

Challenges in Signal Analysis of Resonant-Mass Gravitational Wave Detectors

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An overview of the main points related to data analysis in resonant-mass gravitational wave detectors will be presented. Recent developments on the data analysis system for the Brazilian detector SCHENBERG will be emphasized.

I. INTRODUCTION

As predicted by the theory of relativity and other theories of gravitation, time-dependent gravitational forces are expected to propagate in spacetime in the form of waves [1]. Such gravitational radiation is extremely difficult to detect because gravitation is the weakest of all the fundamental forces of nature.

For instance, a wave of very strong amplitude could generate a displacement of 10^{-18} m in a system (“antenna”) with typical length of 1m. In order to detect such a tiny displacement special sensors must be used and the signal should be sent to computers to be properly analyzed. The path between the antenna and the computer is tricky, though, because many spurious signals - noise - come as well.

When an experiment is performed, normally the signal that is the object of the observation (the “useful” signal, $u(t)$) is accompanied by other, unwanted signals labelled with the generic name of “noise” ($n(t)$). The goal of signal analysis is to retrieve the useful signal out of noise, as illustrated in Figure 1. In the case of gravitational wave experiments, the useful signal is a gravitational wave.

Because signal analysis identifies the signal in the midst of the noise it becomes a fundamental part of the experiment. To do so it is necessary to know as much as possible about the signal and the sources of noise. In this work general features of data analysis in gravitational wave resonant-mass detectors will be presented, so that the reader can become familiar with the latest challenges faced by the Brazilian group that works

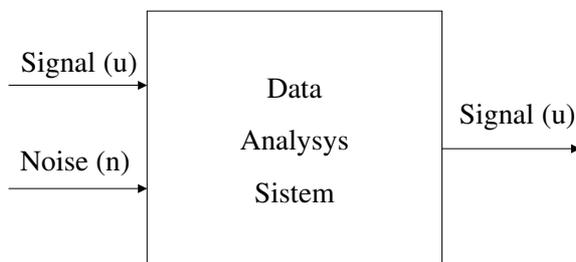


FIG. 1: The main objective of signal analysis is to retrieve the useful signal out of noise.

in the SCHENBERG gravitational wave detector within this important field.

This paper is organized as follows: in Section II the motivation for the research is presented, namely gravitational waves and their sources. Section III is devoted to presenting general features of resonant-mass detectors, while Section IV discusses some noises present in such detectors. Several basic concepts usual in the context of data analysis are presented in Section V while some important challenges that must be faced in the analysis of SCHENBERG’s data are discussed in Section VI. The closing comments are made in the final section.

II. GRAVITATIONAL WAVE SOURCES

Strong gravitational waves (g.w.) are expected to be generated by astrophysical objects [2]. For instance, two very massive stars orbiting each other would emit such waves. In particular, when they are coalescing they emit waves in a large frequency range. Another example is given by a black hole that rings down, also emitting in different wavelengths [3].

Astrophysical sources emit basically three kinds of waves, depending on the waveform: bursts (impulsive signals of short duration, like those produced by supernova explosions), continuous waves (periodic, with long duration, like those emitted by stable binary systems) and stochastic waves (a spectrum composed with the superposition of many sources, as the one expected from cosmological origin).

From the analysis of the detected waves emitted by such sources important information is expected. The very first direct detection will provide a test for one of the predictions of the theory of general relativity. Then, continuous observation will allow testing other theories of gravitation [4, 5], besides initiating gravitational astronomy [6, 7].

In order to make astrophysical observations the following parameters that characterize the g.w. are needed: the amplitudes of the two states of polarization of the wave as functions of time ($h_+(t)$ and $h_\times(t)$), the source direction in the sky (given by the angles θ and ϕ) and the phase of the wave (obtainable from the detailed time dependency of $h_+(t)$ and $h_\times(t)$, usually associated to the polarization angle α). Therefore, gravitational wave observatories must be able to detect at least five independent observables in order to allow for gravitational wave astronomy to start. On the other hand, detection of at least one observable would be a strong evidence of

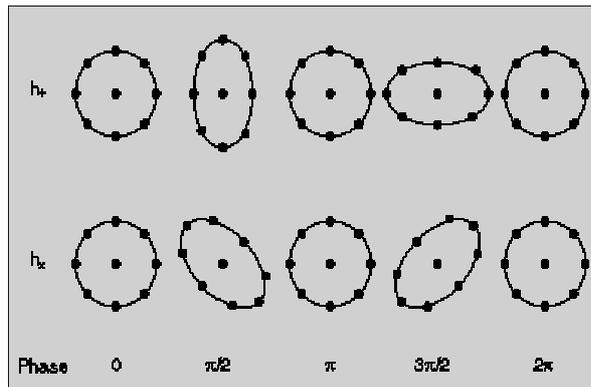


FIG. 2: When a gravitational wave passes through a ring of particle it changes their relative positions, depending on the wave's polarization. The top line shows the motion produced by a wave with polarization "+", while the bottom line shows the motion due to a wave with polarization "x".

the existence of such waves. All the existing detectors are presently aimed at this first detection, while being prepared to become part of g.w. observatories in the future as well. In the next section a brief overview on resonant-mass detectors will be presented.

