The $J/\psi DD^*$ Vertex

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We employ QCD sum rules to calculate the $J/\psi DD^*$ form factors and coupling constant by studying the three-point $J/\psi D^* D$ correlation function. We find that the momentum dependence of the form factor depends on the off-shell meson. We get a value for the coupling which is in agreement with estimates based on constituent quark model.

Hadrons are composites of the underlying quarks whose effective fields describe point-like physics only when all the interacting particles are on mass-shell. When at least one of the particles in a vertex is off-shell, the finite size effects of the hadrons become important. Therefore, the knowledge of the form factors in hadronic vertices is of crucial importance to estimate any hadronic amplitude using hadronic degrees of freedom. This work is devoted to the study of the $J/\psi D^* D$ form factor, which is important, for instance, in the evaluation of the dissociation cross section of J/ψ by pions and ρ mesons using effective Lagrangians [1, 2, 3]. Since a decrease of J/ψ production in heavy ions collisions might signal the formation of a quark-gluon plasma (QGP) [4], a precise evaluation of the background, i.e., conventional J/ψ absorption by co-moving pions and ρ mesons, is of fundamental importance.

The $J/\psi D^* D$ coupling has been studied by some authors using different approaches: vector meson dominance model plus relativistic potential model [1] and constituent quark meson model [5]. Unfortunately, the numerical results from these calculations may differ by almost a factor two. The relevance of this difference can not be underestimated since the cross section is proportional to the square of the coupling constants. In ref. [3] it was shown that the use of different coupling constants and form factors can lead to cross sections that differ by more than one order of magnitude, and that can even have a different behavior as a function of $\sqrt{s}$.

In previous works we have used the QCD sum rules (QCDSR) to study the $D^* D\pi$ [6, 7], $DD\rho$ [8] and $J/\psi DD$ [9] form factors, considering two different mesons off mass-shell. In these works the QCDSR results for the form factors were parametrized by analytical forms such that the respective extrapolations to the off-shell meson poles provided consistent values for the corresponding coupling constant. In this work we use the QCDSR approach to evaluate the $J/\psi D^* D$ form factors and use the same procedure described above to estimate the $J/\psi D^* D$ coupling constant.

The three-point function associated with a $H_1 H_2 H_3$ vertex (see Fig. 1), where H_1 and H_3 are the incoming and outgoing external mesons respectively and H_2 is the off-shell meson, is given by

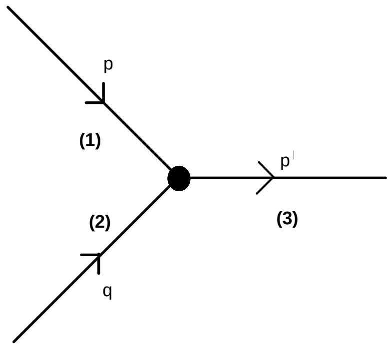

Figure 1. Diagram representing the $H_1(p)H_2(q)H_3(p')$ vertex.

$$\Gamma_{\mu\nu}(p,p') = \int d^4x\, d^4y\; e^{ip'.x}\, e^{-i(p'-p).y} \times \langle 0|T\{j_3(x) j_2^\dagger(y) j_1^\dagger(0)\}|0\rangle\,, \tag{1}$$

where j_i is the interpolating field for H_i. For J/ψ, D^* and D mesons the interpolating fields are respectively $j_\mu^{(\psi)} = \bar{c}\gamma_\mu c$, $j_\nu^{(D^*)} = \bar{q}\gamma_\nu c$ and $j^{(D)} = i\bar{q}\gamma_5 c$ with q and c being a light quark and the charm quark fields.

The phenomenological side of the vertex function, $\Gamma(p,p')$, is obtained by the consideration of H_1 and H_3 state contribution to the matrix element in Eq. (1):

$$\Gamma_{\mu\nu}^{(phen)}(p,p') = \frac{1}{p^2 - m_1^2}\frac{1}{p'^2 - m_3^2}\langle 0|j_3|H_3(p')\rangle \times\langle H_3(p')|j_2^\dagger|H_1(p)\rangle\langle H_1(p)|j_1^\dagger|0\rangle + \text{h. r.}\,, \tag{2}$$

where h. r. means higher resonances.

