Theoretical Methods in the Design of the Poloidal Field Coils for the ETE Spherical Tokamak

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This paper describes the theoretical models and the method used in the design of the poloidal field coils system for the ETE (Experimento Tokamak Esférico) small-aspect-ratio tokamak. The method is illustrated with the equilibrium configurations obtained for ETE.

I. Introduction

As it is usually done in the design of the poloidal field coils for tokamaks, the plasma cross section shape is the given input and the coils currents and positions are the desired output. The solution of this problem requires a mixed approach, combining synthesis and analysis of diverse magnetostatic field sources. In the design of the ETE tokamak, presently under construction in our laboratory, a minimal set of coils was adopted to attain the small-aspect-ratio plasma equilibrium configuration. It consists of: (1) the plasma magnetizing coils system, formed by the ohmic heating solenoid and two pairs of compensation coils; (2) a pair of equilibrium field coils; and (3) a pair of elongation coils. Fig. 1 illustrates the equatorially symmetric poloidal field coils system for ETE.

Since the numerical solution of the Grad-Schlüter-Shafranov equation, that describes the plasma equilibrium, is both computer time consuming and brings difficulties in the treatment of small-aspect-ratio configurations, we used an extension of the semi-analytic direct variational method [1] to solve the equilibrium equation. This method is appropriate for use in personal computers and was implemented with the *Mathematica* package [2]. In the following sections we will firstly describe the technique used in the optimization of the magnetizing coils system, which does not require a solution of the plasma equilibrium equation, and then the integrated design of all the poloidal field coils for the required equilibria.

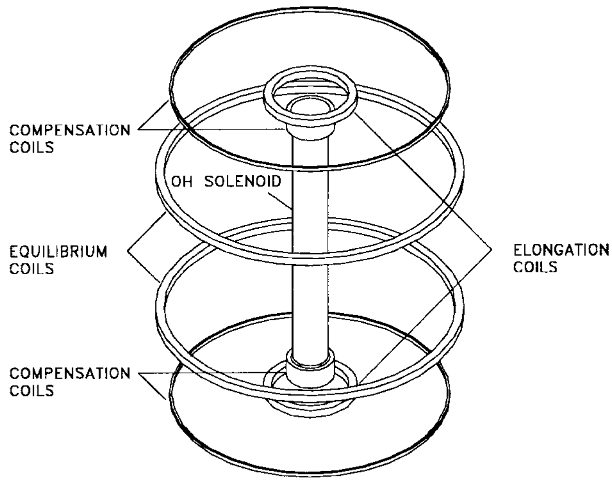

Figure 1. Illustration of the poloidal field coils system for the ETE tokamak.

II. Magnetizing coils system

The purpose of the magnetizing coils system is to produce the poloidal magnetic flux which is necessary to establish the toroidal plasma current by transformer action. During the raise of the plasma current the plasma temperature increases as a result of ohmic heating. In ETE a long central ohmic heating (OH) solenoid produces most of the required flux. However, for successful initial ionization of background neutrals and in order to avoid interference with the plasma position and shape during the discharge, the residual magnetic field produced by the OH solenoid must be reduced to a minimum in the region where the plasma is formed and sustained. The creation of a region of sufficiently small magnetic field near the plasma center is accomplished by means of the two pairs of compensation coils which constitute, with the OH solenoid, the magnetizing coils system.

Since the compensation coils in ETE are in series with the OH solenoid (passive compensation), the reduction of the error field can be attained only by adjusting the coils positions and the integer number of windings per coil. Employing a multipole moment expansion for the magnetizing flux on the geometrical center of the plasma cross section, we can calculate directly the free parameters of the compensation coils that lead to cancelation of the moments to a prescribed order and, therefore, to a reduced error field near the center. Constraints on the problem are imposed by accessibility for diagnostics plus space and engineering limitations, involving the size of the coils and available power supplies.

