Rayleigh Resolution Criterion for Light Sources of Different Spectral Composition*

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The results of very simple experiments to evaluate Lord Rayleigh Resolution Criterion validity are discussed in cases of quasimonochromatic sources of small angular dimensions (LEDs) and monochromatic sources (Lasers), the emissions of which have different or equal spectral compositions. Visual observations as well as color photographs and color video recording were utilized in the experiments. When LEDs and lasers of different color were used, better resolutions than those of Rayleigh Criterion were obtained owing to the non-spectral yellow false color resulting from the overlapping of the red and green spectral colors. Therefore, the observation of the non-spectral false color implies the super-resolution process. The consideration of the non-spectral false color is a new approach in super-resolution studies. Finally, an illumination and reading system of high density CD-ROMs (9 GB) based on the obtained results is suggested.

I. Introduction

In 1879, John William Strutt (Third Baron Rayleigh) investigated the limit of resolution (resolving or separating power) of some optical instruments, such as telescopes and spectroscopes [1]. As the telescope pupil is circular, the image of a star appears as a bright disk surrounded by less intensity rings. This irradiance distribution corresponds to the diffraction pattern of a circular pupil or Airy distribution:

$$I(\theta) = I(0)[2J_1(k \cdot a \cdot \sin\theta)/k \cdot a \cdot \sin\theta]^2,$$

in which $k = 2\pi/\lambda$, where λ is the light wavelength and a the pupil radius. In such case, the angular radius θ of the bright disk, is equal to:

$$\theta = 1.2197\lambda/2a \approx 1.22\lambda/2a.$$

Lord Rayleigh [1] observed that the images produced by two stars emitting in the same spectral region will have their diffraction bright disks in contact when the stars angular separation is equal to 2θ, in which case there will be no doubt that they are resolved (unless their magnitudes are notably different). This diffraction central disks condition of contact is known as the **null first derivative**, in so far as in such situation the

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Airy distribution first zero of one of the images coincides with the central peak of the other. In spite of this interesting result, Lord Rayleigh commented that this rule is convenient on account of its simplicity, and it is sufficiently accurate in view of the necessary uncertainty as to what exactly is meant by resolution. And he added that perhaps in practice somewhat more favourable conditions are necessary to secure a resolution that would be thought satisfactory. In this respect, it can be affirmed that the Sparrow resolution criterion represents an improvement. The Sparrow criterion is based on the **null second derivative** condition to establish that both images of two point light sources affected by diffraction are resolved.

II. Resolution and spectral content

Lord Rayleigh himself stated other factors to be taken into account when analysing the optical signals resolution. Among other factors, he pointed out the importance of color to study the diffraction gratings resolution and the use of color filters to that purpose [2]. He also discussed the influence of lenses and prisms aberrations on the resolution [1] and also the quality of the optical surfaces [2]. In all these cases, such studies are related to light propagation through optical instruments in which the limiting pupils are constituted by the rectangular apertures of the dispersive elements: prisms and diffraction gratings. Besides, the light considered in these cases arises from sources of longitudinal geometries, such as the entrance slits of spectroscopes, spectrographs and monochromators. This study deals with the case of circular sources of small angular diameter observed through pupils which are also circular, such as those of the eyes and the photo and video cameras. It is true that the iris type mechanisms that determine the edges of pupils from polygonals do not produce in general a circular aperture. The effect of this difficulty was pointed out by Sommerfeld [3], who explained the presence of pairs of light diffraction fans, as well as the disguising of other diffractive effects by them. Likewise, Gu and Gan [4] recently studied the effect produced by the serrated or rugose edge of a circular pupil on the diffraction pattern.

They found out that relations of the order of 5% between the rugosity and the radius of the pupil are enough to generate notable perturbations in the diffraction pattern.

