K^* Loops and the Strangeness Radius and Magnetic Moment of the Nucleon

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In this work we extend previous kaon loop calculations, including the vector K^* meson and estimate the strangeness radius of the nucleon and its strange magnetic moment. We find out that, in contrast to naive expectations, this contribution is very important and gives large values for both the radius and the magnetic moment.

In a very recent work [1] it was suggested that it may be very natural to expect the value of the strange magnetic moment of the nucleon to be positive, rather than the generally negative values obtained in model calculations [2-11]. Indeed, preliminary results of the MIT-Bates experiment[12]: $G_M(0) = 0.46 \pm 0.36 \pm 0.08 \pm 0.18$ (the first error is statistical, the second is due to the background and the third takes into account uncertainties in the radiative corrections entering the analysis) seem to corroborate McKeown's suggestion. This measurement apparently favours a sizable positive value of the strange magnetic moment of the nucleon and seems to be in contradiction with all the calculations quoted above.

To our knowledgement there are only three theoretical calculations which predict a positive strange magnetic moment. The oldest is based on an SU(3) chiral hyperbag model [13], one of the others is a resonance saturation model in which the unknown constants arising in chiral perturbation theory are determined by the t-channel exchange of vector mesons [14], and the third one calculates the contribution of all $Y^*\Sigma^*$ hadronic loops using a unquenched non-relativistic quark model [15].

The theoretical situation regarding the strangeness radius of the nucleon is not better and present nucleon model estimates [2-11,13,14] contain large and often uncontrolled theoretical uncertainties. For the Sachs radius, for instance, the predictions vary over one order of magnitude and in their sign. The only existing experimental information about the strangeness radius of the nucleon is a reanalysis of older neutrino scattering data [16], which seems to favour a negative strangeness radius (and also a negative strangeness magnetic moment), but with too poor statistic to be conclusive. However, with the present experimental techniques, the strange vector form factors can be directly measured by parity-violating lepton scattering. Several experiments of this type are in preparation or are already producing data [17-22].

In this paper we extend the loop calculation of refs.[3, 4] with the inclusion of the vector K^* meson. We will show that the inclusion of this meson changes drastically the results from the kaon loop calculation and predicts large values for both the strangeness radius and magnetic moment. In the case of the strangeness radius the K^* loop contribution is one order of magnitude bigger than the kaon loop contribution. In the case of the strange magnetic moment the K^* loop contribution is a number of the order of 4.0 to be compared

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with $\mu_s = -0.24$ obtained with the kaon loop. These results show that inspite of the fact that the kaon is the lightest strange meson, its contribution to the nucleon strangeness moments is not the dominant one in a one loop calculation. They also show that problably one has to include all possible strange intermediate states before being able to extract definitive predictions from loop calculations.

Our starting point is the vector-meson baryon effective lagrangian which is given by

$$\mathcal{L} = -g_v(\bar{\Psi}\gamma_\alpha \Psi b^\alpha + \frac{K}{2M}\bar{\Psi}\sigma_{\alpha\beta}\Psi\partial^\alpha b^\beta) , \qquad (1)$$

where Ψ and b^α are baryon and vector-meson fields respectively, M is the nucleon mass, and $K = g_t/g_v$ where g_t and g_v are the tensor and vector baryon vector-meson coupling constants respectively.

To take into account the size of the hadrons we introduce form factors at the meson-nucleon vertices, used in the determination of the Bonn potential from hyperon-nucleon scattering [23], through the substitution

$$g_v \rightarrow g_v F(k^2) , \qquad (2)$$

with

$$F(k^2) = \frac{m^2 - \Lambda^2}{k^2 - \Lambda^2} , \qquad (3)$$

where m is the vector-meson mass, k is its momentum and Λ is the cut-off. For $k^2 = m^2$ one has $F(m^2) = 1$, therefore, this choice for the form factor is consistent with the values of the mesonic coupling constants extracted from on-shell amplitudes.

