Inclusive Photoproduction of Lepton Pairs in the Parton Model

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In the framework of the QCD parton model, we study unpolarized scattering of high energy real photons from a proton target into lepton pairs and a system of hadrons. For a given parametrization of parton distributions in the proton, we calculate the cross section of this process and show the cancellation of the interference terms.

1 Introduction

The inclusive virtual Compton scattering is the reaction in which a high energy lepton beam bombards a proton target and scatters off the target inelastically

$$l + p \quad \rightarrow \quad l' + \gamma + X, \tag{1}$$

where l and l' represent either an electron or a muon and *X* a system of the final state hadrons. This process was studied in the parton model by Brodsky et al. [1]. The basic idea of the model is the assumption that at high energies the constituents of hadrons behave as if they are free point-like particles. Thus, one picks up only the lowest order electromagnetic contributions to the cross section and neglect all QCD corrections. In the present paper we consider inclusive photoproduction of lepton pairs

$$\gamma\,(k) + p\,(P) \quad \rightarrow \quad l^-\,(p_1) + l^+\,(p_2) + X, \tag{2}$$

which is a crossed process to Eq.(1). At the level of the elementary photon-parton scattering subprocess, the incident photon can either scatter off a parton or split into a lepton pair. We call these contributions the Compton and the Bethe-Heitler process, respectively. Since we imagine the reaction to occur at very high energy, we can assume that all the relevant parton masses are negligible.

2 The Compton Process

There are two Feynman diagrams corresponding to the Compton contribution depicted in Fig. 1. Using $\epsilon_\nu(k)$ and Q_a to denote the polarization vector of the initial photon and the electric charge of the parton of type *a* in units of $|e|$ we write the invariant matrix element as

$$M_C = M_{C1} + M_{C2} = e^3 Q_a^2 \frac{1}{k'^2} \overline{u}(p_1)\gamma_\mu v(p_2)\epsilon_\nu(k)\overline{u}(p') \left[\frac{\gamma^\mu \not{k}\gamma^\nu + 2\gamma^\mu p^\nu}{2\,(p\cdot k)} + \frac{-\gamma^\nu \not{k}'\gamma^\mu + 2\gamma^\nu p^\mu}{-2\,(p'\cdot k)}\right] u(p). \tag{3}$$

Averaging the squared amplitude of Eq.(3) over the initial parton and photon polarizations and summing over the final lepton and parton polarizations, one gets

$$\frac{1}{4}\sum_{polarizations} |M_C|^2 \quad = \quad \frac{e^6 Q_a^4}{4k'^4} L_{\mu\rho} H^{\mu\rho}, \tag{4}$$

where the leptonic and hadronic tensors are

$$\begin{aligned} L_{\mu\rho} &= 4\left[p_{1\mu}p_{2\rho} + p_{1\rho}p_{2\mu} - g_{\mu\rho}\frac{k'^2}{2}\right], \\ H^{\mu\rho} &= 4\left\{\frac{A_C}{(p\cdot k)^2} + \frac{B_C}{(p'\cdot k)^2} + \frac{C_C}{(p\cdot k)\,(p'\cdot k)}\right\}, \end{aligned} \tag{5}$$

