Electrical Current Resonance and Current Echoes: A Proposed new Experimental Technique in Solid State Physics

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The similarity between the electron equation of motion in electric and magnetic fields and the Bloch equations for nuclear magnetic moments in a magnetic field leads to a novel resonance phenomenon involving the electrical current. This suggests a new experimental technique for the investigation of transport properties in conducting materials. In this work we propose a new set of Bloch Equations for the electrical current with different electron-electron and electron-lattice scattering rates and discuss the consequences of their solutions to the study of the electronic properties of conducting materials. We outline the experimental conditions for detecting electrical current resonance and estimate some relevant experimental parameters for the observation of *current echoes* and *free current decays*.

I. Introduction

The dynamics of the magnetization in the presence of an applied magnetic field $\mathbf{B}$ can be described classically through the Bloch equations [1]. These equations are used in the description of several aspects of the phenomenon of Nuclear Magnetic Resonance (NMR); they predict, for example, the formation of spin echoes and free induction decays [1].

In a previous paper we showed that similar phenomena which we have called *current echoes* and *free current decays*, can exist in conducting media [2]. Its deduction, given in details on that reference, follows in a straightforward way from the formalism developed by Jaynes [3] to solve the Bloch equations. Actually, the existence of current echoes can be seen in a very simple way, just by comparing the Bloch Equations for the nuclear magnetization $\mathbf{M}$ in a field $\mathbf{B}$ with that for the electrical current $\mathbf{J}$ in electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$:

$$\frac{\partial \mathbf{M}}{\partial t} + \frac{\mathbf{M} - \mathbf{M}_o}{T_1} + \gamma_n(\mathbf{B} \times \mathbf{M}) = 0 \qquad (1a)$$

$$\frac{\partial \mathbf{J}}{\partial t} + \frac{\mathbf{J} - \mathbf{J}_o}{\tau} + \gamma_e(\mathbf{B} \times \mathbf{J}) = 0 \qquad (1b)$$

Here, $\mathbf{M}_o = (\chi/\mu_o)\mathbf{B}$ is the nuclear equilibrium magnetization, γ_n the nuclear gyromagnetic ratio, T_1 the nuclear relaxation time. For the sake of simplicity we have taken the spin-lattice (T_1) and spin-spin (T_2) relaxation times as being the same. Similarly, $\mathbf{J}_o = \sigma_o \mathbf{E}$ is the steady state electrical current density, $|\gamma_e| = e/m \approx$ 17.6 GHz kGauss^{-1} is the electronic analog of the "gyromagnetic ratio" for free-electrons and τ the electronic relaxation time. The other symbols have their usual meaning. In both cases the relaxation time is linked to fundamental mechanisms of interactions between the nuclear magnetic moments (or, for instance, electrons in a conduction band) among themselves or with the lattice.

The similarity between these two equations is obvious. Consequently, for a certain configuration of fields, if spin echoes and free induction decays (FID) follow from the solutions of the first equation, current echoes and *free current decays* (FCD) must follow in the same way from the second. However, whereas spin echoes and FID's have been observed since 1950 [4], current echoes have not yet been detected.

Current echoes and FCD's arise when static electric and magnetic fields are applied along the same direction (z, for instance) and an oscillating pulsed magnetic field is applied in a perpendicular direction (x, for instance). In ref. [2] it is shown that at the resonance the

FCD signal amplitude after a pulse of duration τ_p and amplitude B_1 is given by

$$J_y = \sigma_o E_o sin\left(\frac{eB_1\tau_p}{m}\right) \tag{2}$$

and the current echo amplitude after 2 pulses with the same duration is:

$$J_y = \sigma_o E_o sin(\frac{eB_1\tau_p}{m}) sin^2\left(\frac{eB_1\tau_p}{2m}\right) \tag{3}$$

In these equations, E_o is the modulus of the static electric field. Note that (2) and (3) represent the signals in the rotating reference frame [1].