Light emitting diodes (LED) of 3 rnm in diameter and in end-on observation were used in the experiments as light sources. Toshiba LEDs emitting in the green and red regions of the visible spectrum were used. These emissions are quasimonochromatic ($\Delta\lambda/\lambda << 1$) and their spectra, observed in the laboratory, are shown in Fig. 1. It is clearly noticed that the spectral content in the intermediate region corresponding to the yellow color is extremely low. The total irradiance for point light sources affected by diffraction through a circular aperture (Airy distribution) was calculated under Rayleigh conditions using the 560 nm (Green) and 650 nm (Red) wavelength values corresponding to the maxima of the LEDs spectra. The results are shown in Fig. 2. It must be mentioned that the Rayleigh condition acquires now a degeneration as there are two ways of making the maxima and the first minima of the respective Airy distribution curves coincide, since this criterion is based on the coincidence of the **null first derivatives.** In Fig. 2 - with the purpose of simplifying the representation and without implying too much loss of rigorousness - the graphics of the irradiance Airy patterns are represented without considering that the emissions are quasimonochromatic. A treatment dealing with such an aspect is being carried out following already known contributions [5]. Thus, as the angular radius of the Airy disk is proportional to θ, the red disk diameter will be larger than the green one. Then, if the green Airy pattern maximum and the red Airy pattern first minimum (Fig. 2, a) overlap, the angular distance between the sources is equal to the angular radius θ of the red Airy disk. Likewise, the brightness corresponding to the minimum between both disks is equivalent to 0,6144, i.e. it is lower than the value of 0,7350 corresponding to two point light sources of equal wavelength and brightness [6]. Its spatial resolu-

tion is thus secured. In the opposite case (Fig. 2, b) the maximum of the red Airy pattern overlaps the minimum of the green Airy pattern, so the angular distance between the sources is less than θ and the brightness between them reaches a minimum of 0,8498, being this value higher than the pertaining to two sources of equal wavelength and brightness. This implies, *a priori*, that the sources could not be distinguished at all. However, the easy visual observation and the photographic and videographic recordings of the non-spectral yellow false color produced by the overlapping of the red and green allow to secure the resolution.

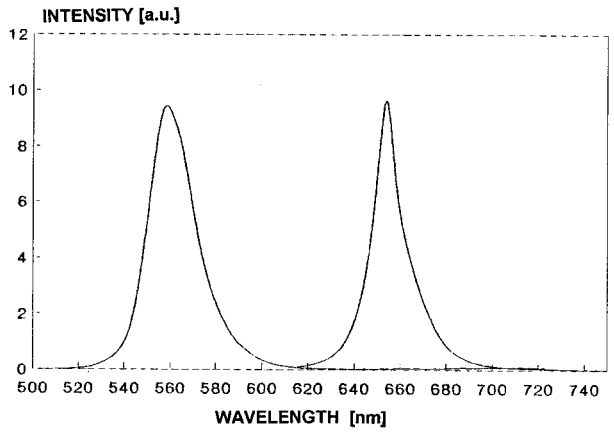

Figure 1. Emission spectra of green (left) and red (right) LEDs. Their irradiances are normalized and expressed in arbitrary units.

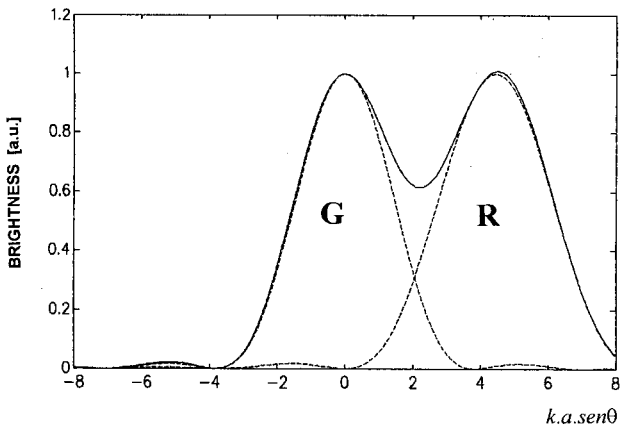

Figure 2. a) Null first derivative condition of Rayleigh Criterion taking into account the central peak of the green Airy pattern and the first zero of the red Airy pattern.

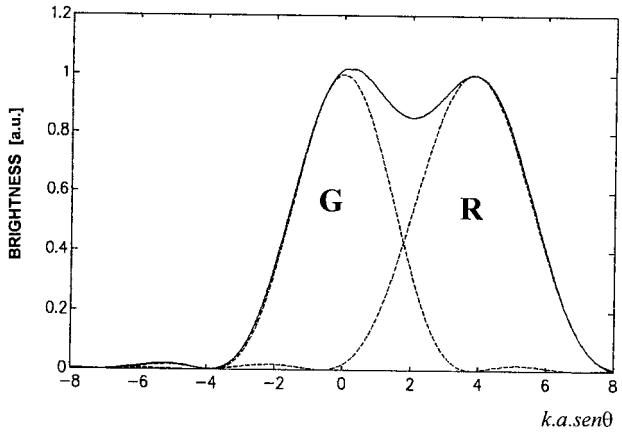

Figure 2. b) Null first derivative condition of Rayleigh Criterion taking into account the central peak of the red Airy pattern and the first zero of the green Airy pattern.

