

# Studies of Weak Interactions with Ultra-Cold Atoms

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Received December 09, 1996

In this paper, we discuss the role of trapped atoms for the next generation of precision experiments looking to unveil important details of the weak interactions. In particular, we concentrate on proposals for studies of beta-decay, atomic parity non-conservation and atomic electric dipole moments. By closely inspecting eventual discrete symmetry violations, these experiments will serve as important paths to search for physics beyond the Standard Model.

## I. Introduction

The Standard Model (SM) of the elementary particles in its broader sense is the theory that explains all of the known interactions, with the exception of the elusive force of gravity. This model has attained a number of relevant achievements, including the prediction of outstanding new phenomena -perhaps the most significant of them being the prediction of the weak neutral currents with its subsequent discovery in the early 80s [1].

In spite of its great success, the SM still leaves many important questions unanswered. To give a few examples, masses of known fermions have to enter the theory as parameters that come from experiments, CP violation is poorly understood. To account for these drawbacks, a number of models that assume physics beyond the SM (and that could hopefully fill up the gaps left by it) have been created. Therefore, it is the task of experimentalists to test these models by performing extremely precise searches for physics beyond

the SM. Many of these experiments require the use of large particle accelerators, whereas some of them (as in studies of atomic parity violation and searches for an atomic electric dipole moment -EDM) can be made in table-top set-ups.

With the advent of laser cooling and trapping technologies during the past decade [2], some of these experiments can be performed with an unprecedented accuracy. More specifically, beta-decay, atomic-parity non-conservation and atomic EDM studies can take advantage of these techniques to extend their precision by about one order of magnitude or more.

In the present work, we analyze the three possible types of experiments mentioned above. In particular, we show why the use of trapped atoms can be advantageous for each case. We will also describe some of the experimental efforts in our laboratories that can be useful for this kind of experiments. In what follows, we concentrate on experiments using neutral atoms; studies involving ions have been thoroughly reviewed

in ref.[3].

## II. Beta-Decay

Beta-decay is the process in which an unstable nucleus (or nucleon, in the case of the free neutron) undergoes a transition into another more energetically favorable isotope. It can be described by the following equations for the cases of  $\beta^+$  and  $\beta^-$ , respectively:

$${}^N_Z A \rightarrow {}^{N+1}_{Z-1} A + \beta^+ + \nu \quad (1)$$

$${}^N_Z A \rightarrow {}^{N-1}_{Z+1} A + \beta^- + \bar{\nu} \quad (2)$$

where  $Z$  is the number of protons,  $N$  is the number of neutrons and  $A = Z + N$ . A related process is the electron capture:

$${}^N_Z A + e \rightarrow {}^{N+1}_{Z-1} A + \nu \quad (3)$$

but for the present work we will be more concerned with processes (1) and (2).

In fact, to better understand beta-decay, we will concentrate for a moment on the decay of the free neutron, which is unstable with a half-life of about 10 minutes:

$$n \rightarrow p + \beta^- + \bar{\nu}. \quad (4)$$

Both the neutron and the proton have the same spin 1/2. Therefore, the emitted electron and anti-neutrino

will have a total spin of either 0 or 1. This can occur in four possible ways [4]:

- anti-symmetric (1): electron and anti-neutrino with anti-parallel spins, forming an anti-symmetric wave-function. The proton and neutron will accordingly have zero total spin. This decay is called a Fermi (F) transition;
- symmetric (3): electron and anti-neutrino with spins in the same direction (both up or down) or with anti-parallel spins, forming a total symmetric wave-function. The proton and neutron will have their total spins 0 or 1 for each case. This type of decay will be spin dependent and is known as a Gamow-Teller (GT) decay.

For nuclei, other transitions-referred to as forbidden transitions- can occur. The properties of all transitions are summarized in Table 1. In it, among other things, we show the order of magnitude for the  $ft$  values of these transitions:  $t$  is the half-life due to the decay; the factor  $f$  involves both the Coulomb effect of the nucleus on the ejected particles and dynamical terms. The more probable a transition, the lower the value of  $ft$ .

Table 1: Types of beta-decay found in nuclear transitions and some of their properties [4].

Transition	Spin: $ \Delta J $	Parity: $\Delta\pi$	$\log(ft)$
Super-allowed (F+GT)	0	0	$\approx 3$
Allowed (GT)	$0_{not\ 0} \rightarrow 0,1$	0	$\approx 4-6$
1 <sup>st</sup> forbidden	0,1,2	$\neq 0$	$\approx 6-9$
2 <sup>nd</sup> forbidden	2,3	0	$\approx 11-13$
3 <sup>rd</sup> forbidden	3,4	$\neq 0$	$\approx 18$

Before we turn to investigate how the Hamiltonian that describes the neutron decay looks, let us inspect the familiar case of electromagnetism. The interaction Hamiltonian can be given in a relativistically invariant form by:

$$H_{EM} = e J_\mu^{EM} A^\mu \quad (5)$$

where  $J_\mu^{EM} = (\rho, \vec{J})$  is the current 4-vector and  $A^\mu = (\phi, \vec{A})$  is the potential 4-vector. Using the Dirac matri-

ces:

$$\gamma^0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}; \quad \gamma^\mu = \begin{bmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{bmatrix} \quad (6)$$

where  $\vec{\sigma}$  are the Pauli matrices, the electronic current can be described by:  $J_\mu^{EM} = -e \bar{\psi}_e \gamma_\mu \psi_e$ . This yields:

$$H_{EM} = -e \bar{\psi}_e \gamma_\mu \psi_e A^\mu. \quad (7)$$

In an analogous way, one would be tempted at first to define a current for neutrons and protons (labeled

