

## Results and Problems in Decoherence Theory

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The main steps in the development of the ideas on decoherence are briefly reviewed, together with their present achievements. Unsolved problems are also pointed out.

### I. EARLY LANDMARKS

The idea of decoherence originated in the problem of macroscopic interferences. This problem, first pointed out by Von Neumann and Schrödinger (hence its current description as the Schrödinger cat problem), was encountered in the theory of quantum measurements. Basically, it came from the fact that a quantum superposition of two microscopic quantum states can be amplified through a measurement interaction into a superposition of two macroscopic states of the measuring instrument. Since no such superposition had ever been observed in a real experiment, a deep difficulty arose in the foundations of quantum mechanics and became the topic of many interrogations and investigations. It was of course closely related to some other problems involving the status of wave function reduction or the uniqueness of physical reality, although not identical with them. Contrary to these two fundamental problems, which only produced speculations till now, the enigma of macroscopic interferences led to a significant progress in quantum theory, essentially through purely technical and non-philosophical investigations.

The first hints of decoherence go back to Heisenberg, who came close to the notion of environment when wondering about the moving frontier separating the domains of quantum and classical laws, according to his views on their relation. A better understanding of microscopic disorder occurred during the period 1950-1970 and studies in magnetic resonance brought forward the idea of relaxation. Relaxation is the name for the irreversible return of quantum degrees of freedom to thermal equilibrium, this phenomenon being of course closely related to other forms of damping. Van Kampen first proposed that these processes could remove macroscopic interferences [1]. Then, some important results on the quantum theory of irreversible processes having been obtained by Van Hove [2], they were applied to the measurement problem by Deneri, Loinger and Prosperi [3].

These works were not completely quantitative however, and their main aim was only to show that relaxation can finally suppress macroscopic interferences. This is true as a matter of principle, but quantitatively insufficient because the effect is too slow.

In a seminal paper, Zeh pointed out in 1970 the basic concepts of decoherence theory [4], namely:

(i). A macroscopic system can be conceived as made of two interacting subsystems, from a theoretical standpoint. As a matter of fact, this distinction raises some tricky problems and I shall use a very simple point of view in a first approach. Let us say that the quantities that can be directly observed or inferred in a specific system during a specific experiment de-

fine the first subsystem, which will be called “relevant” in this lecture. An example of observed quantity is typically the position of a pointer on a voltmeter dial; an example of inferred quantity being a spin component of an atom in a Stern-Gerlach experiment. The other subsystem is the “environment”, which involves everything that is not relevant, typically the “atomic bowels” of the matter in the object, as well as an external environment (consisting for instance of the molecules in a surrounding atmosphere or the photons in surrounding light). An important point is that the environment involves many degrees of freedom (although it was found that 10 can be a large enough number of them in some circumstances [5]). On the other hand, the choice of relevant observables is often somewhat subjective or at least not a priori well defined.

(ii). Interferences occur rapidly in the environment, because of the complexity of the wave function as a function of the environment variables. They are due basically to a loss of coherence among the phases of different parts of the environment wave functions describing different measurement results. A standard example is provided by the wave functions of atoms near the axis of a pointer. Their phases are expected to differ strongly when the pointer starts moving toward the left or right directions and coherence is rapidly lost. In other words, *decoherence is a phase effect*. Its main consequence is the disappearance of a macroscopic quantum superposition, because the environment wave functions become orthogonal.

(iii). Decoherence is a *dynamical quantum effect*, acting extremely rapidly but not immediately.

### II. THE THEORY OF DECOHERENCE

Although the theory of relaxation came from the  $N$ -body problem, the theory of decoherence raised new problems, because  $N$ -body techniques could not predict quantum phase properties. Investigations began therefore with a study of convenient models, before attempting to reach some sort of general theory.

#### *Models of decoherence*

Much was learned initially from the study of models, which is still actively carried on. In most cases, the environment was simply represented by a collection of two-states systems (spins) or harmonic oscillators [6]-[10]. Another interesting model was much closer to the idea of phase coherence loss [11]. It considered a superposition of different positions for a macroscopic object, on which external molecules or photons collide. The random accumulation of scattering phase-shifts

in the state of the environment generates effectively a decoherence effect.

