Recent Developments in Few–Nucleon Physics

E. Epelbaum
Jefferson Laboratory, Theory Division, Newport News, VA 23606, USA

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I discuss recent applications of chiral effective field theory to study the properties of few–nucleon systems.

I. INTRODUCTION

During the past decade, significant progress has been achieved towards an accurate, microscopic description of few– and even many–body systems. Most of the microscopic calculations performed so far are based on phenomenological models for nuclear forces and current operators. Such a scheme has proven to be useful for a microscopic understanding of nuclear structure physics but suffers from the obvious deficiencies such as the lack of consistency and systematics. It is therefore desirable to approach the problem on a more fundamental level. Chiral Effective Field Theory (EFT) has become a standard tool for analyzing the properties of hadronic systems at low energy in a systematic and controlled way. It is based upon the approximate and spontaneously broken chiral symmetry of Quantum Chromodynamics (QCD). The corresponding Nambu–Goldstone bosons can be identified with pions if one considers the two flavor sector of the up and down quarks as done here. Due to the fact that Goldstone bosons do not interact at vanishingly low energies in the chiral limit, the S–matrix in the pion and single–baryon sectors can be calculated perturbatively via simultaneous expansion in energy and around the chiral limit. The situation in the few–nucleon sector is far more complicated. Due to the presence of shallow bound states, an additional non–perturbative resummation is required. One possible way to deal with this difficulty was suggested by Weinberg, who proposed to apply chiral EFT to the kernel of the corresponding scattering equation which can be viewed as an effective nuclear potential [1, 2]. In this manuscript, I will consider recent applications of this method to describe properties of few–body systems. In section II I present our recent results for the two–nucleon (2N) system at next–to–next–to–next–to–leading order (N^3LO) in the chiral expansion. More complex systems with up to six nucleons are discussed in section III. Brief summary is presented in section IV.

II. TWO NUCLEONS

The two–nucleon system has been extensively studied in chiral EFT during the past decade. One starts from the effective Lagrangian for pions and nucleons which is consistent with the approximate and spontaneously broken chiral symmetry of QCD and can be constructed along the lines of [3, 4]. The first few terms read:

$$\begin{aligned}\mathcal{L}_{\rm eff} \;=\; & \frac{1}{2}\partial_\mu\pi\cdot\partial^\mu\pi - \frac{1}{2}M_\pi^2\pi^2 + N^\dagger\left(i\partial_0 + \frac{\vec{\nabla}^2}{2m} + \frac{g_A}{2F_\pi}\vec{\sigma}\tau\cdot\vec{\nabla}\pi - \frac{1}{2F_\pi^2}\tau\cdot(\pi\times\dot{\pi})\right)N - \frac{C_S}{2}(N^\dagger N)^2 - \frac{C_T}{2}(N^\dagger\vec{\sigma}N)^2 \\ & + N^\dagger\left(-\frac{2c_1}{F_\pi}M_\pi^2\pi^2 + \frac{c_3}{F_\pi^2}(\partial_\mu\pi\cdot\partial^\mu\pi) - \frac{c_4}{2F_\pi^2}\epsilon_{ijk}\epsilon_{abc}\sigma_i\tau_a(\nabla_j\pi_b)(\nabla_k\pi_c)\right)N + \ldots\,, \qquad (1)\end{aligned}$$

where I switch to the nucleon rest–frame and list only first few terms. Here, M_π (F_π) refers to the pion mass (decay constant) and g_A, c_i and C_i are the low–energy constants (LECs). Further, σ_i and τ_a denote spin and isospin Pauli matrices, respectively. Following Weinberg [1, 2], the Lagrangian (1) can be applied to derive nuclear forces which might then be used in the corresponding dynamical equation to calculate the S–matrix elements. In the language of old–fashioned time–ordered perturbation theory, nuclear forces are given by a set of irreducible diagrams, i.e. those diagrams which do not contain purely nucleonic intermediate states. The relevant contributions to the nuclear forces can be identified from the infinite set of possible time–ordered diagrams by counting the powers of the low–momentum scale Q associated with external nucleon momenta or M_π. More precisely, the N–nucleon force receives contributions of the order $\sim(Q/\Lambda_\chi)^\nu$ with Λ_χ being the pertinent hard scale and

$$\nu = -2+2N-2C+2L+\sum_i V_i\Delta_i\,, \quad \text{where} \quad \Delta_i = d_i+\frac{1}{2}n_i-2\,. \qquad (2)$$