$H$ , for hadronic) in the same way as in the EM case:

$$J_\mu^H = \bar{\psi}_p \gamma_\mu \psi_n \quad (8)$$

and a current for the electron and neutrino (labeled  $l$ , for leptonic):

$$J_\mu^l = \bar{\psi}_e \gamma_\mu \psi_\nu. \quad (9)$$

Before we proceed to show that the two currents above are incomplete, it is very important to realize a subtle but at the same time drastic change. In eqs. (8) and (9),  $\psi_\alpha$  is not a wave-function for particle  $\alpha$ . Instead, it is an operator that indicates that particle  $\alpha$  was annihilated (as the neutron in eqs. 8 and 4) or an anti- $\alpha$  was created (as the anti-neutrino in eqs. 9 and 4). By the same token,  $\bar{\psi}_\alpha$  indicates that a particle was created (proton in eqs. 8 and 4) or an anti-particle annihilated.

However, the currents defined in eqs. (8) and (9) are not the only possibility. The fact that the currents contain the matrices  $\gamma_\mu$  give them the character of a vector. However, there is no reason *a priori* why not to use a more general form, giving it not only the charac-

ter of a vector, but also that of a scalar, pseudo-scalar, tensor or pseudo-vector. The most general format of the 4-current is then a sum of the following terms (we use the leptonic current as an example):

- scalar:  $\bar{\psi}_e \psi_\nu$ ;
- vector:  $\bar{\psi}_e \gamma_\mu \psi_\nu$ ;
- tensor:  $\bar{\psi}_e \sigma_{\lambda\mu} \psi_\nu$ ;
- pseudo-scalar:  $\bar{\psi}_e \gamma_5 \psi_\nu$ ;
- pseudo-vector or axial:  $\bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu$ ;
- pseudo-tensor:  $\bar{\psi}_e \sigma_{\lambda\mu} \gamma_5 \psi_\nu$ ;

where  $\gamma_5 = i\gamma_0 \gamma_1 \gamma_2 \gamma_3$ .

The Hamiltonian for beta-decay,  $H_\beta$ , comprises a current-current interaction, i.e., instead of involving a current and a potential as in eq. (5) for the EM case, it only involves the hadronic and leptonic currents:  $H_\beta \propto J_\mu^H J^{\mu,l}$ . Then, using the generalized currents, we can define the most general Hamiltonian for the beta-decay [5, 6]:

$$\begin{aligned} H_\beta = & \bar{\psi}_p \psi_n (C_S \bar{\psi}_e \psi_\nu + C'_S \bar{\psi}_e \gamma_5 \psi_\nu) + \bar{\psi}_p \gamma_\mu \psi_n (C_V \bar{\psi}_e \gamma_\mu \psi_\nu + C'_V \bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu) \\ & + \frac{1}{2} \bar{\psi}_p \sigma_{\lambda\mu} \psi_n (C_T \bar{\psi}_e \sigma_{\lambda\mu} \psi_\nu + C'_T \bar{\psi}_e \sigma_{\lambda\mu} \gamma_5 \psi_\nu) - \bar{\psi}_p \gamma_\mu \gamma_5 \psi_n (C_A \bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu + C'_A \bar{\psi}_e \gamma_\mu \gamma_5 \psi_\nu) \\ & + \bar{\psi}_p \gamma_5 \psi_n (C_P \bar{\psi}_e \gamma_5 \psi_\nu + C'_P \bar{\psi}_e \psi_\nu) + H.C. \end{aligned} \quad (10)$$

where the  $C$ s are coupling constants. This Hamiltonian will violate parity P (due to the primed terms) and can also violate time-reversal symmetry, T, if the coefficients have imaginary parts. In fact, the only systems known to violate T (or, similarly CP) are the K mesons [40].

In the SM, the weak interactions are expected to contain only the real part of the terms  $C_A, C'_A, C_V$  and  $C'_V$ . The leptonic current is given by:

$$(J_\mu^l)_{SM} = \bar{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_\nu, \quad (11)$$

which forms the basis of the famous V-A theory postu-

lated simultaneously at Caltech by Feynman and Gellman [7] and at Rochester by Marshak and Sudarshan [8]. The parity violation of the weak interactions (observed dramatically by Wu and collaborators [9]) is naturally included in this theory.

As discussed in the introduction, even though the SM has had great success, it is important to look for physics beyond this theory. Jackson, Treiman and Wyld in as early as 1957 (one year after parity violation was seen) investigated the general form of the Hamiltonian  $H_\beta$ , eq. (10) [6]. They considered only F and GT decays, since for these transitions -unlike the forbidden

ones— the nuclear effects can be relatively easily taken into account. They found that, by careful experimental study of the spectra of the emitted particles, it should

be possible to find out about the possible existence of terms violating time-reversal symmetry. The spectrum of the decaying particles would be given by [6, 10, 11]:

$$\frac{d\Gamma}{d\Omega_e d\Omega_\nu} \cong 1 + A \frac{\vec{J} \cdot \vec{p}_\beta}{E_\beta} + D \frac{\vec{J} \cdot \vec{p}_\beta \times \vec{p}_\nu}{E_\beta} + R \frac{\vec{J} \cdot \vec{p}_\beta \times \vec{\sigma}}{E_\beta} + \dots \quad (12)$$

where  $\vec{J}$  and  $\vec{\sigma}$  stand for the nuclear and beta polarizations, respectively.