A remarkable outcome of the study of models was a close similarity between their various results, namely:

(i). The theory yields a master equation for the time evolution of a reduced density matrix, which contains in principle a complete information about every relevant observable and is defined by

$$\rho_{red}(t) = Trace_{environment}[\rho(t)] \quad (1)$$

(ii). Decoherence appears as an approximate diagonalization of the reduced density, as shown by a typical example of a master equation,

$$\begin{aligned} \frac{\partial}{\partial t} \langle x | \rho_r | x' \rangle &= i \langle x | [H_r, \rho_r] | x' \rangle \\ -\mu(x-x')^2 \langle x | \rho_r | x' \rangle &, \end{aligned} \quad (2)$$

where a unique relevant observable  $X$  with eigenvalues  $x$  is considered. The first term in the right-hand side represents a standard quantum evolution under the effect of a relevant Hamiltonian  $H_r$  whereas the second term represents decoherence and tends to make the density diagonal when the decoherence coefficient  $\mu$  is large. It means that the basis  $|x\rangle$  is very peculiar, since it is the unique one in which diagonalization takes place [12] (it was often chosen by hand in the models, rather from common experience among physicists than the leaning for generality among mathematicians).

(iii). The decoherence coefficient  $\mu$  is actually most often very large. For instance, in the case of an object with mass  $m$  and a relaxation (damping) time  $\tau$ , one gets at a high enough temperature  $T$ ,

$$\mu = \frac{mT}{\hbar^2} \cdot \tau^{-1} . \quad (3)$$

Systems having an infinite damping time (such as ordinary light beams) can therefore be insensitive to decoherence and this remarkable property was observed in some superconducting quantum interference devices [13, 14].

(iv). Decoherence is often followed by a classical behavior of the relevant subsystem (e.g., a pointer). Decoherence was observed experimentally for the first time in 1996, for highly excited atoms crossing a resonating cavity where the photons act as an environment [5]. The theory could be worked out in detail in that case and the experimental results agreed with it beautifully.

### The attempts at a general theory

There are now at least three different approaches to a more general theory of decoherence:

(i). A very efficient one is coarse graining [15, 16]. The relevant observables are averages of a field density over extended regions of space. One may use densities of mass, energy, charge, and so on, and these decohering observables are very similar to the “Newton discrete quantities”, which were used long ago when discretizing a continuous matter to replace it by a set of small finite pieces. Diagonalization holds

holds in that special case, at least in a non-relativistic approximation.

(ii). Zurek proposed the existence of “predictability sieves” as a possible answer to the problem of diagonalization [17]. Some states would be stable under perturbations from the environment and would decohere. The idea is attractive, but has not yet resulted into a constructive theory since the only known examples rely on explicit models.

(iii). One can also apply a standard method in the quantum theory of irreversible processes, the projection method [18], which is easily adapted to the case of decoherence as follows [19].

The total Hamiltonian  $H$  of the system is supposed to be made of three parts: a relevant Hamiltonian  $H_r$  involving only the relevant subsystem, the environment energy  $H_e$  and a coupling term  $H_c$ . The total density operator evolves under the action of  $H$ :

$$i\partial\rho/\partial t = [H, \rho] . \quad (4)$$

The method involves a choice of significant observables  $A^i$ , their average values  $a^i = Tr(\rho A^i)$  being the quantities one wants to know with their evolution in time. One uses most conveniently for the set  $\{A^i\}$  a complete set of commuting relevant observables  $X$  describing for instance a measuring apparatus and a measured microscopic observable, together with the environment energy  $H_e$  and the identity operator  $I$  (for the sake of normalization). A test density operator  $\rho_0$  is then introduced as containing a minimal information while generating the expected average values of these quantities, i.e.

