

# Inelastic Coulomb Scattering Rate of a Multisubband Q1D Electron Gas

M. Tavares and G.-Q. Hai

*Instituto de Física de São Carlos,*

*Universidade de São Paulo, 13560-970 São Carlos, SP, Brasil*

Received February 6, 1999

We study, the Coulomb scattering lifetimes of electrons in two coupled quantum wires by calculating the quasiparticle self-energy within a multisubband model of quasi-one-dimensional (Q1D) electron system. We consider two strongly coupled quantum wires with two occupied subbands. The intrasubband and intersubband inelastic scattering rates are calculated for electrons in different subbands. Contributions of the intrasubband, intersubband plasmon excitations, as well as the quasiparticle excitations are investigated. Our results show that the plasmon excitations of the first subband are the most important scattering mechanism for electrons in both subbands.

## I Introduction

Single-particle properties of electrons in quasi-one-dimensional (Q1D) systems have attracted considerable interest. By calculating the quasiparticle renormalization factor  $Z_k$  [1] and the momentum distribution function  $n_k$  around the Fermi surface, Hu and Das Sarma [2] have clarified that a clean Q1D electron system in a semiconductor quantum wire shows the Luttinger liquid behavior, whereas even slightest amount of impurities restores the Fermi surface and the Fermi-liquid behavior remains. Within a one-band model, they calculated the self-energy due to electron-electron Coulomb interactions for an unclean Q1D system by using the leading-order GW dynamical screening approximation.[2, 3] This self-energy gives rise to the inelastic Coulomb scattering rate that plays a fundamental role in measurements of the quasiparticle lifetime in Q1D doped systems. In the inelastic-scattering processes, the quasiparticle loses energy each time it scatters and its lifetime provides information about the different excitation channels associated with energy relaxation.[4, 5]

This paper is devoted to study the collective excitation and the Coulomb inelastic-scattering rate in a multisubband Q1D doped semiconductor. The numerical calculations are applied to two strongly coupled GaAs/AlGaAs quantum wires in which the occupation of higher subbands provides more scattering channels. We show that the intersubband coupling and intersubband excitations can be important in the energy relaxation process.

## II Theoretical formulation

By assuming zero thickness in the  $z$  direction, the subband energies  $E_n$  and the wave functions  $\phi_{k,n}(y)$  are obtained from the numerical solution of the coupled one-dimensional Schrödinger equation in the  $y$  direction. The Coulomb inelastic-scattering rate of an electron in a subband  $n$  with momentum  $k$  is obtained [1] by the imaginary part of the screened exchange self-energy  $\Sigma_n[k, \xi_n(k)]$ , where  $\xi_n(k) = E_n + \hbar^2 k^2 / 2m^* - \mu$  is the single-particle energy with  $\mu$  being the chemical potential and  $m^*$  the electron effective mass in GaAs. At zero temperature [6], the screened exchange self-energy is given by

$$\Sigma_n[k, \xi_n(k)] = \frac{i}{(2\pi)^2} \int dq \int d\omega' \times \sum_{n_1} V_{nn_1n_1n}^s(q, \omega') G_{n_1}^{(0)}(k + q, \xi_n(k) - \omega'), \quad (1)$$

where  $G_{n_1}^{(0)}(k, \omega)$  is the bare Green's function of non-interacting electrons and  $V_{nn_1n_1n}^s(q, \omega')$  is the dynamically screened electron-electron interaction potential. The self-energy  $\Sigma_n[k, \xi_n(k)]$  was written by taking the leading perturbative term in an expansion in the dynamically screened RPA exchange interaction (GW approximation).

Similarly to the one-band model[7], the self-energy in Eq.(1) can be separated into the frequency-independent exchange and the correlation part,  $\Sigma_n[k, \xi_n(k)] = \Sigma_n^{ex}(k) + \Sigma_n^{cor}[k, \xi_n(k)]$ . The exchange

part is given by

$$\Sigma_n^{\varepsilon x}(k) = -\frac{1}{2\pi} \int dq \sum_{n_1} V_{nn_1n_1n}^b(q) f_{n_1}(\xi_{n_1}(k+q)), \quad (2)$$