The two terms containing the constants  $R$  and  $D$  are of special importance since, if found to be different from zero, they would imply time-reversal violation. These constants are given by:

$$\begin{aligned} R\xi &= 2|M_{GT}|^2 \lambda_{J'J} \text{Im}(C_T C_A'^* + C_T' C_A^*) \\ &+ 2\delta_{J'J} M_F M_{GT} \left(\frac{J}{J+1}\right)^{1/2} \text{Im}(C_S C_A'^* + C_S' C_A^* - C_V C_T'^* - C_V' C_T^*) + O\left(\frac{\alpha Z}{p_\beta}\right) \end{aligned} \quad (13)$$

and:

$$D\xi = 2\delta_{J'J} M_F M_{GT} \left(\frac{J}{J+1}\right)^{1/2} \text{Im}(C_S C_T'^* - C_V C_A'^* + C_S' C_T^* - C_V' C_A^*) + O\left(\frac{\alpha Z}{p_\beta}\right), \quad (14)$$

with

$$\xi = |M_F|^2 (|C_V|^2 + |C_V'|^2 + |C_S|^2 + |C_S'|^2) + |M_{GT}|^2 (|C_A|^2 + |C_A'|^2 + |C_T|^2 + |C_T'|^2), \quad (15)$$

where  $M_F$  and  $M_{GT}$  are the amplitudes for the F and GT processes;  $\lambda_{J'J}$  is a constant that depend on the nuclear spin. Also, the terms on the order of  $\alpha Z/p_\beta$  ( $\alpha$  is the fine structure constant) are the corrections due to the Coulomb interaction that could mimic a time-reversal signal [10]. To make sure that these corrections are small, it is important to use atoms of low  $Z$  whose decays have large energetic differences.

In Table 2, we list the alkali atoms that could possibly be used for this kind of experiments. We restrict ourselves to alkalis since they are the typical protagonists of laser traps (the reason why laser trapped atoms can lead to improvements in the experimental limit will be addressed below). We considered only alkalis of low  $Z$  (Li, Na and K) that decay via super-allowed (F+GT) or allowed (GT) transitions. Also, we required relatively large energetic differences ( $E > 1$  MeV) and suitable half-lives for laser trapping ( $t > 300$  ms). We can estimate the number of trapped atoms by assuming that about  $10^9$  ions/s are produced in the nuclear reactions used and about 1 in  $10^5$  of them get to the trap region. In optimized conditions, about 50% of these can get

captured, so in principle there would be about  $4 \times 10^4$  atoms at any given time at the trap for an isotope with a half-life of 1s.

Experiments to measure the  $D$  coefficient have to use super-allowed transitions (eq. 14). An experiment to detect  $D$  would involve having the parent atoms well polarized and performing measurements of the momenta of the electron and of the neutrino (eq. 12). Since it is almost impossible to measure the momenta of the neutrinos directly, they are accessed by measuring the momenta of the recoiling nuclei. The current experimental limits on  $D$  show that it does not differ from zero to a precision close to  $1/10^4$  [13]. This is already enough to constrain some models that predict physics beyond the SM [14]. Prospects to push this limit even further with the aid of trapped atoms are very encouraging.

The reasons laser trapped atoms are a good alternative for this kind of measurement are: essentially 100% of the atoms can be polarized if, after being initially captured in a magneto-optical trap (MOT) [15], they

are loaded into a magnetic trap [16]. Also, the recoiling nuclei will not be much affected by the dilute atomic gas so their momenta can be better determined. Some of the experimental difficulties that will be encountered include efficiently transporting the ions produced in the nuclear reaction to the trap, neutralizing them and avoiding depolarization through collisions [17]. From Table 2, we see that the candidates for a  $D$  measure-

ment on laser cooled atoms are  $^{21}\text{Na}$ ,  $^{36}\text{K}$  and  $^{37}\text{K}$ . It is important to notice that two of them have already been trapped, namely  $^{21}\text{Na}$  [18] and  $^{37}\text{K}$  [19]. These two isotopes decay through transitions between mirror-nuclei (i.e.,  $Z_i = N_f$ ;  $Z_f = N_i$ ) which makes them very attractive since theoretical complications in their case are negligible.

Table 2: Isotopes of alkali atoms that can in principle be used for beta-decay experiments in laser traps. The half-life of each isotope is shown along with properties of the beta-decay branch of interest: the energetic difference,  $ft$  values, type of transition and its branching ratio. Data from ref. [12].

Isotope	Half-life	$E_\beta$ (MeV)	$\log(ft)$	Transition (B.R.)
$^8\text{Li}$	838 ms	13	5.6	GT (100%)
$^{20}\text{Na}$	447 ms	11.3	5.0	GT (79%)
$^{21}\text{Na}$	22.5 s	2.5	3.6	F+GT (95%)
$^{24}\text{Na}$	15.0 hr	1.4	6.1	GT (100%)
$^{25}\text{Na}$	60.0 s	3.8	5.3	GT (63%)
$^{26}\text{Na}$	1.1 s	7.4	4.7	GT (88%)
$^{27}\text{Na}$	0.3 s	8.0	4.3	GT (84%)
$^{36}\text{K}$	342 ms	$\left\{ \begin{array}{l} 9.9 \\ 5.3 \end{array} \right.$	4.8	F+GT (44%)
$^{37}\text{K}$	1.2 s		3.5	GT (42%)
$^{38m}\text{K}$	924 ms	5.1	3.7	F+GT (98%)
$^{38}\text{K}$	7.6 min	5.0	3.5	F (100%)
$^{38}\text{K}$	7.6 min	2.7	5.0	GT(100%)
$^{44}\text{K}$	22.1 min	2.4	6.1	GT (28%)
$^{45}\text{K}$	17.8 min	2.1	5.7	GT (51%)
$^{47}\text{K}$	17.5 s	4.0	4.8	GT (81%)
$^{48}\text{K}$	6.8 s	5.0	5.2	GT (23%)

