Topological Charge Screening and Pseudoscalar Glueballs

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Topological charge screening in the QCD vacuum is found to provide crucial nonperturbative contributions to the short-distance expansion of the pseudoscalar (0^{-+}) glueball correlator. The screening contributions enter the Wilson coefficients and are an indispensable complement to the direct instanton contributions. They restore consistency with the anomalous axial Ward identity and remedy several flaws in the 0^{-+} glueball sum rules caused by direct instantons in the absence of screening (lack of resonance signals, violation of the positivity bound and of the underlying low-energy theorem). The impact of the finite width of the instanton size distribution and the (gauge-invariant) renormalization of the instanton contributions are also discussed. New predictions for the 0^{-+} glueball mass and decay constant are presented.

1 Introduction

The glueball sector of QCD has remained intriguing and challenging since the early days of QCD [1] and glueball physics offers promising opportunities for the study of low-energy gluon dynamics and of the often elusive gluonic component in hadron wavefunctionals. One such opportunity was recently exploited in OCD sum rule analyses, which found nonperturbative short-distance physics in the form of direct instantons [2] to play a crucial role in the structure and dynamics of the scalar (0^{++}) glueball [3, 4]. Indeed, the instanton-improved operator product expansion (IOPE) of the 0^{++} glueball correlator resolves two long-standing problems of the conventional sum rules (the mutual inconsistency of different Borel moment sum rules and the conflict with the underlying low-energy theorem [5, 6]), generates new scaling relations between fundamental glueball and instanton properties, and leads to improved sum rule predictions for scalar glueball properties [3]. (See also the subsequent gaussian sum rule analysis [7], based on the same instanton contributions.) The Borel sum rule analysis has recently been improved and extended (to realistic instanton size distributions and renormalized instanton contributions) in [4]. Hard nonperturbative contributions to the pseudoscalar glueball IOPE and sum rules were also investigated. On these we will report in the following. A comprehensive analysis of both spin-0 glueball channels can be found in Ref. [4].

The implementation of the direct instanton contributions in the 0^{-+} channel is straightforward: the expressions from the 0^{++} channel [3] can simply be taken over, with their signs inverted. This is a consequence of the (Minkowski) (anti-) self-duality of the (anti-) instanton's field strength, $G_{\mu\nu}^{(I,\bar{I})} = \pm i\tilde{G}_{\mu\nu}^{(I,\bar{I})}$. However, one immediately suspects that adding the dominant and, due to the sign change, strongly repulsive instanton contributions will seriously unbalance the Borel sum rules. Two of the consequences were recently observed in [8]: any reliable signal for a pseudoscalar glueball resonance disappears (in contradiction to lattice evidence), and even the fundamental spectral positivity bound is violated. In addition, the crucial low-energy theorem for the zero-momentum 0^{-+} glueball correlator, and therefore the underlying anomalous axial Ward identity, is strongly violated [4]. As pointed out in [4], these problems have an appealing solution in the form of additional nonperturbative short-range contributions to the IOPE, associated with topological charge screening. Below, we will sketch the implementation of the screening contributions and their impact on the sum rule analysis. We also comment on the effects of realistic instanton size distributions and the renormalization of the instanton-induced Wilson coefficients.

2 Correlator, IOPE and sum rules

Our discussion will be based on the pseudoscalar glueball correlation function

$$\Pi_P(x) = \langle 0|T\, O_P(x)\, O_P(0)\, |0\rangle\,, \qquad (1)$$

where O_P is the standard gluonic interpolating field

$$O_P(x) = \alpha_s G^a_{\mu\nu}(x)\, \tilde{G}^{a\mu\nu}(x) \qquad (2)$$

($\tilde{G}_{\mu\nu} \equiv (i/2)\,\varepsilon_{\mu\nu\rho\sigma} G_{\rho\sigma}$), and its Fourier transform

$$\Pi_P(-q^2) = i\int d^4x\, e^{iqx}\, \langle 0|T\, O_P(x)\, O_P(0)\, |0\rangle\,. \qquad (3)$$

The zero-momentum limit of this correlator is governed by the low-energy theorem (LET) [9]

$$\Pi_P\left(q^2=0\right)=(8\pi)^2\frac{m_u m_d}{m_u+m_d}\langle\bar{q}q\rangle \tag{4}$$

(for three light flavors and $m_{u,d} \ll m_s$) which derives from the axial anomaly and imposes stringent consistency requirements on the sum rule analysis [4].