$$\rho_0 = \exp(-\lambda_i A^i) , \quad (5)$$

$$Tr(\rho_0 A^i) = a^i \equiv Tr(\rho A^i) . \quad (6)$$

It turns out that

$$\rho_0 = \rho_r \oplus \rho_e , \text{ with } \rho_e = Z^{-1} \exp(-H_e/T) . \quad (7)$$

The second equation does not mean that the environment is in thermal equilibrium, but that its average energy is the only significant quantity one needs taking into account for it. A clever trick is then used in the following way [18]. One introduces some “density operators” (i.e. not necessarily positive or normalized linear functionals over observables) through

$$s_i = \partial\rho_0/\partial a^i \quad (8)$$

and a superoperator (acting on density operators) through  $P = \sum_i s_i \otimes A^i$ , so that for instance

$$P\rho = \sum_i s_i \cdot Tr(\rho A^i) = \rho_0 . \quad (9)$$

The superoperator  $P$  is a projection (i.e.  $P^2 = P$ ) as well as  $Q = I \otimes I - P$ . Letting  $P$  and  $Q$  act on the two sides of Eq. (4), one gets two coupled equations for  $\rho_0$  and  $\rho_1 = Q\rho$ .

Eliminating (formally)  $\rho_1$ , one obtains in principle a master equation for the evolution of  $\rho_0$ .

This equation would be of little use if it could not be written down explicitly, so that another important trick is then used. The coupling  $H_c$  between the relevant subsystem and the environment is redefined by subtracting its relevant part from it, i.e.

$$H_c \rightarrow H'_c = H_c - \{Tr_e(\rho_0 H_c)\} \otimes I_e . \quad (10)$$

The physical meaning of this subtraction is that, in most cases, the quantity  $H'_c$  is small and fluctuating: One may consider as an example the motion of a piston in a cylinder containing a gas. The observable  $X$  will be the position of the piston. The coupling term  $H_c$  involves a priori the interaction potentials of the gas molecules with the atoms in the piston, but when one subtracts its relevant part (which is the gas pressure on the piston), the remaining coupling consists only of the fluctuations arising from the collisions of the gas molecules with the piston.

This simple procedure has a very wide domain of applicability and the new coupling  $H'_c$  can be treated in many cases by means of second-order perturbation calculus to yield an explicit master equation generalizing Eq. (2).

### III. SOME THEORETICAL RESULTS

The third method has yielded the wider collection of instructive results and I shall therefore rely on it when stating the main results of theory. It recovers as special cases the results of every known model and its range of validity is certainly very wide, when properly used or extended. I shall restrict however the discussion to two important questions.

#### Diagonalization

Zurek asked how general is the diagonalization property in decoherence [12]. Coarse graining shows that diagonalization holds at least for the “Newton observables”, which have a strong connection with position in space [15, 16]. A mathematical criterion for diagonalization is given by

$$[H'_c, X] = 0 . \quad (11)$$

The eigenvectors of the commuting observables  $X$  provide in that case a diagonalization basis.

This condition is however very strong from a mathematical standpoint and the physical reasons for its occurrence are little understood. In the case of coarse-grained “Newton observables”, they are due to the very simple expression of a non-relativistic basic Hamiltonian (quadratic kinetic energy plus interaction potentials). There is probably a deeper and more direct connection with Galilean invariance, but no proof is known for this conjecture.

The criterion (11) is also satisfied when the system under consideration is a SQUID loop and  $X$  is the magnetic flux through the loop. In that case, the origin of commutation lies in the Maxwell equation for induction and there is no obvious

physical analogy with the previous case.

#### Classical limit

The basic mathematical formalism for investigating the classical limit of quantum mechanics is always microlocal analysis, in one form or another. It relies on associating a symbol  $a(x, p)$ , which is an ordinary function, with every operator  $A$  through a formula such as

$$a(x, p) = \int \langle x' | A | x'' \rangle \delta \left( x - \frac{x' + x''}{2} \right) e^{-ip(x' - x'')/\hbar} dx' dx'' . \quad (12)$$

When the operator  $A$  is the reduced density operator, the associated symbol is the Wigner function  $W(x, p)$ . Similarly, a Hamilton function  $h_r(x, p)$  is associated with the relevant Hamiltonian  $H_r$ .

