Ultrasound Attenuation and Collective Modes in Mixed $d_{x^2-y^2} + id_{xy}$ State of Unconventional Superconductors

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Received on 23 May, 2003.

We discuss two methods used for the study of unconventional superconductors: ultrasound attenuation and collective modes. These two methods, as well as microwave absorption, turn out to be coupled and have become very important now. Within models built by path integration technique we analyze some recent ideas concerning possible realization of the mixture of different d-wave states in high temperature superconductors (HTSC). We specifically consider the mixture of $d_{x^2-y^2}$ and d_{xy} states. We study the collective mode spectrum and the ultrasound attenuation in the mixed state, and show that each of the two methods allows us to distinguish a pure d-wave state from a mixed one. They also allow us to identify the type of pairing and order parameter in unconventional superconductors, including the presence and topology of gap nodes, the magnitude of the gap, and degree of admixture in the mixed state.

1 Introduction

The study of ultrasound attenuation (UA) and collective modes (CM) in unconventional superconductors (USC) has become very important now [1-9]. UA experiments are important to study the topology of nodes of SC gap, structure of order parameter and type of pairing. They also allow estimate of the gap value and the extent of admixture in a possible mixed state [4, 7, 8]. The significance of studying CM arises from their effect on UA and microwave absorption (MWA) experiments, neutron scattering, photoemission and Raman scattering. The large peak in the dynamical spin susceptibility in HTSC arises from a weakly damped spin-density-wave CM. This gives rise to a dip between the sharp low energy peak and the higher binding energy hump in the ARPES spectrum. Also, the CM of amplitude fluctuation of the d-wave gap yields a broad peak above the pair-breaking threshold in the B_{1g} Raman spectrum [7]. The contribution of collective modes to UA and MWA may be substantial. Such compounds as Sr_2RuO_4 show that it will also be necessary in addition to accounting the lattice symmetry and the spin-orbital interaction to take into account the complex topology of the Fermi surface. After decades of search for the collective mode in USC they have now been observed by UA [5] as well as by MWA experiments [4].

2 Collective modes in HTSC under d-pairing

In the path integration method the superconducting state with d-wave pairing is described by the following effective action [1, 2], obtained by integration over fast and slow Fermi-fields:

$$S_{eff} = g^{-1} \sum_{p,ia} c^{\dagger}_{ia}(p) c_{ia}(p) + \frac{1}{2} ln\, det\, \frac{M(c_{ia}, c^{\dagger}_{ia})}{M(c^{(0)}_{ia}, c^{(0)\dagger}_{ia})} \quad (1)$$

where $c^{(0)}_{ia}$ is the condensate value of Bose-fields c_{ia} and $M(c_{ia}, c^{\dagger}_{ia})$ is the 4×4 matrix depending on Bose-fields and parameters of quasi-fermions.

The number of degrees of freedom in the case of d-wave pairing is equal to 10, *i.e*, we must have five complex canonical variables, which can be naturally chosen in the form $c_1 = c_{11}+c_{22}$, $c_2 = c_{11}-c_{22}$, $c_3 = c_{12}+c_{21}$, $c_4 = c_{13}+c_{31}$, $c_5 = c_{23} + c_{32}$. In the canonical variables, the effective action has the form

$$S_{eff} = (2g)^{-1} \sum_{p,j} c^{\dagger}_j(p) c_j(p)(1 + 2\delta_{j1}) + \frac{1}{2} ln\, det\, \frac{M(c^{\dagger}_j, c_j)}{M(c^{(0)\dagger}_j, c^{(0)}_j)}\ , \quad (2)$$

where

$$M_{11} = Z^{-1}[i\omega + \xi - \mu(\vec{H}\vec{\sigma})]\delta_{p1p2}$$

$$
\begin{aligned}
M_{22} &= Z^{-1}[-i\omega + \xi + \mu(H\sigma)]\delta_{p1p2} \\
M_{12} &= (\beta V)^{-1/2}(\frac{15}{32\pi})^{1/2}[c_1(1-3cos^2\theta) + c_2 sin^2\theta cos^2\phi + c_3 sin^2\theta sin2\phi + c_4 sin2\theta cos\phi + c_5 sin2\theta sin\phi] \\
M_{21} &= M_{12}^* \qquad (3)
\end{aligned}
$$

This functional determines all the properties of the model superconducting Fermi system with d-wave pairing. We use this effective action for analyzing the collective mode spectrum in a model of HTSC.