where  $f_n(\xi_n(k))$  is the Fermi-Dirac distribution function and  $V_{nn_1n_1n}^b(q)$  is the multisubband electron-electron bare interaction. Notice that  $\Sigma_n^{\varepsilon x}(k)$  is real because the bare interaction potential  $V_{nn_1n_1n}^b(q)$  is totally real. Therefore, one only needs to analyze the imaginary part of  $\Sigma_n^{\varepsilon or}[k, \xi_n(k)]$ , since it gives rise to the imaginary part of the self-energy which we are interested in. After some algebra, we find that the Coulomb inelastic-scattering rate for an electron in a subband  $n$  with momentum  $k$  is given by

$$\sigma_n(k) = \sum_{n'} \sigma_{n,n'}(k) = -\text{Im} \Sigma_n^{\varepsilon or}[k, \xi_n(k)],$$

with

$$\sigma_{n,n'}(k) = \frac{1}{2\pi} \int dq \times \{ \text{Im} [V_{nn'n'n}^s(q, \xi_{n'}(k+q) - \xi_n(k))] - V_{nn'n'n}^b(q) \} \times \{ \theta(\xi_n(k) - \xi_{n'}(k+q)) - \theta(-\xi_{n'}(k+q)) \}, \quad (3)$$

where  $\theta(x)$  is the standard step function. In the above equation, the frequency integration has already been carried out, since the bare Green's function  $G_{n_1}^{(0)}$  can be written as a delta function of  $\omega$ .

### III Results and discussions

In this section, we will present some numerical results of  $\sigma_{n,n'}(k)$  within a two-subband model ( $n, n' = 1, 2$ ). In order to understand the inelastic Coulomb scattering processes of the Q1D electron gas, we will firstly analyze the collective excitation dispersion relation in this multisubband system to show the different channels of energy relaxation.

We consider two strongly double symmetric GaAs/AlGaAs quantum wires of widths  $l_{w_1} = l_{w_2} = 150 \text{ \AA}$  and barrier height  $V_0 = 228 \text{ meV}$ . Between them there is a potential barrier of width  $l_b = 10 \text{ \AA}$ . The total density of electrons is taken as  $N_e = 1.0 a_b^{*-1}$ , where  $a_b^* = \kappa \hbar^2 / 2m^* e^2$  is the effective Born radius with  $\kappa$  being the dielectric constant of the static lattice and  $e$  the electron charge. In such a system, the electron densities in the first ( $n = 1$ ) and the second ( $n = 2$ ) subbands are  $0.73 a_b^{*-1}$  and  $0.27 a_b^{*-1}$ , respectively. Fig. 1 shows the collective excitation dispersion relation of the Q1D electron gas. We find two intrasubband collective excitation modes indicated by (1, 1) and (2, 2) and two intersubband modes indicated by

(1, 2) and (1, 2)'. The intrasubband plasmon dispersions are approximately linear in the long-wavelength limit, while the intersubband ones have finite energy values at  $q = 0$ . The low-energy intrasubband plasmon is mainly due to the second subband, while the high-energy one is mainly due to the first subband. We also see a quite large depolarization shift of the intersubband plasmon mode (1, 2). The shadow areas indicated by  $QPE_{nn'}$  present the quasiparticle excitation regions which can result in Landau damping of the collective excitation modes. Notice that, due to the symmetry of the system, the intrasubband and intersubband modes do not couple to each other in such a way that the intersubband quasiparticle excitations do not damp the intrasubband plasmon modes and vice versa.[8, 9] The existence of two undamped intersubband collective modes is a particular feature of the Q1D system with two occupied subbands. The occupation of the second subband opens up a gap inside the inter-subband quasiparticle excitations  $QPE_{12}$  region where the intersubband mode (1, 2)' appears.