The multipole expansion for the poloidal flux Φ_M of the magnetizing coils system on the geometrical center $R_0(a)$ of the plasma cross-section is given by (R and Z are the cylindrical coordinates)

$$\begin{aligned}\Phi_M(R,Z) &= \Phi_0 + M_0 + M_1 + M_2 + \ldots \\ &+ \left(\frac{R^2-R_0^2(a)}{R_0^2(a)}\right) M_0 \\ &- \left(\frac{\left(R^2-R_0^2(a)\right)^2-4R^2Z^2}{R_0^4(a)}\right) M_1 \\ &+ \left(\frac{\left(R^2-R_0^2(a)\right)^3-12R^2\left(R^2-R_0^2(a)\right)Z^2+8R^2Z^4}{R_0^6(a)}\right) M_2 + \ldots\end{aligned}$$

where Φ_0 is the flux due to an ideal (infinitely long) ohmic solenoid. The flux due to the finite OH solenoid was obtained by the superposition of the flux produced by an infinite equivalent current sheet minus the fluxes due to two semi-infinite current sheets that represent the effect of the solenoid ends. The inner pair of compensation coils was also modeled by two equivalent current sheets while the outer pair was modeled by circular current loops. In this way the coefficients M_0, M_1, M_2,... were calculated in terms of algebraic and elementary transcendental functions of the coils geometrical parameters (with strengths proportional to the current per turn in the system). Taking into account all the design constraints, it was found that a satisfactory compensation could be obtained by cancelling M_0, M_1 and M_2, respectively the dipole, quadrupole and hexapole moments in the multipole expansion (this involves optimizing only three of the free geometrical parameters). Actually, higher order compensation would require an outer pair of coils located too far from the toroidal field coils and an unattainable precision in the coils positions. Fig. 2 shows the compensated flux contours on the poloidal plane and Fig. 3 shows the corresponding vertical error field on the equatorial plane, near the condition of maximum flux swing operation of the magnetizing system in ETE. Fig. 2 shows also the plasma boundary, vacuum vessel and toroidal field coils outlines.

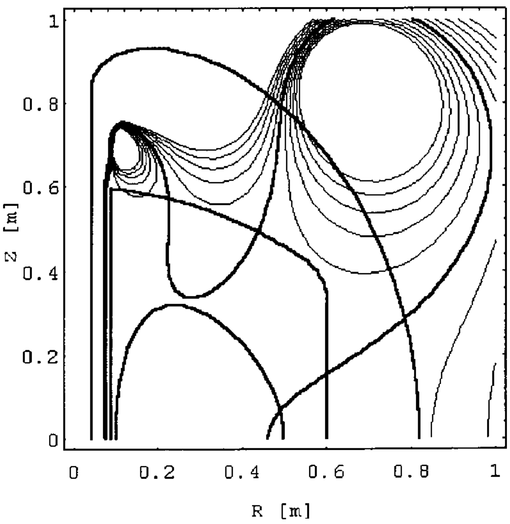

Figure 2. Magnetizing flux contours on the poloidal plane. The heavy contour indicates a poloidal flux of 0.25Wb (2×7.8MA-turns in the OH solenoid for double-swing operation) with a 2% flux increment between contours. The plasma boundary, vacuum vessel and toroidal field coils outlines are also displayed.

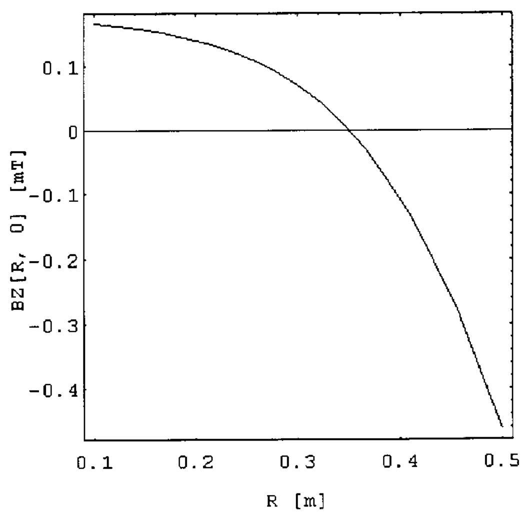

Figure 3. Vertical error field on the equatorial plane, near the condition of maximum flux swing operation of the magnetizing system in ETE.