with the coefficients

$$
\begin{aligned}
A_C &= (p\cdot k)\left[p'^{\mu}k^{\rho}+p'^{\rho}k^{\mu}-g^{\mu\rho}\left(p'\cdot k\right)\right],\\
B_C &= (p\cdot k')\left[p'^{\mu}p^{\rho}+p'^{\rho}p^{\mu}\right]+(p'\cdot k')\left[p^{\mu}k'^{\rho}+p^{\rho}k'^{\mu}-2p^{\mu}p^{\rho}\right]+(p\cdot p')\left[2p^{\mu}p^{\rho}-p^{\mu}k'^{\rho}-p^{\rho}k'^{\mu}\right]\\
&\quad+\frac{k'^2}{2}\left[-p'^{\mu}p^{\rho}-p'^{\rho}p^{\mu}+g^{\mu\rho}(p\cdot p')\right]-g^{\mu\rho}\left(p\cdot k'\right)\left(p'\cdot k'\right),\\
C_C &= (p\cdot k)\left[p'^{\mu}k'^{\rho}+p'^{\rho}k'^{\mu}-p'^{\mu}p^{\rho}-p'^{\rho}p^{\mu}\right]+(p'\cdot k)\left[2p^{\mu}p^{\rho}-p^{\mu}k'^{\rho}-p^{\rho}k'^{\mu}\right]\\
&\quad+(p\cdot k')\left[-p'^{\mu}p^{\rho}-p'^{\rho}p^{\mu}-p'^{\mu}k^{\rho}-p'^{\rho}k^{\mu}\right]+(p'\cdot k')\left[2p^{\mu}p^{\rho}+p^{\mu}k^{\rho}+p^{\rho}k^{\mu}\right]\\
&\quad+(p\cdot p')\left[-p^{\mu}k^{\rho}-p^{\rho}k^{\mu}-p^{\mu}k'^{\rho}-p^{\rho}k'^{\mu}\right]\\
&\quad+g^{\mu\rho}\left[2\left(p\cdot p'\right)\left(p\cdot k'\right)+\left(k\cdot k'\right)\left(p\cdot p'\right)+\left(p\cdot k'\right)\left(p'\cdot k\right)-\left(p\cdot k\right)\left(p'\cdot k'\right)\right].
\end{aligned}
\tag{6}
$$

The next step is to integrate Eq.(4) over the Lorentz-invariant phase space in a specific frame of reference, i.e.

$$
\frac{1}{(2\pi)^5}\int\left(\frac{d^3p'}{2E_{p'}}\right)\int\left(\frac{d^3p_1}{2E_1}\right)\int\left(\frac{d^3p_2}{2E_2}\right)\ \delta^{(4)}\left(p+k-p'-p_1-p_2\right)\frac{e^6Q_a^4}{4k'^4}H_{\mu\rho}L^{\mu\rho}. \tag{7}
$$

Since $L_{\mu\rho}$ depends only on the momenta of the final leptons, we calculate first the integral

$$
\int d^4p_1\int d^4p_2\ \delta^{(4)}\left(k'-p_1-p_2\right)\delta^{+}\left(p_1^2-m^2\right)\delta^{+}\left(p_2^2-m^2\right)\mathsf{L}_{\mu\rho}, \tag{8}
$$

where *m* denotes the lepton mass and then contract it with $H^{\mu\rho}$. The calculation of Eq.(8) is particularly simple in the lepton pair center-of-mass frame. After contraction, we still have to integrate over d^4k' and d^3p' and divide the expression by the flux factor. In terms of the Mandelstam variables for the scattering process at the parton level, the subprocess differential cross section reads

$$
\frac{d^2\sigma_C}{d\hat{t}dM^2} = -\left(\frac{2\alpha^3Q_a^4}{3}\right)\frac{1}{\hat{s}^2}\left[\frac{\hat{s}^2+\hat{u}^2+2M^2\hat{t}}{\hat{s}\hat{u}}\right]\Phi\left(M^2\right), \tag{9}
$$

with M^2 being the invariant mass squared of the lepton pair and $\Phi\left(M^2\right)=\frac{M^2+2m^2}{M^4}\sqrt{1-\frac{4m^2}{M^2}}\ \theta\left(M^2-4m^2\right)$. The cross section for photon-proton inelastic scattering in the parton model is obtained by summing Eq.(9) over all types *a* of partons and all possible longitudinal momentum fractions *x* weighted with the parton distribution functions $f_a(x)$. Furthermore, we write $\hat{t}=-Q^2$ and $\hat{s}=xs$ where *s* represents the square of the photon-proton center of mass energy. Since the mass of the scattered parton vanishes, one gets $x=Q^2/\left[2P\cdot(k-k')\right]\equiv x_B$. The Compton differential cross section for inclusive photoproduction of lepton pairs can be then written as

$$
\frac{d^3\sigma_C\left[\gamma p\to l^+l^-X\right]}{dQ^2dM^2dx_B} = \frac{2\alpha^3}{3}\int_0^1 dx\ \delta\left(x-x_B\right)\ \sum_a f_a(x)\,Q_a^4\frac{1}{(xs)^2}\left[\frac{(xs)^2+\left(M^2+Q^2-xs\right)^2-2M^2Q^2}{xs\left(M^2+Q^2-xs\right)}\right]\Phi\left(M^2\right). \tag{10}
$$