The matrix element of the current j_2 defines the vertex function $V_{\lambda\lambda'}(p,p')$:

$$\langle H_3(p')|j_2^\dagger|H_1(p)\rangle = \langle H_2(q)|j_2^\dagger|0\rangle \frac{V_{\lambda\lambda'}(p,p')}{q^2 - m_2^2}\,, \tag{3}$$

where $q = p' - p$. Calling $p_1,\ p_2$ and p_3 the four momentum of $J/\psi,\ D^*$ and D respectively one has

$$V^{\lambda\lambda'}(p_1,p_2,p_3) = g_{\psi DD*}(q^2)\epsilon^{\alpha\beta\gamma\delta}\epsilon_\alpha^\lambda(p_1)\epsilon_\gamma^{\lambda'}(p_2)p_{3\beta}p_{2\delta}\,. \tag{4}$$

The vacuum to meson transition amplitudes appearing in Eqs. (2) and (3) are given in terms of the corresponding meson decay constants f_{H_i} by

$$\langle 0|j^{(D)}|D\rangle = \frac{m_D^2 f_D}{m_c} \ , \tag{5}$$

and

$$\langle V(p,\epsilon)|j_\alpha^\dagger|0\rangle = m_V f_V \epsilon_\alpha^* \ , \tag{6}$$

for the vector meson $V = J/\psi$ or $V = D^*$. Therefore, using Eqs. (3), (4), (5) and (6) in Eq. (2) we get

$$\Gamma_{\mu\nu}^{(phen)}(p,p') = C\frac{g_{\psi DD*}(q^2)\epsilon_{\alpha\beta\mu\nu}p^\alpha p'^\beta}{(q^2-m_2^2)(p^2-m_1^2)(p'^2-m_3^2)} + \text{h. r.} \ , \tag{7}$$

where

$$C = \frac{m_D^2 m_{D^*} m_\psi f_D f_{D^*} f_\psi}{m_c} \ . \tag{8}$$

The contribution of higher resonances and continuum in Eq. (7) will be taken into account as usual in the standard form of ref. [10].

The QCD side, or theoretical side, of the vertex function is evaluated by performing Wilson's operator product expansion (OPE) of the operator in Eq. (1). Writing $\Gamma_{\mu\nu}$ in terms of the invariant amplitude:

$$\Gamma_{\mu\nu}(p,p') = \Lambda(p^2,p'^2,q^2)\epsilon_{\alpha\beta\mu\nu}p^\alpha p'^\beta \ , \tag{9}$$

we can write a double dispersion relation for Λ, over the virtualities p^2 and p'^2 holding $Q^2 = -q^2$ fixed:

$$\Lambda(p^2,p'^2,Q^2) = -\frac{1}{4\pi^2}\int dsdu \ \frac{\rho(s,u,Q^2)}{(s-p^2)(u-p'^2)} \ , \tag{10}$$

where $\rho(s,u,Q^2)$ equals the double discontinuity of the amplitude $\Lambda(p^2,p'^2,Q^2)$ on the cuts $s_{min} \leq s \leq \infty$, $m_c^2 \leq u \leq \infty$, with $s_{min} = 4m_c^2$ in the case of off-shell D^* or D and $s_{min} = m_c^2$ in the case of off-shell J/ψ. We consider diagrams up to dimension three which include the perturbative diagram and the quark condensate. To improve the matching between the two sides of the sum rules, we perform a double Borel transformation in both variables $P^2 = -p^2 \rightarrow M^2$ and $P'^2 = -p'^2 \rightarrow M'^2$ We get one sum rule for each meson considered off-shell. Calling $g_{\psi DD^*}^M(q^2)$ the ψDD^* form factor for the off-shell meson M, we get the following sum rules:

$$C\ \frac{g_{\psi DD^*}^{(D)}(t)}{(t-m_D^2)}e^{-\frac{m_{D^*}^2}{M'^2}}e^{-\frac{m_\psi^2}{M^2}} = \frac{1}{4\pi^2}\int_{4m^2}^{s_0}\int_{u_{min}}^{u_0} dsdu\rho^{(D)}(u,s,t)e^{-\frac{s}{M^2}}e^{-\frac{u}{M'^2}}\Theta(u_{max}-u), \tag{11}$$

$$C\ \frac{g_{\psi DD^*}^{(D^*)}(t)}{(t-m_{D^*}^2)}e^{-\frac{m_D^2}{M'^2}}e^{-\frac{m_\psi^2}{M^2}} = \frac{1}{4\pi^2}\int_{4m^2}^{s_0}\int_{u_{min}}^{u_0} dsdu\rho^{(D^*)}(u,s,t)e^{-\frac{s}{M^2}}e^{-\frac{u}{M'^2}}\Theta(u_{max}-u), \tag{12}$$

and

$$C\ \frac{g_{\psi DD^*}^{(J/\psi)}(t)}{(t-m_\psi^2)}e^{-\frac{m_D^2}{M'^2}}e^{-\frac{m_{D^*}^2}{M^2}} = \frac{1}{4\pi^2}\int_{m^2}^{s_0}\int_{u_{min}}^{u_0} dsdu\rho^{(J/\psi)}(u,s,t)e^{-\frac{s}{M^2}}e^{-\frac{u}{M'^2}}\Theta(u_{max}-u), \tag{13}$$

with $t = q^2$,

$$\rho^{(D)}(u,s,t) = \rho^{(D^*)}(u,s,t) = \frac{3m_c}{\sqrt{\lambda}}\left(1+\frac{s\lambda_2}{\lambda}\right), \tag{14}$$