III. Equilibrium field and elongation coils

The equilibrium field coils provide the radially inward force that balances the outward force produced by the interaction of the plasma current with its self-field. The dynamics of the plasma requires that the current in the equilibrium coils opposes and varies with the plasma current. Similarly, the elongation coils produce a predominantly vertical force that provides some control of the plasma cross section shape. To stretch the plasma cross section, the currents in these coils must run parallel to the plasma current.

In order to optimize the design of the complete set of poloidal field coils, the plasma equilibrium has to be solved both for a given boundary shape and for specified parameters such as the plasma current, the external toroidal induction and the peak pressure at the plasma center. Then, the coils currents can be adjusted to fit the vacuum poloidal flux function consistently with the assumed plasma boundary and positions of the coils. These positions can also be adjusted to some extent, but are essentially determined by accessibility and engineering constraints. The magnetic flux contribution of all poloidal coils has to be taken into account in these calculations, since there is no clear separation between the coils contributions to equilibrium.

An approximate solution to the plasma fixed boundary equilibrium problem can be effectively obtained using variational techniques and a spectral representation of the flux surfaces [3][4]. Furthermore, the problem can be greatly simplified by the introduction of trial functions for the spectral amplitudes, allowing the use of direct variational methods [1]. The starting point is given by the variational principle which states that the internal energy of the plasma

$$\begin{aligned} U[\Phi_P] &= \int\int\int_{V(a)} \left(\frac{B_P^2}{2\mu_0} + \frac{B_T^2 - B_{T,0}^2}{2\mu_0} + p \right) d^3r \\ &= \int_0^a \left[\frac{K(\rho)}{2} \left(\frac{d\Phi_P}{d\rho} \right)^2 - \frac{L(\rho)}{2} \frac{dI^2}{d\rho} + p(\rho) \frac{dV}{d\rho} \right] d\rho \end{aligned}$$

is stationary under virtual displacements ξ of the topological radius ρ,

$$\rho^* = \rho + \xi,$$

for fixed boundary conditions:

$$\xi(0) = \xi(a) = 0.$$

In this expression $\Phi_P(\rho)$ and $V(\rho)$ are, respectively, the poloidal magnetic flux and the plasma volume enclosed by a magnetic surface denoted by ρ, a is the minor radius of the plasma, $p(\rho)$ is the plasma pressure profile, and B_P, B_T are the poloidal and toroidal components, respectively, of the magnetic induction ($B_{T,0}$ is the external field contribution); $I(\rho)$ is the total poloidal current which flows through a disk centered on the symmetry axis (the poloidal plasma current is $I_P(\rho) = I(0) - I(\rho)$); $L(\rho)$ is the inductance of the toroidal solenoid which coincides with a magnetic surface; and $K(\rho)$ is the inverse kernel to calculate the internal inductance of the plasma loop [5][1]. The Euler equation for the functional $U[\Phi_P]$ leads to the equilibrium equation (flux-surface averaged Grad-Schlüter-Shafranov equation) [5]

$$\frac{d\Phi_P}{d\rho}\frac{d}{d\rho}\left(K(\rho)\frac{d\Phi_P}{d\rho} \right) = -\frac{dL}{d\rho} I(\rho) \frac{dI}{d\rho} - \frac{dV}{d\rho}\frac{dp}{d\rho},$$

while the integral forms of Ampère's law give the relations between the toroidal plasma current profile $I_T(\rho)$ and $\Phi_P(\rho)$

$$I_T(\rho) = K(\rho)\frac{d\Phi_P}{d\rho},$$

and between the toroidal magnetic flux $\Phi_T(\rho)$ and $I(\rho)$

$$\frac{d\Phi_T}{d\rho} = I(\rho)\frac{dL}{d\rho}.$$

We next represent the nested magnetic surfaces by the truncated Fourier expansions for the inverse mapping $(\rho, \theta) \rightarrow (R, Z)$ [6]

$$\begin{aligned} R(\rho,\theta) &= R_0(\rho) + \rho\cos\theta - \frac{\rho T(\rho)}{2}\sin^2\theta, \\ Z(\rho,\theta) &= \rho E(\rho)\left(1 - \frac{T(\rho)}{2}\cos\theta \right)\sin\theta, \end{aligned}$$