III. Study of quasimonochromatic discoidal light sources

The possibility of raising the resolving power of optical instruments (eyes, photographic and television cameras, telescopes, microscopes and projectors) by the non-spectral yellow false color was confirmed when the red and green LEDs were observed with the naked eye as well as photographically. As the visual observation corresponds to the photographic one, only the latter results will be described. Both green and red LEDs were mounted on two orifices performed on a blackened metallic plate, their centers separated 3 mm. They were observed at distances of 2 m, 4 m, and 8 m perpendicularly to the plate. The LEDs electric excitation was controlled by measuring their irradiance with a 1970 PR Spectra luminancemeter of the Pritchard type. An FG-20 Nikon camera with a Nikkor f = 135 mm teleobjective lens was used when photographing them. Kodak films of 400 ASA Elite (Color) and 100 ASA (Black and white) were utilized. With an exposition of tens of minutes, only the red or green LEDs were photographed, one by one or in pairs, at the different distances. With an exposition of fractions of seconds and using films for color slides, only the central peaks of the diffraction patterns, the dimensions of which sometimes correspond to the LEDs geometrical projection, were recorded. Once the red and green LEDs were photographed at the three distances, the films were developed in the laboratory. Image enlargements to 68 X obtained by microscopy are shown in Fig. 3, together with the dimensional

description of the experiment. The enlargements were obtained using a PH Neofluor IM 35 Carl Zeiss microscope. Although the original images of the color slides are equal in size, the same does not occur with the microscopic images shown in Fig. 3, owing to the spectral content of the microscope illumination lamp. In the upper part of Fig. 3, the images of one and a pair of red LEDs (in black and white) can be compared. It is evident that at a distance of 8 m, it cannot be distinguished whether it is a photograph of one or two LEDs. In the case of the red and green LEDs images, the non-spectral yellow false color allows the two sources to be easily distinguished. The electronically processed images obtained from the microscopic results of the color slides of a green and a red LED photograph are shown in the lower part of Fig. 3. (The impossibility of reproducing colors in this paper reduces the direct observation of beautiful original color images in Figs. 3, 4 and 5). The red and green LEDs and their images at a distance of 2 m, 4 m and 8 m enlarged by microscopy are represented in light and dark grey respectively on a black background. The non-spectral yellow false color represented in white becomes visible at four meters.

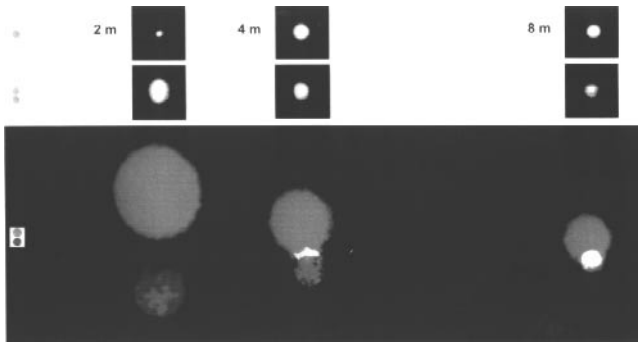

Figure 3. In the upper part, the images of one and a pair of red LEDs can be compared at 2 m, 4 m and 8 m. It is evident that at a distance of 8 m, it cannot be distinguished whether it is a photograph of one or two LEDs. In the case of the red and green LEDs images, the non-spectral yellow false color allows the two sources to be easily distinguished. The electronically processed images obtained from the microscopic results of the color slides of a green and a red LED photograph are shown in the lower part. The red and green LEDs and their images at a distance of 2 m, 4 m and 8 m enlarged by microscopy are represented in light and dark grey respectively on a black background. The non-spectral yellow false color represented in white becomes visible at four meters.

