Intermediate Pure Mixed State of the Quantized Radiation Field

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We introduce the intermediate pure-mixed state of the light field, which interpolates between a pure state $\hat{\rho}^{(P)}$ $[Tr(\hat{\rho}^{(P)})^2 = 1]$and a mixed state $\hat{\rho}^{(M)}$ $[Tr(\hat{\rho}^{(M)})^2 < 1]$, allowing us the investigation of the influence of the mixing effects upon some properties exhibited by a pure state when isolated. Following this strategy we show in what fashion the presence of a thermal state degrades the sub-Poissonian statistics and the squeezing effect exhibited by a pure state. Intervals of parameters preserving these nonclassical effects are discussed.

I. Introduction

Since the pioneer theoretical work by Glauber[1], after the discovering of the maser and the laser, extensive study of the quantized radiation field has been developed in the literature, with remarkable interest in the interaction process between photons and atoms. In this line, considerable effort was done to get a more precise understanding of the structure and dynamics of atoms and molecules, to better control of their internal and external degree of freedom, and also the realization of novel radiation sources[2] allowing one to prepare new states of the quantized radiation field. The research in this area of quantum optics has increased after the detection of various nonclassical properties of the light field, as antibunching[3], sub-Poissonian statistics[4], squeezing[5], etc. There are also other nonclassical effects which emerge from the interaction between light-field and atoms, theoretically treated in the so called Jaynes-Cummings model[6]. Examples of these states are: the collapse and revival effect[7] in the atomic inversion; oscillations in the photon distribution[8]; entanglement of the atom and field states, leading to decoherence of a wavefunction[9]; formation of a macroscopic superposition of quantum states[10] (Schrödinger cat); teleportation[11]; etc.

As the most usually investigated states in quantum optics one can cite: the number state[12], eigenstate of the photon number operator $\hat{n}$; the coherent state[1] $|\alpha\rangle$,eigenstate of the annihilation operator $\hat{a}$; the (Pegg-Barnett) phase state[13] $|\theta_m\rangle$, eigenstate of the phase operator $\hat{\phi}_\theta = \sum_m \theta_m |\theta_m\rangle\langle\theta_m|$; the squeezed-coherent state[5] $|z,\alpha\rangle$, eigenstate of the (quasi-particle) annihilation operator $\hat{b} = (cosh\ r)\hat{a} + e^{i\phi}(sinh\ r)\hat{a}^\dagger$, $z = re^{i\phi}$; etc.

Besides these "standard" states of quantum optics there are also states which are intermediate between two of these states. Among them, one can cite: the binomial state, introduced by Stoler et al[14], which interpolates between the number state $|n\rangle$ and the coherent state $|\alpha\rangle$ in two different limits; the intermediate number-phase state (INPS), introduced recently by Baseia et al[15], which interpolates between the num-

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ber state $|n\rangle$ and the (Pegg-Barnett) phase state $|\theta_m\rangle$; the intermediate number-squeezed state (INSS), more recently introduced by Baseia et al[16], which interpolates between the number state $|n\rangle$ and the squeezed state $|z, \alpha\rangle$; etc.

While these intermediate states are assumed to be generated by a single light source[14] - [18], alternative intermediate states generated by two light sources were also introduced in the literature. Among them one can cite the superposition of two number states, introduced by Wodkiewicz[19]; the superposition of two coherent states, by Schleich et al[20]; the even and odd coherent states, by Malkin et al[21]; the even and odd squeezed states, by Xin et al[22]; the intermediate number-coherent state (alternative to binomial state[14]), by Baseia et al[23]; the intermediate number-squeezed state (alternative to INSS[16]), by Baseia et al[24]; etc. A unified approach to these states was presented in the Ref.[25]

In all these cases, interesting nonclassical properties displayed by such states, as those mentioned before, have been extensively studied in the literature. Also, the interaction of an atom with a light field initially in one of such states is a subject of current investigation[4, 7].

In the present paper we will follow our previous procedure(Refs. [23]-[25]), introducing a convenient intermediate state which interpolates between a pure state $\widehat{\rho}^{(P)} = |\psi\rangle\langle\psi|$ $[tr(\widehat{\rho}^{(P)})^2 = 1]$, and a mixed state $\widehat{\rho}^{(M)}$ $[tr(\widehat{\rho}^{(M)})^2 < 1]$. We call it as Intermediate Pure Mixed State (IPMS) of the quantized radiation field. Besides constituting a theoretical tool allowing a compact treatment incorporating both components, $\widehat{\rho}^{(P)}$ and $\widehat{\rho}^{(M)}$, the IPMS allows one to investigate the possible occurrence of new properties of a field in such state, as shown in Refs.[14]-[24]. It should be emphasized here that intermediate states may exhibit interesting properties, even if they are not exhibited in their limiting states. For example, the binomial state shows quadrature squeezing, althought its extrem states, $|n\rangle$ and $|\alpha\rangle$, do not show this effect.