The effective baryon-meson interaction is nonlocal for extended baryons and this induces an electromagnetic vertex current coupling if the photon field is present. The gauge invariance of this interaction can be restored by making use of the minimal substitution. This generates the seagull vertex [24]

$$i\Gamma^{(s)}_{\mu\alpha}(k,q) = ig_v Q_M (q \pm 2k)_\mu \frac{F(k^2) - F((q \pm k)^2)}{(q \pm k)^2 - k^2}\left[\pm\gamma_\alpha + \frac{iK}{2M}\sigma_{\alpha\beta}k^\beta\right] , \qquad (4)$$

where the upper and lower signs correspond to an incoming or outgoing vector meson (with index α) respectively, $Q_M = -1$ is the K^* strangeness charge, and q is the photon momentum.

Due to the presence of the derivative in eq.(1), the minimal substitution also generates a current-vertex coupling even if the meson-nucleon form factors are not present:

$$i\Gamma^{(v)}_{\mu\alpha}(k) = \pm\frac{g_v Q_M K}{2M} F((q \pm k)^2)\sigma_{\alpha\mu} , \qquad (5)$$

where the sign convention is the same as explained above.

The diagrams we are considering are shown in figures 1 to 3, where we have represented the electric coupling by a circle and the magnetic coupling by a cross. In the diagrams (d) in Fig. 2 and (c) in Fig. 3 we have two possibilities of vertices: the seagull vertex given by eq.(4) and the current-vertex coupling given by eq.(5).

The expressions for the different diagrams are

$$\Gamma^{(1a)}_\mu(p',p) = ig_v^2 Q_\Lambda \int \frac{d^4k}{(2\pi)^4}(F(k^2))^2 D^{\alpha\beta}(k)\gamma_\alpha S(p'-k)\gamma_\mu S(p-k)\gamma_\beta , \qquad (6)$$

$$\begin{aligned}\Gamma^{(1b)}_\mu(p',p) &= -ig_v^2 Q_M \int \frac{d^4k}{(2\pi)^4} F((k+q)^2)F(k^2)D^{\alpha\lambda}(k+q)D^{\sigma\beta}(k)[(2k+q)_\mu g_{\sigma\lambda} + \\ &- (k+q)_\sigma g_{\lambda\mu} - k_\lambda g_{\sigma\mu}]\gamma_\alpha S(p-k)\gamma_\beta , \end{aligned} \qquad (7)$$

$$\Gamma_\mu^{(1c)}(p',p) = ig_v^2 Q_M \int \frac{d^4k}{(2\pi)^4} F(k^2) D^{\alpha\beta}(k) \left[\frac{(q+2k)_\mu}{(q+k)^2 - k^2} \left(F(k^2) - F((k+q)^2) \right) \times \right.$$
$$\left. \gamma_\alpha S(p-k)\gamma_\beta - \frac{(q-2k)_\mu}{(q-k)^2-k^2} \left(F(k^2) - F((k-q)^2) \right) \gamma_\alpha S(p'-k)\gamma_\beta \right] , \quad (8)$$

$$\Gamma_\mu^{(2a)}(p',p) = -\frac{g_v^2 K Q_\Lambda}{2M} \int \frac{d^4k}{(2\pi)^4} (F(k^2))^2 D^{\alpha\beta}(k) k^\nu [\sigma_{\alpha\nu} S(p'-k)\gamma_\mu S(p-k)\gamma_\beta +$$
$$- \gamma_\alpha S(p'-k)\gamma_\mu S(p-k)\sigma_{\beta\nu}] , \quad (9)$$