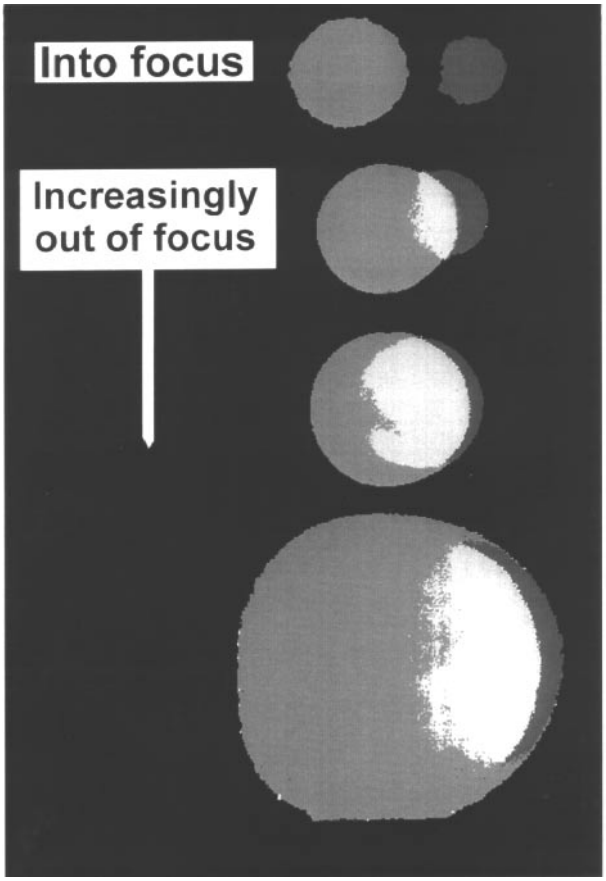

Figure 4. Slides obtained in the experiment were observed microscopically introducing an incorrect focusing in the operation. The original image used corresponds to the one taken at a distance of 2 m as shown in Fig. 3. A sequence of four photographs at 68 X, the first one in focus and the others progressively out of focus, is shown. As in Fig. 3, the original color images were reproduced in a grey scale by means of an electronic process. The red and green LEDs are perfectly separated in the in focus microscope image, while in the out of focus images, the non-spectral yellow false color allows distinguishing the two light sources.

Likewise, the slides obtained in the experiment were observed microscopically introducing an incorrect focusing in the operation. The original image used is the one taken at a distance of 2 m shown in Fig. 3. A sequence of four photographs at 68 X, the first one **in focus** and the others progressively **out of focus,** is shown in Fig. 4. In this Fig. 4 as in Fig. 3, the original color images were reproduced by means of an electronic process. The red and green LEDs are perfectly separated in the **in focus** microscope image, while in the out of focus images, the non-spectral yellow false color allows distinguishing the two light sources.

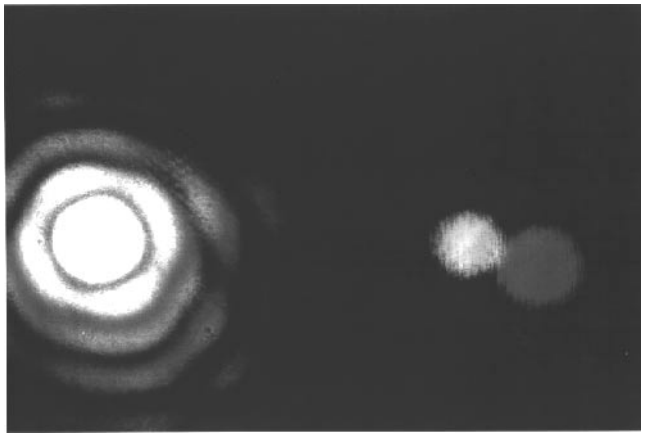

Figure 5. The non-spectral yellow false color Airy pattern is observed in the diffraction zero order (Left). This Airy diffraction pattern seems to be produced by a circular pupil illuminated by yellow light. However, in the diffraction first order (Right), the Airy patterns of each of the spectral components -red and green- are clearly resolved. As in Fig. 3, the original color images were reproduced in a grey scale by means of an electronic process.

IV. Conclusions, projects and proposal

The non-spectral false color observation makes it possible to resolve light sources beyond the limit of resolution established by the Rayleigh Criterion, i.e. the super-resolution observation is achieved. Experiments can be performed to allow the visualization of multijunction LEDs and a collection of bounded LEDs of different color emissions, the observation of biological specimens stained by many fluorocromes and of electrophoretogram with the overlapping of different color bands, the detection of images of stars with different spectral distributions and the recording of atmospheric disturbances due to the overlapping of light from sources of different spectral content can be mentioned among other cases.