The hadronic information in the glueball correlators is most directly accessible in the dispersive representation

$$\Pi_P\left(Q^2\right)=\frac{1}{\pi}\int_{s_i}^{\infty}ds\frac{\operatorname{Im}\Pi_P(-s)}{s+Q^2} \tag{5}$$

where the necessary number of subtractions is implied. The QCD sum-rule description of the spectral functions contains one or two resonances (in zero-width approximation) and the local-duality continuum, i.e.

$$\begin{aligned}\operatorname{Im}\Pi_P^{(ph)}(s)&=\pi\sum_{i=1}^{2}f_{Pi}^2 m_{Pi}^4\delta\left(s-m_{Pi}^2\right)\\&\quad+\theta(s-s_0)\operatorname{Im}\Pi_P^{(IOPE)}(s)\end{aligned} \tag{6}$$

The continuum representation covers the invariant-mass region "dual" to higher-lying resonances and multi-hadron continuum, starting at an effective threshold s_0. It is obtained from the discontinuities of the IOPE, i.e. the expansion of the correlator at large, spacelike momenta $Q^2 \equiv -q^2 \gg \Lambda_{QCD}$ into condensates of operators $\hat{O}_D$ of increasing dimension D,

$$\Pi_P(Q^2)=\sum_{D=0,4,\ldots}\tilde{C}_D^{(P)}\left(Q^2;\mu\right)\left\langle 0\left|\hat{O}_D\right|0\right\rangle_\mu . \tag{7}$$

"Hard" field modes with momenta $|k|>\mu$ contribute to the momentum-dependent Wilson coefficients $\tilde{C}_D\left(Q^2\right)$ while "soft" modes with $|k|\le\mu$ generate the condensates $\left\langle\hat{O}_D\right\rangle$.

In order to write down the sum rules, the IOPE - with the continuum subtracted and weighted by powers of $-Q^2$ - is Borel-transformed,

$$\begin{aligned}\mathcal{R}_{P,k}^{(IOPE)}(\tau;s_0)&=\hat{B}\left[\frac{\left(-Q^2\right)^k}{\pi}\int_0^{s_0}ds\frac{\operatorname{Im}\Pi_P^{(IOPE)}(-s)}{s+Q^2}\right]\\&=-\delta_{k,-1}\Pi_P^{(IOPE)}(0)\\&\quad+\frac{1}{\pi}\int_0^{s_0}ds s^k\operatorname{Im}\Pi_P^{(IOPE)}(s)e^{-s\tau},\end{aligned} \tag{8}$$

for $k\ge -1$. The hadronic parameters m_{Pi}, f_{Pi}, s_0 are then determined by matching these moments in the fiducial τ-region to their resonance-induced counterparts (and a subtraction constant for $k=-1$, fixed by the LET (4)). The resulting IOPE sum rules are

$$\mathcal{R}_{P,k}^{(IOPE)}(\tau;s_0)=\sum_{i=1}^{2}f_{Pi}^2 m_{Pi}^{4+2k}e^{-m_{Pi}^2\tau}-\delta_{k,-1}\Pi_P^{(ph)}(0). \tag{9}$$

The perturbative contributions to the IOPE coefficients, i.e. the conventional OPE, can be found in [10, 11, 8, 4]. In the following we will focus on the nonperturbative contributions due to direct instantons and topological charge screening.