The  $R$  term requires for its determination a measurement of the polarization of the beta-particles, which entails more elaborate experimental techniques. It is presently limited by 1 part in 100. [13]. As stated by Häusser, a measurement of  $R$  at a level of precision of 1% for  $^{37}\text{K}$  would usefully constrain the constants  $C_s$  of eq.( $H_\beta$ ) [17].

Inspecting Table 2, we find the peculiar transition of  $^{38m}\text{K}$ . It decays through a pure Fermi transition, which is of special importance since it reveals the value of  $V_{ud}$  without theoretical difficulties. Here,  $V_{ud}$  is the matrix element of the CKM matrix that describes the mixing of  $u$  and  $d$  quarks [4]. Its measurement is important to inspect for the unitarity of the CKM matrix ( $V_{ud}^2 + V_{cs}^2 + V_{ib}^2 = 1$ ), which reveals the number of generations of particles -thought to be three in the

SM [20]. The metastable isotope  $^{38m}\text{K}$  has also been recently trapped at TRIUMF in Canada [19].

Before we conclude this section, we would like to mention two recent experimental proposals. One of them involves the stimulation of beta-decay in a Bose condensate [21]. It was predicted that a condensate with  $10^{14}$  beta-unstable atoms mixed with an ion trap with about 1000 ions can enhance the beta-decay rate by about a factor of two, in a process somewhat analogous to the stimulated emission on lasers. The other one, developed by two of the authors (R.E. and N.P.B.), showed that it would be possible to obtain limits on the chemical potential of cosmological neutrinos using a sample of atoms in a condensate [22]. These neutrinos were created shortly after the Big Bang and nowadays form a fermion gas that permeates the Universe with a

temperature  $T_\nu \cong 2$  K. It is usually assumed that the chemical potential associated with this gas is zero, but a non-zero value could indicate the existence of an asymmetry on the number of neutrinos and anti-neutrinos –which can only be ruled out by experimental analysis. It is surprising that the conditions found currently in Bose condensation experiments [15] can already set constraints close to the ones given by nuclear physics experiments. However, to arrive at limits that can compete with other indirect evidences it would be necessary to have a condensate with as many as  $10^{16}$  atoms. The possibility to realize both experiments mentioned above in a near future is not evident at the moment.

### III. Atomic Parity Non-Conservation

The phenomenon of parity non-conservation in atoms shows up as a consequence of the weak interac-

tion between the electrons and the nucleus. The Hamiltonian that describes this interaction is basically the same as the one describing the beta-decay (eq. 10). However, to describe atomic parity non-conservation (APNC), we can limit ourselves to the terms that appear in the V-A theory (terms containing  $C_A, C'_A, C_V$  and  $C'_V$ ). Also, it is more customary to replace the coupling constants of eq. (10) by ones that describe the interaction between the electrons and the quarks that constitute the nucleons: a neutron is made out of 1  $u$  and 2  $d$  quarks; a proton is 1  $d$  and 2  $u$  quarks. The parity violating part of the Hamiltonian will then be given by [23]:

$$H_{APNC} = H_1 + H_2, \quad (16)$$

with:

$$H_1 = \frac{G_F}{\sqrt{2}} \sum_i \bar{\psi}_e \gamma_\mu \gamma_5 \psi_e (C_{1u} \bar{\psi}_{u,i} \gamma_\mu \psi_{u,i} + C_{1d} \bar{\psi}_{d,i} \gamma_\mu \psi_{d,i}) \quad (17)$$

and:

$$H_2 = \frac{G_F}{\sqrt{2}} \sum_i \bar{\psi}_e \gamma_\mu \psi_e (C_{2u} \bar{\psi}_{u,i} \gamma_\mu \gamma_5 \psi_{u,i} + C_{2d} \bar{\psi}_{d,i} \gamma_\mu \gamma_5 \psi_{d,i}), \quad (18)$$

where  $G_F$  is the Fermi coupling constant of the weak interactions ( $G_F \cong 10^{-5} \text{ GeV}^{-2}$ ), the sum is over all quarks in the nucleus and the  $C$ s are the new coupling constants mentioned above. These constants can be given to first order as a function of a single argument:

$$C_{1u} = \frac{1}{2} \left(1 - \frac{8}{3} \sin^2 \theta_w\right) \quad (19)$$

$$C_{1d} = -\frac{1}{2} \left(1 - \frac{4}{3} \sin^2 \theta_w\right) \quad (20)$$

$$C_{2u} = -C_{2d} = \frac{1}{2} \left(1 - \frac{8}{3} \sin^2 \theta_w\right). \quad (21)$$

Here,  $\theta_w$  is the weak mixing angle or Weinberg angle that can be expressed as:

$$\sin^2 \theta_w = 1 - (M_W/M_Z)^2. \quad (22)$$

The quantities  $M_Z$  and  $M_W$  are the masses of the bosons that intermediate the weak interaction. It is

remarkable that the experiments on APNC deal with atomic transitions (with energies of the order of 1 eV) and can yield information on particles of mass of the order of 100 GeV!