3 The Bethe-Heitler process

The Bethe-Heitler contribution can be calculated from Feynman diagrams shown in Fig. 1. The amplitude reads

$$
M_{BH}=M_{BH1}+M_{BH2} = -e^3Q_a\frac{1}{(k-k')^2}u(p')\gamma_\mu u(p)\epsilon_\nu(k)\overline{u}(p_1)\left[\frac{2p_1^{\nu}\gamma^{\mu}-\gamma^{\nu}\not{k}\gamma^{\mu}}{-2\left(p_1\cdot k\right)}+\frac{\gamma^{\mu}\not{k}\gamma^{\nu}-2p_2^{\nu}\gamma^{\mu}}{-2\left(p_2\cdot k\right)}\right]v(p_2), \tag{11}
$$

and moreover

$$
\begin{aligned}
\frac{1}{4}\sum_{polarizations}|M_{BH}|^2 &= \frac{e^6Q_a^2}{4\left(k-k'\right)^4}H_{\mu\rho}L^{\mu\rho},\\
H_{\mu\rho} &= 4\left[p_\mu p'_\rho + p_\rho p'_\mu - g_{\mu\rho}\left(p\cdot p'\right)\right],\\
L^{\mu\rho} &= 4\left\{\frac{A_{BH}}{\left(p_1\cdot k\right)^2}+\frac{B_{BH}}{\left(p_2\cdot k\right)^2}+\frac{C_{BH}}{\left(p_1\cdot k\right)\left(p_2\cdot k\right)}\right\},\\
A_{BH} &= \left(p_1\cdot k\right)\left[p_2^\mu k^\rho + p_2^\rho k^\mu - g^{\mu\rho}\left(p_2\cdot k\right)\right] - m^2\left[p_1^\mu p_2^\rho + p_1^\rho p_2^\mu - g^{\mu\rho}\left(p_1\cdot p_2\right)\right]\\
&\quad + m^2\left[p_2^\mu k^\rho + p_2^\rho k^\mu - g^{\mu\rho}\left(p_2\cdot k\right)\right] - m^2\left(p_1\cdot k\right)g^{\mu\rho} + m^4 g^{\mu\rho},\\
B_{BH} &= \left(p_2\cdot k\right)\left[p_1^\mu k^\rho + p_1^\rho k^\mu - g^{\mu\rho}\left(p_1\cdot k\right)\right] - m^2\left[p_1^\mu p_2^\rho + p_1^\rho p_2^\mu - g^{\mu\rho}\left(p_1\cdot p_2\right)\right]\\
&\quad + m^2\left[p_1^\mu k^\rho + p_1^\rho k^\mu - g^{\mu\rho}\left(p_1\cdot k\right)\right] - m^2\left(p_2\cdot k\right)g^{\mu\rho} + m^4 g^{\mu\rho},\\
C_{BH} &= 2\left(p_1\cdot p_2\right)\left[p_1^\mu p_2^\rho + p_1^\rho p_2^\mu - g^{\mu\rho}\left(p_1\cdot p_2\right)\right] - \left(p_1\cdot k\right)\left[p_1^\mu p_2^\rho + p_1^\rho p_2^\mu - 2p_2^\mu p_2^\rho\right]\\
&\quad - \left(p_2\cdot k\right)\left[p_1^\mu p_2^\rho + p_1^\rho p_2^\mu - 2p_1^\mu p_1^\rho\right] - \left(p_1\cdot p_2\right)\left[p_1^\mu k^\rho + p_1^\rho k^\mu + p_2^\mu k^\rho + p_2^\rho k^\mu\right]\\
&\quad + 2g^{\mu\rho}\left(p_1\cdot p_2\right)\left[\left(p_1\cdot k\right)+\left(p_2\cdot k\right)-m^2\right] - 2m^2k^\mu k^\rho. \qquad (12)
\end{aligned}
$$

Following the same procedure as in the previous section and after some tedious algebra one obtains the subprocess differential cross section,

$$
\frac{d^2\sigma_{BH}}{d\hat{t}dM^2} = \left(\alpha^3Q_a^2\right)\frac{1}{\hat{s}^2\hat{t}^2\left(M^2-\hat{t}\right)}\left[-C_1\hat{u}\hat{s}+C_2\hat{t}\right]. \qquad (13)
$$

