Electronic Properties of Coupled Quantum Rings in the Presence of a Magnetic Field

Gonzalo Fuster, Mónica Pacheco, and Zdenka Barticevic

Departamento de Física, Universidad Técnica Federico Santa María

Av. España 1680, Casilla 110-V, Valparaíso, Chile

We have studied the energy spectrum of a system formed by two concentric, coupled, semiconductor quantum rings. We have investigated the effect of a uniform magnetic field applied along the rings axis, on the energy spectrum of the system. We found that the spectrum of the coupled rings with similar confinement length corresponds roughly to the superposition of the spectra of two separate rings, although modified by the anticrossings produced among different states of the individual rings with same angular momentum. The Aharonov-Bohm oscillations do not have a definite period.

1 Introduction

Quantum structures of annular symmetry have been the object of interest since the application of self-assembling techniques to the fabrication of semiconductor quantum rings. The ground state and excitations in rings made of InGaAs, in the presence of a magnetic field parallel to the ring axis have been studied by Lorke et al[1], by means of spectroscopic techniques. They found Aharonov-Bohm type oscillations in the energy spectrum as a function of the number of magnetic flux quanta enclosed by the ring. Also, the magneto-optical spectrum of a ring in the presence of an external electric field, and in the presence of impurities has been studied [2,3,4]. In this work we calculated the properties of a system formed by two concentric coupled semiconductor rings in the presence of an axial magnetic field. We studied the effect on the spectrum of varying the coupling between the rings, by changing the rings radii. Rings with different confinement parameters show Aharonov-Bohm type oscillations associated with the wider ring. When both rings have the same confinement parameter the ground state oscillations do not have a definite period, and interesting anticrossings can be observed.

2 Model

We performed our calculations within the independent-particle formalism using the effective mass approximation. The potential of every individual ring is modeled as the superposition of a quantum well potential along the axial direction, with a lateral confinement potential on the ring plane. This potential is generated by a rotation, around the ring axis, of a one-dimensional parabolic potential centered at a distance of one ring radius from the ring center:

$$v_i\left(\rho\right) = \frac{1}{2}\, m^* \omega_{g,\,i}^2 \left(\rho - \rho_i\right)^2 \;\; ;\; i\,=\,1,\,2 \qquad (1)$$

where m* is the electronic effective mass of the material of the ring, ρ_i is the ring radius and ω_i is a characteristic frequency of the lateral geometric confinement which defines the degree of confinement. The system formed by two concentric coupled rings is then modeled as the superposition of the potentials of each ring with their minima at different radii, and truncated at the intersection circle of radius:

$$V_{lat}\left(\rho\right) = v_1\left(\rho\right)\theta\left(\rho - \tilde{\rho}\right) + v_2\left(\rho\right)\theta\left(\tilde{\rho} - \rho\right) \qquad (2)$$

where $\tilde{\rho} = \rho_1\, \frac{\omega_{g,1}}{\omega_{g,1}+\omega_{g,2}} + \rho_2\, \frac{\omega_{g,1}}{\omega_{g,1}+\omega_{g,2}}$, and

$$\theta\left(\rho - \tilde{\rho}\right) = \left\{ \begin{array}{ll} 0\; ; & \rho < \tilde{\rho} \\ 1\; ; & \rho > \tilde{\rho} \end{array} \right\}$$

The two-dimensional potential used to describe the coupled rings is shown in Fig. 1.

For the in-plane motion the Hamiltonian can be expressed as:

$$H = \frac{1}{2m^*}\left(\vec{p} + \frac{e}{c}\,\vec{A}\right)^2 + V\left(\vec{\rho}\right) \qquad (3)$$

The symmetric gauge has been used for the potential vector: $\vec{A} = \frac{B}{2}\left(-y, x, 0\right)$. We solve the eigenvalue problem for the Hamiltonian of equation (3) by expanding the corresponding envelope function as a linear combination of two sets of basis functions associated to each ring respectively. The method is similar to the one described in [2].

$$f_{i,\,n}^m\left(\rho,\,\phi\right) = \sum_j C_{i,\,n}^{j,\,m} e^{i\,m\,\phi} \rho\, e^{-(\rho - \rho_i)^2/2} H_j\left(\rho - \rho_i\right)$$
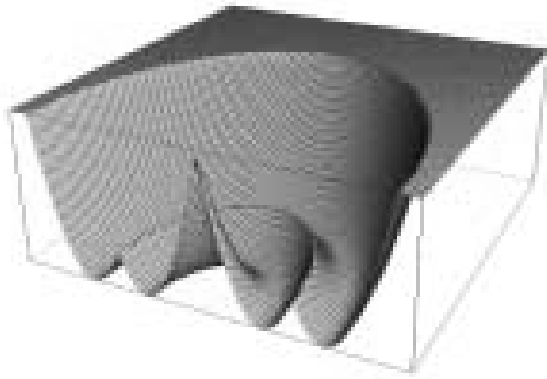
Figure 1. Lateral potential profile of the coupled rings system.

