Production Cross Sections of Super Heavy Elements

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This report describes a method for calculating fusion and decay probabilities in reactions leading to the production of transfermium elements. The competition between quasi-fission and fusion is described on the basis of the Dinuclear System Concept (DNSC). Both the competition between fusion and quasi-fission and statistical decay of heavy highly fissionable excited compound nuclei is described in an approach based on the Monte-Carlo method.

1 Introduction

In the last few years in the Laboratory of Nuclear Reactions, JINR, a lot of experimental information was obtained concerning the formation of superheavy elements (see [1, 2] and references therein) as well as the mass distribution of fragments, produced in reactions which are used for the synthesis of superheavy elements [3]. In [3] for the $^{48}Ca + ^{238}U, ^{244}Pu, ^{248}Cm$ reactions the fragment mass distributions were measured. It was found out that in all cases a maximum in the distribution corresponding to the doubly magic nucleus ^{208}Pb occurred. This maximum was observed in a rather large energy range of the bombarding ions ^{48}Ca; for instance, this was demonstrated for the ^{48}Ca+ ^{244}Pu reaction. We shall try to explain this experimental fact using our approach.

In our calculations of the reactions used to synthesize superheavy elements, we consider them to proceed in three stages: formation of the double nuclear system, formation of the compound nucleus (in competition to quasi-fission and complete fusion) and statistical decay of the excited compound nucleus.

2 The fundamentals of the Dinuclear System Concept

The motivation for the Dinuclear System Concept (DNSC) and a comparison of DNSC with existing models for the fusion of massive nuclei have been already given in our previous papers, see for example [4, 5]. Therefore here we shall discuss only its main features, applied to the analysis of fusion reactions used for the production of transfermium elements.

In the best known models for complete fusion of nuclei, the production cross section for compound nuclei, σ_{CN}, is not different from the capture cross section, σ_c. In other words, after the capture stage, a compound nucleus is formed with a 100% probability. In our approach, complete fusion is the final stage of the evolution of a DNS, at which all the nucleons of one nucleus have already gradually been transferred to the other nucleus. Thus, the complete fusion cross section, σ_{CN}, is part of the capture cross section, σ_c, and, competing with fusion, there is also quasi-fusion. Therefore the complete fusion cross section can be written as

$$\sigma_{CN}(E^*) = \sigma_c \cdot P_{CN} \approx \pi\lambda_0^2 \sum_{l=0}^{l_{cr}} (2l+1) T(l, E_{CM}) \cdot P_{CN},$$

where P_{CN} is the probability that complete fusion occurs, l_{cr} - the limiting value of the compound nucleus angular momentum, and T - the penetrability of the barrier.

In the evolution of the DNS, each nucleus of the DNS retains its Individuality. This is a consequence of the influence of the shell structure of the partner nuclei since the kinetic energy of the bombarding ion, and, thus, the resultant excitation energy, as a rule, is low in these reactions.

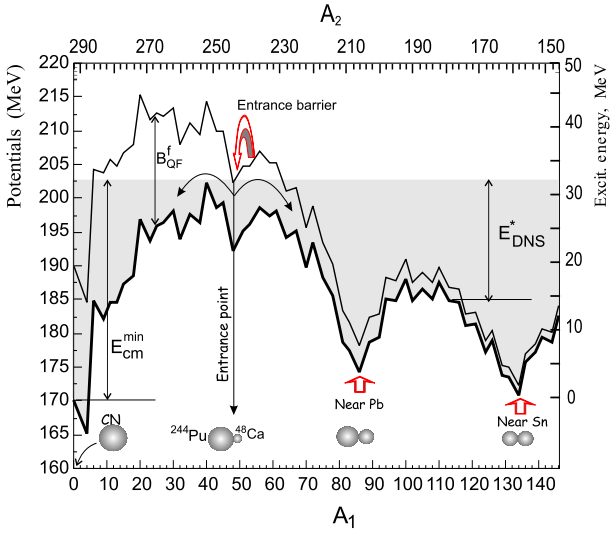

Figure 1. Driving potential (lower curve) of the dinuclear system as a function of the atomic number of one of the fragments for the reaction $^{48}Ca + ^{244}Pu$.

The Macroscopic Dynamic Model (MDM) [6, 7] description of the coalescence of two nuclear drops does not take into account the nuclear shell structure, and complete fusion does not compete with quasi-fission. These two processes are considered to be separated in energy space. An essential characteristic of the DNS that dictates its evolution is the system's potential energy V(Z,L). In this work we took Proximity as the nuclear potential $V_N(R)$ (for details, see [7]).