Differently from our previous strategies[23]-[25], the main aim here is to investigate the influence of mixing effects upon the properties of a pure state. Mixing originates, e.g., from thermal influences coming from the environment that always surrounds a cavity. These thermal influences disappear only in the limit of zero temperature where thermal excitation vanishes: $\bar{n}_T = 1/[exp(\hbar\omega/kT) - 1] \to 0$, if $T \to 0$. The present treatment allows us to investigate in what fashion the (controlled) presence of mixing effects erase nonclassical properties exhibited by a pure state. It also allows us to determine the intervals of the involved parameters in which the degradation of some nonclassical property is negligible, thus being preserved. For each property investigated we choose the (pertinent) pure state, which exhibits this property when isolated.An interpolating parameter $\xi \in [0, 1]$, stands for the partial contribution of each component state, $\widehat{\rho}^{(P)}$ and $\widehat{\rho}^{(M)}$ [Cf. Eq.(1)].

At first sight it would seem that the present approach constitutes a (very) simplified alternative to other treatments in the literature, as that employing the Caldirola-Kanai Hamiltonian[26], or that employing a Hamiltonian including a thermal reservoir[27]. In theses cases the treatment is much more complicated since the state describing a system (acted upon by thermal influences) is time-dependent, its stationary solution being obtained for assymptotic times, $t >> \tau_o$, where τ_o is a characteristic relaxation time of the system. However, in these treatments the thermal presence enters as a consequence of the inclusion of dissipation in the system - which is not the case in the present scenario. As an exemple, a paper by Voudras et al[28] studied the destruction of oscillatory feature in photon-number distribution caused by small amount of noise. An alternative approach, by Takahashi et al[29], was applied by Lee[30] to study the influence of thermal noise on 2-mode squeezing. This latter approach is similar to ours in the sense that no dynamical evolution is considered. Another similar procedure in this respect, introducing thermal noise, can be seen in Ref.[31] in wich a classical field is written as $\vec{E}(\vec{r}, t) = \vec{E}_0(\vec{r}, t) + \vec{E}_T(\vec{r}, t)$, where $\vec{E}_0(\vec{r}, t)$ stands for a classical field at temperature $T = 0$, whereas $\vec{E}_T(\vec{r}, t)$ represents a thermal field component, at $T \neq 0$. Related topics and treatments can also be found in Ref.[32].

This paper is arranged as follows. In the Sect.2 we introduce the IPMS, defined as in the Eq.(1). In the Sect.3 we investigate the (destructive) influence of mixing effects upon the sub-Poissonian statistics shown by a pure state, when isolated. In the Sect. 4 we investigate the same for squeezing. The Sect. 5 contains the comments and conclusion.

II. Intermediate pure mixed state(IPMS)

We introduce the IPMS as the superposition

$$\widehat{\rho}(\xi)=\xi\widehat{\rho}^{(M)}+(1-\xi)\widehat{\rho}^{(P)} \tag{1}$$

where $\xi\in[0,1]$ is the interpolating parameter: $\widehat{\rho}(0)=\widehat{\rho}^{(P)}$ and $\widehat{\rho}(1)=\widehat{\rho}^{(M)}$. The component states $\widehat{\rho}^{(M)}$ and $\widehat{\rho}^{(P)}$ stand for the mixed state and the pure state, respectively:

$$\widehat{\rho}^{(M)}=\sum_{n=0}^{\infty}p_n|n\rangle\langle n|, \tag{2}$$

$$\widehat{\rho}^{(P)}=\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}C_nC_{n'}^*|n\rangle\langle n'|. \tag{3}$$

Note that $\widehat{\rho}^{(M)}$ is diagonal in the basis $\{|n\rangle\}$ whereas off-diagonal terms of $\widehat{\rho}^{(P)}$ are not zero. Also, note that $tr(\widehat{\rho}(\xi))=1$ if $tr(\widehat{\rho}^{(M)})=tr(\widehat{\rho}^{(P)})=1$. The probability of finding n photons in the IPMS is given by

$$p_n(\xi)=\xi p_n^{(M)}+(1-\xi)p_n^{(P)} \tag{4}$$

with

$$\sum_{n=0}^{\infty}p_n(\xi)=\sum_{n=0}^{\infty}p_n^{(M)}=\sum_{n=0}^{\infty}p_n^{(P)}=1.$$

An useful formula is obtained from the average value of an arbitrary operator $\widehat{O}$, calculated in the IPMS. We have,

$$\begin{aligned}\langle\widehat{O}\rangle(\xi)&=tr\ [\widehat{\rho}(\xi)\widehat{O}]=\xi\ tr[\widehat{\rho}^{(M)}\widehat{O}]+(1-\xi)\ tr\ [\widehat{\rho}^{(P)}\widehat{O}]\\&=\xi\ \langle\widehat{O}\rangle_{(M)}+(1-\xi)\ \langle\widehat{O}\rangle_{(P)}.\end{aligned} \tag{5}$$