It should be necessary to study the photographic and videographic materials used and the processing of them, as well as the reproduction systems. Experience has demonstrated that small variation in the original characteristics of materials or processes produce considerable variation in the non-spectral false color, disguising the real relative contribution of the original spectral colors. Thus, the psychophysical analysis of the non-spectral false color is still useful to recognize such contributions. An observer without training is able to distinguish easily up to seven non-spectral false colors between red and green, while one with training is able to distinguish up to thirteen colors.

When the image corresponding to a luminous object that contains a non-spectral false color is analysed with a spectral apparatus, the images of its original spectral components are observed. In fact, a reproduction of the spectral analysis by diffraction of the non-spectral false color signal is shown in Fig. 5. The non-spectral yellow false color Airy pattern is observed in the diffraction zero order. This Airy diffraction pattern seems to be produced by a circular pupil illuminated by yellow light. However, in the diffraction first order, the Airy patterns of each of the spectral components -red and green- are clearly resolved.

This experiment supports a new proposal to illuminate high density CD - ROMs (9 GB) based on the overlapping technology of two medium density CD-ROMs (4,5 GB). Presently, the proposed solution to illuminate high density CD-ROMs [7] is conceived by simply doubling the illumination and reading optical system, using one of them for each side of the CD-ROM. However, it is possible to conceive a design utilizing only one optical system illuminated by two LEDs or by two LDs lasers emitting in the red and green. The red and green light beams will overlap by means of a dichroic beam combiner and the focusing of both beams on the recorded marks in the first and second CD-ROM reflection layers will be performed by a single optical system. However, special care should be taken in the selection of materials to build-up this optical system. This must be the case because the focus difference produced by the material chromatic aberration will allow the red beam to be focused on the recorded marks of one side of the CD-ROM, while the green beam will be focused on the recorded marks of the other side. Then, the red and green light signals reflected on both sides of the high density CD-ROM will be aligned as it occurs at the zero diffraction order in the experimental result shown in Fig. 5. However, as it was demonstrated in this experiment, both spectral signals were resolved at the first diffraction order. Nevertheless, to analyse the combined red and green light reflected at both CD-ROM layers the diffraction grating can be replaced by other type of spectral analyser device, it is proposed to use a dichroic beamsplitter. So, the spectral and spatial unresolved red and green signals will be sepa-

rated by the dichroic beamsplitter and focused on the respective detectors by the single optical system. Fig. 6 is a simplified diagram of the proposed optical system to illuminate the recorded marks on both sides of high density CD-ROMs and to read the recorded signals by using a single optical system.

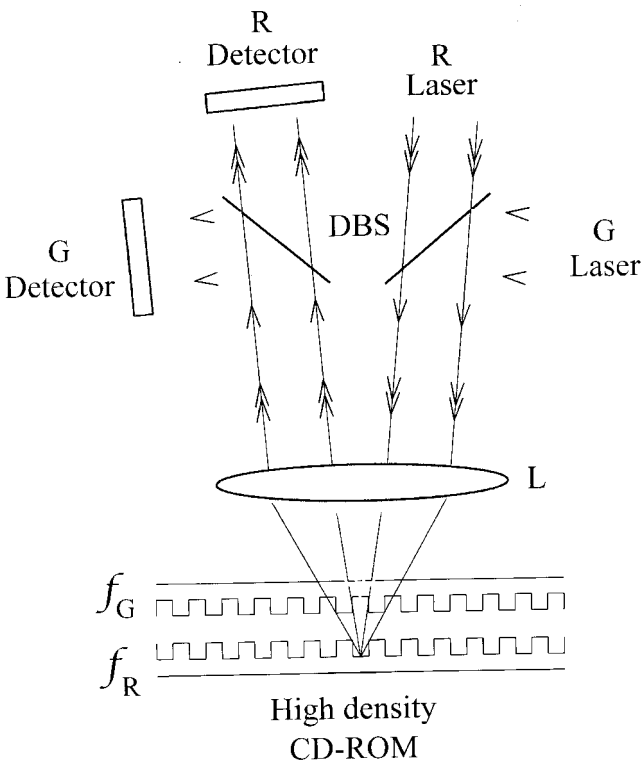

Figure 6. Simplified diagram of the proposed optical system to illuminate the grooves of both sides of high density CD-ROMs and to read the recorded signals. The red (R) and green (G) light beams will overlap by means of a dichroic beamsplitter (DBS) and the focusing of both beams on the recorded marks at each side of the high density CD-ROM will be performed by a single optical system with larger focal length in the red than in the green spectral bands.

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