II. Steady state solution

If the oscillating field is applied continuously, it can be shown [2] that the stationary solution for $\mathbf{J}$ is given by (with $\mathbf{J}(0) = 0$):

$$\mathbf{J}(\infty) = \frac{\sigma}{\tau}\left(\frac{1}{\tau} + \tilde{\beta}\right)^{-1}\mathbf{E} \tag{4}$$

where $1/\tau + \tilde{\beta}$ is a matrix given [3] by:

$$\begin{pmatrix} 1/\tau & -\gamma B_z & \gamma B_y \\ \gamma B_z & 1/\tau & -\gamma B_x \\ -\gamma B_y & \gamma B_x & 1/\tau \end{pmatrix} \tag{5}$$

The inverse of the above matrix can be easily calculated:

$$\left(\frac{1}{\tau} + \tilde{\beta}\right)^{-1} = \frac{\tau^3}{1+\gamma^2\tau^2(B_x^2+B_y^2+B_z^2)}\times$$

$$\times\begin{pmatrix} 1/\tau^2+\gamma^2B_x^2 & -(\gamma B_z/\tau+\gamma^2B_xB_y) & -(\gamma^2B_xB_z+\gamma B_y/\tau) \\ -(\gamma^2B_xB_y-\gamma B_z/\tau) & 1/\tau^2+\gamma^2B_y^2 & -(\gamma^2B_yB_z+\gamma B_x/\tau) \\ \gamma^2B_xB_z+\gamma B_y/\tau & -(\gamma^2B_yB_z-\gamma B_x/\tau) & 1/\tau^2+\gamma^2B_z^2 \end{pmatrix} \tag{6}$$

From (4) and (6) we can easily work out the components of $\mathbf{J}$ for an arbitrary configuration of fields $\mathbf{B}$ and $\mathbf{E}$. For instance, for a static magnetic field along the z-axis and an oscillating magnetic field along x we obtain *absorption* and *dispersion* relations for the components of the current, similar to those in the magnetic case [2].

Consider now the field components in the laboratory frame:

$$\mathbf{B} = (2B_1 cos\omega t, 0, B_o)$$

$$\mathbf{E} = (0, 2E_1 sin\omega t, E_o)$$

that is, oscillating electric and magnetic fields applied along y and x directions, respectively, and static electric and magnetic fields applied along z.

In the laboratory coordinate system the total electric and magnetic fields depend on t and (4) cannot be used [3]. We can, however, make a transformation to a *reference frame* rotating with angular frequency ω, where $\mathbf{E}$ and $\mathbf{B}$ are static [1]:

$$\mathbf{B} = (B_1, 0, \omega/\gamma - B_o)$$

$$\mathbf{E} = (0, E_1, E_o)$$

The "fictitious field" ω/γ appears as a consequence of the transformation to the rotating frame [1]. On this frame, (4) and (6) are again valid. Replacing the above components for the vectors $\mathbf{E}$ and $\mathbf{B}$, it is straightforward to obtain the solutions. At the *resonance*, i.e., when $\omega = |\gamma|B_o$ [2], we get:

$$J_x(\infty) = 0$$

$$J_y(\infty) = \frac{\sigma(E_1+\omega_1\tau E_o)}{1+\omega_1^2\tau^2}$$

$$J_z(\infty) = \frac{\sigma(E_o-\omega_1\tau E_1)}{1+\omega_1^2\tau^2}$$

where $\omega_1 = \gamma B_1$. Note that the above expressions were obtained without any approximation concerning the relative magnitudes of the fields. It is interesting to notice the fact that both the longitudinal and transverse components J_y and J_z are proportional to E_1 and B_1. This is an intuitive result since there are two ways of

changing these components: either we change the electric field along a particular direction (z or y), or we change the magnitude of the *torque* in J_z by modifying B_1 along x.

For free electrons, $|\gamma| \approx 17.6GHz \quad kGauss^{-1}$. If $\tau = 10ns$ and $B_1 = 50Gauss$, $\omega_1\tau \approx 1$ and the component $J_z(\infty) \approx (\sigma/2)(E_o - E_1)$. Since these two components of the electric field (E_o and E_1) are mutually independent, the z-component of the current will be zero when they are made equal, irrespective of the value of σ.

