$ and electrons are considered massless particles.

Charge neutrality condition and the baryon number density are

\begin{equation}
2n_u - n_d - n_s - 3n_e = 0.
\end{equation}

\begin{equation}
n_B = \frac{1}{3} (n_u + n_d + n_s). \tag{4}
\end{equation}

In Eqs.(3) and (4) we replace the quarks densities $n_i$ by its correspondent expressions in terms of chemical potentials. For a given value of $n_B$ these equations form a nonlinear system with two independent chemical potentials $\mu$ and $\mu_e$. Numerical solution of this system gives us the evolution of the fermions densities, showed in Fig(1a).
We describe the CFL phase by the pressure

$$P_{\text{CFL}} = \sum_{i=u,d,s} P_i + \frac{3\Delta^2 \mu^2}{\pi^2} - B + P_{\text{GB}}^{\text{CFL}},$$

where the first term gives the pressure of the noninteracting quarks, second term is the contribution from the formation of the condensate with $\mu = \frac{1}{3}(\mu_u + \mu_d + \mu_s)$, the third is the vacuum contribution and the last term is due to the Goldstone bosons.

In a normal quark matter, the fact that $m_s \neq 0$ shifts the energy of strange quarks near the Fermi surface by $\sim \frac{m_s^2}{2\mu}$ and it leads to the decay $s \rightarrow u + d + \pi$ or $s \rightarrow u + e + \nu_e$. This decay will reduce the number of strange quarks and build up a Fermi sea of electrons until $\mu_e \sim \frac{m_s^2}{2\mu}[7]$. 

In superfluid quark matter, the system can also gain energy $\frac{m_s^2}{2\mu}$ by introducing an extra up and strange hole. This process require the breaking of a pair and an energy cost that will be equivalent to the Kaon mass $m_K$. Note that an

upordownparticlewithastrangehole has the quantum numbers of a Kaon.

Taking $\mu_e \sim 0$ in the CFL phase, the strange quark will decay into a configuration with the quantum numbers of a $K^0$. A schematic picture is showed in Fig.(1d).

At low temperature and high density a stable configuration of the bubble is reached with a strange quark component. We do not study the additional stability given by color superconductivity[8]. We remark that taking into account surface effects, the condition of charge neutrality of the bubble may not be realistic.

According to our study the bubbles of quark matter surviving the confinement transition may reach such high values of chemical potential, larger than those in neutron stars, that could enter in the colour superconducting CFL phase. We have analized this phase without electron chemical potential. However, if the electron chemical potential is not zero, other Goldstone bosons appears in order to mantain the charge neutrality[7].

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