Vacuum Polarization Effects in Relativistic Nuclear Pairing

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In the present work we discuss the contribution of the vacuum polarization on the nuclear pairing, in the context of the relativistic Hartree-Fock-Bogoliubov (HFB) approximation. The polarization effects on the pairing, as a function of Fermi momentum, is shown with the scalar and vector meson-nucleon coupling constants scaled by a parameter x. We have obtained that the nuclear pairing is affected by the vacuum polarization.

1 Introduction

Relativistic models of nuclear pairing have been developed within the framework of mean field theory through the BCS and the Hartree-Fock-Bogoliubov (HFB) approximations. A precise relativistic description of pairing correlations, the Dirac-Hartree-Fock-Bogoliubov (DHFB) approximation, was developed by Bailin and Love[1] and, later applied to nuclear matter[2, 3]. In these calculations the pairing gaps obtained were found to be much larger than those furnished in nonrelativistic models by using realistic potentials. These discrepancies were resolved in Ref.[4], by associating the pairing vertex function with the low-energy two-nucleon $^1S_0$ virtual state. On the other hand, the HFB theory is a relatively simple approximation neglecting any contribution to the nucleon-nucleon interaction beyond the bare potential. In order to account for many nuclear properties, it is necessary to go beyond the pure BCS or HFB approximations and add other effects to the nucleon-nucleon interaction, such as medium effects on the meson propagation.

In the present work, we develop an effective relativistic $\sigma - \omega$ nucleon-nucleon interaction that includes RPA medium modifications of the meson propagator and use it to study their effect on pairing in the $^1S_0$ channel. In norelativistic calculations the medium effects modify the effective nucleon-nucleon interaction by adding a repulsive term, thus causing a reduction in the $^1S_0$ gap. In Ref.[5] we have described part of this calculation which did not include the pure vacuum contribution. This term is divergent and we must to use a renormalization scheme to render it finite. We found in that paper that the results were opposite to the nonrelativistic calculations. Here we have included the vacuum terms and have studied their contributions to the meson propagator and their consequencies on the pairing gap.

2 Relativistic random-phase approximation - RPA

In this section, we calculate the full meson propagator for spacelike momenta in the one-loop approximation. Much of the formalism described here was developed by Chin[6] and by Horowitz et al.[7]. Due to its lack of covariance, the nuclear medium allows the mixture of mesonic fields of different Lorentz structure through the nucleon loops. This effect is included in the meson propagator as an infinite sum over ring diagrams, which consist of repeated insertions of the lowest-order one-loop proper polarization. This diagramatic representation including the polarization tensor can be written analytically using Dyson's equation for an extended meson propagator which is given by,

$$\mathfrak{D}^{a}{}_{b} = \mathfrak{D}_0^{a}{}_{b} + \mathfrak{D}_0^{a}{}_{c}\, \Pi^{c}{}_{d}\, \mathfrak{D}^{d}{}_{b} \tag{1}$$

where $\mathfrak{D}_0^{a}{}_{b}$ is the noninteracting meson propagator with indices extending from one to five and Π is the renormalized proper polarization tensor. Since we have both scalar and vector mesons in our model, besides being affected by the presence of the nuclear matter, the meson propagator must take into account the mutual interaction of the respective fields, called scalar-vector mixing. This effect is forbidden in the vacuum due to covariance. However, it is permitted in the medium and we will see later that it is important in the study of nuclear stability. Using the Feynman rules, we obtain the polarization tensor from the ring diagram. It can be written as,

$$\Pi(q)^{a}{}_{b} = -i\lambda \int \frac{d^4k}{(2\pi)^4} Tr[\Gamma^a G(k)\Gamma_b G(k+q)] \tag{2}$$

where λ is the isospin degeneracy factor. The vertex functions are represented by the Γ's that, in the $\sigma - \omega$ model are given by, $\Gamma_s = g_s I$ and $\Gamma_v = -g_v\gamma_\mu$ with g_s and g_v the scalar and vector coupling constants. The nucleon propagator G(k) may be written, in general, as a sum of Feynman $G^F(k)$ and density-dependent $G^D(k)$ contributions as,