Here, L (C) is the number of loops (separately connected pieces), V_i refers to the number vertices of type i and Δ_i is the corresponding chiral dimension. Further, n_i is the number of nucleon field operators and d_i the number of derivatives and/or insertions of M_π. For example, the leading interactions given in the first line of eq. (1) have dimension $\Delta_i = 0$, while the subleading ones in the second line correspond to $\Delta_i = 1$. No interactions with $\Delta_i < 0$ are allowed by spontaneously broken

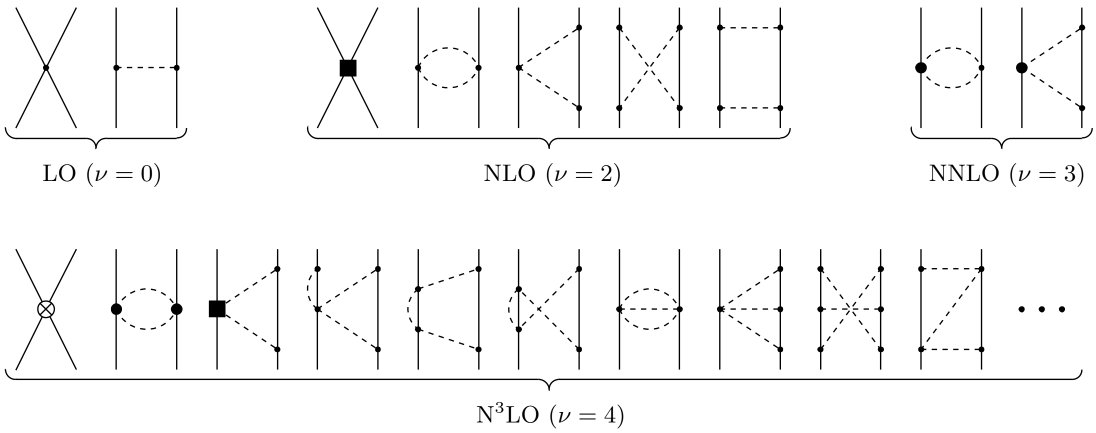

FIG. 1: Two–nucleon force up to N^{3}LO. Solid (dashed) lines denote nucleons (pions). Solid dots, filled circles, filled rectangles and crossed circles refer to vertices with $\Delta_i = 0, 1, 2$ and 4, respectively.