In Fig. 1, two profiles of the potential energy are shown: the profiles along the minimum and the maximum of the potential surface calculated for the reaction ^{48}Ca+^{244}Pu. The curve $V(Z, L = 0)$ (for the value of R corresponding to the pocket) has a few local minima, which reflect the shell structure in the interacting nuclei.

Among them most pronounced are: the minimum corresponding to the compound nucleus (Z=0) and three others, the first one being at Z of the light fragment equal to 20 and corresponding to the entrance channel (the projectile ^{48}Ca) and the other two at Z=82 (the complementary heavy fragment – the doubly magic ^{208}Pb nucleus) and Z=52 (the complementary fragment corresponding to the magic Sn-nucleus), respectively. In this way, there is evidence of shell structure in the driving potential, which will manifest itself (as it will be shown below) in the fragment mass distributions as well.

Heavy and superheavy elements (SHE) are typically produced at projectile energies, at which the obtained compound nucleus would have excitation energy as low as possible. This ensures higher survival probability for the compound nucleus while it de-excites. In Fig.**??**, the DNS energy that corresponds to the minimally possible excitation energy of the compound nucleus is shown by cross-hatching. As follows from Fig.**??**, while descending from the Businaro-Gallone point (B.G.) to the point of compound nucleus formation, the system undergoes highest heating. Only at this stage of the DNS evolution most of the potential energy of the dinuclear system will be transformed into excitation of the compound nucleus. This peculiarity of the DNS evolution, characteristic of SHE fusion reactions, required the use of experimental masses in the calculation of the potential energy. In addition, on the way to the compound nucleus, the DNS has to overcome the inner potential barrier B^*_{fus}, which is the difference between the values of the potential at the B.G. point and at the reaction entrance point.

The inner potential barrier B^*_{fus} is due to the endothermic nature of the nucleon transfer in the massive DNS and makes the system move in the direction to the compound nucleus. The motion of the DNS in the reverse direction, to greater symmetry, might result in its leaving the potential pocket (with a break-up into two fragments, which takes place during the motion in the direction to increasing R) after overcoming the QF barrier, which we define as the difference between the values of the driving potential for the entrance channel and the point of the break-up into two fragments. The energy necessary to overcome these barriers is deduced from the excitation energy E^* of the dinuclear system, which is an essential feature of our approach. A compound nucleus is unlikely to be formed if the DNS excitation energy is smaller than the value of B^*_{fus}. The more symmetric the combination of the nuclei in the entrance channel, the higher is the inner fusion barrier B^*_{fus}, which the dinuclear system has to overcome on the way to the compound nucleus, and the lower the quasi-fission barrier B_{QF}. Hence, QF offers stronger competition.

As the nucleon transfer between the nuclei in the DNS is of statistic nature, there is a possibility that the system may reach and overcome the B.G. point. Thus a compound nucleus may be formed. The alternative to that process is the break-up of the system into two fragments (quasi-fission). In the calculation of the probability of proton transfer from one nucleus to the other in a dinuclear system we applied the expression from ref. [8] and assumed that the macroscopic nucleon transfer probability P_z can be found from the microscopic probability λ_z and level density ρ_z as P_z=$\lambda_z\rho_z$. The level density can be written in terms of the DNS potential energy as $\rho_z = \rho$(E-V(Z, 1)), where E^* is the excitation energy of the dinuclear system. Finally, the proton capture P^+ and stripping P^- probabilities can be written as follows:

$$P^+ = \left\{1 + \exp\left[\frac{V(Z+1,L)-V(Z-1,L)}{2T}\right]\right\}^{-1},$$
$$P^- = \left\{1 + \exp\left[\frac{V(Z-1,L)-V(Z+1,L)}{2T}\right]\right\}^{-1},$$

where $T = (E^*/a)^{1/2}$ is the nuclear temperature and $a = 0.093A$ [10] is the level density parameter. Knowing these relative $(P^+ + P^- = 1)$ probabilities and using a random value uniformly distributed over the interval between 0 and 1, we simulate the direction for the motion of the DNS: either in the direction to a symmetric system or in the direction to the compound nucleus. We repeat this procedure as many times as needed to obtain the necessary statistics.