As another example of application of (4), consider a problem of cylindrical symmetry. For instance, a static magnetic field along z with field gradient $\partial B_z/\partial z$ and a *radial* electric field $E = E_\rho$, produced, for instance, by a charged wire placed along z. It can be shown that if the field gradient is small, the radial component of the magnetic field will be: $B_\rho = -(R/2)\partial B_z/\partial z$, where R is the radius of the orbit of the particle in the field [5]. On matrix (6) we now must replace B_y by $B_\theta = 0$ and B_x by B_ρ, given above. By doing so, and calculating the matrix product (4) one finds:

$$J_\rho(\infty) = \sigma\frac{1 + [(\gamma\tau R/2)\partial B_z/\partial z]^2}{1 + [B_z^2 + (R/2)\partial B_z/\partial z)^2]\gamma^2\tau^2}E_\rho$$

$$J_\theta(\infty) = \sigma\frac{\gamma\tau B_z}{1 + [B_z^2 + (R/2)\partial B_z/\partial z)^2]\gamma^2\tau^2}E_\rho$$

$$J_z(\infty) = -\sigma\frac{(\gamma^2 R/2)B_z\partial B_z/\partial z}{1 + [B_z^2 + (R/2)\partial B_z/\partial z)^2]\gamma^2\tau^2}E_\rho$$

Note that if the magnetic field is considered homogeneous, that is, $\partial B_z/\partial z = 0$, we obtain expressions to the components of the Hall current in cylindrical coordinates. The axial current is produced by the field gradient. Also note that if the electric field configuration is changed, by making, for instance, $\mathbf{E}$ along z, all we have to do is to pickup the elements of the last column of (6) as the numerators of the above expressions.

We see from the above results that the sign of J_θ depends upon the sign of the particle charge q, but not that of J_z, which is always opposite in sign to the magnetic field gradient. This reflects the well known tendency for the confinement of charged particles, such as in a plasma, by a magnetic field, in the presence of a field gradient, creating the so-called *magnetic mirrors* [5].

The observation of current echoes and electrical current resonance (or ECR, for short) would lend all the experimental potential of usual NMR to the study of transport phenomena in electrical conductors. This paper is devoted to the discussion of these potentialities and under which conditions current echoes should be detected.

III. What can we learn from ECR and current echoes?

Equation (1b) for the electrical current was written assuming a single relaxation time. As in the magnetic case, there are no *a priori* reasons for this supposition; electron scattering can occur either via interactions with the lattice or with other electrons. We will call the respective scattering rates τ_{el}^{-1} and τ_{ee}^{-1} and *propose*, in analogy with the magnetic equations, the following Bloch equations for the components of the current in the rotating frame:

$$\begin{aligned}\frac{\partial J_z}{\partial t} &= -\gamma_e^* J_y B_1 + \frac{\sigma_o E_o - J_z}{\tau_{el}}\\ \frac{\partial J_x}{\partial t} &= +\gamma_e^* J_y b_o - \frac{J_x}{\tau'_{ee}}\\ \frac{\partial J_y}{\partial t} &= \gamma_e^*(J_z B_1 - J_x b_o) + \frac{J_y}{\tau'_{ee}}\end{aligned} \tag{7}$$

where $b_o = B_o - \omega/\gamma_e^*$ and $|\gamma_e^*| = e/m^*$, with m^* the electron effective mass. Here τ'^{-1}_{ee} includes contributions from electron-electron and electron-lattice scattering rates [1]:

$$\frac{1}{\tau'_{ee}} = \frac{1}{\tau_{ee}} + \frac{1}{2\tau_{el}} \tag{8}$$

In a two- or three-pulse experiment, if the relaxation between the pulses can be neglected [6], one should expect the following dependences of the current echo amplitudes at resonance with the pulse separation:

$$J_y(2\Delta\tau) = J_y(0)e^{-2\Delta\tau/\tau'_{ee}} \qquad (2 \text{ pulses})$$

where $\Delta\tau$ is the time interval between the two pulses, and

$$J_z(\Delta\tau) = J_z(0)(1 - 2e^{-\Delta\tau/\tau_{el}}) \qquad (3 \text{ pulses})$$

where in this case $\Delta\tau$ represents the time interval between the first and second pulses [7].