$$\Gamma_\mu^{(2b)}(p',p) = +\frac{g_v^2 K Q_M}{2M} \int \frac{d^4k}{(2\pi)^4} F((k+q)^2) F(k^2) D^{\alpha\lambda}(k+q) D^{\sigma\beta}(k) \times$$
$$((2k+q)_\mu g_{\sigma\lambda} - (k+q)_\sigma g_{\lambda\mu} - k_\lambda g_{\sigma\mu})[(k+q)^\nu \sigma_{\alpha\nu} S(p-k)\gamma_\beta +$$
$$- \gamma_\alpha S(p-k)\sigma_{\beta\nu} k^\nu] , \quad (10)$$

$$\Gamma_\mu^{(2c)}(p',p) = +\frac{g_v^2 K Q_M}{2M} \int \frac{d^4k}{(2\pi)^4} F(k^2) D^{\alpha\beta}(k) k^\nu \left[\frac{(q+2k)_\mu}{(q+k)^2-k^2} \left(F(k^2) - F((k+q)^2) \right) \times \right.$$
$$(\gamma_\alpha S(p-k)\sigma_{\beta\nu} - \sigma_{\alpha\nu} S(p-k)\gamma_\beta) + \frac{(q-2k)_\mu}{(q-k)^2-k^2} \left(F(k^2) + \right.$$
$$\left. - F((k-q)^2) \right) (\sigma_{\alpha\nu} S(p'-k)\gamma_\beta - \gamma_\alpha S(p'-k)\sigma_{\beta\nu}) \Big] , \quad (11)$$

$$\Gamma_\mu^{(2d)}(p',p) = +\frac{g_v^2 K Q_\Lambda}{2M} \int \frac{d^4k}{(2\pi)^4} F(k^2) D^{\alpha\beta}(k) [F((k+q)^2)\sigma_{\alpha\mu} S(p-k)\gamma_\beta +$$
$$- F((k-q)^2)\gamma_\alpha S(p'-k)\sigma_{\beta\mu}] , \quad (12)$$

$$\Gamma_\mu^{(3a)}(p',p) = \frac{ig_v^2 Q_\Lambda K^2}{4M^2} \int \frac{d^4k}{(2\pi)^4} (F(k^2))^2 k^\gamma k^\nu D^{\alpha\beta}(k) \sigma_{\alpha\nu} S(p'-k)\gamma_\mu S(p-k)\sigma_{\beta\gamma} , \quad (13)$$

$$\Gamma_\mu^{(3b)}(p',p) = -\frac{ig_v^2 Q_M K^2}{4M^2} \int \frac{d^4k}{(2\pi)^4} F((k+q)^2) F(k^2) D^{\alpha\lambda}(k+q) D^{\sigma\beta}(k) [(2k+q)_\mu g_{\sigma\lambda} +$$
$$- (k+q)_\sigma g_{\lambda\mu} - k_\lambda g_{\sigma\mu}](k+q)^\nu k^\gamma \sigma_{\alpha\nu} S(p-k)\sigma_{\beta\gamma} , \quad (14)$$

$$\Gamma_\mu^{(3c)}(p',p) = \frac{ig_v^2 Q_M K^2}{4M^2} \int \frac{d^4k}{(2\pi)^4} F(k^2) D^{\alpha\beta}(k) k^\nu k^\gamma \left[\frac{(q+2k)_\mu}{(q+k)^2-k^2} \left(F(k^2) - F((k+q)^2) \right) \times \right.$$
$$\left. \sigma_{\alpha\nu} S(p-k)\sigma_{\beta\gamma} - \frac{(q-2k)_\mu}{(q-k)^2-k^2} \left(F(k^2) - F((k-q)^2) \right) \sigma_{\alpha\nu} S(p'-k)\sigma_{\beta\gamma} \right] , \quad (15)$$

$$\Gamma_\mu^{(3d)}(p',p) = -\frac{ig_v^2 Q_M K^2}{4M^2} \int \frac{d^4k}{(2\pi)^4} F(k^2) D^{\alpha\beta}(k) k^\nu \left[F((k+q)^2)\sigma_{\alpha\mu} S(p-k)\sigma_{\beta\nu} + \right.$$
$$\left. + F((k-q)^2)\sigma_{\alpha\nu} S(p'-k)\sigma_{\beta\mu} \right] , \quad (16)$$

In the above equations $D_{\alpha\beta}(k) = (-g_{\alpha\beta} + k_\alpha k_\beta / m^2)(k^2 - m^2 + i\epsilon)^{-1}$ is the K^* propagator, $S(p-k) = (\not{p} - \not{k} - M_\Lambda + i\epsilon)^{-1}$ is the Λ propagator, $Q_\Lambda = 1$ is the Λ strangeness charge and $p' = p + q$.