From this result we easily obtain,

$$\langle\widehat{O}^2\rangle(\xi)=\xi\ \langle\widehat{O}^2\rangle_{(M)}+(1-\xi)\ \langle\widehat{O}^2\rangle_{(P)}. \tag{6}$$

and, from Eqs.(5) and (6), we obtain the dispersion (variance) in the arbitrary operator $\widehat{O}$,

$$\Delta\widehat{O}^2(\xi)=\xi\Delta\widehat{O}^2_{(M)}+(1-\xi)\Delta\widehat{O}^2_{(P)}+\xi(1-\xi)[\langle\widehat{O}\rangle_{(M)}-\langle\widehat{O}\rangle_{(P)}]^2\ . \tag{7}$$

Note that the dispersion in an arbitrary operator $\widehat{O}$ is not only the (weigthed) sum of the dispersions in the components $\widehat{\rho}^{(M)}$ and $\widehat{\rho}^{(P)}$. There is also the presence of an "interference" term (last term in Eq.(7)), which is proportional to the squared difference of the averages $\langle\widehat{O}\rangle_{(M)}$ and $\langle\widehat{O}\rangle_{(P)}$. When this difference tends to zero the total dispersion tends to the (weighted) sum of the partial dispersions. From the general expression(7) we obtain the dispersions in the number operator $\widehat{n}$ and in the quadrature operators, $\widehat{X}_i$,

$$\widehat{X}_1=(\widehat{a}+\widehat{a}^\dagger)/2\ , \tag{8}$$

$$\widehat{X}_2=(\widehat{a}-\widehat{a}^\dagger)/2i\ . \tag{9}$$

The dispersion in the number operator $\widehat{n}$ allows us to investigate the occurrence, or not, of sub-Poissonian statistics, whereas the dispersions in the quadrature operators $\widehat{X}_1$ and $\widehat{X}_2$ allows us to investigate the occurrence of the squeezing effect. In the present case we will take a pure state $\widehat{\rho}^{(P)}$ exhibiting one of such nonclassical effects and investigate in what fashion the presence of a mixed state degrades this effect. In the next section we will study the sub-Poissonian effect.

III. Influence of mixing upon sub-Poissonian statistics

One operational ingredient showing sub-Poissonian statistics is the Fanos's factor F, defined as[4]

$$F(\xi) = \Delta\widehat{n}^2(\xi)/\langle\widehat{n}(\xi)\rangle \tag{10}$$

where

$$\Delta\widehat{n}^2(\xi) = \langle\widehat{n}^2(\xi)\rangle - \langle\widehat{n}(\xi)\rangle^2 \tag{11}$$

is the dispersion in the number operator $\widehat{n}$. If $F(\xi) < 1$ the state is said to be sub-Poissonian. For a single-mode and stationary field, as in the present case, sub-Poissonian and antibunching are concomitant effects, hence the study of only one of them is sufficient in the present context.

The application of the Eqs. (5) and (6) for $\widehat{O} = \widehat{n}$, in the Eq. (10), gives

$$F(\xi) = \frac{\xi(\Delta n_M)^2 + (1-\xi)(\Delta\widehat{n}_P)^2 + \xi(1-\xi)(\langle\widehat{n}\rangle_M - \langle\widehat{n}\rangle_P)^2}{\xi\langle\widehat{n}\rangle_M + (1-\xi)\langle\widehat{n}\rangle_P} \ . \tag{12}$$

Note that $F(0) = F^{(P)}$ and that $F(1) = F^{(M)}$, with $F^{(P)} = \Delta\widehat{n}_P^2/\langle\widehat{n}\rangle_P$, $F^{(M)} = \Delta\widehat{n}_M^2/\langle\widehat{n}\rangle_M$, respectively - as it should. At this point we will assume a pure state component $\widehat{\rho}^{(P)}$ which exhibits the sub-Poissonian effect, i.e., $F^{(P)} < 1$. We take

$$\widehat{\rho}^{(P)} = |N_0\rangle\langle N_0| \tag{13}$$

since an arbitrary number state exhibits maximum sub-Poissonian ($F = 0$). Also, to be more specific, we will assume that the mixed state component $\widehat{\rho}^{(M)}$, is a thermal state [see Eq.(2)]

$$\widehat{\rho}^{(M)} = \widehat{\rho}^{(T)} = \sum p_n^{(T)}|n\rangle\langle n| \tag{14}$$

where $p_n^{(T)}$ is the Bose-Einstein distribution

$$p_n^{(T)} = \frac{1}{1+\bar{n}_T}(\frac{\bar{n}_T}{1+\bar{n}_T})^n \tag{15}$$

with[33],

$$\bar{n}_T = \frac{1}{e^\eta - 1}, \quad \eta = \frac{\hbar\omega}{k_B T} \ , \tag{16}$$

and

$$\Delta\widehat{n}_T{}^2 = \bar{n}_T + (\bar{n}_T)^2. \tag{17}$$

For the number state component [Eq.(13)], we have that

$$\langle\widehat{n}\rangle_P = \bar{n}_P = N_0; \quad \Delta\widehat{n}_P{}^2 = 0 \ . \tag{18}$$