$$\begin{aligned}
G(k) &= G^F(k) + G^D(k), \\
G^F(k) &= (\gamma^\mu k^\star_\mu + M^\star(k)) \frac{1}{k^{\star 2}_\mu - M^{\star 2}(k) + i\varepsilon}, \\
G^D(k) &= (\gamma^\mu k^\star_\mu + M^\star(k)) \\
&\times \frac{i\pi}{E^\star_k}\delta(k^\star_0 - E^\star_k)\theta(k_F - |\vec{k}|)
\end{aligned} \tag{3}$$

where $k^\star_\mu = (k_0 - g_v V^0, \vec{k})$ and $E^\star_k = \sqrt{\vec{k}^2 + M^{\star 2}}$. The Feynman propagator describes the propagation of virtual particles and antiparticles while the density-dependent term corrects G^F for the propagation of holes inside the Fermi sea. This term also corrects $G^F(k)$ for the Pauli exclusion principle and vanishes at zero baryon density. With the above form of the baryon propagator, each polarization insertion can be divided into two parts: the Feynman term or vacuum polarization part and a density-dependent part. The scalar and vector parts of the vacuum polarization are given by,

$$\begin{aligned}\Pi^F_s(q) &= -ig_s^2\lambda\int\frac{d^4k}{(2\pi)^4}Tr[G_F(k)G_F(k+q)],\\ \Pi^\mu_{F\ \nu}(q) &= -ig_v^2\lambda\int\frac{d^4k}{(2\pi)^4}Tr[\gamma^\mu G_F(k)\gamma_\nu G_F(k+q)]\end{aligned}\quad(4)$$

This term describes the self-energy correction to the meson propagator due their coupling to nucleon-antinucleon excitations. This contribution to the polarization insertion is divergent and must be renormalized. The density-dependent part, on the other hand, is finite and is formed of products of Feynman and density-dependent terms. Its scalar, vector and mixed parts are given repectively by,

$$\begin{aligned}\Pi_s(q) &= -ig_s^2\int\frac{d^4k}{(2\pi)^4}Tr[\,G_D(k)\,G_F(k+q)\\ &+\ G_F(k)G_D(k+q)], &(5)\\ \Pi^\mu{}_\nu(q) &= -ig_v^2\int\frac{d^4k}{(2\pi)^4}Tr[\gamma^\mu G_D(k)\gamma_\nu G_F(k+q)\\ &+\ \gamma^\mu G_F(k)\gamma_\nu G_D(k+q)], &(6)\\ \Pi^M_\mu(q) &= -ig_sg_v\int\frac{d^4k}{(2\pi)^4}Tr[\gamma_\mu G_D(k)G_F(k+q)\\ &+\ \gamma_\mu G_F(k)G_D(k+q)]. &(7)\end{aligned}$$

These describe the coupling of the meson to particle-hole excitations and also correct the $N\bar{N}$ contribution for Pauli blocking. The interacting modes are obtained from the poles in the meson propagator by solving Eq.(1). Solving formally the meson propagator by inverting Eq.(1) we obtain,

$$(\mathfrak{D}^{-1})^a{}_b = (\mathfrak{D}_0^{-1})^a{}_b - \Pi^a{}_b. \quad (8)$$

Since we want to include σ and ω mesons as well as their mixing, it is convenient to use a meson propagator in the form of a 5×5 matrix. The non-interacting meson propagator is given by a block-diagonal matrix as,

$$\mathfrak{D}_0{}^a{}_b = \begin{pmatrix} D_0^\mu{}_\nu & 0\\ 0 & \Delta_0\end{pmatrix}, \quad (9)$$

where the noninteracting propagators for the σ and ω mesons are given respectively by,

$$\begin{aligned}\Delta_0(q) &= \frac{1}{q_\mu^2 - m_s^2 + i\varepsilon}, &(10)\\ D_0^\mu{}_\nu(q) &= \left(\delta^\mu{}_\nu - \frac{q^\mu q_\nu}{m_v^2}\right)\frac{-1}{q_\mu^2 - m_v^2 + i\varepsilon}. &(11)\end{aligned}$$

We can write the polarization tensor in a matrix form similar to that of Eq.(9).

$$\Pi^a{}_b(q) = \begin{pmatrix}\Pi^\mu{}_\nu & \Pi^m_\nu\\ \Pi^\mu_m & \Pi_s\end{pmatrix}, \quad (12)$$

Baryon current conservation implies the following constraints on the one-loop polarization insertions,

$$\begin{aligned}q_\mu\Pi^\mu{}_\nu &= \Pi^\mu{}_\nu q^\nu = 0\\ q_\mu\Pi^\mu_m &= 0.\end{aligned}\quad(13)$$

