A Dirac Description of $^1S_0 +^3 S_1 -^3 D_1$ Pairing in Nuclear Matter

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We develop a Dirac-Hartree-Fock-Bogoliubov description of nuclear matter pairing in $^1S_0$ and $^3S_1 -^3 D_1$ channels. Here we investigate the density dependence ot the $^1S_0$ and $^3S_1 -^3 D_1$ pairing fields in asymmetric nuclear matter, using a Bonn meson-exchange interaction between Dirac nucleons. In this work, we present preliminary results.

1 Introduction

Nonrelativistic calculations of $^1S_0 + {^3S_1} -^3 D_1$ in symmetric nuclear matter, using standard nucleon-nucleon interactions, yeald a $^1S_0$ pairing gap of about the expected size, but an extremely large pairing gap, of the order of 10 MeV, in the $^3S_1 -^3 D_1$ channel [1, 2, 3, 4, 5]. It has been suggested that relativistic effects substantially reduce this pairing gap [6], as is the case for $^1S_0$ pairing at densities near saturation [7]. However, the size of the pairing gap has also been found to be related to the energy of the virtual/bound state in the vacuum of the channel under consideration [7]. This would imply a much larger pairing gap in the $^3S_1 -^3 D_1$ channel, corresponding to the deuteron in the vacuum, than that in the $^1S_0$ channel, which corresponds to the two-nucleon virtual state in the vacuum.

2 The Formalism

We take the hamiltonian form of the HFB equation to be

$$\begin{pmatrix} h_k - \mu_k & \bar{\Delta}_k^\dagger \\ \bar{\Delta}_k & -h_{Tk} + \mu_k \end{pmatrix} \begin{pmatrix} U_{kn} \\ V_{kn} \end{pmatrix} = \varepsilon_{kn} \begin{pmatrix} U_{kn} \\ V_{kn} \end{pmatrix},$$

where

$$h_k = \vec{\alpha} \cdot \vec{k} + \beta M + \beta \Sigma_k$$

and

$$h_{Tk} = B h_{-k}^T B^\dagger = \vec{\alpha} \cdot \vec{k} + \beta M + \beta \Sigma_{Tk}$$

with

$$\Sigma_{Tk} = B \Sigma_{-k}^T B^\dagger.$$

The index n denotes the 16 solutions to the HFB equation.

The self-consistency equations may be written as

$$\beta\Sigma_k = \sum_{j,\alpha\beta} \Bigg(\gamma_0 \Gamma_{j\alpha}(0) D_j^{\alpha\beta}(0) \int \frac{d^3q}{(2\pi)^3} \mathrm{Tr}\left[\gamma_0 \Gamma_{j\beta}(0) g(q)\right] - \int \frac{d^3q}{(2\pi)^3} \gamma_0 \Gamma_{j\alpha}(k-q) D_j^{\alpha\beta}(k-q) g(q)\, \gamma_0 \Gamma_{j\beta}(q-k)$$

and

$$\bar{\Delta}_k = \int \frac{d^3q}{(2\pi)^3} B \left(\gamma_0 \Gamma_{j\alpha}(k-q)\right)^T B^\dagger D_j^{\alpha\beta}(k-q) \times \bar{f}(q)\, \gamma_0 \Gamma_{j\beta}(q-k),$$

where the index j refers to the different mesons exchanged and the indices α and β are their Lorentz/isospin indices. The normal and anomalous densities, $g(q)$ and $f(q)$, respectively, are given by

$$g(q) = \sum_n U_{qn} U_{qn}^\dagger \quad \text{and} \quad \bar{f}(q) = \sum_n V_{qn} U_{qn}^\dagger,$$

with the sum over n running over the appropriate set of solutions of the HFB equation.