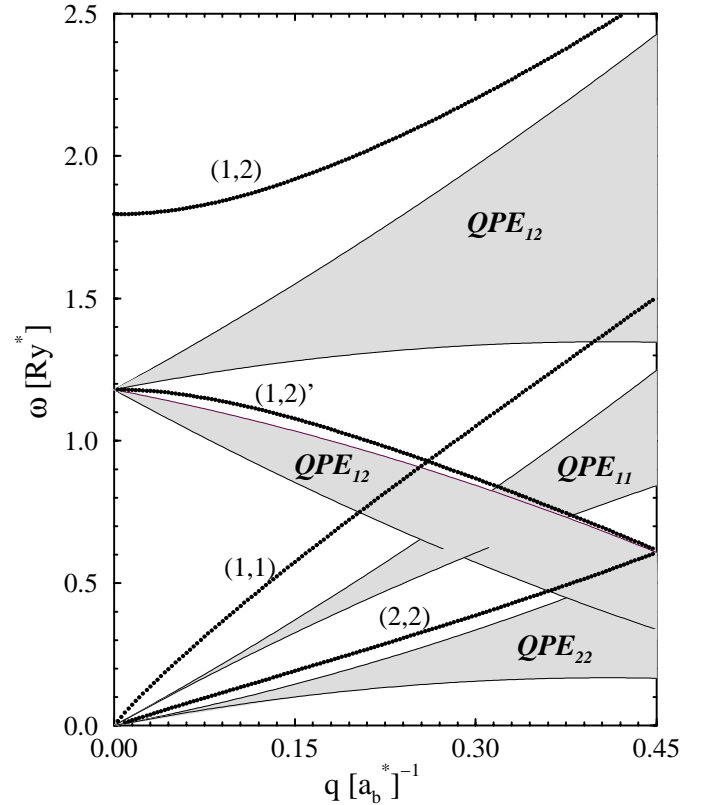


Figure 1. The multisubband collective excitation spectra for two strongly coupled symmetric GaAs/Al<sub>0.3</sub>Ga<sub>0.7</sub>As quantum wires of widths  $l_{w_1} = l_{w_2} = 150 \text{ \AA}$  and barrier height  $V_0 = 228 \text{ meV}$ . The two quantum wires are separated by a barrier of  $l_b = 10 \text{ \AA}$ . The electron density is  $N_e = 1.0 a_b^{*-1}$ . The plasmon dispersions (dotted lines) are indicated by  $(n, n')$ . The shadow areas present the quasiparticle excitation regions indicated by  $QPE_{nn'}$ .

In Fig. 2, we show the Coulomb inelastic-scattering rate  $\sigma_{nn'}(k)$  as a function of the electron wave vector  $k$  in this two subband coupled quantum wire system. The parameters of the system are the same as in Fig. (1). The solid curves denote the intrasubband scattering rate for which the initial and final states of the electron are in the same subband. The dotted ones correspond to the intersubband scattering rate. Fig. 2(a) shows the scattering rate for an electron initially in the first conduction subband. The numerical results show that the intrasubband scattering rate  $\sigma_{nn}(k) = 0$  at  $k = k_{F1}$  and  $k_{F2}$  ( $k_{F1} = 1.15a_b^{*-1}$  and  $k_{F2} = 0.42a_b^{*-1}$ ) as it should be. The scattering rate  $\sigma_{11}(k)$  presents a very pronounced peak at about  $k = 1.9 a_b^{*-1} > k_{F1}$  corresponding to the contribution from the plasmon excitation of the first subband. Such a contribution opens up a channel through which the electron energy can relax by emitting one plasmon. This strong peak is due to the onset of the scattering of the plasmon mode (1, 1), restricted by the conservation of energy and momentum, joining up with the divergency of the density of states at the bottom of the 1D subband. We also observe the contributions from the quasiparticle excitation of the second subband at small wave vectors ( $k < k_{F2}$ ), and from the plasmon excitation of the second subband, peaked at  $k \simeq 0.5a_b^{*-1}$ . The intersubband scattering rate  $\sigma_{12}(k)$  (dotted line) has contributions from the intersubband plasmon excitation (high-energy), at  $k \simeq 3.35 a_b^{*-1}$ , and from quasiparticle intersubband excitation (region  $QPE_{12}$ ), at small values of wave vectors. The scattering rate  $\sigma_{12}(k)$  is much smaller than  $\sigma_{11}(k)$  because the corresponding intersubband transition is from lower to higher subband and, moreover, intersubband plasmon has higher energy. The sum of the two curves in Fig. 2(a) yields the total scattering rate for an electron in the first subband. Fig. 2(b) shows the scattering rate for electrons in the second subband. The intrasubband scattering rate  $\sigma_{22}(k)$  (solid curve) also shows a pronounced peak at  $k \simeq 1.9 a_b^{*-1}$  corresponding to the scattering of the intrasubband plasmon mode (1, 1), while the scattering from the plasmon mode (2, 2) of the second subband only leads to a small peak at  $k \simeq 0.7 a_b^{*-1}$ . There is a small shoulder between the first and the second subband plasmon excitation. This shoulder denotes the contribution of the intrasubband quasiparticle excitation (region  $QPE_{11}$ ) to the  $\sigma_{22}(k)$ . The strong scattering of the plasmon (1, 1) of the first subband to the  $\sigma_{22}(k)$  indicates a strong

coupling of the in-plane motion of the collective excitations from the different subbands. From  $\sigma_{22}(k)$  and  $\sigma_{11}(k)$ , we also observe a relatively large contribution from  $QPE_{11}$  and  $QPE_{22}$ , respectively. This is because the virtue intersubband scattering (coupling) makes the energy-momentum conservation less restrictive in the scattering processes. The dotted curve in Fig. 2(b) shows the contribution from the intersubband plasmon excitation. One sees that the intersubband scattering  $\sigma_{21}(k)$  has a peak located at lower momentum  $k$  comparing to that of  $\sigma_{12}(k)$ . In the scattering process from the second to the first subband, the electron transfer its potential energy to kinetic energy which facilitates the scattering.