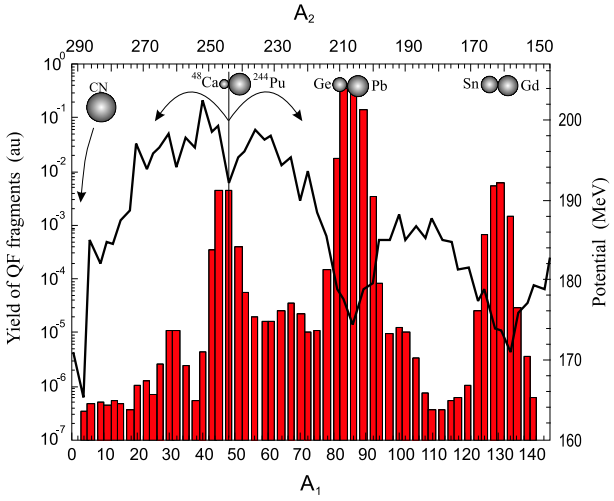

Figure 2. Driving potential of the dinuclear system as a function of the mass number of one of the reaction fragments. Presented also calculations of the total spectrum in the mass fragment distribution for the reaction $^{48}Ca + ^{244}Pu$.

Figure 2 shows the calculated results - the mass distribution of QF products for the reaction $^{48}Ca + ^{244}Pu$ for the excitation energy E* = 35 MeV. It is seen from this figure that the spectrum for the mass distribution of reaction

fragments correlates with the structure of the driving potential. The maxima of the mass distribution are matched by the position of the local minima of the potential. This is a reflection of the nature of our approach to calculation in the case that there exist quasi-stationary states (local minima), which gives grounds to apply the statistical model.

3 Survival probability for the statistical decay of the compound nucleus

The formula for the evaporation residue cross section in the stage of the de-excitation of the fissioning excited compound nuclei can be written as:

$\sigma_{ER}(E^*) \approx \pi\lambda^2 \sum_{l=0}^{l_{cr}} \sigma_{CN}(E^*, l) \cdot W_{sur}(E^*, l)$, where σ_{CN} is the production cross section for the compound nucleus.

There are several approaches within the framework of the statistical model used to calculate the survival factor W_{sur} for the compound nucleus when competition takes place between fission and particle evaporation. This depends on the particular calculating algorithm used.

4 Scheme of calculating the de-excitation process for an excited compound nucleus

In our case, we calculate the value W_{sur} within the framework of the standard statistical model that takes into account the evaporation of neutrons as well as of charged particles, and also the emission of γ-quanta. Since we study the decay of compound nuclei of considerable angular momentum, we make use of a quasi-classical version of the statistical model [9]. In the present work, the nucleus de-excitation process was calculated by applying an approach based on gambling the random value – the Monte-Carlo method. Such an approach was successfully used to calculate the decay of heavy nuclei in [11]. In our opinion, it reflects the random, statistical nature of particle evaporation or fission in the most adequate way.

The angular momenta of the compound nuclei produced in a complete fusion reaction will be distributed over the value L, the vector $\overrightarrow{L}$ lying in the plane perpendicular to the ion beam. With the help of these random numbers, the value of the angular momentum and its orientation in space were simulated. Further, the maximum residual energies of all the processes, particle and γ-ray emission and fission, were calculated for this nucleus: $E_\nu^{(\max)*} = E^* - E_{rot} - B_\nu - V_\nu$; $E_f^{(\max)*} = E^* - E_{rot} - B_f$, where E_{rot} is the rotational energy, V_ν - the Coulomb barrier for a charge particle, B_f- the fission barrier, B_ν - the particle binding energy in the nucleus, $(V_\nu, B_\gamma = 0)$.

For all values $E_\nu^* > 0$, the type of emitted particle – ν or γ-quantum was also simulated . After the type of the de-excitation process (the decay channel) was found and the characteristics of the evaporated particles $\nu = n, p, d, t,^3 He,^4 He$ or γ-quanta were simulated, in case no fission occurred. So their kinetic energy carried-away e_υ, orbital momentum $\overrightarrow{l}$ and angle θ were found. For a given type of evaporation particle, ν or γ-quanta, the values of kinetic energy e_υ, $\overrightarrow{l}$, and $\cos\theta$ were simultaneously specified using three random numbers and then rejection was performed with the help of a fourth random number according to the three-dimensional probability density given by the expression [12]

$$W(e_\nu, l_\nu, \cos\theta) \sim$$
$$l \cdot \exp\left[2\sqrt{a(E^* - E_\nu - (L^2 + l^2)/2J + Ll\cos\theta/J)}\right]$$

In a coordinate frame with a Z axis parallel to $\overrightarrow{L}$, the azimuthal angle of the vector was simulated. The azimuthal angle of the escaping particle was simulated in a coordinate frame with a Z axis parallel to $\overrightarrow{l}$.

