Experimental Status of Corrections to Newton's Gravitational Law Inspired by Extra Dimensional Physics

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Corrections to Newton's gravitational law inspired by extra dimensional physics and by the exchange of light and massless elementary particles between the atoms of two macrobodies are considered. These corrections can be described by the potentials of Yukawa-type and by the power-type potentials with different powers. The strongest up to date constraints on the corrections to Newton's gravitational law are reviewed following from the Eötvos- and Cavendish-type experiments and also from the measurements of the Casimir and van der Waals force.

1 Introduction

It is common knowledge that the gravitational interaction is described on a different basis than all the other physical interactions. Up to the present there is no unified description of gravitation and gauge interactions of the Standard Model which would be satisfactory both physically and mathematically. Gravitational interaction persistently avoids unification with the other interactions. In addition, there is an evident lack of experimental data in gravitational physics. Newton's law of gravitation, which is also valid with high precision in the framework of the Einstein General Relativity Theory, is not verified at the separations less than 1 mm. Surprisingly, at the separations less than $1\,\mu$m corrections to the Newton's gravitational law are not excluded experimentally that are many orders of magnitude greater than the Newtonian force itself. What this means is the general belief, that the Newton's law of gravitation is obeyed up to Planckean separation distances, is nothing more than a large scale extrapolation. It is meaningful also that the Newton's gravitational constant G is determined with much less accuracy than the other fundamental physical constants. In spite of all attempts the results of recent experiments on the precision measurement of G are contradictory [1].

Prediction of non-Newtonian corrections to the law of gravitation comes from the extra dimensional unification schemes of High Energy Physics. According to this schemes, which go back to Kaluza[2] and Klein [3], the true dimensionality of physical space is larger than 3 with the extra dimensions being spontaneously compactified at the Planckean length-scale. At the separation distances several times larger than a compactification scale, the Yukawa-type corrections to the Newtonian gravitational potential must arise. This prediction would be of only academic interest if to take account of the extreme smallness of the Planckean length $l_{Pl} = \sqrt{G} \sim 10^{-33}\,$cm (we use units with $\hbar = c = 1$) and the excessively high value of the Planckean energy $M_{Pl} = 1/\sqrt{G} = 10^{19}\,$GeV. Recently, however, the low energy (high compactification length) unification schemes were proposed [4, 5]. In the framework of these schemes the "true", multidimensional, Planckean energy takes a moderate value $M_* = 10^3\,$GeV=1 TeV and the value of a compactification scale belongs to a submillimeter range. It is amply clear that in the same range the Yukawa-type corrections to the Newtonian gravitation are expected [6, 7] and this prediction can be verified experimentally.

Much public attention given to non-Newtonian gravitation is generated not only by the extra dimensional physics. The new long-range forces which can be considered as corrections to the Newton's law of gravitation are produced also by the exchange of light and massless hypothetical elementary particles between the atoms of closely spaced macrobodies. Such particles (like axion, scalar axion, dilaton, graviphoton, moduli, arion etc.) are predicted by many extensions to the Standard Model and practically inavoidable in the modern theory of elementary particles and their interactions [8]. The long-range forces produced due to the exchange of hypothetical particles can be considered as some corrections to the Newton's gravitational law leading to the same phenomenological consequences as in the case of extra spatial dimensions.

In the present report we summarize the best constraints on the corrections to Newton's gravitational law obtained from the recent laboratory experiments (we do not consider the astrophysical constraints or satellite experiments in preparation). The main attention is paid to the gravitational experiments of the Eötvos- and Cavendish-types. The new constraints following from the Casimir and van der Waals force measurements are briefly discussed (see also Ref. [9]). The paper is organized as follows. In Sec. II the types of potentials describing the corrections to Newton's gravitational law are briefly outlined. In Sec. III the best constraints on the parameters of these potentials following from the gravitational experiments of Eötvos- and Cavendish-type are summarized. In Sec. IV the constraints following from the Casimir and van der Waals force measurements are collected. In Sec. V several conclusions are formulated. The laboratory experiments are demonstrated to have the potential for obtaining more strong constraints on the corrections to Newtonian gravitation in near future.