$\lambda = (u+s-t)^2 - 4us$, $\lambda_2 = u+t-s+2m_c^2$ and

$$u_{min}^{max} = \frac{1}{2m_c^2}\left[-st + m_c^2(s+2t) \pm \sqrt{s(s-4m_c^2)(t-m_c^2)^2}\right], \tag{15}$$

in the case of off-shell D or D^*. In the case of an off-shell J/ψ we get:

$$\rho^{(J/\psi)}(u,s,t) = \frac{3m_c}{\lambda^{3/2}}\left[(u-s)^2 - t(u+s-2m_c^2)\right] - 4\pi^2 <\bar{q}q> \delta(s-m_c^2)\delta(u-m_c^2), \tag{16}$$

and

$$u_{min}^{max} = \frac{1}{2m_c^2}\left[-st + m_c^2(2s+t) \pm \sqrt{t(t-4m_c^2)(s-m_c^2)^2}\right]. \tag{17}$$

In the Eqs. (11), (12) and (13) we have transferred to the QCD side the higher resonances contributions through the introduction of the continuum thresholds s_0 and u_0.

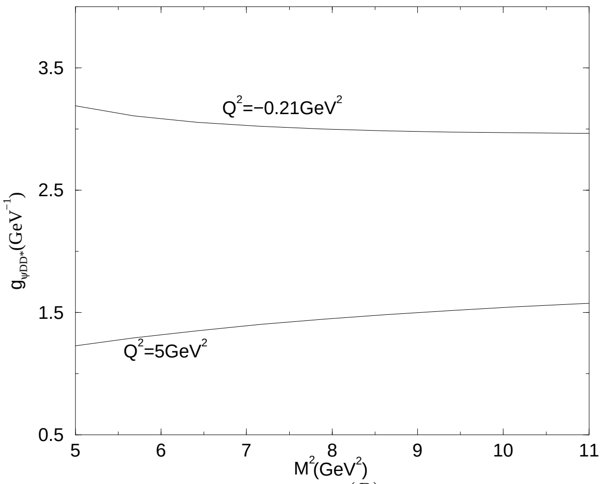

Figure 2. M^2 dependence of $g^{(D)}_{J/\psi DD^*}(Q^2)$ for $Q^2 = -0.21$ GeV^2 and $Q^2 = 5.0$ GeV^2.

The parameter values used in all calculations are $m_c = 1.3$ GeV, $m_D = 1.87$ GeV, $m_{D^*} = 2.01$ GeV, $m_\psi = 3.1$ GeV, $f_D = (170 \pm 10)$ MeV, $f_{D^*} = (240 \pm 20)$ MeV, $f_{J/\psi} = (405 \pm 15)$ MeV, $\langle \bar{q}q \rangle = -(0.23)^3$ GeV^3. The continuum thresholds for the sum rules are $s_0 = (m_1 + \Delta_s)^2$ and $u_0 = (m_3 + \Delta_u)^2$ with $\Delta_s = \Delta_u = 0.5$ GeV.

We first discuss the $J/\psi DD^*$ form factor with an off-shell D meson. In Fig. 2 we show the behavior of the form factor $g^{(D)}_{J/\psi DD^*}(Q^2)$ at $Q^2 = 5.0$ GeV^2 and $Q^2 = -0.21$ GeV^2, as a function of the Borel mass M^2 using $M'^2 = M^2 \frac{m^2_{D^*}}{m^2_\psi}$. We can see that the QCDSR results are rather stable in the interval $7 \leq M^2 \leq 11$ GeV^2. In Fig. 3 we show $g^{(D)}_{J/\psi DD^*}(Q^2 = -0.21\,\text{GeV}^2)$ as a function of M^2 and M'^2.

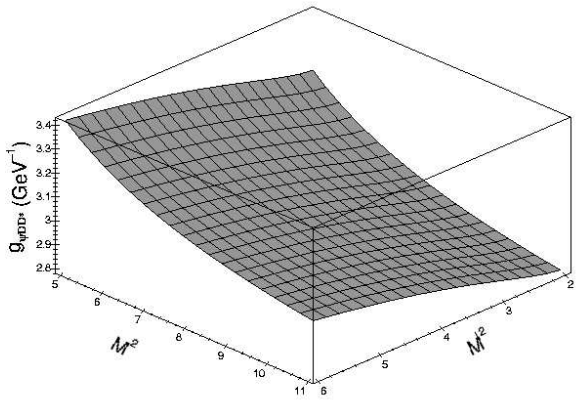

Figure 3. M^2 and M'^2 dependence of $g^{(D)}_{\psi DD^*}(Q^2 = -0.21\,\text{GeV}^2)$.