The nucleon matrix element of the strange-quark vector current, $< p'|\bar{s}\gamma_\mu s|p >$, can be parametrized in terms of the strange Dirac and Pauli nucleon form factors

$$< p'|\bar{s}\gamma_\mu s|p >= \bar{U}(p') \left[F_1^{(s)}(q^2)\gamma_\mu + i\frac{\sigma_{\mu\nu}q^\nu}{2M}F_2^{(s)}(q^2)\right] U(p) \ , \tag{17}$$

where $U(p)$ denotes the nucleon spinor. It is often convenient to use the electric and magnetic Sachs form factors

$$G_E^{(s)}(q^2) = F_1^{(s)}(q^2) + \frac{q^2}{4M^2}F_2^{(s)}(q^2) \ , \tag{18}$$

$$G_M^{(s)}(q^2) = F_1^{(s)}(q^2) + F_2^{(s)}(q^2) \ , \tag{19}$$

which describe the strangeness charge and current distribution respectively. The strangeness radius, r_s^2, and magnetic moment, μ_s, are defined as the first nonvanishing moments of the Sachs form factors

$$r_s^2 = 6\frac{d}{dq^2}G_E^{(s)}(q^2)\bigg|_{q^2=0} \ , \tag{20}$$

$$\mu_s = G_M^{(s)}(0) = F_2^{(s)}(0) \ . \tag{21}$$

The last relation in eq.(21) is due to the normalization of the strange Dirac form factor, $F_1^{(s)}(0) = 0$, which follows from strangeness conservation and the zero overall strangeness charge of the nucleon.

Table 1. Contributions for the strangeness radius and magnetic moments from the diagrams shown in figs.1 to 3

Diagrams in	$r_s^2(\text{fm}^2)$	μ_s
Fig.1	-1.62×10^{-2}	-5.33×10^{-2}
Fig.2	-2.20×10^{-1}	-3.28
Fig.3	-8.48×10^{-3}	-0.82

In table I we give the results for r_s^2 and μ_s. The values of the coupling constants, masses and cut-off used are [23]$M = 939\,\text{MeV}$, $M_\Lambda = 1116\,\text{MeV}$, $m = 895\,\text{MeV}$, $g_v/\sqrt{4\pi} = -1.588$, $K = 3.26$ and $\Lambda = 2.2\,\text{GeV}$. We have divided the results in three kinds of contributions: the first line gives the results from the diagrams in Fig. 1, where only the electric kind of vertex between the vector-meson and the baryon was considered. The second line gives the contribution from the diagrams in Fig. 2, where we have one electric and one magnetic vertex. Finally the third line gives the contribution from two magnetic vertices (diagrams in Fig. 3). From this table we see that the largest contribution comes from the diagrams with one electric and one magnetic vertex. The contribution from diagrams with two electric vertices is completely negligible for the magnetic moment. Adding the three contributions the final result for the K^* loop is

$$r_s^2 = -0.244(-0.212)\,\text{fm}^2 \ , \tag{22}$$

$$\mu_s = -4.15(-3.62) \ . \tag{23}$$

The numbers between parenthesis refer to $\Lambda = 2.1\,\text{GeV}$.