III. RESONANT-MASS DETECTORS

When a gravitational wave passes through a ring of particles it changes their relative positions [8], as shown in Figure 2. Similarly, solid bodies are distorted in the presence of such waves due to the changes in spacetime. This effect is the basis for resonant-mass gravitational wave detectors. For instance, a massive cylinder would oscillate longitudinally in the presence of a g.w., in a frequency resonant with the wave.

The first of such detectors was built in the 1960's by Joseph Weber. It was a massive, cylindrical aluminum bar at room temperature, 1.5m long, monitored by piezoelectric transducers. Nowadays there are improved bar detectors, sometimes cooled down to millikelvin temperatures, longer and using more sensitive transducers [9]. Those that are operational at the moment are: ALLEGRO (EUA), EXPLORER (Switzerland), NAUTILUS and AURIGA (both in Italy)

Bar detectors are able to determine only one observable, and due to their geometry there are directions in which they are more sensitive than others. For these reasons several bar detectors, appropriately positioned, would have to be used if one wished to build a gravitational observatory with only this kind of detector. In fact, investigations have already been done in this direction [10].

Besides cylindrical geometry, it has been known for some time that spherical, solid objects could be used as g.w. resonant-mass detectors. In principle this geometry has no preferred direction of observation (i.e., it is omnidirectional) and the five observables needed for gravitational as-

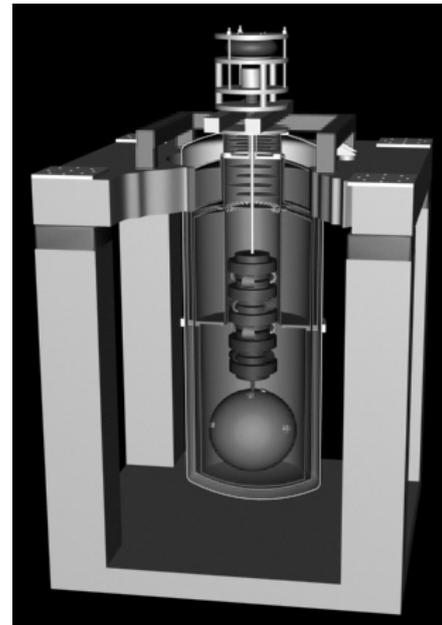


FIG. 3: Schematics of the SCHENBERG gravitational wave detector.

tronomy could be obtained from only one detector appropriately equipped [6, 11]. These are major advantages, but for many years bars were preferred because they were easier to machine and equip.

Under the rationale that spherical geometry is the best, in the last decade a lot of effort has been invested in investigating and building resonant-mass g.w. detectors with this design [9, 12]. The Brazilian *Mario SCHENBERG* detector is one of these last generation detectors [13] (see Figure 3). When fully operational it will be able not only to acknowledge the presence of a g.w. within its bandwidth: it will be able to inform the direction of its source in sky, the wave's amplitude and its polarization - one only antenna working as a gravitational wave observatory in a bandwidth between 3000 and 3400Hz, sensitive to displacements around 10^{-20} m. To this end at least 6 transducers will continuously monitor displacements of the antenna's surface. The data collected will be sent to be analyzed and, as mentioned above, it will certainly be accompanied by some noise. In the next section some of the main noise sources in SCHENBERG will be presented.

IV. NOISE SOURCES IN SCHENBERG

People walking around the detector may generate undesired vibrations on the resonant-mass antenna which become *seismic noise* in the signal detected by the transducers. This kind of noise can be minimized with the use of an appropriate suspension system, as the one used in SCHENBERG [13, 14]: the spherical antenna is carefully suspended from its center of mass by a rod attached to a system of "springs" specially designed to attenuate seismic noise.