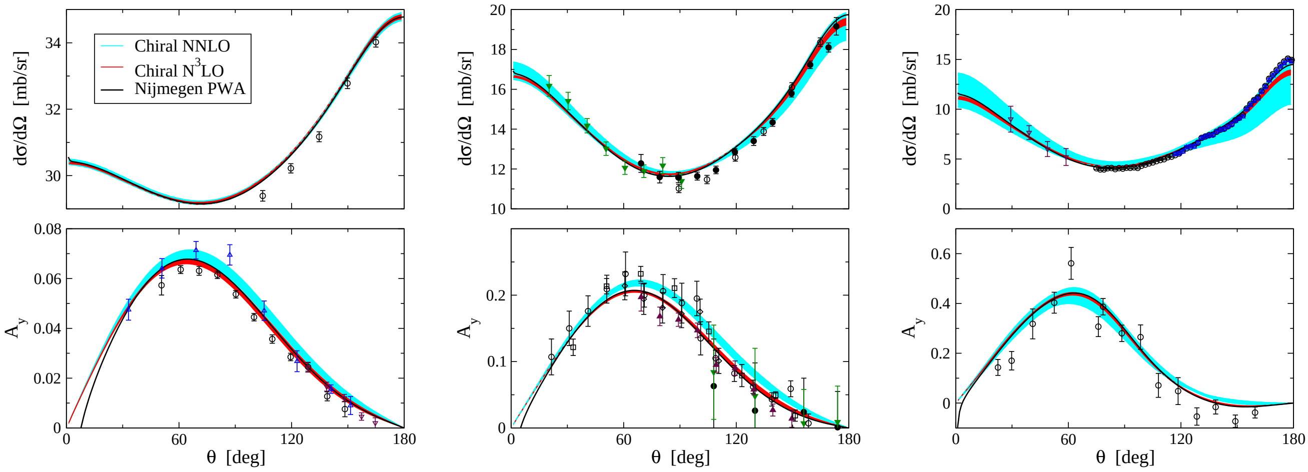

FIG. 2: *np* differential cross section and vector analyzing power at $E_{\rm lab} = 25$ MeV (left panel), $E_{\rm lab} = 50$ MeV (middle panel) and $E_{\rm lab} = 96$ MeV (right panel). The light (dark) shaded bands show the NNLO (N^{3}LO) results. The Nijmegen PWA result is taken from [17]. For data see [15].

chiral symmetry. Consequently, the chiral order ν is bounded from below and for any given ν only a finite number of diagrams needs to be taken into account. Notice further that the boundary $\nu \geq 2N - 4$, which follows from eq. (2) for connected diagrams, implies a rather natural picture, in which nucleons interact mainly via 2N forces while many–body forces provide small corrections.

As shown in Fig. 1, the general structure of the NN force in the chiral EFT approach can be expressed as

$$V_{2N} = V_{\rm NN} + V_{1\pi} + V_{2\pi} + V_{3\pi} + \ldots\,, \qquad (3)$$

where the NN contact terms $V_{\rm NN}$ and the pion–exchange contributions can be obtained order–by–order, see eqs. (1) and (2):

$$\begin{aligned} V_{\rm NN} &= V_{\rm NN}^{(0)} + V_{\rm NN}^{(2)} + V_{\rm NN}^{(4)} + \ldots\,, \\ V_{1\pi} &= V_{1\pi}^{(0)} + V_{1\pi}^{(2)} + V_{1\pi}^{(3)} + V_{1\pi}^{(4)} + \ldots\,, \\ V_{2\pi} &= V_{2\pi}^{(2)} + V_{2\pi}^{(3)} + V_{2\pi}^{(4)} + \ldots\,, \\ V_{3\pi} &= V_{3\pi}^{(4)} + \ldots\,. \end{aligned} \qquad (4)$$

Here the superscript means the chiral order ν. The NN potential was first worked out up by Ordóñez, Ray and van Kolck [5], who derived an energy–dependent, non–hermitian two–nucleon (2N) potential up to next–to–next–to–leading order (NNLO) in the chiral expansion and applied it to the nucleon–nucleon system. The explicit energy dependence of the potential is a severe complication for applications in three– (3N) and more–nucleon systems. Energy–independent expressions for the chiral potential at NNLO have been derived by several groups independently using different methods [6–8] and

	NLO	NNLO	N^3LO	Exp
E_d [MeV]	$-2.171\ldots-2.186$	$-2.189\ldots-2.202$	$-2.216\ldots-2.223$	$-2.224575(9)$
A_S [$\mathrm{fm}^{-1/2}$]	$0.868\ldots0.873$	$0.874\ldots0.879$	$0.882\ldots0.883$	$0.8846(9)$
η_d	$0.0256\ldots0.0257$	$0.0255\ldots0.0256$	$0.0254\ldots0.0255$	$0.0256(4)$

TABLE I: Deuteron observables at NLO, NNLO and N^3LO in chiral EFT in comparison to the data.

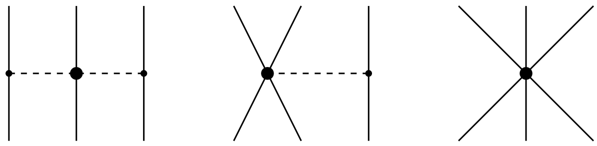

FIG. 3: 3N force at NNLO. For notation see Fig. 1

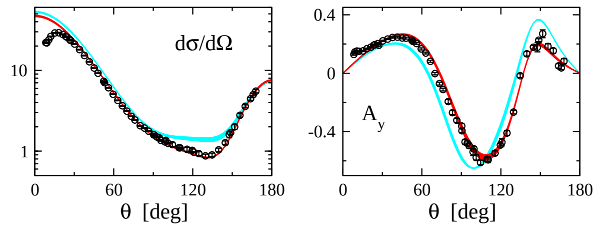

FIG. 4: Nd elastic observables at 65 MeV.