In the first approximation, the spectrum is determined by the quadratic form of the effective action obtained as a result of the shift, $c_{ia}(p) \to c_{ia}(p) + c_{ia}^{(0)}(p)$, in Bose fields by a condensate wave function $c_{ia}^{(0)}(p)$ whose form is determined by the SC phase. The spectrum can be found from the equation $det\ Q = 0$ where Q is the matrix of the quadratic form.

Let us consider the results of calculation of the CM spectrum for the SC phases appearing in the symmetry classification of HTSC. We consider the following states: $d_{x^2-y^2}$, d_{xy}, d_{xz}, d_{yz} and $d_{3z^2-r^2}$. For each SC phase, five high frequency modes were determined as well as five modes whose energies are either zero or small, $\ll \Delta_0$.

The high-frequency modes are as follows (measured in units of Δ_0). For $3z^2 - r^2$: $E_1 = 2.0 - i1.65$, $E_{2,3} = 1.85 - i0.69$, and $E_{4,5} = 1.64 - i0.50$, for $d_{x^2-y^2}$ and d_{xy}: $E_1 = 1.88 - i0.79$, $E_2 = 1.66 - i0.50$, $E_3 = 1.14 - i0.68$, $E_4 = 1.13 - i0.71$, and $E_5 = 1.10 - i0.65$, for d_{xz} and d_{yz}: $E_1 = 1.76 - i1.1$, $E_2 = 1.7 - i0.48$, $E_3 = 1.14 - i0.68$, $E_4 = 1.13 - i0.73$, and $E_5 = 1.04 - i0.83$.

The results on high-frequency modes can be useful in determining the order parameter and the type of pairing in HTSC as well as for interpreting the ultrasound and microwave absorption experiments in these systems. We should note here that collective modes are damped more strongly in case of d-pairing than in case of p-pairing. This is connected with the nodal structure of energy gap. As a rule one has point nodes under p-pairing and lines of nodes under d-pairing.

3 Collective modes in the mixture of two d-wave states

Recent experiments [10] and theoretical considerations [11, 12] suggest that in HTSC the mixture of different d-wave states is realized. We have calculated for the first time the CM spectrum in a mixed $d_{x^2-y^2} + id_{xy}$ state of HTSC. We used the model of d-pairing for superconductors (HTSC, heavy fermions *etc*) created by P. N. Brusov and N. P. Brusova within path integration technique earlier [13]. We have shown that in spite of the spectra in both $d_{x^2-y^2}$ and d_{xy} states being identical the spectrum in the mixture $d_{x^2-y^2}+id_{xy}$ state turns out to be quite different from them. Thus, ultrasound and/or microwave absorption experiments could be used to distinguish the mixture of two d-wave states from pure d-wave states.

While it is widely believed that there is d-wave pairing in oxide superconductors, there is no resolution to whether it is a pure d-wave state or a mixed one. This uncertainty arises because it is not known convincingly whether we have exact zero gap along some chosen lines in momentum space (like the case of $d_{x^2-y^2}$) or the gap is anisotropic but nonzero everywhere (except maybe some points). Existing experiments (tunneling etc.) do not provide a definite answer. There are some experiments [10] which could be explained [11] assuming the realization in HTSC of a mixed states, like $d_{x^2-y^2} + id_{xy}$. Annett *et al.* [12] considered the possibility of mixture of different d-wave states in HTSC and came to the conclusion that $d_{x^2-y^2} + id_{xy}$ is the most likely state. We suggest here one of the possible ways to distinguish the mixture of two d-states from pure d-states. For this we considered the mixed $d_{x^2-y^2} + id_{xy}$ state and calculated the spectrum of collective modes in this state. The comparison of this spectrum with the spectrum of the pure d-wave states of HTSC shows that they are significantly different and could be the probe of the symmetry of the order parameter in HTSC.

We have used the model of d-pairing in HTSC and HFSC, described by Eqs. (2) and (3) and considered the mixed $d_{x^2-y^2} + id_{xy}$ state. The order parameter in this state takes the following form

$$
\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + i \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (4)
$$

and the gap $\Delta(T) = \Delta_0(T) sin^2\theta$. The gap equation has the following form

$$
g^{-1} + \frac{\alpha^2 Z^2}{2\beta V} \sum_p \frac{sin^4\Theta}{\omega^2 + \xi^2 + \Delta_0^2 sin^4\Theta} = 0 \qquad (5)
$$

where $\Delta_0 = 2cZ\alpha$ and $\alpha = (15/32\pi)^{1/2}$.