Other kinds of noise are harder to minimize. The Brownian motion of the atoms of the antenna generate *thermal noise*, which can be reduced by cooling the massive sphere to temperatures as low as possible. SCHENBERG is expected to be cooled down to 4.2K (liquid Helium temperature) in its first test run. After the antenna is cooled down, the standard procedure to reduce its thermal noise has been to apply a threshold in the detected signal above which a signal may be considered a candidate for a g.w. event. It has been typical to set the threshold to amplitude signal-to-noise ratio between 3 and 5 [15]. In the future it is planned to cool SCHENBERG down to temperatures of the order of millikelvin, decreasing the antenna's thermal noise in about one order of magnitude [16].

Other relevant noises in SCHENBERG are related to the transducers used to translate the mechanical motion of the antenna into an electrical signal. They are microwave parametric transducers [17] that generate noises that can be divided into two groups: narrow band and broadband noises. The narrow band noise includes the back action and the Brownian noises. The broadband noise can be divided into two components: one due to the amplifier and another due to the phase noise in the pump microwave source. All these noises have been modelled mathematically [16] so that the corresponding expressions can be used in the development of digital filters that help to minimize these noises in the detected signal. The importance of the determination of such mathematical expressions will become more clear within the context of data analysis, as follows.

V. BASIC CONCEPTS IN DATA ANALYSIS

The electric signal that leaves the transducers carries information of both the gravitational wave and the different noises. Data analysis then tries to extract the g.w. signal as clean as possible out of noise. In other words, one is concerned with improving the signal-to-noise ratio of measurements: an accurate measurement can be made when the g.w. signal causes an output of the detector that is large compared to the random variations of the output when no g.w. signal is present. In what follows some basic concepts important do data analysis will be presented, including signal-to-noise ratio. The theory of signal detection, much of it invented in the context of the development of the radar during World War II, can be found in a number of books [18, 19].

The output of a g.w. detector is expected to be continuously recorded, and this allows one to know this output as a function of time: $s(t)$. Such mathematical object is called a *time series*. In principle one hopes to know as much of the g.w. signal as possible, making it a deterministic time series of a predetermined form, $u(t)$, or a *template*. For instance, burst signal are commonly associated to Dirac's delta function. On the other hand, noise is usually associated to time series, $n(t)$, that randomly varies from one realization to the next. Before data analysis one then has $s(t) = u(t) + n(t)$. Notice that in the absence of any g.w. signal the detector output is just plain noise: $s(t) = n(t)$.

These distinctions between the time series of the g.w. signal and noise imply different mathematical operations to charac-

terize the regularities of these series. A useful way to express the time series is in terms of its Fourier transform, $S(f)$, a function of frequency which contains the same information of the time series $s(t)$, defined by

$$S(f) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} s(t) e^{-2\pi f t} dt.$$

This puts the data in the *frequency domain*. Notice that since data is sampled in discrete sets in the practical world, the discrete analog of the above equation is a more appropriate way to define the Fourier transform for real data sets. In fact, there exists a powerful algorithm for calculating discrete Fourier transforms, the FFT or Fast Fourier Transform, which is very useful in modern laboratory instrumentation. But for this brief presentation of the subject the continuous approach will be preferred.

The value of the Fourier transform of $s(t)$ at the frequency f is a measure of the degree to which $s(t)$ varies like a sinusoid of frequency f . Putting it in a better way, the Fourier transform gives the contribution of a sinusoid of frequency f to a sum of sinusoids that equals the function of interest, $s(t)$ in this example.

For a deterministic signal as the one expected for g.w. its Fourier transform can be calculate quite straightforwardly from the above definition. The characterization of a random time series (noise) in the frequency domain demands an extra step, which is the definition of the *autocorrelation function* of the noise:

$$n * n(\tau) \equiv \int_{-\infty}^{\infty} n(t) n(t + \tau) dt.$$

This function of a time offset τ is a way of measuring how $n(t)$ is related to itself, at different time offsets between two copies of $n(t)$. When $\tau = 0$ the time series is aligned to itself so the autocorrelation function will always have a maximum at this time offset. The width of $n * n$ indicates how rapidly the noise changes with time.

The translation of the random time series $n(t)$ into the frequency domain is then given by the the *power spectrum* of this series (also known as power spectral density), defined as the Fourier transform of the autocorrelation function:

$$P_s(f) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} n * n(\tau) e^{-2\pi f \tau} d\tau.$$

$P_s(f)$ is a measure of the amount of time variation in the time series that occurs in the specific frequency f .