where θ is the poloidal angle coordinate. The Fourier coefficient $R_0(\rho)$ corresponds to the geometric centers of the flux surfaces, $E(\rho)$ to the elongation and $T(\rho)$ to the triangularity. It can be shown that the geometric coefficients $V(\rho)$, $L(\rho)$ and $K(\rho)$ of the equilibrium equation can be calculated analytically for an arbitrary number of terms in the spectral representation for $R(\rho,\theta)$ and $Z(\rho,\theta)$, and for an arbitrary dependence of the spectral amplitudes on ρ, effectively reducing the fixed boundary equilibrium problem to a one dimensional variational problem.

Following the approach for direct variational problems (the Ritz procedure), we introduce trial functions for the Shafranov shift, elongation and triangularity profiles. The trial function for the geometric centers of the flux surfaces has the simple parabolic form

$$R_0(\rho) \cong R_m - [R_m - R_0(a)]\,(\rho/a)^2$$

and the triangularity coefficient has a linear dependence on ρ

$$T(\rho) \cong T(a)\,(\rho/a)\,.$$

The elongation coefficient is approximated by the binomial form

$$\begin{aligned} E(\rho) \quad &\cong \quad \frac{E_m}{1-(E(a)/E_m)^\gamma}\left[1-\left(\frac{E(a)}{E_m}\right)^{(1+\gamma)}\right. \\ &\left. -\frac{(1-E(a)/E_m)\,(E(a)/E_m)^\gamma}{\left[1-(1-E(a)/E_m)\,(\rho/a)^2\right]^\gamma}\right] \end{aligned}$$

which reproduces many different profiles for various values of γ. It was found that the results for small and large aspect ratio tokamaks are best reproduced by values of $\gamma \sim 50$ that do not allow large variations of the elongation. These approximations improve previous results [1] and lead to a problem with two variational parameters, namely, the position $R_0(0) = R_m$ of the magnetic axis and the value $E(0) = E_m$ of the elongation at the axis. In this way, the variational procedure consists in the determination of a stationary point for the plasma internal energy U as a function of the parameters R_m and E_m. This semi-analytic approach allows simple computations of all the flux surface quantities, such as the safety factor and the macroscopic plasma quantities related to the specified pressure and current density profiles. In particular, the plasma equilibrium parameters of ETE listed in Table 1 were calculated for the profiles

$$\begin{aligned} p(\rho) \quad &= \quad p(0)\left[1-(\rho/a)^2\right]^{\alpha_p}, \\ I_T(\rho) \quad &= \quad I_T(a)\left(\frac{\rho}{a}\right)^2\left[\left(1+\frac{1}{\alpha_I}\right)-\frac{1}{\alpha_I}\left(\frac{\rho}{a}\right)^2\right]^{\alpha_I}, \end{aligned}$$

with $\alpha_p = 2$ and $\alpha_I = 1/2$. Work in progress indicates that more appropriate forms for the elongation coefficient can be attained, and better fittings obtained by the introduction of quadrangularity corrections in the flux surfaces. These corrections must lead to consistent expansions of the spectral amplitudes near the magnetic axis when the elongation is varied.

The direct variational procedure gives an approximate global solution of the plasma fixed boundary equilibrium problem. A further advantage in the present problem is that the equivalent surface current density on the plasma boundary has a simple analytic form, suggesting a direct application of the vector analogue of Green's theorem [7] to calculate the external field for equilibrium (this is equivalent to the virtual casing principle [5]). The surface current density can be calculated from the solution of the internal problem according to the formula ($\widehat{n}$ denotes the normal direction to the plasma surface)

$$\overrightarrow{K} = \widehat{n} \times \overrightarrow{B}_{(-)}/\mu_0,$$

which leads to the expression for the toroidal component

$$K_T = \frac{|\nabla\rho|}{2\pi\mu_0 R}\frac{d\Phi_P}{d\rho} = \frac{1}{2\pi\mu_0}\left(\frac{h_\theta}{\sqrt{g}}\right)\frac{I_T(\rho)}{K(\rho)},$$

where h_θ is the metric coefficient

$$h_\theta = \sqrt{\left(\frac{\partial R}{\partial\theta}\right)^2 + \left(\frac{\partial Z}{\partial\theta}\right)^2}$$

and $\sqrt{g}$ is the Jacobian of the transformation $(\rho, \theta) \rightarrow (R, Z)$

$$\sqrt{g} = R\left(\frac{\partial R}{\partial\rho}\frac{\partial Z}{\partial\theta} - \frac{\partial R}{\partial\theta}\frac{\partial Z}{\partial\rho}\right).$$