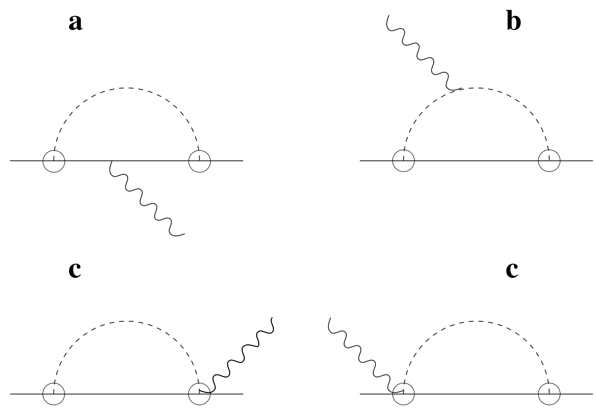

Figure 1. Diagrams contributing to $K^* - \Lambda$ loop. The circles refer to electric couplings, the dashed line represents the vector-meson K^* and the solid line represents the baryons.

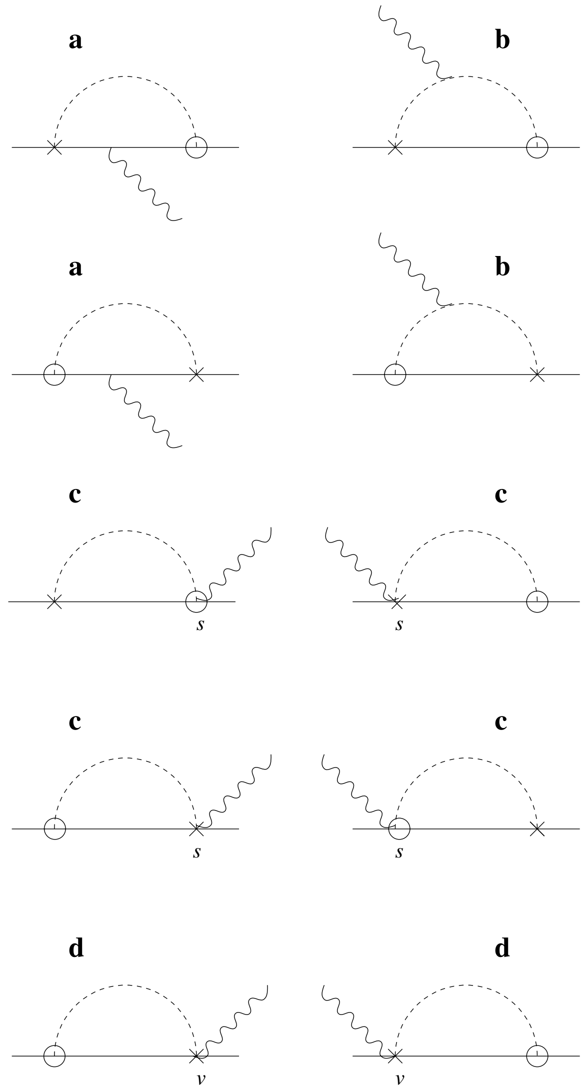

Figure 2. Same as Fig.1. The crosses refer to magnetic couplings.

At a first look it seems surprising that the loop contribution from K^* is bigger than the loop contribution from the kaon. However, if we notice that the cut-off in the Bonn form factor for $N\Lambda K^*$ is much bigger than the cut-off for kaons (1.2GeV), and also that the tensor coupling is much bigger than the $N\Lambda K$ coupling constant the differences in magnitude are not so unexpected. A negative value for the radius is compatible with a naive expectation. This expectation follows from a geometric interpretation of the nucleon strange radius. Since K^* is lighter than Λ it lives, on average, further from the nucleon center of mass than the strange baryon. Therefore, since K^* has a negative strangeness charge (in our convention), it results in a negative strangeness squared radius.