The fission process was accounted for with the help of the weight functions, $FU = \prod_{i=1}^{x} [1 - \Gamma_f/\Gamma_{tot}]_i$ (where Γ_f and Γ_{tot} are the fission and total partial widths) which is especially convenient for highly fissionable nuclei and significantly saves computing time. All the values thus found were transformed into the centre-of-mass system of the colliding nuclei, and then the characteristics of the daughter (final) nucleus were calculated

$\overrightarrow{L}_d = \overrightarrow{L}_m - \overrightarrow{l}$; $A_d = A_m - A_\nu$; $Z_d = Z_m - Z_\nu$.
$E_d^* = E_m^* - B_\nu - e_\nu - (L_d^2 - l^2)/2J$; where the index m denotes the mother (initial) nucleus.

Then for that residual nucleus, the maximum residual energies were calculated for all the decay channels: particle emission, γ-quantum emission and fission. Among all the processes energetically allowed, the de-excitation of the nucleus was again simulated and so on for as long as $E_d^* > 0$.

In Fig. 3 the calculated results for the probability of neutron evaporation as a function of excitation energy are presented for the ^{258}No nucleus, which is close to our case. The solid curve corresponds to calculations taking account of the energy dependence of the shell correction; the dotted curve shows calculations with the liquid-drop barrier. Also shown in the figure are the experimental values derived from the neutron evaporation cross sections by the so-called pair reaction method (for details and references see [13]). It is seen from the data presented in the figure, that for the small excitation energy of the nucleus, it is necessary to take account of the shell correction being dependent on E^*. For $E^* > 35MeV$, shell effects in such a heavy nucleus as ^{256}No fade out almost completely. As one can see from Fig. 4 and 5 our approach allows description of the formation of SHE both in "cold" and "hot" fusion reactions. Our estimation for the formation cross section of element 116 in the $^{48}Ca + ^{248}Cm$ reaction with evaporation of 3 or 4 neutrons is about *0.5 pb*.

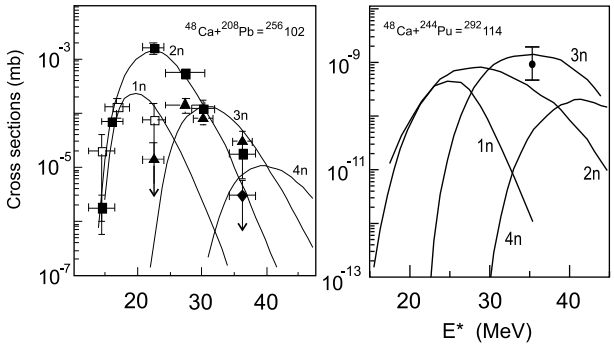

Figure 4. Excitation functions for Z=102,114 compound nuclei formation (calculations) for the ^{48}Ca+^{208}Pb,^{244}Pu reactions. Points are experimental data from [1, 14].

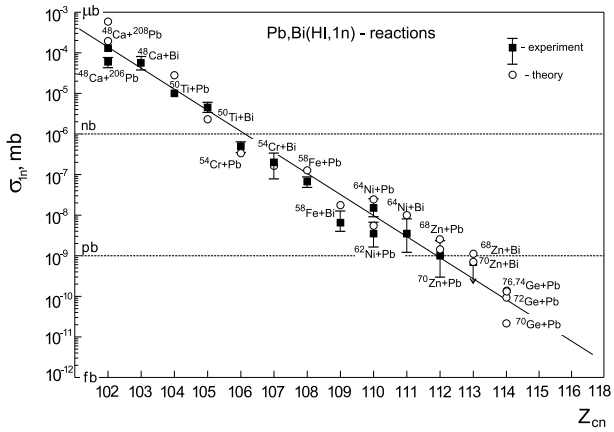

Figure 5. Experimental data (black squares) and theoretical calculations (open circles) for synthesis of elements from 102 to 114 in cold fusion reactions (HI,1n). References on experimental data one can see, for example, in [15].

Summing up, in our approach based on the concept of the DNS, it is possible to describe quite well the experimental data on the formation of superheavy elements in reactions accompanied by neutron evaporation, as well as data on the fragment mass distributions resulting from the contribution of the process of quasi-fission in these reactions.

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