2 Description of the Corrections to Newtonian Gravitation in Terms of Potentials

The usual Newton's law of gravitation is only valid in a 4-dimensional space-time. If the extra dimensions exist, it will be modified by some corrections. In models with large but compact extra dimensions (like those proposed in Ref. [4]) the gravitational potential between two point particles with masses m_1 and m_2 separated by a distance $r \gg R_*$, where R_* is a compactification scale, is given by [6, 7]

$$V(r) = -\frac{Gm_1m_2}{r}\left(1+\alpha_G e^{-r/\lambda}\right). \tag{1}$$

The first term in the right-hand side of Eq. (1) is the Newtonian contribution, whereas the second term represents the Yukawa-type correction. Here G is the Newton's gravitational constant, α_G is a dimensionless constant depending on the nature of extra dimensions and λ is the interaction range of a correction.

At small separation distances $r \ll R_*$ the usual Newton's law of gravitation should be generalized to

$$V(r) = -\frac{G_{4+n}m_1m_2}{r^{n+1}} \tag{2}$$

in order to preserve the continuity of the force lines in a $(4+n)$-dimensional space-time. Here G_{4+n} is the underlying multidimensional gravitational constant connected with the usual one by the relation $G_{4+n} \sim GR_*^n$.

In fact the characteristic energy scale in multidimensional space-time is given by the multidimensional Planckean mass $M_* = 1/G_{4+n}^{1/(2+n)}$, and the compactification scale is given by[4]

$$R_* = \frac{1}{M_*}\left(\frac{M_{Pl}}{M_*}\right)^{2/n} \sim 10^{\frac{32}{n}-17}\,\text{cm}, \tag{3}$$

where $M_{Pl} = 1/\sqrt{G}$ is the usual Planckean mass, and $M_* = 10^3$ GeV as was told in Introduction. Then, for $n = 1$ (one extra dimension) one finds from Eq. (3) $R_* \sim 10^{15}$ cm. If to take into account that, as was shown in Refs. [6, 7], $\alpha_G \sim 10$ and $\lambda \sim R_*$, this possibility must be rejected on the basis of solar system tests of Newton's gravitational law [10]. If, however, $n = 2$ one obtains from Eq. (3) $R_* \sim 1\,$mm, and for $n = 3$ $R_* \sim 5\,$nm. For these scales the corrections of form (1) to Newton's gravitational law are not excluded experimentally.

The other type of multidimensional models considers noncompact but warped extra dimensions. In these models the leading contribution to the gravitational potential is given by[5, 11]

$$U(r) = -\frac{Gm_1m_2}{r}\left(1+\frac{2}{3k^2r^2}\right), \tag{4}$$

where $r \gg 1/k$ and $1/k$ is the so-called warping scale. Here the correction to the Newton's gravitational law depends on the separation distance inverse proportionally to the third power of separation.

As was mentioned in Introduction, many extensions to the Standard Model predict the hypothetical long-range forces, distinct from gravitation and electromagnetism, caused by the exchange of light and massless elementary particles between the atoms of macrobodies. Under appropriate parametrization of the interaction constant these forces also can be considered as some corrections to the Newton's gravitational law. The velocity independent part of the effective potential due to the exchange of hypothetical particles between two atoms can be calculated by means of Feynman rules. For the case of massive particles with mass $\mu = 1/\lambda$ (λ is their Compton wavelength) the effective potential takes the Yukawa form

$$V_{Yu}(r) = -\alpha N_1N_2\frac{1}{r}e^{-r/\lambda}, \tag{5}$$

where $N_{1,2}$ are the numbers of nucleons in the atomic nuclei, α is a dimensionless interaction constant. If to introduce a new constant $\alpha_G = \alpha/(Gm_p^2) \approx 1.7\times10^{38}\alpha$ (m_p being a nucleon mass) and consider the sum of potential (5) and Newton's gravitational potential one returns back to the potential (1).

For the case of exchange of one massless particle the effective potential is just the usual Coulomb potential which is inverse proportional to separation. The effective potentials inverse proportional to higher powers of a separation distance appear if the exchange of even number of pseudoscalar particles is considered. The power-type potentials with higher powers of a separation are obtained also in the exchange of two neutrinos, two goldstinos or other massless fermions [12]. The resulting interaction potential acting between two atoms can be represented in the form[13]

$$U(r) = -\Lambda_l N_1N_2\frac{1}{r}\left(\frac{r_0}{r}\right)^{l-1}, \tag{6}$$

where $r_0 = 1\,\text{F}=10^{-15}\,$m is introduced for the proper dimensionality of potentials with different l, and Λ_l with $l = 1, 2, 3, \ldots$ are the dimensionless constants.