The spectrum of collective excitations in the first approximation is determined by the quadratic part of S_h, obtained after shift $c_j \to c_j + c_j^0$ where c_j^0 are the condensate values of c_j, which take the following form, $c_j^0(p) = (\beta V)^{1/2} c\delta_{p0} b_j^0$ and $b_2^0 = 2$, $b_3^0 = 2i$ with all remaining components of b_j^0 equal to zero. Excluding terms involving g^{-1} by gap equation, one obtains the following form for the quadratic part of S_h:

$$\begin{aligned} S_{eff} &= \frac{\alpha^2 Z^2}{8\beta V}\sum_p \frac{[c^0Y^*][c^{+0}Y]}{\omega^2+\xi^2+[c^0Y^*][c^{+0}Y]}\sum_j (1+2\delta_{j1})c_j^+(p)c_j(p) \\ &+\frac{Z^2}{4\beta V}\sum_{p1+p2=p}\frac{1}{M_1M_2}\{(i\omega_1+\xi_1)(i\omega_2+\xi_2)\left([c^+(p)Y(p_2)]\right. \\ &[c(p)Y^*(p_1)]+[c^+(p)Y(p_1)][c(p)Y^*(p_2)]\left.\right)-\Delta^2[c^+(p)Y(-p_1)] \\ &[c^+(-p)Y(-p_2)]-\Delta^{+2}[c(p)Y^*(-p_1)][c(-p)Y^*(-p_2)]]\ \} \ . \end{aligned} \quad (6)$$

Here $[cY^*] = c_1(1-3cos^2\theta) + c_2 sin^2\theta cos2\phi + c_3 sin^2\theta sin2\phi + c_4 sin2\theta cos\phi + c_5 sin2\theta sin\phi$. The coefficients of the quadratic form are proportinal to the sums of the products of Greens functions of quasifermions. At low temperature, $T \ll T_c$, we can go from a summation to an integration. After calculating all integrals except over angular variables and equating the determinant of the resulting quadratic form to zero, one gets the following set of equations. These determine the whole spectrum of collective modes for the $d_{x^2-y^2}+id_{xy}$ state. The index i refers to the collective mode branch number.

$$\begin{aligned} &\int_0^1 dx\int d\phi\{\frac{\sqrt{\omega^2+4f}}{\omega}ln\frac{\sqrt{\omega^2+4f}+\omega}{\sqrt{\omega^2+4f}-\omega}g_1+(g_1-\frac{3}{2}f_1)lnf\}=0, \quad i=1 \\ &\int_0^1 dx\int d\phi\{\frac{\omega}{\sqrt{\omega^2+4f}}ln\frac{\sqrt{\omega^2+4f}+\omega}{\sqrt{\omega^2+4f}-\omega}g_1+(g_1-\frac{3}{2}f_1)lnf\}=0, \quad i=1 \\ &\int_0^1 dx\int d\phi\{\frac{\sqrt{\omega^2+4f}}{\omega}ln\frac{\sqrt{\omega^2+4f}+\omega}{\sqrt{\omega^2+4f}-\omega}g_i+(g_i-\frac{1}{2}g)lnf\}=0, \quad i=2,3,4 \\ &\int_0^1 dx\int d\phi\{\frac{\omega}{\sqrt{\omega^2+4f}}ln\frac{\sqrt{\omega^2+4f}+\omega}{\sqrt{\omega^2+4f}-\omega}g_i+(g_i-\frac{1}{2}g)lnf\}=0, \quad i=2,3,4 \end{aligned} \quad (7)$$

Here $g_1 = (1-3x^2)^2$, $g_2 = (1-x^2)^2cos^2 2\phi$, $g_3 = g = 4(1-x^2)x^2cos^2\phi$, $g_4 = 4(1-x^2)x^2sin^2\phi$, $g_5 = (1-x^2)^2sin^2\phi$, $f_1 = (1/4)[(1-3x^2)^2+3(1-x^2)^2cos^2 2\phi]$, $f = (1-x^2)^2$. We have used the substitutions $cos\theta = x$, $\omega = \omega/\Delta_0$.