We take for the Dirac and isospin structure of the mean fields

$$\begin{aligned} \beta\Sigma_k &= \Big(\beta\Sigma_{s0}(k) + \Sigma_{00}(k) + \vec{\alpha}\cdot\hat{k}\Sigma_{v0}(k)\Big) \otimes 1 \\ &+ \Big(\beta\Sigma_{si}(k) + \Sigma_{0i}(k) + \vec{\alpha}\cdot\hat{k}\Sigma_{vi}(k)\Big) \otimes \tau_i \end{aligned}$$

and

$$\begin{aligned} \bar{\Delta}_k &= [\bar{\Delta}_{si}(k) + \beta\bar{\Delta}_{0i}(k) + \vec{\alpha}\cdot\hat{k}\bar{\Delta}_{Ti}(k)] \otimes \tau_i \\ &+ [\bar{\Delta}_1(k)\gamma_5\vec{\gamma}\cdot\hat{\xi} + \bar{\Delta}_2(k)\,\gamma_5\vec{\gamma}\cdot Y_2(\hat{k})\cdot\hat{\xi} \\ &+ \bar{\Delta}_3(k)\gamma_0\gamma_5\vec{\gamma}\cdot\hat{\xi} + \bar{\Delta}_4(k)\,\gamma_0\gamma_5\vec{\gamma}\cdot Y_2(\hat{k})\cdot\hat{\xi} \\ &+ \bar{\Delta}_5(k)\hat{k}\cdot\hat{\xi}\,\gamma_5 + \bar{\Delta}_6(k)(\hat{\xi}\times\hat{k})\cdot\vec{\gamma}] \otimes 1\,, \end{aligned}$$

where the rank two tensor $Y_2(\hat{k})$ is given by

$$Y_2(\hat{k})_{ij} = \frac{1}{\sqrt{2}}(3\,\hat{k}_i\,\hat{k}_j - \delta_{ij})\,.$$

The densities can be decomposed similarly, where the component densities can be obtained with the appropriate traces,

$$g_{s0}(q) = \frac{1}{8}\sum_n U_{qn}^\dagger \beta \otimes 1 U_{qn}, \bar{f}_{si} = \frac{1}{8}\sum_n U_{qn}^\dagger 1 \otimes \tau_i V_{qn}, \text{ etc.}$$

The vertices and propagators of the mesons are given in Table 1, where we have defined

$$\tilde{d}_\alpha(q) = \frac{1}{m_\alpha^2 - q^2}.$$

We will neglect retardation effects in the following so that the composite form-factor/reduced propagator will take the form

$$d_\alpha(q) = \frac{1}{\vec{q}^2 + m_\alpha^2}\left(\frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 + \vec{q}^2}\right)^2 .$$

We will denote the remaining factors of the vertices as the bare vertices.

To obtain the reduced self-consistency equations, we substitute these decompositions in the unreduced equations, calculate and take traces. The equations for the components of $^1S_0$ pairing field that result are uncoupled integral equations of the form

$$\begin{aligned}
\bar{\Delta}_{si}(k) \quad = \quad & -\int \frac{d^3q}{(2\pi)^3}\left[g_\sigma^2 d_\sigma(k-q) + g_\delta^2 d_\delta(k-q)\right. \\
& 4g_\omega^2 d_\omega(k-q) - 4g_\rho^2 d_\rho(k-q) \\
& -\left(\vec{k}-\vec{q}\right)^2\left(\left(\frac{f_\pi}{m_\pi}\right)^2 d_\pi(k-q)\right. \\
& \left.\left. + \left(\frac{f_\eta}{m_\eta}\right)^2 d_\eta(k-q)\right)\right]\bar{f}_{si}(q)
\end{aligned}$$

The equations for the $l = 0$ and $l = 2$ components of $^3S_1 -^3 D_1$ pairing field reduce to coupled equations. To uncouple these further, we make an additional approximation - we replace the components of the mean field and pairing field by their (spherically symmetric) angular averages.

TABLE 1. Meson-nucleon vertices and propagators. The factor $F_i(q)$ is the vertex form factor.

meson	vertex	propagator
σ	$g_\sigma F_\sigma(q)\, 1 \otimes 1$	$D_\sigma(q) = -\tilde{d}_\sigma(q)$
δ	$g_\delta F_\delta(q)\, 1 \otimes \tau_k$	$D_\delta^{kl}(q) = -\delta^{kl}\tilde{d}_\delta(q)$
ω	$g_\omega F_\omega(q)\, \gamma_\mu \otimes 1$	$D_\omega^{\mu\nu}(q) = g^{\mu\nu}\tilde{d}_\omega(q)$
ρ	$g_\rho F_\rho(q)\, \gamma_\mu \otimes \tau_k$	$D_\rho^{\mu k,\nu l}(q) = g^{\mu\nu}\delta^{kl}\tilde{d}_\rho(q)$
π	$\frac{f_\pi}{m_\pi} F_\pi(q)\, i\gamma_\mu q^\mu \gamma_5 \otimes \tau_k$	$D_\pi^{kl}(q) = -\delta^{kl}\tilde{d}_\pi(q)$
η	$\frac{f_\eta}{m_\eta} F_\eta(q)\, i\gamma_\mu q^\mu \gamma_5 \otimes 1$	$D_\eta(q) = -\tilde{d}_\eta(q)$