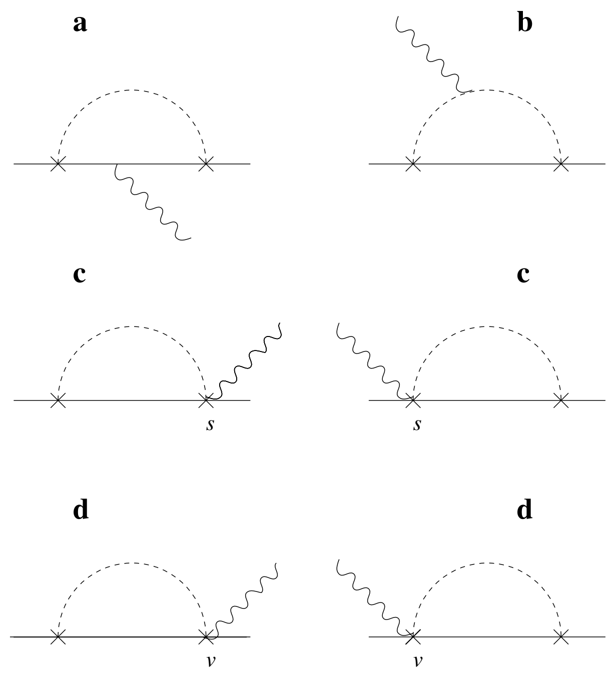

Figure 3. Same as Fig.1 for two magnetic vertices.

Adding the $N\Lambda K$ and $N\Lambda K^*$ loop contributions we finally get

$$r_s^2 = -0.266(-0.240)\,\mathrm{fm}^2 \ , \tag{24}$$

$$\mu_s = -4.39(-3.94) \ . \tag{25}$$

Due to the drastic change in the results from the loop calculation when the vector meson K^* is included, we don't feel that we can draw definitive predictions as long as we don't take into account all possible strange intermediate states in the calculations [25].

The other possible strange intermediate states are: ΣK, $\Sigma^* K$ and $\Sigma^* K^*$. It was argued in refs.[3, 4] that due to the very small coupling contant [23] the contribution from the ΣK intermediate state is much smaller than the ΛK contribution and can, therefore, be neglected. The $\Sigma^* K$ intermediate state was already considered in a different context, to evaluate the intrinsic strangeness distribution of the nucleon [26]. In ref.[26] it was shown that the $\Sigma^* K$ contribution to the intrinsic strangeness of the nucleon is of the same order as the ΛK contribution. Therefore, it can't be neglected. As the $N\Sigma^* K^*$ coupling constant is bigger than the $N\Sigma^* K$

coupling constant [23] we expect the $\Sigma^* K^*$ contribution to be bigger than the ΛK contribution. There is also one more contribution which should be taken into account: a $K - K^*$ loop where the strange current induces a spin flip of the pseudoscalar to vector meson. Work in this direction is under consideration and will be presented elsewhere [25].

So far there is only one calculation of the strangeness radius and magnetic moment in which a sum over all meson-baryon loops is performed. This is a non-relativistic quark model calculation [15] where it is shown that when one works at the one loop order, and sums to high excitation in the hyperon-strange-meson spectrum, there exist delicate cancellations within various subsets of the spectrum, which lead to small values for the strangeness radius and magnetic moment. However, their results depend on several model asssumptions, as for instance the use of the simple non-relativistic quark model even for high-lying excitations; the use of simple harmonic oscillator wavefunctions for the baryons and mesons (which lead to a crude description of the couplings between $h_{s\bar{s}}|p>$ and the meson-hyperon states), and the use of the ${}^3P_0$ flux-tube breaking mode for $s\bar{s}$ pair creation (essential, e.g., for the sub-cancellations in SU(6) multiplets). In light of the mentioned uncertainties, the quantitative results of ref.[15] (small values for the strangeness radius and magnetic moment) can neither be expected to be generic nor reliable (as the authors concede in their conclusions).

In summary, from our calculation and from the calculation presented in ref.[15] we can conclude that one can not expect to get a reliable result for the strangeness radius and magnetic moment by considering only the low-lying intermediate states.

Acknowledgements

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