The first term of eq. (16) can be reduced to a simple form in the non-relativistic limit:

$$H_1 = \frac{G_F}{2\sqrt{2}} Q_W \rho(\vec{r}) \psi_e^\dagger \gamma_5 \psi_e \quad (23)$$

where  $\rho(\vec{r})$  is the probability density of the nucleons and  $Q_W$  is the so-called weak charge. It is analogous to the more common electric charge on the EM case, except that the weak interaction does not possess a static form (as in electrostatics). The weak charge is related to the strength of the APNC interaction and is simply given by a sum of the electron-quark coupling constants, weighted by the number of  $u$  and  $d$  quarks in the nucleus:

$$Q_W = 2[(2Z + N)C_{1u} + (Z + 2N)C_{1d}] = Z(1 - \sin^2\theta_W) - N. \tag{24}$$

The Hamiltonian  $H_1$ , due to its dependence on  $Q_W$  and  $\psi_e^\dagger \psi_e$ , will grow as  $Z^3$ . For this reason, experiments that look for APNC signals use heavy atoms such as Cs [24, 25], Tl [26] and, ultimately, Fr [27].

The second term of the Hamiltonian,  $H_2$ , is smaller than  $H_1$  by about two orders of magnitude. This term is spin dependent and will be zero for nuclei with even number of protons and neutrons. It is predicted that a major contribution to this term would come from a nuclear anapole moment, which is a P- odd multipole of the nucleus that arrives from parity violating nuclear forces [28]. In the non- relativistic limit,  $H_2$  will be proportional to the constant  $k_a$  which by its turn is proportional to a power of the nuclear anapole moment  $a$ :  $k_a \propto a^{2/3}$ . For  $^{133}\text{Cs}$ , it was predicted that  $k_a = 0.25$  to  $0.33$ , whereas the APNC experiment performed by Wieman's group at Colorado (to be described below)

yielded  $k_a = 0.76(39)$  [24]. An experiment with higher precision by one order of magnitude should be able to confirm the existence of anapole moments.

In what follows, we will briefly describe the experiments that use the Stark effect performed at Colorado [24, 25]. Many important details will not be discussed here [29]. The experiments that make use of optical rotation will also not be described here since they can not be readily adapted for use with laser trapped atoms. They are reviewed in ref. [30].

The Stark experiments rely on the fact that the atomic energy eigenstates are not eigenstates of the  $H_{APNC}$  Hamiltonian. Hence, one can use perturbation theory (taking  $H_{APNC}$  as the perturbing Hamiltonian) and find that a state of principal quantum number  $n$  and null angular momentum  $|\psi\rangle$  will be transformed into the perturbed state  $|\widetilde{\psi}\rangle$ :

$$|\widetilde{nS}\rangle = |nS\rangle + \sum_{n',l' (n' \neq n, l' \neq 0)} \frac{\langle nS | H_{APNC} | n'l' \rangle}{E_{n'l'} - E_{nS}} |n'l'\rangle. \tag{25}$$

From the above, it can be seen that the electromagnetically forbidden transition  $nS \rightarrow (n+1)S$  is now possible. Its amplitude:

$$E_{APNC} = \langle (n+1)S | e\mathbf{x} | \widetilde{nS} \rangle + \langle \widetilde{(n+1)S} | e\mathbf{x} | nS \rangle \tag{26}$$

would be however extremely small. If we were to send light resonant with  $nS \rightarrow (n+1)S$ , the excitation rate would be proportional to  $|E_{APNC}|^2$ , a signal too tiny to be detected. By using an external electric field, the states  $|nS\rangle$  will be further mixed due to the Stark effect and the transition rate for a  $nS \rightarrow (n+1)S$  would occur due to both processes, yielding a measurable transition rate of:

$$|E_{st} + E_{APNC}|^2 = E_{st}^2 \pm 2E_{st}E_{APNC}, \tag{27}$$

where  $E_{st}$  is the Stark effect amplitude which is pro-

portional to the applied field.

In the Colorado experiments, cesium atoms are sent to a region of uniform electric-field  $E$ , where a strong laser beam drives the atoms from the  $6S$  to the  $7S$  state. After being excited, the atoms decay through the transitions  $7S \rightarrow 6P$  and then  $6P \rightarrow 6S$ , when characteristic photons are emitted. By detecting these photons, the excitation rate is deduced. Finally, by reversing the E-field, the interference term changes sign and the value of  $E_{APNC}$  can be found.

Once the value of  $E_{APNC}$  is known, we still need

to extract physics from it. To do that, one uses atomic theory calculations to find how the weak charge  $Q_W$  is related to the amplitude  $E_{APNC}$ . It is found that [31]:

$$E_{APNC} \cong -0.9 \times 10^{-11} i |e| a_0 (-Q_W/N). \quad (28)$$

From the results of Noecker et al for  $E_{APNC}$  [24] and theoretical results as above, it is found that:

$$(Q_W)_{Cs} = -71.04 \pm (1.58)_{exp} \pm (0.88)_{th}. \quad (29)$$

The first uncertainty comes from the experiment (statistical and systematic) and the second from uncertainties in the theoretical calculation. From the result above and eq. (22) one can precisely obtain  $\theta_W$ .