To simplify the calculations, it is convenient to work in a frame in which **q** is along the z-axis, that is, $q = (0,0,q_3,q_4)$ with $q_\mu^2 = q_4^2 - q_3^2$. Within this reference frame and with the symmetry of the loop integrals, the polarization matrix is constituted of two blocks. A diagonal one, called the transverse block, and a non-diagonal one, called the scalar-longitudinal block. Substituting this in Eq.(8) we obtain a meson propagator matrix where the mass of the transversal modes is obtained directly from the first block which has a simple form given by,

$$m_t^{\star 2} = m_v^2 + \Pi_t. \quad (14)$$

For the scalar and longitudinal modes mass it is necessary to diagonalize the second block,. The modes are the eigenvalues given by,

$$\left.\begin{matrix}\mathfrak{m}_l^{\star 2}\\ \mathfrak{m}_s^{\star 2}\end{matrix}\right\} = \frac{m_\omega^{\star 2} + m_\sigma^{\star 2} \pm \sqrt{(m_\omega^{\star 2} - m_\sigma^{\star 2})^2 - 4\Pi_m^2}}{2} \quad (15)$$

where $m_\sigma^{\star 2} = m_s^2 + \Pi_s$ and $m_\omega^{\star 2} = m_v^2 + \Pi_l$ are the scalar and longitudinal mass respectively, and

$$\Pi_l = -\left(\Pi^3{}_3 + \Pi^4{}_4\right) \quad \text{and} \quad \Pi_m = \frac{\sqrt{-q^2}}{q_3}\Pi^4. \quad (16)$$

Next we will express the full meson propagator in its spectral form using the eigenvectors obtained obtained above,

$$\begin{aligned}\mathcal{D}^a{}_b &= -\frac{e_1^a\ e_{1b}}{q^2 - m_t^{\star 2}} - \frac{e_2^a\ e_{2b}}{q^2 - m_t^{\star 2}} - \frac{q_\mu^a\ q_{\nu b}/q^2}{q^2 - m_v^2}\\ &- \frac{\eta_1^a\ \eta_{1b}}{q^2 - \mathfrak{m}_l^{\star 2}} + \frac{\eta_2^a\ \eta_{2b}}{q^2 - \mathfrak{m}_s^{\star 2}}.\end{aligned}\quad(17)$$

To complete these expressions we must evaluate the one-loop polarization contribution and obtain the modified meson masses. Integrating in the angular variables and taking the momentum q to be purely, space-like, $q^\mu = (0, q_s, 0, 0)$, we find

$$\begin{aligned}
\Pi_s(q_s) &= \frac{2g_s^2\lambda}{\pi^2}\int_0^{k_f}\frac{k^2\,dk}{E_k}\left\{\frac{1}{2}-\frac{q_s^2+4M^{\star 2}}{8kq_s}\ln\frac{q_s+2k_f}{q_s-2k_f}\right\}\\
\Pi_l(q_s) &= \frac{2g_v^2\lambda}{\pi^2}\int_0^{k_f}\frac{k^2\,dk}{E_k}\left\{\frac{1}{2}-\frac{q_s^2-4E_k^2}{8kq_s}\ln\frac{q_s+2k_f}{q_s-2k_f}\right\}\\
\Pi_t(q_s) &= -\frac{g_v^2\lambda}{\pi^2}\int_0^{k_f}\frac{k^2\,dk}{E_k}\left\{\frac{1}{2}-\frac{q_s^2+4k^2}{8kq_s}\ln\frac{q_s+2k_f}{q_s-2k_f}\right\}\\
\Pi_m(q_s) &= \frac{g_s g_v\lambda M^\star}{q_s\pi^2}\int_0^{k_f} k\,dk\,\ln\frac{q_s+2k_f}{q_s-2k_f}. \qquad (18)
\end{aligned}$$

The $N\bar{N}$ contributions to the one-loop polarization involve integrals that are divergent. They can be made finite by subtracting the appropriate counterterms[7],

$$\Pi^{RF}(q)=\Pi^F(q)-CTC. \qquad (19)$$

For the scalar vacuum polarization the counterterms necessary to render this component finite are given by[8],

$$\Pi_s^{RF}(q)=\Pi_s^F(q)-\alpha_2-\alpha_3\phi_0-\frac{1}{2}\alpha_4\phi_0^2-\zeta_s q^2. \qquad (20)$$