According to the Green's theorem, the internal poloidal flux in the vacuum region (produced by the plasma current) is given by the integral over the plasma surface current density

$$\Phi_{int}(\overrightarrow{r}) = -\mu_0 \oint K_T(\overrightarrow{r}')G(\overrightarrow{r}, \overrightarrow{r}')d\ell(\theta')$$

and the external flux (produced by the poloidal field coils) is given by the sum over the coils currents

$$\Phi_{ext}(\overrightarrow{r}) = \Phi_M - \mu_0 \sum_k I_k G(\overrightarrow{r}, \overrightarrow{r_k}),$$

where G is the Green's function of the Grad-Schlüter-Shafranov equation (ζ is the toroidal angle coordinate)

$$G(\overrightarrow{r}, \overrightarrow{r}') = -\left\langle \frac{\pi R R'}{\left|\overrightarrow{r} - \overrightarrow{r}'\right|} \right\rangle_{\zeta'}.$$

Finally, the total poloidal flux at the plasma edge is given by

$$\Phi_P(a) = \Phi_{int}(\overrightarrow{r}(a)) + \Phi_{ext}(\overrightarrow{r}(a)),$$

where $\Phi_P(a)$ is known from the solution of the fixed boundary equilibrium problem. This explicit expression for the poloidal flux function allows the calculation of Φ_M and I_k by means of a least squares approximation and without any iterative procedure, simplifying the determination of the external currents distribution necessary to sustain the given plasma shape. In the present paper we adopted the usual representation of the Green's function in terms of elliptic integrals [8]. Alternatively, an expansion in terms of toroidal multipoles [9] can be utilized which, coupled with the spectral representation for the flux surfaces, leads to an analytic approximation of the ideal external field for equilibrium. This latter approach can be used with advantage in a free boundary formulation of the plasma equilibrium problem for magnetic reconstruction purposes.

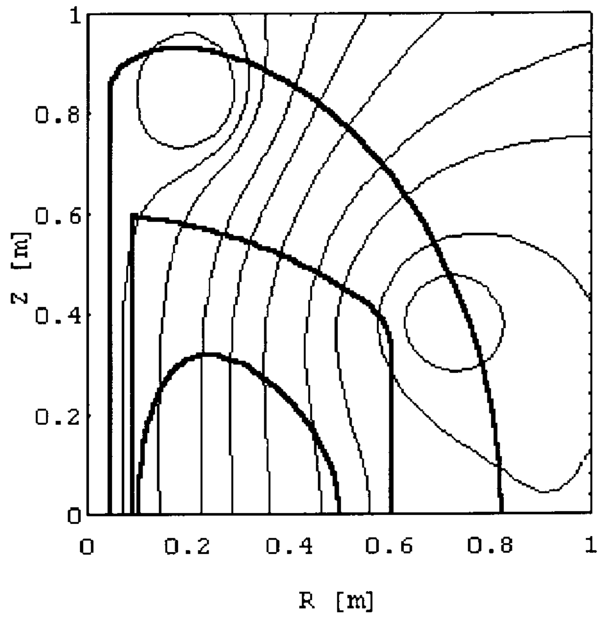

Figure 4. Vacuum flux produced by the poloidal field coils in ETE.

The method briefly described in the previous paragraphs was applied to the small-aspect-ratio configuration of ETE. The equilibrium and elongation coils were modeled by circular current loops and the magnetizing coils system was modeled by an ideal transformer, since the error field in the plasma region (according with the calculation in Section II) is much smaller than the vertical equilibrium field. In this way, the best fit of the currents in the coils gives, besides the equilibrium field, the Ampere-turns in the magnetizing system necessary to drive the plasma current (neglecting the resistive losses). Fig. 4 shows the vacuum poloidal flux contours generated by the coils and Fig. 5 shows the equilibrium flux contours for the initial phase of operation of ETE [10].