It has been recently shown by Rosner [32] that increasing the accuracy of  $Q_W$  by a factor of 4 (0.5% relative accuracy) would noticeably constrain the parameters related to physics beyond the SM. From the value above, we see that if the experiment is made better by a factor of 4, the value of  $Q_W$  would be limited by the theoretical calculations. Even though there are tantalizing prospects that theory will also break the 0.1–0.5% accuracy barrier, it would be necessary to have these theories “calibrated” at these levels, by checking them against other well-measured quantities, which is not a trivial matter.

Experimentalists can help on this battle front by employing more than one isotope of the same element in their experiments [33]. The signal for different isotopes could be used to extract ratios of  $Q_W$ , in which the theoretical calculations would be nearly washed out.

In the case of cesium, there is only one stable isotope ( $^{133}\text{Cs}$ ). Experiments using beams of the other unstable isotopes would not be possible since the fluxes would be severely low. This is the main reason to use laser trapped atoms for this kind of experiments: the unstable atoms can be produced in an accelerator and then accumulated in a MOT where the APNC measurements would be performed (in fact, while the measurements are accomplished the MOT should be off to avoid problems due to the laser light and non-uniform magnetic fields). A further advantage is that the cold atoms in the trap will have a narrow linewidth, which increases transition rates and enables better results.

Although it still remains to be shown that APNC experiments can be performed at high-precision levels

in magneto-optical traps, it has been shown in a number of laboratories that short-lived isotopes can be captured in these traps [18, 19, 27, 34]. A particularly interesting case is the experiment with Fr that is predicted to have an APNC signal 18 times larger than Cs.

While the short-lived isotopes needs to be trapped right after being produced (on-line), longer-lived isotopes can be prepared in a nuclear facility and later be transported to a laboratory where they would be loaded into a MOT for the APNC measurements. In the case of Cs, there are three isotopes that have half-lives of more than a year:  $^{134}\text{Cs}$  ( $T_{1/2}=2$  yr),  $^{135}\text{Cs}$  ( $T_{1/2}=2 \times 10^6$  yr),  $^{137}\text{Cs}$  ( $T_{1/2}=30$  yr). Since their relative yield as a fission by-product is similar [35], a 1 mg sample produced three decades ago would roughly contain no  $^{134}\text{Cs}$ , equal amounts of  $^{133}\text{Cs}$  and  $^{135}\text{Cs}$  and about half as much  $^{137}\text{Cs}$ . This would correspond to an activity of the order of 10 mCi (due to  $^{137}\text{Cs}$ ). Even though this corresponds to a highly active material, it should be possible to use such a sample if careful procedures are followed.

At the University of São Paulo, two tests were realized to check how feasible it would be to trap these long-lived isotopes. First, we verified that low amounts of CsCl could be efficiently transformed into the atomic form of Cs with minimal manipulation required. This is a useful test, since the irradiated material would not come in the chemically reactive metallic form, but in some other form, possibly dissolved in HCl. Second, we tested if it was possible to trap  $^{133}\text{Cs}$  atoms departing from small samples, which would be necessary due to the high specific activity of the material. We prepared an ampoule containing 3 mg of stable Cs that was used to successfully load a MOT. Together, these tests demonstrate that the prospects to load long-lived radioactive isotopes in a MOT in an off-line manner are reasonable. A more detailed account of these experiments is being prepared.

We are also inspecting the possibility of realizing these kind of experiments by detecting the signal via ion- instead of photon-detection. The atoms excited to the  $7S$  state would interact with a second laser source that would ionize them. Since ion detection can occur

with efficiencies of virtually 100%, the signal should increase considerably. Furthermore, there would be basically no background -in contrast to the case of photon detection, where filters have to be used to eliminate the light that drives the atoms to the  $7S$  state from the relevant signal. Efforts towards these goals are under way at São Carlos, where experiments to study spectroscopy of highly-excited levels of Cs and their ionization rates are being prepared.

#### IV. Electric dipole moment

The first experiment attempting to find a permanent electric dipole moment (EDM) was performed in 1950 [36], years before the violation of discrete symmetries was theoretically predicted [5]. However, it was not until parity violation was verified [9] that the results were finally published [37]. This first search tried to find a non-zero EDM for the neutron. Subsequent experiments have improved the obtained limit by about 6 orders of magnitude. A review of the experiments on neutron EDM appears in ref. [38].

Other systems frequently used for searches of permanent EDMs are atoms and molecules. In this section, we will focus on the experiments with atoms and, in particular, on how *laser cooled* atoms can help to improve the current experimental effort. A thorough review of the experiments on EDM for neutrons, molecules and atoms appears in ref. [39].

It is fairly straightforward to see how a permanent EDM of a physical system would have to violate two discrete symmetries, namely parity and time-reversal. The violation of parity can be seen from the following integral, describing a dipole matrix element:

$$d_{ij} = e \int_{-\infty}^{+\infty} \psi_i(x) x \psi_j^*(x) dx, \quad (30)$$

which in principle should vanish for permanent dipole moments  $d_{ii}$ . However, if the states  $\psi_i(x)$  are actually mixed with other states of opposite parity, the integral can be non-zero and parity is violated.

The time-reversal non-conservation can be seen with the help of Fig.1 [39]. In Fig. 1a, we see a schematic representation of an electron, e.g., with its spin pointing up. We choose the electron's EDM to be pointing up as

well. Note that the EDM has to be in the same direction as spin, otherwise there could be a new quantum axis in which the projections of EDM up and down could be found. The Pauli exclusion principle would then enable the existence of atoms with twice as many electrons, which is not the case. Next, we perform a time-reversal operation (Fig. 1b), followed by a rotation (Fig. 1c). Since we know from the conservation of angular momentum that physics is invariant under rotations, the only way to explain the discrepancy between Figs. 1a and 1c is by realizing that the permanent EDM violates the time-reversal operation [41].