The substitution of Eqs. (13), (18) in the (12) gives,

$$F(\xi) = \frac{\xi\bar{n}_T(1+\bar{n}_T) + \xi(1-\xi)(\bar{n}_T - N_0)^2}{\xi\bar{n}_T + (1-\xi)N_0} \ . \tag{19}$$

The most favourable situation to maintain sub-Poissonian statistics in the IPMS, as we find for $\xi = 0$, comes from the condition $\bar{n}_T = N_0$, in which both components have the same (average) excitation. In this case we obtain

$$F(\xi) = \xi(1+N_0), \tag{20}$$

hence $F(\xi) < 1$, if $0 < \xi < 1/(1+N_0)$. For example, for $N_0 = 1$ the IPMS will be sub-Poissonian if $0 < \xi < 1/2$. Fig. 1 shows the Fano's factor $F(\xi)$ as function of the involved parameters, ξ, N_0 and $\bar{n}_T$.

IV. Influence of Mixing upon the squeezing effect

The operational ingredient showing squeezing is the dispersion in the (Hermitean) quadrature operators, $\widehat{X}_1$ and $\widehat{X}_2$, as mentioned before[cf. Eqs.(8) and (9)],

$$\Delta\widehat{X}_i{}^2 = \langle\widehat{X}_i^2\rangle - \langle\widehat{X}_i\rangle^2, \quad , i = 1, 2. \tag{21}$$

Squeezing occurs when $\Delta\widehat{X}_i{}^2 < 1/4$, with $i = 1$ or 2(not both simultaneously, to preserve the Heisenberg uncertainty relation). The application of Eqs. (5) and (6), with $\widehat{O} = \widehat{X}_i$, gives for the quadrature $\widehat{X}_1$, say,

$$\Delta\hat{X}_1^{\,2}(\xi) = \xi\ (\Delta\hat{X}_1^{\,2})_M + (1-\xi)\ (\Delta\hat{X}_1^{\,2})_P + \xi\ (1-\xi)(\langle\hat{X}_1\rangle_M - \langle\hat{X}_2\rangle_P)^2\ . \tag{22}$$

Note that $\Delta\hat{X}_1^{\,2}(0) = (\Delta\hat{X}_1^{\,2})_P$ and $\Delta\hat{X}_1^{\,2}(1) = (\Delta\hat{X}_1^{\,2})_M$, as it should, since the IPMS interpolates between the limiting states, $\hat{\rho}^{(M)}$and $\hat{\rho}^{(P)}$.

At this point we will assume a pure state component, $\hat{\rho}^{(P)}$, which exhibits the squeezing effect. Without loss of generality, we will also assume that the quadrature $\hat{X}_1$ is squeezed, namely, $(\Delta\hat{X}_1^{\,2})_P < 1/4$. Now, a natural candidate for a pure state exhibiting squeezing is the squeezed-coherent state $|z,\alpha\rangle = \hat{S}(z)|\alpha\rangle$, $\hat{S}(z) = \exp[(\hat{z}\hat{a}^{\dagger 2} - \hat{z}^*\hat{a}^2)/2]$. Hence we set

$$\hat{\rho}^{(P)} = |z,\alpha\rangle\langle z,\alpha| \tag{23}$$

with,

$$z = re^{i\phi}\ \ ;\ \ \alpha = |\alpha|e^{i\theta}. \tag{24}$$

The mixed state component is assumed to be the same as in Sect. 3[Cf. Eqs. (14)-(16)]. In this configuration, we have in the Eq.(22), for z real [subscript $T(z,\alpha)$ stands for thermal(squeezed-coherent)],

$$(\Delta\hat{X}_1^{\,2})_M = (\Delta\hat{X}_1^{\,2})_T = \frac{1}{4}(2\bar{n}_T + 1)\ , \tag{25}$$

$$(\Delta\hat{X}_1^{\,2})_P = (\Delta\hat{X}_1^{\,2})_{z,\alpha} = \frac{1}{4}e^{-2r}\ , \tag{26}$$

$$\langle\hat{X}_1\rangle_M = \langle\hat{X}_1\rangle_T = 0\ , \tag{27}$$

$$\langle\hat{X}_1\rangle_P = \langle\hat{X}_1\rangle_{z,\alpha} = |\alpha|\ \cos(\theta)\ e^{-r}\ \ ;\ \ z\ real. \tag{28}$$

The substitution of Eqs.(25)-(28) in the Eq.(22) gives,

$$\Delta\hat{X}_1^{\,2}(\xi,\bar{n}_T,Re(\alpha),r) = \frac{1}{4}\{\xi(2\bar{n}_T+1) + [(1-\xi)(1+4\xi(Re(\alpha))^2)]e^{-2r}\}\ . \tag{29}$$