Thus, with the appropriate pulse sequence, the electron-electron and electron-lattice scattering rates can be measured independently. We can further split τ_{el}^{-1} into two main contributions:

$$\frac{1}{\tau_{el}} = \frac{1}{\tau_{el}^{ph}} + \frac{1}{\tau_{el}^{imp}} \tag{9}$$

where the first term represents the phonon scattering rate and the second the residual impurities and crystal defects contributions [8]. Then, at temperatures well below the Debye temperature Θ_D, the phonon contribution can be neglected and the second term can be measured separately. In short, from the dependence of $J_z(\Delta\tau)$ with the temperature one should be able to identify the two contributions to τ_{el}, whereas from $J_y(2\Delta\tau)$ one can have the value of τ'_{ee}.

The analogous of τ_{ee} and τ_{el} in usual NMR are the spin-spin and spin-lattice relaxation times, T_2 and T_1, respectively. The relation $T_1 \geq T_2$ is always verified in the magnetic case [10] and whereas T_1 is temperature dependent, T_2 usually does not depend on T. In the case we are analyzing, both τ_{ee} and τ_{el}^{ph} are temperature dependent, with $\tau_{ee} \propto T^{-2}$ and $\tau_{el}^{ph} \propto T^{-3}$ [8]. Pauli Exclusion Principle limits the number of electrons which can participate on electron-electron scattering to those which are in an energy range $\Delta\varepsilon \sim k_B T$ about the Fermi energy ε_F. The result is that at relatively high temperatures τ_{ee} can become several orders of magnitude larger than τ_{el}. Even at low temperatures, it is difficult to say whether these two quantities will become comparable. Thus, we should expect the lattice contribution to dominate the "transverse" relaxation process in equation (8). However, it may happen that in systems showing strong electronic correlation phenomena, such as Kondo lattices and heavy-fermions, we would have $\tau_{ee} \leq \tau_{el}$, as in the magnetic case.

Concerning ECR spectroscopy, for a fixed value of B_o, the position of the resonance peaks will be directly related to the effective mass m^* of conduction electrons, since the resonance frequency can be written as

$$\nu_c = \left|\frac{\gamma_e^*}{2\pi} B_o\right| = \left|\frac{e}{2\pi m^*}\right| B_o \tag{10}$$

from which m^* can be calculated for each line. In this sense m^* plays the same role for the electron in a conduction band as γ_n for different isotopes in NMR. Then, an ECR spectrum should provide the effective masses and proportions of the current carriers in different conduction bands.

In the next section we turn to the discussion of the experimental conditions for the observation of ECR, FCD's and current echoes.

IV. Experimental Parameters

The key question concerning the observation of current echoes in conducting media is how fast is the relaxation. As in the magnetic case, in order that the transient effects can be observed in metals, relaxation should be slow during the application of the pulses and between them (in a more than one-pulse experiment). If pulses of about 1 ns width are used, it is likely that the detection of the free current decay requires minimum relaxation times of about 0.5 ns.

Longer relaxation times can be achieved in high-purity single-crystals at low temperatures. For a metal like copper, for instance, with Debye temperature of approximately 315 K, below 4.2 K the electron-lattice relaxation will be dominated by impurity scattering. Then, the question of how big can be made τ_{el}^{imp} is the same as asking how pure can a single-crystal be grown.

The authors do not know of recent publications where relaxation times have been measured in single-crystalline metals at these temperatures. We believe that with modern ultra-high-vacuum techniques of atomic deposition, single-crystals could be grown sufficiently pure in order to meet the relaxation time requirements.

The operating frequency range of ECR experiments can be estimated from $\nu_c(GHz) \approx 2.8 B_o(kGauss)$ [8]. In a field of $0.2 kGauss$, for instance, the signal should appear at about $0.56 GHz$, a standard value for commercial oscillators. We remark that this value refers to the frequency in the *laboratory frame*. In the rotating coordinate system, at the resonance, the electrical current "feels" only the AC field, whose amplitude should be typically one order of magnitude smaller, leading to lower precession frequencies. Then the Hall current in the rotating frame will appear reduced in respect to the

laboratory system, and this fact can be used to detect the resonance in a CW experiment.