3 Results

To illustrate the effect of the coupling between the rings we present, in Fig. 2, the electronic energy spectrum for a system of two rings with radii $\rho_1 = 400$[Å] and $\rho_2 = 450$[Å]. In figure 2(a), both rings have the same width $w_1 = w_2 = 30$ [Å], while in figure 2(b) the correponding widths are: $w_1 = 30$ [Å] and $w_2 = 31$ [Å] respectively. The width of the ring is defined in terms of the characteristic frequency of the confinement potential: $w_i = \sqrt{\hbar/(m^*\omega_i)}$.

It can be observed that, the energy spectrum as a function of the magnetic field consists of two sets of levels, which in the case of rings with different widths are clearly separated in energy. The ground state energy is determined by the ring with the smaller confinement. The radius of this ring determines the position in energy of the lower group of excited states.

The effect of the ring coupling is emphasized in Fig. 3(a) where we have displayed an enlarged view of the magnetic dependence of the spectrum near the ground state energy, for the same radii and confinement parameters as it is shown in Fig. 2(a). For comparison, Fig. 3(b) shows the corresponding spectra of two isolated rings. It can be seen that the coupling produces anticrossings among different states of the corresponding isolated rings, with the same quantum number of angular momentum. It is apparent that the Aharonov-Bohm oscillations of the ground-state energy for the two-ring system do not present a well defined periodicity. This is so, even in the case of non interacting rings as a consequence of the finite width of the rings.

In summary, we found that the energy spectrum of a system of two concentric coupled rings, with similar confinement length, corresponds roughly to the superposition of the spectra of two independent rings, although interesting anticrossings among different states of the individual rings with same angular momentum can be observed. The Aharonov-Bohm oscillations do not have a definite period. For rings with different confinement parameters the ground state energy is roughly equal to the ground state energy of the wider ring.
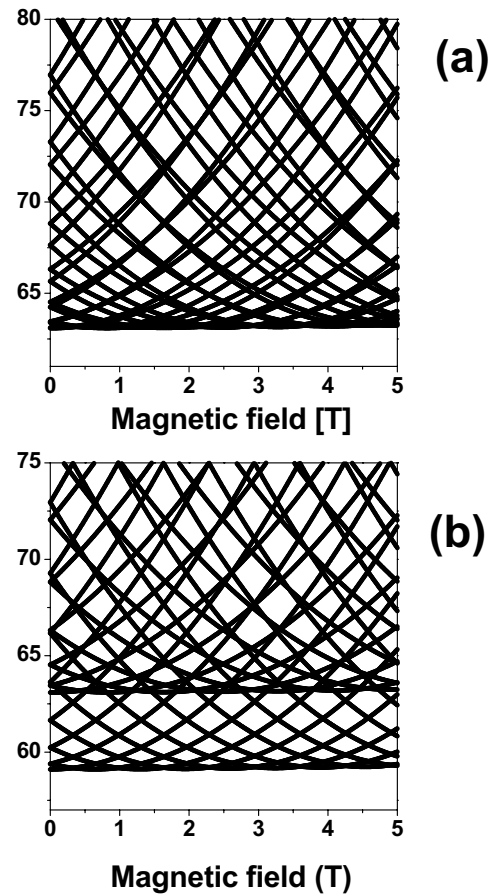

Figure 2. Energy spectrum as a function of the magnetic field of a coupled concentric-rings system. a) Both rings have the same widths. (b) The rings have slightly differents widths.
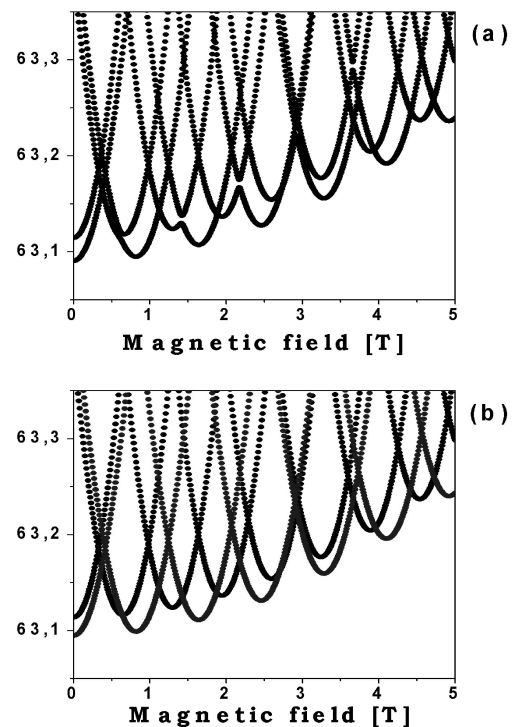

Figure 3. Energy spectrum as a function of the magnetic field of a system of: (a) two coupled rings and (b) two isolated rings.

References

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