This result shows that $\Delta\hat{X}_1^{\,2}(0) < 1/4$ for $r > 0$, as it should, since we have chosen the pure state component as a squeezed-coherent state[Eq.(23)]. If $\xi \neq 0$ the IPMS shows squeezing if $\bar{n}_T$ and α are sufficiently small. For example, for $\alpha = 0$ (squeezed vacuum) and $\bar{n}_T << 1$, such that $2\bar{n}_T + 1 \simeq 1$, we have

$$\Delta\hat{X}_1^{\,2}(\xi) \simeq \frac{1}{4}[\xi + (1-\xi)e^{-2r}] < 1 \tag{30}$$

for any ξ in the interval $0 \leq \xi < 1$. Fig.2 is for variance $\Delta\hat{X}_{1\xi}^{\,2}$ as function of the involved parameters, $\xi, \bar{n}_T, \alpha$ and r.

V. Comments and conclusion

We have introduced the IPMS, an intermediate state of the quantized radiation field, which interpolates between a pure state $\hat{\rho}^{(P)}$ and a mixed state $\hat{\rho}^{(M)}$ [Eq.(1)]. The treatment is general since the components $\hat{\rho}^{(M)}$ and $\hat{\rho}^{(P)}$ are arbitrary. We calculate general expressions for the Fano's factor $F(\xi)$[see Eq.(12)] and for the expression of variances of quadrature operators $\hat{X}_i$, $\Delta\hat{X}_i^{\,2}(\xi)$ [see Eq. (29)], the first (latter) allowing us to study the influence of mixing upon sub-Poissonian (squeezing) effect. In both cases the greater or smaller presence of mixing is controlled by the parameter ξ. For sub-Poissonian we have taken the pure state component $\hat{\rho}^{(P)}$ as a number state [Cf. Eq.(13)], due to the fact that this state is the one exhibiting maximum sub-Poissonian ($F = 0$). On the other hand, in the case for squeezing we have taken the pure state component $\hat{\rho}^{(P)}$ as a squeezed-coherent state [cf. Eq. (23)], since it is the "natural" state exhibiting quadrature squeezing.

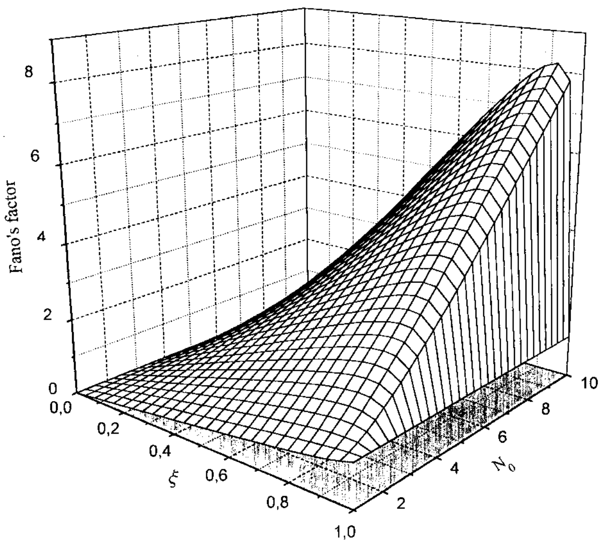

Figure 1a. Fanos's factor as function of the interpolating parameter ξ and the excitation number N_0, for $\bar{n}_T = 0.1$.

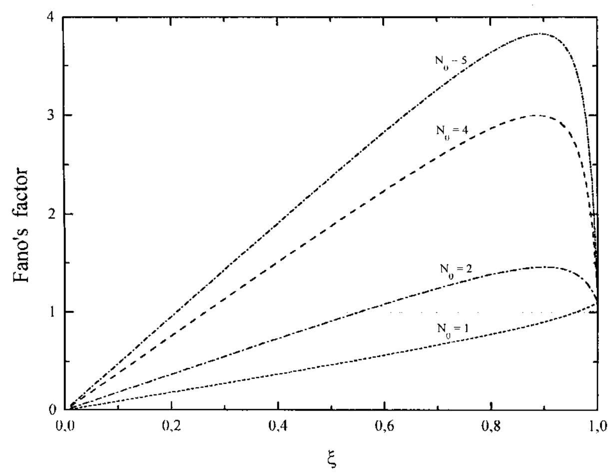

Figure 1b. Vertical cuts in the Fig. 1a, for various values of N_0.

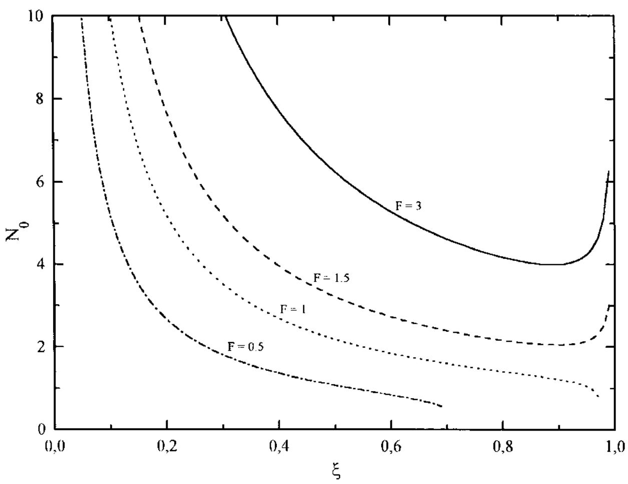

Figure 1c. Horizontal cuts in Fig.1a for various values of Fano's factor.