3 Direct instantons

Dominant direct instanton contributions to the 0^{-+} glueball correlator are received by the unit-operator IOPE coefficient, $\tilde{C}_0^{(P,I+\bar{I})_D}=\Pi_P^{(I+\bar{I})}$. They are best calculated in x-space, with the result

$$\tilde{C}_0^{(P,I+\bar{I})_D}\left(x^2\right)=-\frac{2^8 3}{7}\int d\rho n(\rho)\frac{1}{\rho^4}{}_2F_1\left(4,6,\frac{9}{2},-\frac{x^2}{4\rho^2}\right). \tag{10}$$

The further evaluation requires the (anti-) instanton distribution $n_{I,\bar{I}}(\rho)$ with its two leading moments, the instanton density $\bar{n}$ and average size $\bar{\rho}$, as input. All previous studies of direct instanton effects have relied on the simplest possible, spike-like approximation $n(\rho)=\bar{n}\delta(\rho-\bar{\rho})$. In Ref.[4] a realistic finite-width distribution was implemented instead, which is fully determined by $\bar{n}$, $\bar{\rho}$ and the known small- and large-ρ behavior [12], (for $N_c=N_f=3$)

$$n_g(\rho)=\frac{2^{18}}{3^6\pi^3}\frac{\bar{n}}{\bar{\rho}}\left(\frac{\rho}{\bar{\rho}}\right)^4\exp\left(-\frac{2^6}{3^2\pi}\frac{\rho^2}{\bar{\rho}^2}\right). \tag{11}$$

From the Fourier transform of (10) one finds the direct-instanton-induced Borel moments [3]

$$\begin{aligned}\mathcal{L}_{k-1}^{(I+\bar{I})}(\tau)&=-2^6\pi^2\frac{-\partial^k}{\partial\tau^k}\int d\rho n(\rho)x^2e^{-x}\\&\quad\times\left[(1+x)K_0(x)+\left(2+x+\frac{2}{x}\right)K_1(x)\right],\end{aligned} \tag{12}$$

($x=\rho^2/2\tau$, $k\ge 0$) and from the imaginary part [3]

$$\operatorname{Im}\Pi_P^{(I+\bar{I})}(-s)=-2^4\pi^4\int d\rho n(\rho)\rho^4 s^2 J_2\left(\sqrt{s}\rho\right)Y_2\left(\sqrt{s}\rho\right) \tag{13}$$

at timelike momenta one then has

$$\begin{aligned}\mathcal{R}_{P,k}^{(I+\bar{I})}(\tau)&=-2^7\pi^2\delta_{k,-1}\int d\rho n(\rho)-2^4\pi^3\int d\rho\\&\quad\times n(\rho)\rho^4\int_0^{s_0}ds s^{k+2}J_2\left(\sqrt{s}\rho\right)Y_2\left(\sqrt{s}\rho\right)e^{-s\tau}.\end{aligned} \tag{14}$$

The direct-instanton contributions (with subtracted continuum) are an important complement to the corresponding perturbative ones.

The realistic (finite-width) instanton size distribution tames the rising oscillations of the imaginary part (13) at large s (an artefact with misleading impact on the s_0 dependence of the moments) into a strong decay $\propto s^{-5/2}$. Furthermore, it allows for a gauge-invariant renormalization of the instanton-induced coefficients by excluding contributions from instantons with size $\rho > \mu^{-1}$, i.e. by replacing

$$n(\rho) \to \tilde{n}_{\mu}(\rho) \equiv \theta_{\beta}\left(\rho - \mu^{-1}\right) n(\rho) \quad (15)$$

with a "soft" step function θ_β (e.g. $\theta_\beta\left(\rho - \mu^{-1}\right) = \left[\exp\left(\beta\left(\rho - \mu^{-1}\right)\right) + 1\right]^{-1}$). The instanton-induced μ-dependence turns out to be relatively weak for $\mu < \bar{\rho}^{-1}$, in complicance with the other sources of μ dependence. Note that the standard spike distribution (with $\bar{\rho} < \mu^{-1}$) misses the reduction of the total instanton density to the direct instanton part,

$$\bar{n} = \int_0^\infty d\rho n(\rho) \to \int_0^\infty d\rho \tilde{n}_\mu(\rho) \equiv \bar{n}_{direct}. \quad (16)$$