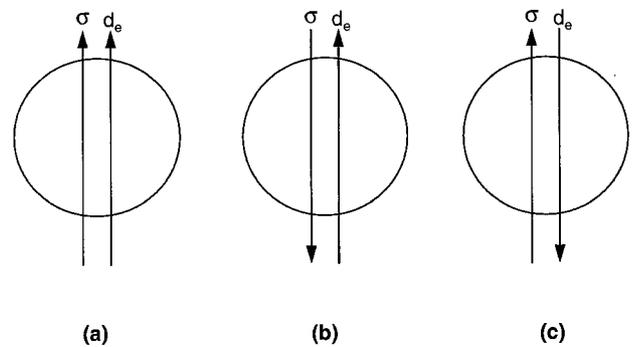


Figure 1: a) An electron is schematically shown with its spin pointing up and with a permanent dipole moment also pointing up; b) after a T-reversal operation, the spin points down, the EDM still points up; c) after a rotation by  $180^\circ$ , the spin points up as in (a), but the EDM now points down [39].

There are four possible sources for an atomic EDM ( $d_a$ ) [23]:

- an electron EDM;
- a nucleon EDM;
- a P,T-odd interaction between the electrons and nucleons. The Hamiltonian of interaction would be the similar to eq. (10);
- a P,T-odd interaction between the nucleons.

The experiments we will concentrate on are primarily concerned with a possible electron EDM ( $d_e$ ). At first thought, one could expect that even if the electrons were to possess a finite EDM, the atom would not have an EDM due to screening effects. However, by taking relativistic effects into account, Sandars [43] showed that atoms not only can have a finite EDM ( $d_a$ ), but also that  $d_a$  can be enhanced with respect to the

electron EDM for heavy paramagnetic atoms. Remarkably, the enhancement ratio  $R = d_a/d_e$  can be quite high:  $R_{Cs} = 114$ ,  $R_{Tl} = -585$  [42].

The experimental searches for atomic EDM look for an energy difference  $\Delta E = 2 \vec{d}_a \cdot \vec{E}$  between the cases of permanent dipole moment  $\vec{d}_a$  parallel or anti-parallel to an applied field  $\vec{E}$ . Also, many of them use in a way or another the method of separated oscillatory fields [44].

One could think of performing an EDM experiment in the following way: an atom is initially prepared in a state  $m$ . It then passes through an interaction region where a  $\pi/2$  pulse is applied [45] such that the atom undergoes the following transition:

$$|m\rangle \rightarrow \frac{1}{\sqrt{2}}(|m\rangle + |-m\rangle). \quad (31)$$

After that, the atom passes through a region where uniform electric ( $E$ ) and magnetic ( $B$ ) fields exist which are perpendicular to the original spin of the atom and also carefully aligned parallel to each other. Due to the magnetic moment  $\mu$  of the atomic nucleus and the eventual permanent EDM, the atom with a velocity  $v$ , after transversing this region of length  $L$  will be left in the state:

$$|\Phi_1\rangle = \frac{1}{\sqrt{2}}[e^{iL/\hbar v (\mu B \pm d_a E)}|m\rangle + e^{iL/\hbar v (\mu B \mp d_a E)}|-m\rangle], \quad (32)$$

where the signs depend on the orientation chosen for the  $E$  field with respect to  $d_a$ .

Afterwards, the atom passes through a second interaction region where a second  $\pi/2$  pulse is applied. Now, the state of the atom will become:

$$|\Phi_1\rangle = \frac{1}{\sqrt{2}}\{[e^{iL/\hbar v (\mu B \pm dE)} - e^{iL/\hbar v /(\mu B \mp dE)}]|m\rangle + [e^{iL/\hbar v (\mu B \pm dE)} + e^{iL/\hbar v (\mu B \mp dE)}]|-m\rangle\}.$$

Therefore, the probability of finding the atom in state  $|-m\rangle$ , e.g., will change from  $\cos^2[\frac{L}{\hbar v}(\mu B + dE)]$  to  $\cos^2[\frac{L}{\hbar v}(\mu B - dE)]$  from the case of  $E$ -field pointing one way or another. Hence, in practice, one would search for a change in the effective Larmor frequency of  $2dE/\hbar$ .

Of course, the real experiment is more complicated than this simple “model” experiment. The main technical challenge is to keep all sources of statistical uncertainties as small as possible. One of the major sources

of such errors is the fact that the atom is in motion with a velocity  $v$ . In its frame of reference, it will feel a motional magnetic field from the applied electric field given by:  $\vec{B}_{mot} = -\vec{v}/c \times \vec{E}$ . Therefore, any misalignment between  $\vec{B}$  and  $\vec{E}$  will change the Zeeman frequency and mimic the EDM effect when the direction of the  $E$ -field is reversed.

The most frequently used method to avoid this problem is to use cells instead of beams [46]. In them, the average velocity of the atoms is zero and the motional effect becomes very small. The RF pulses described in the model experiment above would then be separated by time, instead of space. Usually, two cells are used side by side, to account for possible stray magnetic fields. The systematic uncertainties can be further reduced by introducing a lighter atom for which the EDM effect (that grows with  $Z^3$ ) is small. This atom would then serve as a B-field monitor while a heavier atom provides the EDM signal.