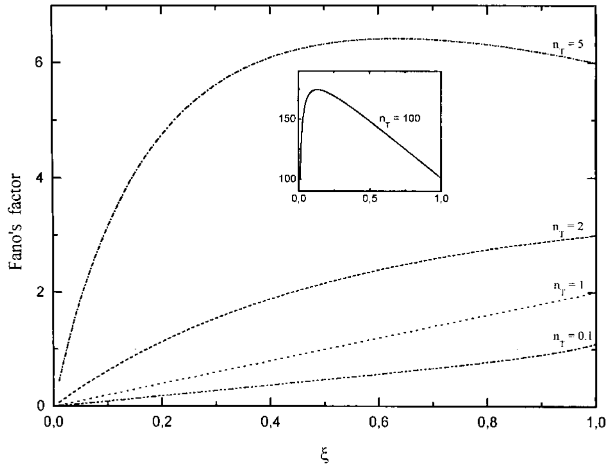

Figure 1d. Fano's fa ctor as function of the interpolating parameter ξ, for $N_0 = 1$ and various values of the thermal excitation $\bar{n}_T$.

Figs. 1 show the plots of Fano's factor [see Eq.(19)] as function of the involved parameters ξ, N_0 and $\bar{n}_T$. Fig. 1a shows the Fano's factor as function of ξ and N_0, for $\bar{n}_T = 0.1$. In this figure we can note that: (i) for $\xi = 0 \Longrightarrow F(0) = 0$, corresponding to the limit of number state; (ii) for $\xi = 1 \Longrightarrow F(1) = 1 + \bar{n}_T = 1.1$, corresponding to the limit of thermal state, for $\bar{n}_T = 0.1$. Fig. 1b is the Fano's factor as function of ξ, for various values of N_0 and $\bar{n}_T = 0.1$, showing vertical cuts in the Fig.1a, for various values of N_0. Note that in this figure the interval of ξ in which the system is sub-Poissonian diminishes when N_0 increases. Also, note that for $N_0 > 1$ all curves for Fano's factor have a maximum about $\xi \simeq 0.9$ (maximum super-Poissonian statistics). For $N_0 = 1$, maximum super-Poissonian is attained at the limit $\xi = 1$, the statistics being sub-Poissonian in the most part of the interval. Fig. 1c shows horizontal cuts in the Fig. 1a, for $F = 0.5, F = 1.0, F = 1.5$ and $F = 3$. Note that these curves decrease monotonically for each fixed value $F < 1$ (sub-Poissonian) whereas reaching a minimum at $\xi \simeq 0.9$, when $F > 1$ (super-Poissonian). Fig. 1d is the Fano's factor as function of ξ, for various values of the parameter $\bar{n}_T$ and N_0 fixed ($N_0 = 1$), showing the influence of temperature, or the average photon-number $\bar{n}_T$, on the statistics of the field. Note that in the limit $\bar{n}_T \to 0$ then $F(\xi) \to \xi N_0$, the IPMS being sub-Poissonian for $0 < \xi < 1/N_0$, $N_0 = 1, 2, 3...$. For $\bar{n}_T \simeq N_0$ then $F(\xi) \to \xi(1 + N_0)$. In this case the IPMS is sub-Poissonian for $0 < \xi < 1/(1 + N_0)$. We see that in both cases ($\bar{n}_T \to 0$, $\bar{n}_T \simeq N_0$) the the Fano's factor increases linearly with ξ, and when we pass from $\bar{n}_T \to 0$ to $\bar{n}_T \simeq N_0$ the interval in which ξ is sub-Poissonian is reduced from $0 < \xi < 1/N_0$ to $0 < \xi < 1/(1+N_0)$. This difference is relevant if N_0 is small ($N_0 < 5$). For $\bar{n}_T \gg N_0$ the Fano's Factor exhibits a maximum about $\xi = \sqrt{2N_0/n}$. In this case the statistics is sub-Poisonian only for very small values of ξ.