If to introduce a new set of constants $\Lambda_l^G = \Lambda_l/(Gm_p^2)$ and consider the sum of (6) and Newton's gravitational potential one obtains

$$U_l(r) = -\frac{Gm_1m_2}{r}\left[1+\Lambda_l^G\left(\frac{r_0}{r}\right)^{l-1}\right]. \tag{7}$$

This equation represents the power-type hypothetical interaction as a correction to the Newton's gravitational law. The potential (4) following from the extra dimensional physics is obtained from Eq. (7) with $l = 3$. Note that the case $l = 3$ corresponds also to two arions exchange between electrons [12].

3 Constraints from Gravitational Experiments

Constraints on the corrections to Newton's gravitational law can be obtained from the experiments of Eötvos- and Cavendish-type. In the Eötvos-type experiments the difference between inertial and gravitational masses of a body is measured, i.e. the equivalence principle is verified. The existence of an additional hypothetical force which is not proportional to the masses of interacting bodies can lead to the appearance of the effective difference between inertial and gravitational masses. Therefore some constraints on hypothetical interactions emerge from the experiments of Eötvos type.

The typical result of the Eötvos-type experiments is that the relative difference between the accelerations imparted by the Earth, Sun or some laboratory attractor to various substances of the same mass is less than some small number. Many Eötvos-type experiments were performed (see, e.g., Refs. [14-17). By way of example, in Ref. [16] the above relative difference of accelerations was to be less than 10^{-11}.

The results of the most precise Eötvos-type experiments can be found in Refs. [18, 19]. They permit to obtain the best constraints on the constants of hypothetical long-range interactions inspired by extra dimensions or by the exchange of light and massless elementary particles (see Fig. 1).

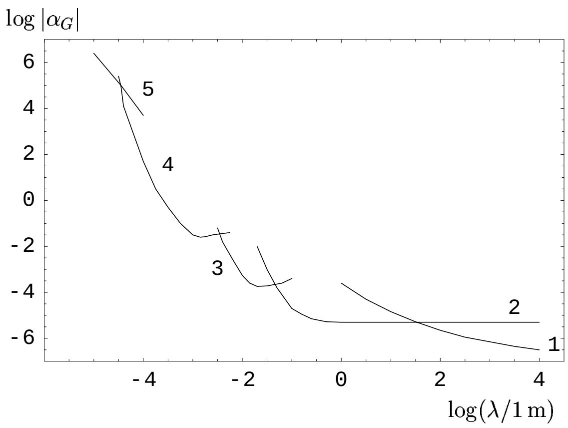

Figure 1. Constraints on the Yukawa-type corrections to Newton's gravitational law. Curves 1, 2 follow from the Eötvos-type experiments, and curves 3, 4 follow from the Cavendish-type experiments. The beginning of curve 5 shows constraints from the measurements of the Casimir force. Permitted regions on $(\lambda,\, \alpha_G)$-plane lie beneath the curves.

The constraints under consideration can be obtained also from the Cavendish-type experiments. In these experiments the deviations of the gravitational force F from Newton's law are measured (see, e.g., Refs. [20-25]). The characteristic value of deviations in the case of two point-like bodies a distance r apart can be described by the parameter

$$\varepsilon = \frac{1}{rF}\frac{d}{dr}\left(r^2 F\right), \tag{8}$$

which is equal exactly to zero in the case of pure Newton's gravitational force. For example, in Refs. [22, 23] $|\varepsilon| \leq 10^{-4}$ at the separation distances $r \sim 10^{-2} - 1\,\mathrm{m}$. This can be used to constrain the size of corrections to the Newton's gravitational law. The results of the most recent Cavendish-type experiment can be found in Ref. [26].

Let us now outline the strongest constraints on the corrections to Newton's gravitational law obtained up to date from the gravitational experiments. The constraints on the parameters of Yukawa-type correction, given by Eq. (1), are presented in Fig. 1. In this figure, the regions of $(\lambda,\, \alpha_G)$-plane above the curves are prohibited by the results of the experiment under consideration, and the regions below the curves are permitted. By the curves 1 and 2 the results of the best Eötvos-type experiments are shown (Refs. [19] and [18], respectively). Curve 4 represents constraints obtained from the Cavendish-type experiment of Ref. [26]. At the intersection of curves 2 and 4 the better constraints are given by curve 3 following from the results of older Cavendish-type experiment of Ref. [25]. As is seen from Fig. 1, rather strong constraints on the Yukawa-type corrections to Newton's gravitational law ($\alpha_G < 10^{-5}$) are obtained only within the interaction range $\lambda > 0.1\,\mathrm{m}$. With decreasing λ the strength of constraints falls off, so that at $\lambda = 0.1\,\mathrm{mm}$ $\alpha_G < 100$. By the beginning of curve 5 the constraints are shown following from the Casimir force measurements (see Sec. IV).