The coefficients in Eq.(13) are the functions of the invariants m^2, M^2 and $(k\cdot k') = \left(M^2-\hat{t}\right)/2$ given by

$$
\begin{aligned}
C_1 &= \left[-4+\frac{6M^2}{\left(k\cdot k'\right)}-\frac{4M^4}{\left(k\cdot k'\right)^2}+\frac{M^6}{\left(k\cdot k'\right)^3}-\frac{20m^2M^2}{\left(k\cdot k'\right)^2}-\frac{4m^4M^2}{\left(k\cdot k'\right)^3}+\frac{6m^2M^4}{\left(k\cdot k'\right)^3}+\frac{16m^2}{\left(k\cdot k'\right)}+\frac{8m^4}{\left(k\cdot k'\right)^2}\right]\\
&\quad\times\ln\left[\frac{1-\sqrt{1-4m^2/M^2}}{1+\sqrt{1-4m^2/M^2}}\right],\\
C_2 &= \left[4\left(k\cdot k'\right)-4M^2+\frac{2M^4}{\left(k\cdot k'\right)}+\frac{4m^2M^2}{\left(k\cdot k'\right)}-\frac{8m^4}{\left(k\cdot k'\right)}\right]\ln\left[\frac{1-\sqrt{1-4m^2/M^2}}{1+\sqrt{1-4m^2/M^2}}\right]\\
&\quad+\left[4\left(k\cdot k'\right)-8M^2+\frac{4M^4}{\left(k\cdot k'\right)}+\frac{4m^2M^2}{\left(k\cdot k'\right)}\right]\sqrt{1-\frac{4m^2}{M^2}}.
\end{aligned}
$$

Finally, one finds the Bethe-Heitler differential cross section for inclusive photoproduction of lepton pairs namely,

$$
\frac{d^3\sigma_{BH}\left[\gamma p\to l^+l^-X\right]}{dQ^2dM^2dx_B} = \alpha^3\int_0^1 dx\,\delta\left(x-x_B\right)\sum_a f_a\left(x\right)Q_a^2\frac{1}{\left(xs\right)^2Q^4\left(M^2+Q^2\right)}\left[C_1\left(M^2+Q^2-xs\right)xs+C_2Q^2\right]. \qquad (14)
$$

4 The interference terms

Four Feynman diagrams give us eight interference terms. It turns out, that they mutually cancel each other after being integrated over the final lepton momenta. To prove this, let us focus only on the integrals over the first two terms, i.e. $M_{C1}M^\star_{BH1}$ and $M_{C1}M^\star_{BH2}$. They contain the following expressions

$$
\begin{aligned}
&\int d^4p_1\int d^4p_2\,\delta^{(4)}\left(k'-p_1-p_2\right)\frac{1}{\left(p_1\cdot k\right)}\mathrm{Tr}\left[\left(\not{p}_1+\mathrm{m}\right)\gamma_\mu\left(\not{p}_2-\mathrm{m}\right)\gamma^\xi\frac{\not{p}_1-\not{k}+\mathrm{m}}{(\mathrm{p}_1-\mathrm{k})^2-\mathrm{m}^2}\gamma_\nu\right],\\
&\int d^4p_1\int d^4p_2\,\delta^{(4)}\left(k'-p_1-p_2\right)\frac{1}{\left(p_2\cdot k\right)}\mathrm{Tr}\left[\left(\not{p}_1+\mathrm{m}\right)\gamma_\mu\left(\not{p}_2-\mathrm{m}\right)\gamma_\nu\frac{\not{k}-\not{p}_2+\mathrm{m}}{(\mathrm{k}-\mathrm{p}_2)^2-\mathrm{m}^2}\gamma^\xi\right]. \qquad (15)
\end{aligned}
$$

First, we use the delta function to integrate over d^4p_2. After we perform the transposition of the trace and the momentum shift $p_1 - k' = -\tilde{p}$ of the second integrand in Eq.(15), the latter assumes the form

$$\int d^4\tilde{p}\ \frac{1}{(\tilde{p}\cdot k)}\mathrm{Tr}\left[\left(\not{k}' - \tilde{\not{p}} + \mathrm{m}\right)^{\mathrm{T}} \gamma^{\xi^{\mathrm{T}}} \frac{\left(\not{k} - \tilde{\not{p}} + \mathrm{m}\right)^{\mathrm{T}}}{(\mathrm{k}-\tilde{\mathrm{p}})^2 - \mathrm{m}^2}\gamma_\nu^{\mathrm{T}} \left(\tilde{\not{p}} - \mathrm{m}\right)^{\mathrm{T}} \gamma_\mu^{\mathrm{T}}\right]. \tag{16}$$