4 Topological charge screening

As argued above, it is not surprising that the dominant and strongly repulsive direct instanton contributions, when added as the *sole* nonperturbative contributions to the IOPE coefficients, upset the 0^{-+} glueball sum rules and have the mentioned, detrimental effects. In fact, this suggests that additional important contributions, which should predominantly affect the pseudoscalar IOPE, are still amiss. And indeed, the 0^{-+} glueball correlator is proportional to the topological charge correlators and therefore maximally sensitive to the short-distance topological charge (probably mainly instanton - antiinstanton) correlations in the QCD vacuum. Their impact on the pseudoscalar glueball correlators was found to be exceptionally strong in the instanton liquid model [2, 13]. Topological charge correlations are created by light-quark loops or, equivalently at low energies, by the attractive (repulsive) η'-meson exchange forces between opposite-sign (equal-sign) topological charges. They lead to Debye screening of the topological charge with "screening mass" $m_{\eta'}$ and the corresponding (small!) screening length $\lambda_D \sim m_{\eta'}^{-1} \sim 0.2$ fm [14, 15]. Since $m_{\eta'} > \mu$, the screening correlations contribute to the Wilson coefficients of the pseudoscalar glueball correlator.

The screening contributions can be obtained from the coupling of the η' mesons to the topological charge density, as dictated by the axial anomaly [16]. In the chiral limit, where $\eta' = \eta_0$ is purely flavor-singlet, and for $k \lesssim m_{\eta'}$ the coupling to the topological charges in the vacuum medium (approximated for simplicity as concentrated in pointlike instantons) is governed by the effective lagrangian [14, 15]

$$\mathcal{L} = \frac{1}{2}\left(\partial\eta_0\right)^2 - 2\bar{n}\cos\left(\gamma_{\eta_0}\eta_0 + \theta\right) \quad (17)$$

where $\bar{n}$ is the global topological charge density and where we have introduced a source $\theta(x)$ for the local topological charge density $Q(x)$. Note the screening mass $m_{scr}^2 = 2\bar{n}\gamma_{\eta_0}^2$ of the η_0. Taking two derivatives of the corresponding generating functional with respect to θ leads (for small amplitudes η_0) to the topological charge correlator

$$\langle Q(x) Q(0)\rangle = \Pi_P(x)/(8\pi)^2 \simeq -2\bar{n}\delta^4(x) - \left(2\bar{n}\gamma_{\eta_0}\right)^2 \langle \eta_0(x)\eta_0(0)\rangle . \quad (18)$$

The first term is just the pointlike approximation to the direct-instanton contribution evaluated above. The second one is the screening correction, which modifies the nonperturbative contributions to the pseudoscalar IOPE coefficient into

$$\Pi_P(x) = \Pi_P^{(I+\bar{I})}(x) - \left(\bar{n}\gamma_{\eta_0}\right)^2 \frac{m_{\eta_0}}{\pi^2 x} K_1\left(m_{\eta_0}x\right). \quad (19)$$

After implementing finite quark-mass effects (i.e. η_0-η_8 mixing), the (Minkowski) Borel moments associated with the screening contributions become

$$\mathcal{R}_{P,k}^{(scr)}(\tau) = -\delta_{k,-1}\left(\frac{F_{\eta'}^2}{m_{\eta'}^2} + \frac{F_\eta^2}{m_\eta^2}\right) + F_{\eta'}^2 m_{\eta'}^{2k} e^{-m_{\eta'}^2\tau} + F_\eta^2 m_\eta^{2k} e^{-m_\eta^2\tau}. \quad (20)$$