Now we consider constraints on the power-type corrections to Newton's gravitational law given by Eq. (7). The best of them follow from the Eötvos- and Cavendish-type experiments. They are collected in Table 1.

Constraints on the constants of power-type potentials.

l	$\vert\Lambda_l\vert_{\max}$	$\vert\Lambda_l^G\vert_{\max}$	Source
1	6×10^{-48}	1×10^{-9}	Ref. [27]
2	2.4×10^{-30}	4×10^{8}	Ref. [18]
3	7×10^{-17}	1.2×10^{22}	Refs. [25, 28, 29]
4	7.5×10^{-4}	1.3×10^{35}	Refs. [23, 29]
5	1.2×19^{9}	2×10^{47}	Refs. [23, 29]

For $l = 1,\, 2$ the constraints presented in Table 1 are obtained from the Eötvos-type experiments, and for $l = 3,\, 4,\, 5$ from the Cavendish-type ones. It is seen that the strength of constraints falls greatly with the increase of l.

4 Constraints from Casimir and van der Waals Force Measurements

As is seen from Sec. III, for larger interaction distances the best constraints on the corrections to Newton's gravitational law follow from the Eötvos-type experiments and for lesser interaction distances from the Cavendish-type ones. With the further decrease of the characteristic interaction distance the strength of constraints following from the gravitational experiments greatly reduces. Within a micrometer separations, the Casimir and van der Waals force [30-32] becomes the dominant force between two macrobodies. As was shown first in Ref. [33] for the case of Yukawa-type interactions in the interaction range $\lambda < 10^{-4}$ m and in Ref. [34] for the power-type ones, the measurements of the van der Waals and Casimir forces lead to the strongest constraints on non-Newtonian gravity (see the discussion about the Casimir effect as a test for non-Newtonian gravitation in Ref. [35]).

Currently a lot of precision experiments on the measurement of the Casimir and van der Waals force has been performed (see Ref. [36] for a review). Also the extensive theoretical study of different corrections to the Casimir force due to surface roughness, finite conductivity of a boundary metal and nonzero temperature gave the possibility to compute the theoretical value of this force with high precision. At the moment the agreement between theory and experiment at a level of 1% is achieved for the smallest experimental separation distances [36]. This permitted to obtain stronger constraints on the corrections to Newton's gravitational law from the results of the Casimir force measurements [37-44]. Here we briefly present the strongest constraints of this type (see Ref. [9] for more detailed discussion of different experiments and prospects for future).

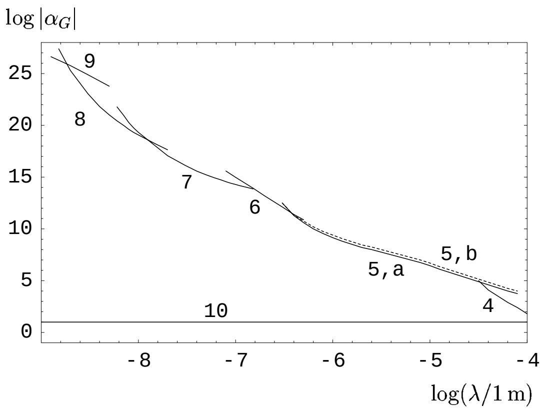

Figure 2. Constraints on the Yukawa-type corrections to Newton's gravitational law. Curves 5–8 follow from the Casimir, and curve 9 from the van der Waals force measurements. The typical prediction of extra dimensional physics is shown by curve 10.

In Fig. 2 the strongest constraints on the Yukawa-type correction to Newton's gravitational law in the interaction range $\lambda < 10^{-4}$ m are presented. This figure is complementary to Fig. 1 and coveres the interaction ranges with smaller λ. The numeration of curves continues Fig. 1. By this means curve 4 is the end of the curve 4 in Fig. 1, and curves 5a,b obtained for $\alpha_G > 0$, respectively, $\alpha_G < 0$ are the continuations of curve 5 in Fig. 1. Curve 6 which bridges a gap between modern experiments follows from old Casimir force measurements between dielectrics[36]. Curve 7 was obtained[42] by the use of the most precision new experiment of Ref. [46]. Curve 9 follows[41] from the experiment of Ref. [47], and curve 9 presents the constraints obtained from old van der Waals force measurements between dielectrics[36]. By a straight line 10 a typical prediction of extra dimensional theories is shown. Remind that for three extra dimensions Eq. (3) gives $R_* \sim 5$ nm, and the interaction range λ is of the same order.