Since the property $\hat{C}\gamma_\mu \hat{C}^{-1} = -\gamma_\mu^{\mathrm{T}}$ holds, where $\hat{C}$ represents the charge conjugation operator, one ends up with

$$-\int d^4\tilde{p}\ \frac{1}{(\tilde{p}\cdot k)}\mathrm{Tr}\left[\gamma_\mu \left(\not{k}' - \tilde{\not{p}} - \mathrm{m}\right) \gamma^\xi \frac{\tilde{\not{p}} - \not{k} + \mathrm{m}}{(\mathrm{k}-\tilde{\mathrm{p}})^2 - \mathrm{m}^2}\gamma_\nu \left(\tilde{\not{p}} + \mathrm{m}\right)\right]. \tag{17}$$

The last expression is exactly equal but opposite in sign to the first term in Eq.(15). Thus, by adding them we get zero. In fact, the result of cancellation is known in general as the Furry's theorem: Feynman diagrams containing a closed fermion loop with an odd number of photon vertices can be omitted in the calculation of physical processes.

5 Kinematics and Figures

As $x \to 0$, the differential cross sections in Eqs.(10) and (14) become singular. However, since $s - M^2 \geq (k - k' + P)^2 \geq M_p^2$, one finds in the laboratory frame, which is the rest frame of the proton with mass M_p,

$$\left[1 + \left(2M_pE - M^2\right)/Q^2\right]^{-1} \quad \leq x \leq \quad 1, \tag{18}$$

where E denotes the initial photon energy and $Q^2 = 2EE' - 2E\sqrt{E'^2 - M^2}\ \cos\vartheta_\gamma - M^2$ is the invariant momentum transfer. The energy of the pair and the angle between photons are denoted by E' and ϑ_γ, respectively. In Fig. 2 both differential cross sections are plotted against ϑ_γ at fixed values of $E = 40\,\mathrm{GeV}$, $M = 3\,\mathrm{GeV}$ and $E' = 10\,\mathrm{GeV}$. Muons are taken as leptons and the following simplified parametrization of parton distributions in the proton is used: $u_{val}(x) = 1.89x^{-0.4}(1-x)^{3.5}(1+6x)$, $d_{val}(x) = 0.54x^{-0.6}(1-x)^{4.2}(1+8x)$ and $sea(x) = 0.5x^{-0.75}(1-x)^7$ [2].

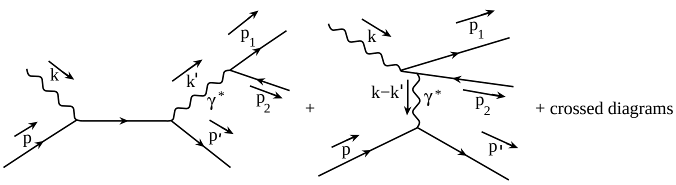

Figure 1. Feynman diagrams for Compton and Bethe-Heitler process. In two crossed diagrams the real and virtual photons are interchanged.

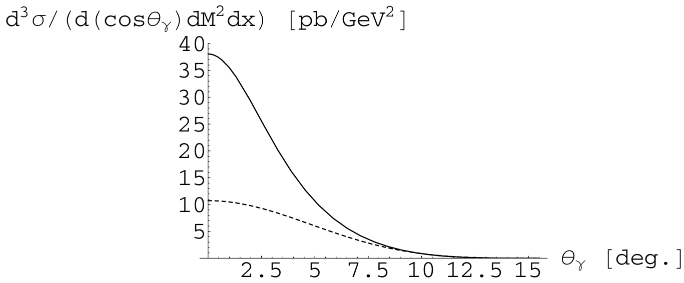

Figure 2. Differential cross section for Compton (solid line) and Bethe-Heitler (dashed line) contributions.

6 Conclusions

In this paper we have applied the parton model formalism to the inclusive photoproduction of lepton pairs from a proton target. We have calculated the unpolarized cross section for Compton and Bethe-Heitler subprocess at particular kinematics. Furthermore, we have illustrated the cancellation of the interference terms.

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