For the vector vacuum polarization integral we proceed as in the previous case where only one counterterms is necessary to render this term finite,

$$\Pi_{\mu\nu}^{RF}(q)=\Pi_{\mu\nu}^F(q)-q^2\frac{\partial}{\partial q^2}\Pi_{\mu\nu}^F|_{q^2=0,M^\star=M}. \qquad (21)$$

All counterterms used here were evaluated by choosing the renormalization point at $q^2=0$. This choice is suitable to describe the effective interaction in the ground state of nuclear matter, where q^2 is small. After evaluating the integrals in Eq.(4) we obtain,

$$\begin{aligned}
\Pi_s^{RF}(q_s) &= \lambda\frac{g_s^2}{4\pi^2}\left[3M^2+13M^{\star 2}-12M^\star M+\frac{4}{3}q_s^2\right.\\
&- \frac{1}{2}(q_s^2+6M^{\star 2})\ln\frac{M^{\star 2}}{M^2}\\
&- \left.\frac{(q_s^2+4M^{\star 2})^{3/2}}{2q_s}\ln\frac{q_s+\sqrt{q_s^2+4M^{\star 2}}}{q_s-\sqrt{q_s^2+4M^{\star 2}}}\right] \qquad (22)
\end{aligned}$$

and,

$$\begin{aligned}
\Pi_v^{RF}(q_s) &= -\lambda\frac{g_v^2}{36\pi^2}\left[12M^{\star 2}-5q_s^2-3q_s^2\ln\frac{M^{\star 2}}{M^2}\right.\\
&+ \frac{3(q_s^2-2M^{\star 2})\sqrt{q_s^2+4M^{\star 2}}}{q_s}\\
&\times \left.\ln\frac{q_s+\sqrt{q_s^2+4M^{\star 2}}}{q_s-\sqrt{q_s^2+4M^{\star 2}}}\right] \qquad (23)
\end{aligned}$$

where we take $q_0 = 0$.

The sum over ring diagrams to all orders produces an extremely strong polarization, causing an instability in the ground state of nuclear matter. We introduce a free parameter x, which scales the coupling constants in the polarization equations and thereby reduces its effect. For the free mesons masses we use

$$m_s = 550 \text{ MeV} \qquad \text{and} \qquad m_v = 773 \text{ MeV} \tag{24}$$

which reproduce nuclear saturation at a density corresponding to a Fermi momentum of 1.36 fm^{-1} and nuclear a matter binding energy of 15.75 MeV per nucleon.

We want to analyze the effects on nuclear pairing of the effective interaction obtained by using the extended meson propagator of Eqs. (1) and (17). We use the pairing model developed on Refs.[3] and [4]. This model uses a Dirac-HFB approximation for symmetric nuclear matter that with Hermiticity and transposition invariance conditions, as well as the requirements of invariance under Lorentz and parity transformation, reduce the possible form of the self-energy in symmetric nuclear matter to

$$\Sigma(k) = \Sigma_S(k) - \gamma_0\Sigma_0(k) + \vec{\gamma}.\vec{k}\Sigma_T(k); \tag{25}$$

while the $^1S_0$ pairing field takes the form

$$\Delta(k) = [\Delta_S(k) - \gamma_0\Delta_0(k) - i\gamma_0\vec{\gamma}.\vec{k}\Delta_T(k)]\vec{\tau}.\hat{n}, \tag{26}$$

where the orientation in isospin $\hat{n}$, is arbitrary. The Dirac pairing field can be reduced to an effective nonrelativistic pairing gap function of the form

$$\Delta = \Delta_g\vec{\tau}.\hat{n}, \tag{27}$$

where the gap function Δ_g is given by

$$\Delta_g = \frac{M^\star}{E}\Delta_0 - \Delta_s - i\frac{k}{E}\Delta_T. \tag{28}$$

An important feature of the pairing self-consistency equation is its reduction to a Beth-Salpeter equation in the vacuum. It has been shown in [4] that the vacuum solution dominates the behavior of the pairing gap function at low densities. The $^1S_0$ channel of the two-nucleon system, does not possess a bound state in the vacuum. Instead it has a virtual state at $K_v \approx -0.05i$ fm^{-1}. We constrain our calculation to be consistent with this. We introduce a cutoff Λ in the momentum integrals which we fix, so as to obtain the virtual state in the correct position.