applied to the 2N system in [9]. Recently, N^3LO corrections to the 2N force have been calculated by Kaiser [10–13] and applied to study the properties of the 2N system in [14, 15]. In our N^3LO analysis [15], a novel regularization scheme for pion loop integrals in the 2π–exchange potential is applied, which is based on the spectral–function representation [16] and allows for a better separation between the long– and short–distance contributions compared to dimensional regularization. Within this scheme, we found the 3π–exchange contribution to the potential to be negligibly small. We have fixed 24 LECs related to contact interactions with up to four derivatives from a fit to *np* phase shifts in S–, P– and D–waves and the corresponding mixing angles.

The resulting potential at N^3LO leads to an accurate description of the phase shifts and the low–energy observables in the 2N system. In Fig. 2 we show the NNLO and N^3LO results for *np* differential cross section and vector analyzing power at three different energy. The bands correspond to the variation of the cut–offs in the spectral–function representation of the potential and in the Lippmann–Schwinger equation. They may serve as a rough estimation of the theoretical uncertainty, which at N^3LO is expected to be of the order $\sim 0.5\%$, 7% and 25% at laboratory energy ~ 50, 150 and 250 MeV, respectively, see [15] for more details.

In Table I we show our predictions for the deuteron binding energy, asymptotic S–wave normalization A_S and asymptotic D/S ratio at various orders in chiral EFT. All these observables are well described at N^3LO.

III. THREE AND MORE NUCLEONS

3N and 4N systems have been studied at NLO [18] and NNLO [19] in the chiral EFT framework solving rigorously the Faddeev–Yakubovsky equations in momentum space. Chiral 3N force starts formally to contribute at NLO ($\nu = 2$), see eq. (2). It is, however, well known that the leading 3N force at this order vanishes provided one uses an energy–independent formulation such as the method of unitary transformation [8, 20], see also [21–23]. Consequently, only the 2N interaction needs to be taken into account at NLO, which is already completely fixed from the 2N system. The first nonvanishing 3N forces appear at NNLO and are given by the diagrams shown in Fig. 3 [19, 22]. While the 2π–exchange contribution is parameter–free, the 1π–exchange and contact interactions depend on one parameter each. These two parameters cannot be determined in the 2N system and were fixed from the triton binding energy and the *nd* doublet scattering length. Our prediction for the α–particle binding energy based upon the resulting parameter–free 3N Hamiltonian, $\mathrm{BE}(^4\mathrm{He}) = -29.51\ldots-29.98$ MeV, agrees well with the empirical (corrected for missing *nn* and *pp* forces) number, -29.8 MeV.

We also observe good description of the 3N scattering data at NNLO at low and intermediate energies. For example, differential cross section and vector analyzing power for elastic *Nd* scattering at $E_{\mathrm{lab}} = 65$ MeV are shown at NLO (light shaded band) and NNLO (dark shaded band) in Fig. 4.

Recently, first and very promising parameter–free results for the 1^+ ground and 3^+ excited states of ^{6}Li were obtained using chiral forces at NLO and NNLO within the no–core shell model framework [24]. At NNLO both the ground and excited state energies are reproduced within the theoretical uncertainty of 5.7% and 7.6% (based on the cut–off variation), respectively.

IV. SUMMARY AND OUTLOOK

Chiral EFT provides a systematic framework to study the low–energy dynamics of hadronic systems. Recent applications in the few–nucleon sector show promising results. The two–nucleon system has been studied at N^3LO. Accurate results for the deuteron and low–energy scattering observables have been obtained. 3N, 4N and 6N systems have been analyzed at NNLO. For the first time, the chiral 3N force has been included in few–body calculations. In the future, N^3LO analysis of the 2N system should be extended to heavier systems. One should also consider reactions with external electroweak and/or pionic probes.

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