Recently, the new physical phenomenon, the lateral Casimir force, was demonstrated first [48, 49] acting between a sinusoidally corrugated gold plate and large sphere. This force acts in a direction tangential to the corrugated surface. The experimental setup was based on the atomic force microscope specially adapted for the measurement of the lateral Casimir force. The measured force oscillates sinusoidally as a function of the phase difference between the two corrugations in agreement with theory with an amplitude of 3.2×10^{-13} N at a separation distance 221 nm. So small value of force amplitude measured with a resulting absolute error 0.77×10^{-13} N [49] with a 95% confident probability gives the opportunity to obtain constraints on the respective lateral hypothetical force which may act between corrugated surfaces.

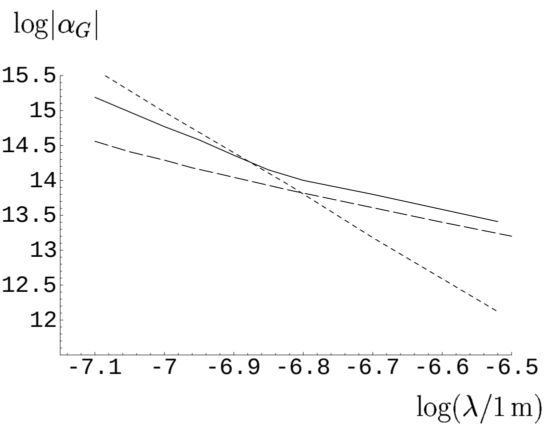

Figure 3. Constraints on the Yukawa-type corrections to Newton's gravitational law from the measurement of the lateral Casimir force between corrugated surfaces (solid curve). For comparison the short-dashed and long-dashed curves reproduce curves 6 and 7 of Fig. 2, respectively, obtained from the meausrements of the normal Casimir force between dielectrics and between gold surfaces.

The obtained constraints [9, 49] are shown in Fig. 3 as the solid curve. In the same figure, the short-dashed curve indicates constraints obtained from the old Casimir force measurements between dielectrics (curve 6 in Fig. 2), and the long-dashed curve follows from the most precision measurement of the normal Casimir force between gold surfaces [46] (these constraints were already shown by curve 7 in Fig. 2). The constraints obtained by means of the lateral Casimir force measurement are of almost the same strength as the ones known previously in the interaction range 80 nm$< \lambda <$150 nm. However, with the increase of accuracy of the lateral Casimir force measurements more promising constraints are expected.

As is seen from Figs. 2, 3, the present strength of constraints is not sufficient to confirm or to reject the predictions of extra dimensional physics with the compactification scale $R_* < 0.1$ mm. However, Fig. 2 gives the possibility to set constraints on the parameters of light hypothetical particles, moduli, for instance. Such particles are predicted in superstring theories and are characterized by the interaction range from one micrometer to one centimeter [50].

5 Conclusions

The above discussion permits to conclude that the laboratory experiments of the Eötvos- and Cavendish-type, and also on measurement of the Casimir and van der Waals force give the possibility to constrain corrections to Newton's gravitational law. Until recent times rather strong constraints were obtained within the interaction range $\lambda > 1$ mm. For smaller λ large work should be done in order to obtain stronger constraints. In this respect the experiments on the Casimir and van der Waals force measurements deserve more attention. So far these experiments were not especially designed to obtain stronger constraints on the corrections to Newton's gravitational law. The obtained strengthening up to 4500 times [42] is only a by-product of the recent Casimir force measurements.

A great deal needs to be done before more strong constraints could be gained from the Casimir force measurements. The most evident suggestion is to use the test bodies of larger size, made of heavier metals at increased separation distance. Also a new dynamical experiment was proposed [35, 42] designed specifically to search for the new forces rather than to test the Casimir force. There is evidently a great potential in the possibility to obtain stronger constraints on the corrections to Newton's gravitational law from the laboratory experiments of different kinds.

Thus, recently the new measurement of the Casimir force was performed by means of a microelectromechanical torsional oscillator [51]. By the results of this experiment the constraints on the Yukawa-type hypothetical interactions were strengthened in more than 10 times [51].

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