When the diagonalization criterion (11) is valid, the master equation has a form similar to Eq.(2), with a decoherence term behaving more or less like  $-\mu(x - x')^2 \langle x | \rho_r | x' \rangle$ . When one uses the Wigner function in place of the density matrix, one gets a diffusion equation

$$\frac{\partial W}{\partial t} = \dots + \mu \frac{\partial^2 W}{\partial p^2} , \quad (13)$$

where the contribution arising from the first (dynamical) term in Eq.(2) has not been written down explicitly.

When the criterion condition (11) does not hold, one gets a diffusion equation in phase space:

$$\frac{\partial W}{\partial t} = \dots + \mu \frac{\partial^2 W}{\partial p^2} + \mu' \frac{\partial^2 W}{\partial x^2} . \quad (14)$$

(Nota: generally, there are also mixed derivatives in  $x$  and  $p$ ). The result is not then diagonalization but a diffusive smoothing of the Wigner function in every direction of phase space. It is then easily understood why the classical approximation becomes valid after decoherence, because a smooth Wigner function behaves essentially as a classical distribution function [19].

### IV. SOME CONSEQUENCES OF DECOHERENCE

The main consequences of decoherence are well known. The most important one for the foundations of physics is an understanding of the transmutation of quantum mechanics into classical physics. The word “transmutation” has been coined here as an opposition to more standard eliminations of irrelevant degrees of freedom. When for instance many degrees of freedom of quantum electrodynamics are eliminated for yielding an elementary theory of the hydrogen atom, these degrees of freedom are “integrated out” (in the Feynman sum over histories), but the unitary rules of quantum mechanics remain valid. In the case of decoherence, when the irrelevant degrees of freedom associated with the environment are eliminated, unitarity is broken. A quantum superposition is

replaced by alternatives obeying standard probability calculus. Determinism can replace quantum randomness, in spite of their apparent incompatibility (as a matter of fact, classical behavior is valid with very small probabilities of error, which means that the probabilistic character of quantum mechanics remains fundamental, however attenuated it may be [20, 21]).

Decohering histories [22], extending Griffiths' idea of consistent histories, can be used to exhibit the very large domain of validity of standard logic in quantum mechanics [23] Griffiths has shown how they remove every paradox from quantum mechanics [24]. Interesting results have also been obtained about the generation of a definite direction of time under decoherence [25]. Finally, it has been found that decoherence allows a derivation of the Copenhagen rules in measurement theory as so many theorems resulting directly from the basic quantum principles [21, 23].

## V. SOME REMAINING PROBLEMS

Quantum computing has raised a problem opposite to the older ones encountered in measurement theory, namely how to avoid or delay the action of decoherence. Whether or not this aim is possible remains an open problem.

The extension of decoherence theory to a relativistic framework (in special or general relativity) remains also an open question.

An objective definition of the relevant observables undergoing decoherence is also a deep problem. There is certainly a hierarchy of observables with different rates of decoherence, but their construction from first principles is poorly understood.

Decoherence is, formally, never complete. There always remain exponentially small non-diagonal terms in the reduced density matrix, reminding us that an initial pure state remains pure according to basic quantum mechanics. Does it mean that decoherence is only a phenomenological theory [26], or is there some deeper way of interpreting very small probabilities [25]?

An attractive answer to this last question would be that decoherence is not a final effect, but is followed by a true reduction effect finishing the job, insuring the uniqueness of physical reality and getting rid of tiny probabilities of entanglement. If this were the right answer, one might expect two properties of reduction. It should act on a state that is already prepared by decoherence and be closely linked to this preparation. It should be also expressible – at least in a phenomenological way – in the framework of ordinary quantum field theory, even if its origin lies deeper, because one sees plainly reduction in ordinary circumstances. The problem of reduction could be therefore the next step in the investigation of the foundations of quantum mechanics. This is why I proposed recently a model of reduction with these apparently reasonable constraints [27].

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