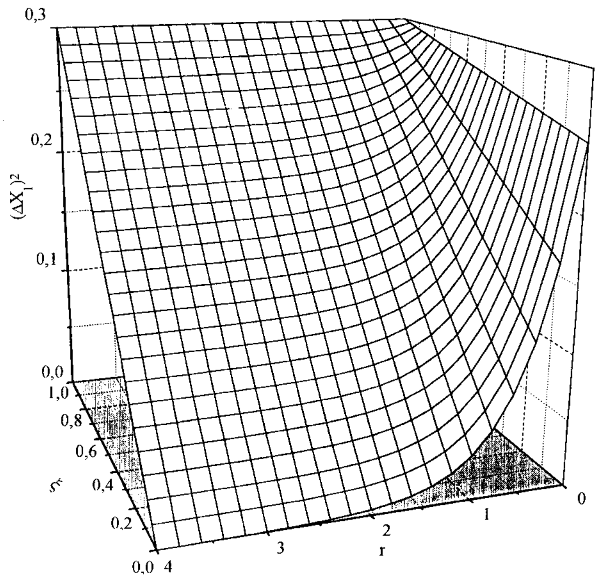

Figure 2a. Plot of variance $\Delta\widehat{X}_1^2$ as function of ξ and the squeezing parameter r, for $\alpha = 0$ (vacuum).

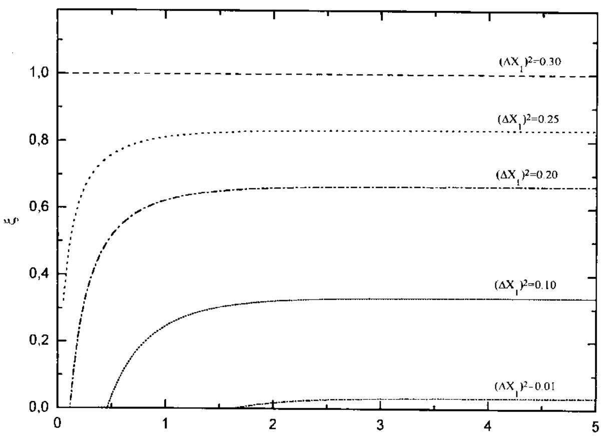

Figure 2b. Horizontal cuts in the Fig. 2a, for various values of variance $\Delta\widehat{X}_1^2$.

Figs.2 show the plots of dispersion in the (squeezed) quadrature $\widehat{X}_1$, as function of the interpolating parameter ξ, for various values of the parameters $\bar{n}_T$, α and r. These figures, obtained from the Eq.(29), are for the IPMS with components $\widehat{\rho}^{(M)}$ and $\widehat{\rho}^{(P)}$ given in Eqs.(14) and (23), namely, for a thermal state component and a squeezed-coherent state component. Note in the Eq.(29) that the mixed state $\widehat{\rho}^{(T)}$ introduces two terms which destroys squeezing: one of them (first term in Eq.(29)) being proportional to the average photon-number $\bar{n}_T$ in the termal state; the second (last term in Eq.(29)), being independent of the average photon-number $\bar{n}_T$. For low thermal excitations ($\bar{n}_T \ll 1$) in such a way that $2\bar{n}_T + 1 \simeq 1$, and α not too large ($\alpha e^{-r} < 1$) the IPMS preserves squeezing: we must have $\alpha < e^r$, which gives $\alpha < 2.7$ for $r = 1$; $\alpha < 150$ for $r = 5$; etc. In this case squeezing is preserved for certain ranges of the parameter ξ. Fig. 2a shows the plot of variance $\Delta\widehat{X}_1^2(\xi)$ as function of ξ and r, for $\alpha = 0$(vacuum) and $\bar{n}_T = 0.1$. Note that for $\xi = 0$ and r running from $r = 0$ to $r = 4$ the variance $\Delta\widehat{X}_1^2(\xi = 0)$ becomes squeezed, as it should; on the other hand, for r=0 and ξ running from $\xi = 0$ to $\xi = 1$ the variance $\Delta\widehat{X}_1^2(\xi)$ increases from 0.25 to 0.3, then anti-squeezing occurs due to the presence of the thermal state, as expected; for $r > 0$, say $r = 2$, the variance $\Delta\widehat{X}_1^2(\xi)$ goes from $\Delta\widehat{X}_1^2(\xi) \simeq 0.01$ to $\Delta\widehat{X}_1^2(\xi) = 0.3$, in the interval $\xi \in [0, 1]$. In this case squeezing is maintained if $\xi < 0.85$. This can be seen in Fig.2b, which corresponds to horizontal cuts in the Fig.2a, for some fixed values of the variance $\Delta\widehat{X}_1^2(\xi)$. Note in Fig.2b that, for very large squeezing ($\Delta\widehat{X}_1^2(\xi) = 0.01 \ll 0.25$), the effect is mantained in the presence of thermal noise only for very small values of ξ ($\xi \leq 0.04$) and for large values of the squeeze parameter r ($r > 1.5$). For squeezing not too large($\Delta\widehat{X}_1^2(\xi) = 0.1$) the effect is maintained for $\xi < 0.25$, if $r \simeq 1$ and for $\xi < 0.3$, if $r \geq 2$. For a small amount of squeezing ($\Delta\widehat{X}_1^2(\xi) = 0.2$) the effect is mantained for $\xi < 0.6$, if $r > 1$. These results show that to maintain a certain amount of squeezing in the presence of the thermal component, this requires increasing the squeeze parameter r. It should also be mentioned that, for $\xi \geq 0.85$ no squeezing results: a thermal field with weight $\xi = \xi_0 = 0.85$ and average photon number $\bar{n}_T = 0.1$ will destroy all squeezing effect in the squeezed-vacuum. When $\bar{n}_T$ increases, this upper bound ξ_0 decreases, thus diminishing the interval ξ in which squeezing effect is preserved.