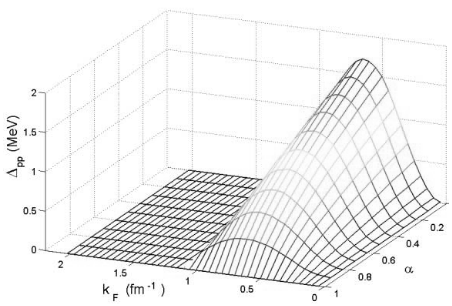

Figure 1. Proton-proton gap as a function of the asymmetry parameter α and density for pure standard pairing.

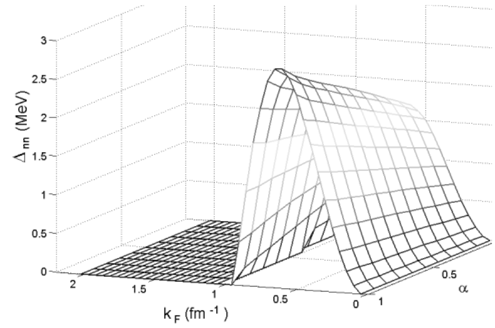

Figure 2. The same as Fig. 1 for neutron-neutron pair.

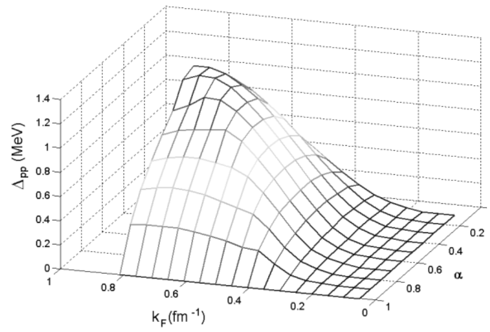

Figure 3. The same as Fig. 1 for proton-proton pair, in calculations for $^1S_0 +^3 S_1 -^3 D_1$ pairing.

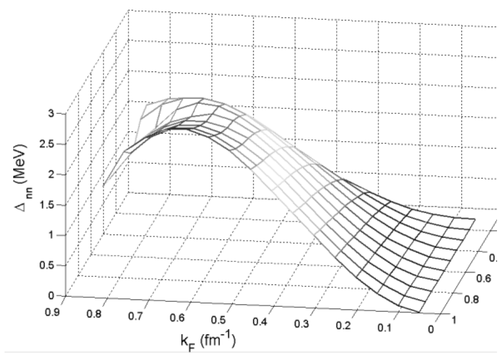

Figure 4. The same as Fig. 3 for neutron-neutron pair.

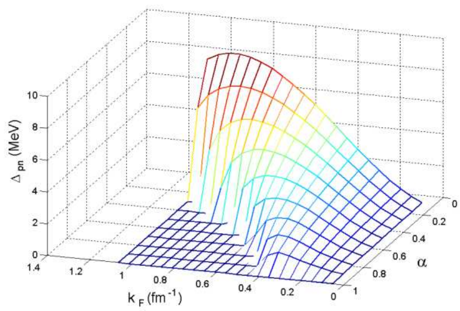

Figure 5. The same as Fig. 3 for quasi-deuteron pair.

3 Results and Conclusions

In Figs. 1 and 2 we show the pairing gap for the pp and nn pairs as a function of asymmetry and density, calculated numerically for pure standard pairing in nuclear matter. In Figs. 3 to 5, we show pp, nn and quasi-deuteron pairings gaps for standard plus quasi-deuteron pairing. We have found that pairing in these two channels coexists in asymmetric nuclear matter. This mixed state is the ground state of nuclear matter in the region in which it exists.

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