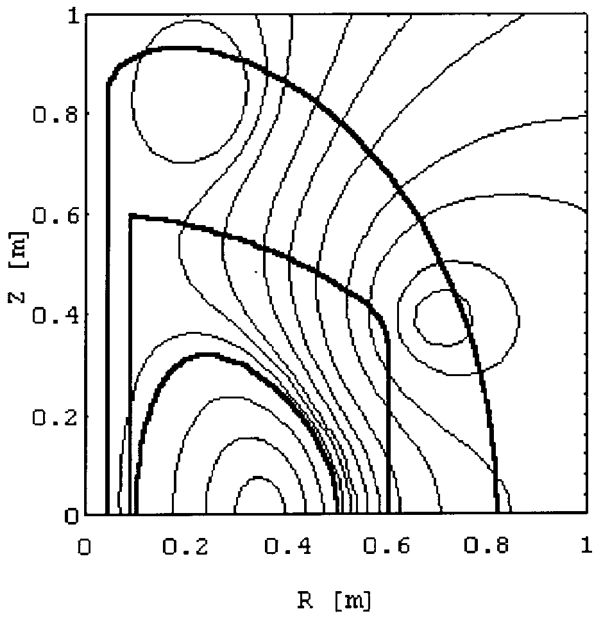

Figure 5. Equilibrium flux for a -220kA plasma current in ETE. The separatrix lies between 1.1 and 1.2 times the poloidal flux at the plasma edge.

The minimal set of coils in ETE fits the constant flux requirement at the plasma boundary within $\pm 1.5\%$. From Fig. 5 we verify that this error, which is larger at the outer plasma edge, can be reduced by the introduction of quadrangularity corrections in the plasma boundary shape. This improvement will be implemented in a future free boundary version of the present model.

IV. Results

Table 1 lists the main plasma parameters determined by the direct variational solution for the ETE tokamak equilibrium in the initial (ohmic) and extended (auxiliary heated) phases of operation. Table 2 lists the geometrical parameters and currents of the poloidal field coils consistent with the plasma equilibria and optimized using the method described in this paper.

Plasma parameter	Initial operation	Extended operation
Major radius $R_0(a)$ [m]	0.30	0.30
Minor radius a [m]	0.20	0.20
Elongation $\kappa(a)$	1.6	1.8
Triangularity $\delta(a)$	0.3	0.3
External toroidal induction B_0 [T]	0.4	< 0.8
Toroidal plasma current $I_T(a)$ [kA]	220	440
Pressure on the magnetic axis $p(0)$ [kPa]	8	80
Internal inductance ℓ_i	0.57	0.53
Current diamagnetism μ_I	0.47	0.16
Current beta β_I	0.20	0.55
Plasma beta β	0.036	0.092
Toroidal beta $\beta_{T,0}$	0.047	0.118
Safety factor on the magnetic axis $q(0)$	0.98	0.98
Safety factor at the plasma edge $q(a)$	5.55	7.04

Table 1: Parameters of the ETE tokamak equilibrium configurations. The extended operation lists maximum parameters that can be attained with auxiliary heating and near the Troyon and Greenwald limits.

Coil denomination	$\overline{R}$ [m]	$\overline{Z}$ [m]	ΔR [m]	ΔZ [m]	$NR \times NZ$	$I \times N$ [kA-turns]
Ohmic Heating Solenoid	0.0725	0	0.021	1.300	2×130	3640 (6760)
Internal Compensation Coils	0.1025	± 0.707	0.021	0.100	2×10	280 (520)
External Compensation Coils	0.650	± 0.871	0.010	0.020	1×2	28 (52)
Equilibrium Coils	0.700	± 0.390	0.040	0.040	4×4	90 (202)
Elongation Coils	0.200	± 0.830	0.040	0.040	4×4	-24 (-299)

Table 2: Geometrical parameters and currents of the poloidal field coils in ETE. The values of the currents in parenthesis correspond to preliminary results for the extended operation.

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