Solving these equations numerically we have found five high frequency modes in each state obtained from the second equations, while the first one appears to give either Goldstone modes or modes with very low energy, $\sim 0.03\Delta_0 - 0.08\Delta_0$. Below we give the results for high frequency modes (E_i is the energy of the i'th branch). $E_{1,2} = \Delta_0(T)(1.93-i0.41)$, $E_3 = \Delta_0(T)(1.62-i0.75)$, $E_{4,5} = \Delta_0(T)(1.59-i0.83)$. This should be compared with the spectrum in the pure $d_{x^2-y^2}$ and d_{xy} states obtained by us: $E_1 = \Delta_0(T)(1.88-i0.79)$, $E_2 = \Delta_0(T)(1.66-i0.50)$, $E_3 = \Delta_0(T)(1.14-i0.68)$, $E_4 = \Delta_0(T)(1.13-i0.71)$, $E_5 = \Delta_0(T)(1.10-i0.65)$

We see that in spite of the spectra in the parent d states being identical, the spectrum of the mixed state is different. In pure state all modes are non degenerate while in the mixed state two high frequency modes are twofold degenerate. The energies of high frequency modes in the pure state are ranged between $1.1\Delta_0$ and $1.88\Delta_0$, while in the mixed state they are between $1.59\Delta_0$ and $1.93\Delta_0$ (the collective modes have higher frequencies). Note also that the damping of collective modes in pure d-states is more than in the mixed state. It can be easily understood, because in pure states the gap vanishes along chosen lines while in mixed state it vanishes just at two points (poles). The difference between the CM spectrum in pure and mixed d-wave states provide a possibility to probe the symmetry by ultrasound or microwave absorption. Note that while there is no restriction in principle on UA or MWA frequencies, real experiments will typicaly require frequencies $\sim$ tens of GHz to stay in the collisionless regime.

Overall, by study of collective modes we have a chance to address two important issues: (i) Does the gap disappear along some chosen lines? (ii) Do we have a pure or mixed d-wave state in HTSC ?

4 Ultrasound attenuation in mixed state

We have also calculated UA for the mixed $d_{x^2-y^2} + id_{xy}$ state. We have used the following expression [9]:

$$\begin{aligned}\alpha_\lambda(\vec{q},\omega) &= 2\pi M^2 \int \frac{d^3p}{(2\pi)^3}\{ [n_F(E) - n_F(E')][(u^2u'^2 - \frac{\Delta^2}{4EE'})\delta(\omega + E - E') \\ &\quad - (\nu^2\nu'^2 - \frac{\Delta^2}{4EE'})\delta(\omega - E + E')] \\ &\quad + [1 - n_F(E) - n_F(E')][(\nu^2u'^2 + \frac{\Delta^2}{4EE'})\delta(\omega - E - E') \\ &\quad - (u^2\nu'^2 + \frac{\Delta^2}{4EE'})\delta(\omega + E + E')] \} \qquad (8)\end{aligned}$$

Here E, u, ν depend on $\vec{p}$, while E', u', ν' depend on $\vec{p}+\vec{q}$. The last two terms describe pairbreaking processes and under standard UA experimental condition $\omega \ll \Delta$ and they are negligible for states with nonvanishing gap. While all authors consider the first two terms only, the detailed analysis of the role of the pairbreaking processes in anisotropic SC with gap vanishing along some directions, as in case of pure d-wave state, is still lacking. For the mixture of two d-wave states this problem is less relevant since there are just two poles where gap disappears. Because of this we use only the first two terms which describe scattering of phonons by unpaired electrons.

Eqn (8) represents the most general result and is complicated. Approximation $\omega \ll \Delta$ allows some simplification. Evaluating integration with respect to electron momentum we have obtained the ultrasound attenuation coefficient.

The UA in the mixed state is different from UA in the pure state. In pure $d_{x^2-y^2}$ state the UA along nodal direction is linear at low temperature while along antinodal directions UA is qualitatively close to exponential behavior (for 2D models) [6]. We have found for 3d systems that in case of equal admixture of the d_{xy} state (for which gap $\propto sin^2\theta$) the UA has a maximum for $\theta = 0, \pi$, but not along the nodal direction. This occurs because the transfer of phonon momentum to Cooper pair along this direction leads to decay of Cooper pair into initial electrons with momenta along the nodes. This is the cause of additional sound attenuation and this contribution turns out to be significant. Comparison of our results with experiments on UA allow to distinguish mixed state from pure one as it has been noted above for collective modes.

5 Conclusions

Two methods to distinguish mixed d-wave states from pure d-wave states have discussed, namely collective mode with frequency spectrum and ultrasound attenuation. The results on high frequency modes can be useful in determining the order parameter and type of pairing in USC as well as for interpretation of ultrasound and microwave absorption experiments in these systems. The 2D p-wave case has been considered earlier [3, 14, 15], and the 2D ‘clapping’ mode $\omega \sim \sqrt{2}\Delta$ has been obtained by us [2] for the first time.

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