From Fig. 3 we see that the stability is still good even considering the two independent Borel parameters. The same kind of stability is obtained for other values of Q^2 and for the other two form factors.

Fixing $M^2 = m_1^2$ and $M'^2 = m_3^2$ we show, in Fig. 4, the momentum dependence of the QCDSR results for the form factors $g^{(D)}_{\psi DD^*}$, $g^{(D^*)}_{\psi DD^*}$ and $g^{(J/\psi)}_{\psi DD^*}$ through the circles, squares and triangles respectively. Since the present approach cannot be used at $Q^2 << 0$, to extract the $g_{\psi DD^*}$ coupling from the form factors we need to extrapolate the curve to $Q^2 = -m_2^2$: the mass of the off-shell meson.

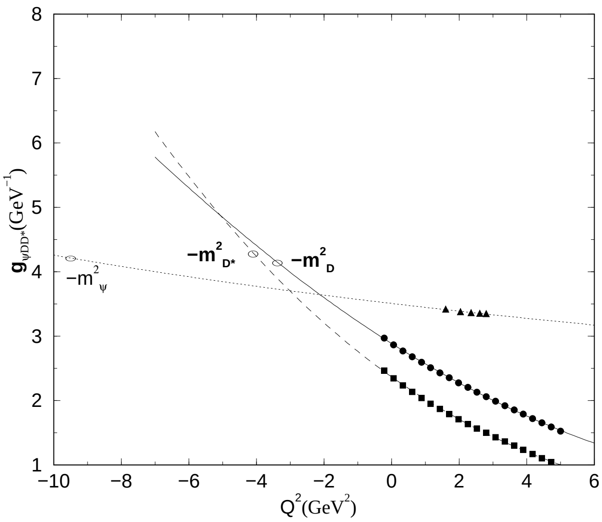

Figure 4. Momentum dependence of the $J/\psi DD^*$ form factors. The dotted, dashed and solid lines give the parameterization of the QCDSR results (triangles, squares and circles) through Eqs. (18), (19) and (20) respectively.

In order to do this extrapolation we fit the QCDSR results with an analytical expression. We tried to fit our results with a mono-pole form, since this is very often used for form factors, but the fit was only good for $g^{(J/\psi)}_{\psi DD^*}$. For $g^{(D)}_{\psi DD^*}$ and $g^{(D^*)}_{\psi DD^*}$ we obtained good fits using a Gaussian form. We get:

$$g^{(J/\psi)}_{\psi DD^*}(Q^2) = \frac{199.2}{Q^2 + 56.8}, \quad (18)$$

$$g^{(D^*)}_{\psi DD^*}(Q^2) = 19.9\text{exp}\left[-\frac{(Q^2+27)^2}{345}\right], \quad (19)$$

$$g^{(D)}_{\psi DD^*}(Q^2) = 12.7\text{exp}\left[-\frac{(Q^2+25.8)^2}{450}\right]. \quad (20)$$

These fits are also shown in Fig. 4 through the dotted, dashed and solid lines respectively. From Fig. 4 we see that all three form factors lead to compatible values for the coupling constant when the form factors are extrapolated to the off-shell meson mass (shown as open circles in Fig. 4). Considering the uncertainties in the continuum threshold, and the difference in the values of the coupling constants extracted when the D, D^* or J/ψ mesons are off-shell, our result for the $J/\psi DD^*$ coupling constant is:

$$g_{\psi DD^*} = (3.48 \pm 0.76)\text{GeV}^{-1}. \quad (21)$$

In Table I we show the results obtained for the same coupling constant using different approaches.in refs. [1] and [5].

Table I: Values of the coupling constant $g_{\psi D D^*}$ in GeV^{-1} evaluated using different approaches.

this work	ref. [1]	ref. [5]
3.48 ± 0.76	8.02 ± 0.62	4.05 ± 0.25

While our result is compatible with the coupling obtained using constituent quark meson model [5], it is half of the value obtained with the vector meson dominance model plus relativistic potential model [1].

To summarize: we have used the method of QCD sum rules to compute form factors and coupling constant in the $J/\psi D D^*$ vertex. Our results for the coupling show once more that this method is robust, yielding numbers which are approximately the same regardless of which particle we choose to be off-shell and depending weakly on the choice of the continuum threshold. As for the form factors, we obtain a harder form factor when the off-shell particle is J/ψ, when compared with the form factors obtained when the off-shell particles are D or D^*.

Acknowledgments

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