Instead of thinking in terms of complex integrals of both positive and negative frequencies, experimentalists often prefer to think in terms of sines and cosines of positive frequencies. So it is common to use the *single sided power spectrum*, $s^2(f)$, defined by

$$s^2(f) \equiv \begin{cases} 2P_s(f), & \text{if } f \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Often the expression "power spectrum" refers to this definition within the experimental context. Also, an object derived

from this quantity is commonly used, the *amplitude spectral density*, defined simply by $s(f) \equiv \sqrt{s^2(f)}$. Details on the advantages and disadvantages of the use of these quantities can be found in [20].

After presenting how signal and noise can be characterized in the theory of signal detection, now the concept of signal-to-noise ratio (or SNR) will be introduced. This is a dimensionless figure of merit for a measurement. In order to create such a dimensionless quantity one must keep in mind that signal detection is the process of searching for a pattern resembling the template in the middle of a noisy record. Such a pattern should occur with a strength unlikely to be due to noise alone.

The match between the time record and the template can be estimated by the *cross-correlation integral* between the template $u(t)$ and the time record $s(t)$, evaluate for all possible times at which the signal could have arrived:

$$s * u(t) \equiv \int_{-\infty}^{\infty} s(\tau)u(t + \tau)d\tau.$$

This definition is similar to the definition of the autocorrelation function, given previously. The cross-correlation function indicates how related the functions $s(t)$ and $u(t)$ are to each other. Then one way to characterize the strength S^2 of the signal present in any time t is using the cross-correlation between the expected form of the output (the template $u(t)$) and the output $s(t)$:

$$S^2 \equiv |s * u(t)|.$$

As for the noise, it can be characterized by N^2 , the mean square value of the cross-correlation between the output $s(t)$ in the absence of g.w. (i.e., noise) and a given template:

$$N^2 \equiv \langle (s * u(t))^2 \rangle.$$

The brackets $\langle \rangle$ indicate averaging over time.

With these characterizations one defines the signal-to-noise ratio as the square root of the ratio of the measure of the amount of signal present (S^2) to the expected value due to noise alone (N^2):

$$SNR \equiv \sqrt{S^2/N^2}.$$

A large SNR indicates that something is present in the time series $s(t)$ other than noise. In practice $SNR \approx 1$ is not of much use, but $SNR \gtrsim 10$ indicate detection of some confidence. Therefore the goal is to maximize SNR in order to detect a g.w. This can be accomplished in a number of ways. For instance, $n(t)$ for antenna's thermal noise can be reduced by cooling the antenna down. As for transducer's noise, a digital filter is useful. Filtering is such an important part of signal analysis that it will be briefly reviewed next.

Suppose there is a device possessing one single input ($i(t)$) and a single output($s(t)$). Such device will be considered a *linear system* if there is some linear relationship between the input and the output: $s(t) = ai(t)$. When the relationship between the input and the output does not change with time,

then the device is a *linear time-invariant system* (or just linear system for short).

The *filters* considered here are linear time-invariant devices in which the input and output are quantities with the same dimensions. On the other hand, the term *transducer* is used as a general name for a linear system whose input and output have different physical units.

One way to specify the input-output relationship in a linear system is to give the "impulse response", $g(t)$. This function is the output obtained when a single unit impulse is applied to the input at $t = 0$. In the frequency domain the Fourier transform of the impulse response, $G(f)$, is called the *frequency response* (or sometimes *transfer function*). This is a complex-valued function of the frequency f whose real part represents the response "in phase" with a sinusoidal input of frequency f , while the imaginary part corresponds to the "quadrature" component. One can show that if $I(f)$ is the Fourier transform of the input and $S(f)$ is the Fourier transform of the output, then $G(f) = S(f)/I(f)$. This equation implies also that in Fourier space the output of a linear system is simply the product of the input and the frequency response, with no need to calculate convolution integrals.

VI. ASPECTS OF SCHENBERG'S DATA ANALYSIS

The theory presented in the last section shows that the knowledge of SNR for SCHENBERG demands the determination of the time series of: the detector's output, the g.w. signal (template) and the noise.

In the particular case of SCHENBERG it is necessary to combine the outputs of several transducers to create the time series of the output, $s(t)$. This combination is part of the mathematical modelling of the detector. There are 6 transducers planned to monitor the antenna surface's motion and there are mathematical models for the detector for the case that all these transducers operational. Such models have investigated two situations: one in which the transducers are perfectly uncoupled [11, 21, 22] and another in which the transducers are somehow coupled to each other [23], an instance that still offers possibilities of investigation.

One of the challenges presently faced by the data analysis group within the GRAVITON project [24] (the one SCHENBERG is part of) is to develop a model of the detector with less than 6 transducers. In this case there is a break in the convenient buckyball symmetry [21] and the consequences of this fact must be investigated. Two approaches are now under investigation: one considers the model already developed for 6 transducers [25] and simply reduces their number; the other considers the fewer transducers as independent devices and redesign the mathematical model. For the study of the last approach several references in the literature may be used as a starting point [26–30].