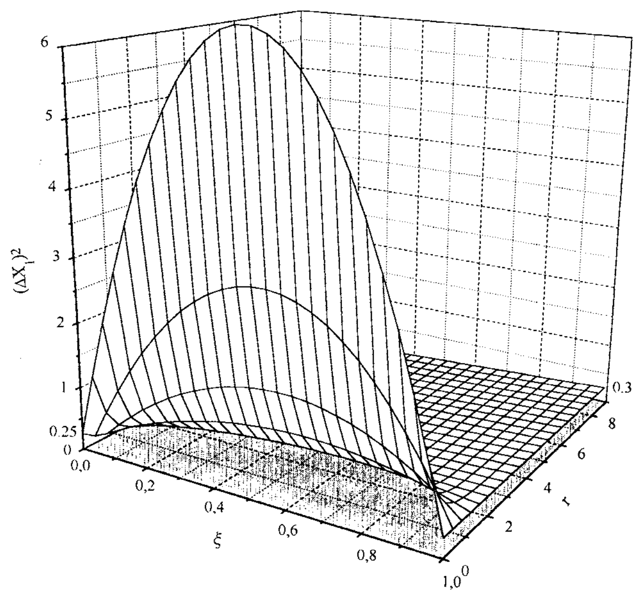

Figure 3a. Same as in Fig.2a, for $\alpha = 5$.

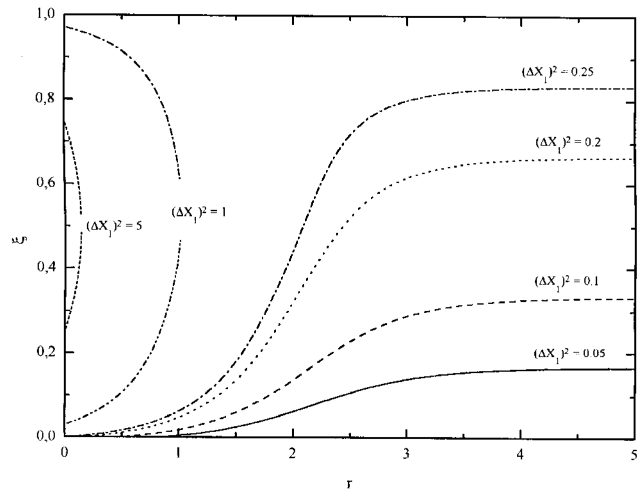

Figure 3b. Same as in Fig.2b, for $\alpha = 5$.

Figs.3 are the same as in Figs.2, except that $\alpha = 5$. Note that for $r = 0$ the varience $\Delta\widehat{X}_1^2(\xi)$ runs from $\Delta\widehat{X}_1^2(0) = 0.25$ to $\Delta\widehat{X}_1^2(1) = 0.3$, as it should. On the other hand, for $\xi = 0$, this variance runs from $\Delta\widehat{X}_1^2(r = 0) = 0.25$ to $\Delta\widehat{X}_1^2(r \gg 1) = 0$, as it should. For $\xi = 0.5$ variance reaches its maximum, for each fixed value of r. When r increases, this maximum in variances decreases. Fig.3b corresponds to horizontal cuts in Fig.3a, for some fixed values of variance $\Delta\widehat{X}_1^2(\xi)$. A comparison between Figs.(2b) and (3b) shows that, in both cases a given squeezing, say $\Delta\widehat{X}_1^2(\xi) = 0.2(0.1)$, is mantained if $\xi < 0.6(\xi < 0.3)$ the limitation in the interval ξ being the same in both figures, the difference being that, to maintain the same squeezing when $\alpha \neq 0$ requires a larger value of r. This results comes from the condition $\alpha e^{-r} < 1$, built-in the Eq.(29), as mentioned before. Another difference between Fig.2b $(\alpha = 0)$ and Fig.3b $(\alpha \neq 0)$ comes from the fact that, for $\alpha \neq 0$ the curves distinguish two regions: one which $\Delta\widehat{X}_1^2(\xi) < 0.25$; the other in which $\Delta\widehat{X}_1^2(\xi) > 0.25$.

As we have seen, the IPMS constitutes an easy tool to investigate the (erasing) influence of mixing upon nonclassical effects, as sub-Poissonian (or antibunching) and squeezing. Other nonclassical effects in quantum optics, such as the collapse and revival effect in the atomic inversion, $\langle\widehat{\sigma}_z\rangle$, can also be investigated throught the application of the IPMS. Such studies are in progress and will be published elsewere.

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