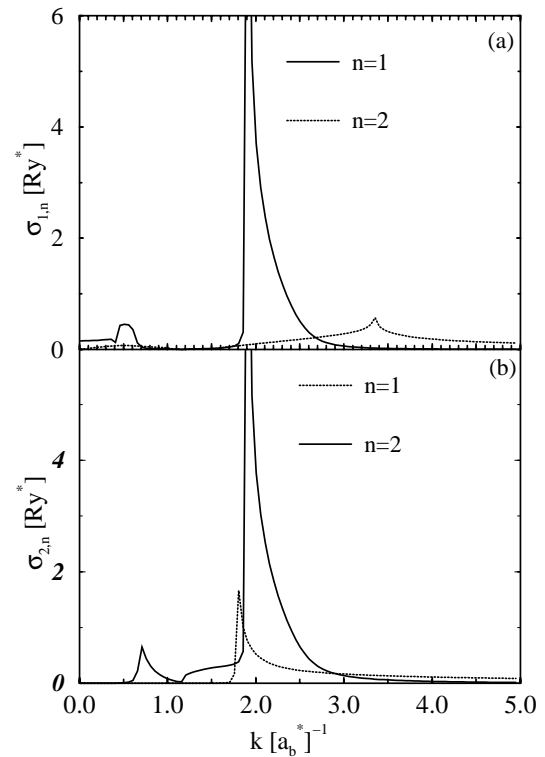


Figure 2. The Coulomb inelastic-scattering rates in the coupled quantum wires for an electron initially in (a) the first subband and (b) the second subband. Solid (dotted) curves give the intrasubband (intersubband) scattering rates. The parameters are the same as in Fig.(1).

Our results show that the plasmon excitations of the first subband are the most important scattering mechanism for electrons in both subbands. This is so because: (i) the strong intersubband coupling and (ii) there are more occupied states in the first subband on which the self-energy effects are stronger and the scattering becomes more pronounced.

## IV Summary

In summary, we have calculated the multisubband collective excitations and the Coulomb inelastic-scattering rate of a quasi-one-dimensional electron gas in two strongly coupled quantum wires with two occupied subbands. We have analyzed the different modes of collective excitations and the corresponding scattering channels through which the electrons, in the different subbands, lose their energies. The role of each excitation mode in the inelastic-scattering relaxation process is clarified. We find that the plasmons in the lowest subband are responsible for the most important scattering channel even for the electrons in higher subbands. The intersubband plasmon modes open up more scattering channels, but their scattering rates are about one order of magnitude smaller than that of the main scattering mechanism.

## Acknowledgments

This work is supported by FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo) and CNPq

(Conselho Nacional de Desenvolvimento Científico e Tecnológico).

## References

- [1] For a review, see G. D. Mahan, *Many-Particle Physics*, Plenum, New York, (1981).
- [2] Ben Yu-Kuang Hu and S. Das Sarma, *Phys. Rev. Lett.* **68**, 1750 (1992); *Phys. Rev. B* **48**, 5469 (1993).
- [3] J.J. Quinn and R. A. Ferrel, *Phys. Rev.* **112**, 812 (1958).
- [4] L. Zheng and S. Das Sarma, *Phys. Rev. B* **54**, 13908 (1996).
- [5] C. R. Bennett and B. Tanatar, *Phys. Rev.* **55**, 7165 (1997).
- [6] B. Vinter, *Phys. Rev. B* **15**, 3947 (1977).
- [7] E.H. Hwang and S. Das Sarma, *Superlattices and Microstructures* **21**, 1 (1997); *Phys. Rev. B* **52**, R8668 (1995).
- [8] M. Tavares and G.-Q. Hai, unpublished.
- [9] G.-Q. Hai, N. Studart, and G.E. Marques, *Phys. Rev. B* **55**, 1554 (1997).