An actual time series of the detector's output is expected be known as soon as it is collecting data, in the next months. At least three transducers are expected to be installed then, so the investigation mentioned above should be conclude soon.

The work on the knowledge of the time series of the tem-

plate has started long ago and is one of the more active fields in data analysis [31], so that several templates already exist that can be used [3, 32, 33]. But still many challenges remain to be faced in the field of g.w. sources. At the moment one of the investigations been carried out by SCHENBERG's data analysis group in this way refers to the detection of astrophysically unmodelled bursts of gravitational radiation using wavelets [34].

A significant work has already been done to determine the time series of the noise in SCHENBERG, as mentioned in Section IV. Since all time series needed are available one is able to translate them into the frequency domain and determine SCHENBERG's SNR, as is done in [3]. Also, based on the model with 6 transducers operational simulations of the detector in the presence of noise have already been made (see Costa and Aguiar in [9]).

The improvement of SNR can be accomplished by using appropriate digital filters, for instance. When noise is white (broadband, spread over spectrum and stationary, implying a spectral density that does not depend on the frequency, which is commonly the case) it can be shown that the best filter for a given template is the *matched filter*. This linear system has an impulse response which is the time reverse of the signal one is interested in: $g(t) = s(-t)$. This filter is presently under investigation for the case of SCHENBERG within GRAVITON's data analysis group.

When noise is not white (either by not being broadband nor stationary, or both) other strategies are used. For instance, the monitoring of the environment with seismographs help vetoing seismic noises. Monitors for cosmic rays work similarly. The possible influence of lightings on the data has been investigated as well (see Magalhaes, Marinho, Jr. and Aguiar in [9]).

In the particular case of SCHENBERG, which will be monitored by several transducers simultaneously, one may wonder if filtering should be performed at the transducers outputs to veto non-white, non-stationary noise before combining them to extract the wave's parameters. This is the case when several bar detectors work in coincidence [15], spaced around the world with some different characteristics among themselves. Maybe such a procedure would eliminate some noise due to local disturbances in individual transducers so that the combined data could be less noisy.

However it can be argued [35] that since SCHENBERG's identical transducers are collecting data simultaneously at the same site there is no need to risk loss of information by filtering their data before combining them, a combination that will be possibly done in real time and with all characteristics under control. The data analysis system is then expected to be able to optimize SNR using the combined outputs without intermediate filtering.

Besides the matched filter another kind of filter is under investigation for SCHENBERG, namely an *adaptative filter*, one that changes with the variations of the power spectrum

of the noise. This is a device useful in the presence of non-stationary noise, like electric and seismic noises. This kind of filter has already been investigated within the g.w. detection context [36].

Finally, an aspect that still deserves investigation is the use of the Bayesian statistics in SCHENBERG's data analysis. Such kind of study is already been carried out related to other detectors (see L.S. Finn in [9]).

VII. CONCLUDING REMARKS

In this work a brief overview of data analysis in resonant-mass gravitational wave detectors was presented, with emphasis on issues involving the SCHENBERG detector. This detector, installed at the Physics Institute of the University of Sao Paulo (Sao Paulo city), is expected to be collecting data soon. It will be able to run in coincidence with other detectors around the world, particularly the broadband interferometric ones (see links for the several groups at [24]). Such kind of coincidence is important in the first place to increase the credibility on the detection of a g.w. For this reason it is interesting to consider the development of a common protocol for information exchange between SCHENBERG and these detectors.

It is worth pointing out the particular feature of SCHENBERG of working as an observatory of g.w. by itself due to its capability of several simultaneous measurements. Monitored by three transducers, as it is planned for the near future, this detector will be able to determine the squared amplitude and the direction of propagation of a g.w. sufficiently strong within its frequency band. Only spherical detectors like SCHENBERG are able to have such versatility. This is the case of the MiniGrail, another spherical detector built in The Netherlands (see de Waard in [9]) sensitive to frequencies smaller than SCHENBERG's. Also, an Italian group is interested in building a large spherical detector (see V. Fafone in [9]).

There is the belief within the international community that works with g.w. detection that the first direct detection of gravitation radiation from an astrophysical source will become a reality in the near future. This will open a new window to the universe, bringing new information about known objects and about fairly unknown things, like dark matter. In order to extract such information from the huge amount of data that is expected to be generated from the detectors' outputs (which has already started) a lot of work will be demanded in the field of data analysis. This is a promising field.

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