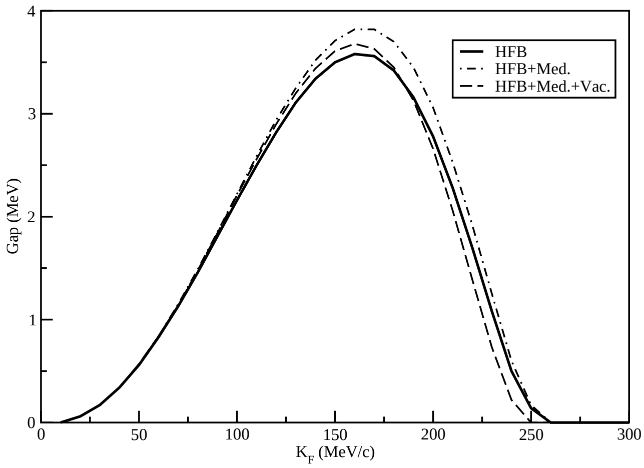

Figure 1. Pairing gap including the medium plus the vacuum contribution to the polarization tensor.

In Fig. 1, we plot the pairing gap as a function of the Fermi momentum for values of the free parameter $x = 3.4$. The curve with $x = 0$ corresponds to the HFB calculation with no medium corrections. We note that the medium polarization contribution causes a slight increase of the pairing gap. This is because, in the $\sigma - \omega$ model, the polarization effects enhance the attraction and diminish the repulsion between the nucleons. This make the effective nucleon-nucleon interaction more attractive and yields a larger pairing gap. On the other hand, we note that the vacuum polarization partially cancels this effect, decreasing the medium contribution We also note in Fig. 1 that the pairing gap including vacuum polarization drops more rapidly than the pairing gap with the medium contribution alone at high density. This is also a result of the partial cancellation of the medium contribution by the vacuum one.

The changes in the pairing gap due to the polarization are opposite those found in the nonrelativstic case. Nonrelativistic calculations [9, 10] have consistently found the polarization effect to diminish the pairing gap. To see whether this is possible here, we analyze the low-density limit of the polarization equations at $q_s = 0$, given by,

$$\Pi_s(0, k_f) = -\frac{g_s^2\lambda}{\pi^2}k_f M^\star \tag{29}$$

$$\Pi_l(0, k_f) = \frac{2}{3}\frac{g_v^2\lambda}{\pi^2}k_f M^\star \tag{30}$$

The scalar contribution is negative while the longitudinal and mixed ones are positive. Substituting these expressions on Eq.(15) and expanding the square root, considering the Π_m term small when compared to $m_v^2 - m_s^2$, we obtain,

$$\begin{aligned} \mathfrak{m}_l^{\star 2} &= m_v^2 + \Pi_l - \frac{\Pi_m^2}{m_v^2 + \Pi_l - m_s^2 - \Pi_s} \\ \mathfrak{m}_s^{\star^2} &= m_s^2 + \Pi_s + \frac{\Pi_m^2}{m_v^2 + \Pi_l - m_s^2 - \Pi_s}. \end{aligned} \tag{31}$$

Assuming the meson masses to take their usual values, the last term on the two expressions is quadratic in the Fermi momentum and thus negligible when compared to the Π_l and Π_s terms at low densities. We thus conclude that the vector meson mass will increase while the scalar meson mass will always decrease, at least at low density. To reproduce the nonrelativistic results in our model, we would have to change the signs of the polarization contributions thus permitting the scalar meson mass to increase and the vector meson mass to diminish. Thus, the polarization corrections of the $\sigma - \omega$ model cannot reproduce the nonrelativistic results under any circumstances.

We conclude that particle-hole excitations of the effective meson propagator make the effective nucleon-nucleon interaction in symmetric nuclear matter more attractive, and in consequence, increasing the pairing gap. On the other hand, nucleon-antinucleon excitaton partially cancels this contribution. Thus, we show that the simple sum of ring diagrams of the RPA is insufficient to reproduce the nonrelativistic results. This might be due the pointlike σ meson which make the effective nucleon-nucleon interaction too atractive. To solve this problem we intend to subtitute the σ meson by a two pion exchange model including other nucleonic degrees of freedon such the delta resonance. Another soluction would be to consider nucleonic substructure using a quark-meson coupling model.

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