The τ-independent term in Eq. (20) is the screening-induced subtraction constant $-\Pi_P^{(scr)}(0)$. It shows that inclusion of the screening contributions is mandatory in order to satisfy the axial $U(1)$ Ward identity and the ensuing LET (4). Indeed, direct instantons generate a large subtraction constant $\Pi_P^{(I+\bar{I})}(0) = -2^7\pi^2\bar{n}$ while the LET demands the zero-momentum limit of the physical correlator to be of the order of the light quark masses, i.e. much smaller. The screening contribution (20) is necessary to cancel most of it and to restore consistency with the LET. In the chiral limit, the screening contribution turns into

$$\Pi_P^{(scr)}(0) = \frac{F_{\eta'}^2}{m_{\eta'}^2} = \frac{\left(16\pi\bar{n}\gamma_{\eta_0}\right)^2}{2\bar{n}\gamma_{\eta_0}^2} = 2^7\pi^2\bar{n} \quad (21)$$

and the cancellation becomes exact (due to the infinite-range interactions mediated by massless Goldstone bosons). The above argument provides compelling evidence for the screening contributions to be an indispensable complement to the direct instantons.

The cancellation between the subtraction terms suggests a simple strategy for renomalizing the screening contributions. Since the large-ρ cutoff μ^{-1} amounts to replacing $\bar{n}$ by $\bar{n}_{dir} = \zeta\bar{n}$ with $\zeta < 1$ (cf. Eq. (16)), consistency with the LET requires the same replacement in the screening contributions (19).

Besides restoring the axial Ward identity, inclusion of the screening contributions resolves the positivity-bound violation and creates a strong signal for both the η' and the pseudoscalar glueball resonances in the corresponding Borel sum rules. The screening contributions (20) are of substantial size, even relative to the direct-instanton contributions, and they are largest at small and intermediate $\tau \lesssim \lambda_{scr}^2 \sim 1$ GeV^2. Moreover, they modify qualitative features of the Borel moments (e.g. the sign of the slope) to which the sum rule fits are very sensitive. Hence, trustworthy sum rule results cannot be obtained even at small and intermediate τ when the screening contributions are ignored [8]. Even the bound obtained in [8] has therefore to be regarded as invalid.

After including the screening contributions, all four Borel-moment sum rules (9) are stable and yield consistent results. Note that previous analyses of the 0^{-+} sum rules have discarded the $k = -1$ sum rule and therefore missed the chance to implement the first-principle information from the low-energy theorem, as well as a very useful consistency check. The two-resonance fit is clearly favored over the one-pole approximation, i.e. the IOPE provides clear signals for the η' resonance (with small η admixtures due to mixing) in addition to a considerably heavier 0^{-+} glueball. The 0^{-+} glueball mass is found to be $m_P = 2.2 \pm 0.2$ GeV, and the decay constant $f_P = 0.6 \pm 0.25$ GeV. (For a complete discussion see [4].)

5 Summary

We have reported recent developments in understanding and evaluating nonperturbative glueball physics in the pseudoscalar channel on the basis of the operator product expansion [4]. Contrary to naive expectation, much of this nonperturbative physics takes place at surprisingly short distances $|x| \sim 0.2 - 0.3$ fm, and consequently shows up in the Wilson coefficients of the IOPE. Direct instantons are the paradigm for such physics, and their contributions to the spin-0 glueball correlators are indeed exceptionally large. We have improved on the previous evaluation of these contributions [3] by implementing a realistic instanton size distribution and the renomalization of the instanton-induced coefficients (both of these improvements should be useful in other hadron channels as well). A further new development is very specific to the 0^{-+} glueball channel: we have found compelling evidence for topological charge screening to provide crucial contributions to the unit operator coefficient of the pseudoscalar glueball IOPE. The screening contributions form an indispensable complement to the direct instantons, roughly speaking “unquenching” them and thereby restoring the axial Ward identity. Moreover, they balance the strong repulsion of the direct instantons, correct the otherwise gross violation of the LET, resolve the violation of the spectral positivity bound and generate a strong (and otherwise absent) signal for the 0^{-+} glueball resonance. With screening included, all Borel moment sum rules provide consistent and stable predictions for the fundamental 0^{+-} glueball properties ($m_P = 2.2 \pm 0.2$ GeV, $f_P = 0.6 \pm 0.25$ GeV).

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