Another important practical parameter is the *skin-depth*, which measures the length of penetration of the radiofrequency field in the metal. This is given by:

$$\delta = \frac{1}{\sqrt{\nu_c \pi \mu \sigma}}$$

where μ and σ are the magnetic permeability and the electrical conductivity, respectively. For copper at 4.2 K, for instance, $\mu \approx 10^{-6} H/m$, and $\rho = 1/\sigma \approx 10^{-12} \Omega m$. At $\nu_o = 0.6 GHz$ we find $\delta \approx 200 \mathring{A}$. This means that, likewise in usual NMR, in an ECR experiment we will be dealing essentially with a surface current. Obviously, the skin-depth can be increased by decreasing the magnitude of the static magnetic field. The "bulk" current will represent a "background" upon which the resonance has to be measured. In this sense, thinner samples will present a smaller background. Another aspect to be pointed out is the fact that the above numbers were estimated using the free-electron value for γ_e^*. From equation (10) we see that the larger m^*, the lower will be ν_c. In a case like the heavy-fermion compound $CeAl_3$, for instance, $m^* \approx 1600m$ [11] and ν_c will be in the range of MHz for the same field B_o.

The experimental setup for ECR should be much the same as that for NMR [9]. Passive electronic components such as double-balanced-mixers, fast switches, phase-shifters, etc., operating from a few kHz to several GHz are commercially available. The characteristics of the power amplifier can be estimated from the AC field amplitude necessary to tip the current by 90°: $B_1 = \pi/2\gamma_e^* \tau_p \approx 50$ Gauss, for $\tau_p = 2ns$. For a cavity of 1 cm^3 with quality factor $Q = 100$ this would correspond to a power of a few kW.

In a CW experiment the sample should have four terminals, two for generating the current $\mathbf{J}$ and two for detecting the transverse components of the current at the resonance. The whole arrangement must be inserted into a solenoid or cavity where the AC field is generated perpendicularly to the static field. In a pulsed experiment a "pickup coil" can be used to detect the transient effects (FCD's or current echoes), as in usual NMR. In this case there is no need for the sample to have extra connections for the detection of the signal.

V. Conclusions

In this paper we proposed a new set of Bloch equations for describing the behavior of the electrical current in electric and magnetic fields and discussed the experimental consequences of their solutions. We have called the resonance phenomena derived from these equations *current echoes* and *free current decays, or FCD's*, and the experimental technique *Electrical Current Resonance*, or *ECR*. The main observable physical quantities in an ECR experiment would be the electron-electron and electron-lattice relaxation times and the electron effective mass. With ECR it would then be possible to study fundamental mechanisms of interactions involving charge carriers and the band structure in conducting materials.

Although the discussion in this paper has been given a classical treatment and directed to conducting materials in Solid State Physics, where relaxation is present, we have recently recognized that the phenomenon of free current decays and current echoes have a quantum-mechanical analog [12]. The eigenstates of a charged particle in a time-independent homogeneous magnetic field are the well known *Landau levels*, sometimes also referred as *Landau tubes* [8]. For a magnetic field applied along the z-axis, these are given by

$$E_n(p_z) = \left(n + \frac{1}{2}\right) \hbar\omega_c + \frac{p_z^2}{2m} \tag{11}$$

In reference [12] it is shown that if we superimpose, perpendicularly to the static field, an oscillating field with magnitude B_1 and frequency ω, in the rotating frame these levels become:

$$E'_n(p'_z) = \left(n' + \frac{1}{2}\right) \hbar\omega'_c + \frac{p'^2_z}{2m} \tag{12}$$

where $\omega'_c = (q/m)B_e$ is the particle *cyclotron frequency about the effective field* B_e . Here the " prime " refers a coordinate system where $\mathbf{B}_e$ is axial. Then, as we sweep over the resonance frequency, the Landau levels are turned by 90°. The pulsed case is also analyzed in ref. [12], and we show that the results are consistent with the classical treatment. These results clearly broaden the possibilities of applications of the effect,

for instance in the de Haas-van Alphen effect, or in a possible resonant method for particle spectroscopy.

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