More recently, Commins and collaborators at Berkeley introduced a technique in which an atomic beam is used, but that has a relatively small motional systematic uncertainty [42]. Their apparatus is symmetric with respect to the beam direction and, by opening and closing valves, the beam goes one way or another: on average, the motional effect is much smaller than that of a single beam. This experiment yielded the best current experimental limit in the electron EDM of  $d_e < 4 \times 10^{-27} e \text{ cm}$ , which already constrains a few models that predict physics beyond the SM, as shown in Fig.2.

Even though the experimental methods mentioned above reduced the motional effects and helped improve the experimental limits by orders of magnitude since the first generation of experiments [39], the  $B_{mot}$ -field is still one of the major problems in the search for an atomic EDM. It is precisely in this aspect that cold atoms can play an important role –their ultra-low velocities can virtually make the motional systematic uncertainty negligible. If a shot-noise limited experiment can be performed on laser cooled Cs, the systematic uncertainty will be given by [47]:

$$\sigma(d_e) = \frac{h}{2R_{Cs}E} \sigma(\nu_{SN}). \quad (34)$$

where  $E$  is the applied field. A shot-noise uncertainty for a single interrogation of  $\sigma(\nu_{SN})=10 \text{ Hz}$ , could lead

to an uncertainty below  $10^{-28} e$  cm after many interrogations. This would improve the limit by over one order of magnitude which would be sufficient to rule out more than one model that predicts physics beyond the SM (Fig.2). Furthermore, according to work by Bijlsma and collaborators, second-order Stark effects can be made negligible in laser cooled Cs [47]. It remains to be seen if other sources of systematic uncertainty can be reduced below shot-noise.

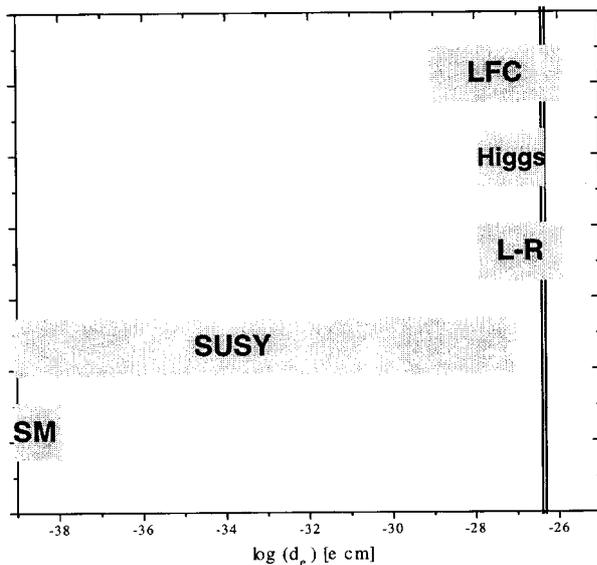


Figure 2: The current best limit on the electron EDM is shown as a dashed line along with the allowed range for  $d_e$  for the standard (SM), super-symmetric (SUSY), left-right symmetric (L-R), Higgs and lepton-flavor-changing (LFC) models. Data from ref. [42].

In an experiment using laser cooling and trapping techniques, atoms would initially be loaded in a MOT. However, since in this environment magnetic and laser fields are present, one would have to take extra steps to realize the EDM measurements. We will consider three different alternatives: free expansion, atomic fountains and dipole traps. The first case would be characterized by periods with the MOT on, when atoms would be captured and cooled down, and MOT off, when the EDM measurement would be performed. Since the atoms would be allowed to fall, the interrogation times would be small (30 ms for a field that is homogenous in a region of 5 mm) and the atoms would attain relatively high velocities in the direction of the fall. A way to improve this scenario is to use micro-gravity environments in space. This would however make the experiment much more costly and complicated.

The two other alternatives seem to be more attractive. The atomic fountain [48] is attained with a strong laser beam resonant with atoms loaded in a MOT: once the laser hits the atoms from the bottom of the trap, they will be pushed up. These atoms will enter an interaction region at the top of their trajectory, when their velocities are extremely small. RF fields can be applied on the way up and then on the way down enabling a Ramsey interference measurement.

In the dipole trap case, the atoms can interact with far-off resonance light which means that AC-Stark shifts and loss of coherence due to spontaneous emission should be small. For red-detuned dipole traps [49], for which atoms are attracted to field maxima, this effect can still be non-negligible and lead to possible systematic errors. For blue-detuned traps, the atoms are repelled by the laser fields and can therefore accumulate in the dark. To our knowledge, only two approaches have successfully trapped atoms in an all-optical dark trap via the dipole force. One of them uses several sheets of light from an Ar<sup>+</sup> laser and was used to show Raman cooling below the recoil limit [50]. Another type of trap, called RODiO (Rotating Off-resonant Dipole Optical trap) was developed at Rochester [51] and is a promising alternative for EDM experiments. It consists of a focused blue-detuned laser beam that is scanned at high frequencies (2–5 kHz) around an atomic cloud loaded from a MOT. Confinement times of 25 ms were observed at a non-optimized version. Efforts to push this limit further are under way and consist mainly of creating end-caps for the trap (the current set-up provides only 2-D confinement) and using higher scan frequencies.

As mentioned above, another experimental artifice to reduce systematic errors is to use a mixture of atoms: one of low  $Z$  (B-field monitor) and another of high  $Z$  (EDM monitor). In recent experiments in our laboratories, it has been demonstrated that such mixtures can exist for laser cooled alkalis: a mixture of Na and Cs has been studied at Rochester [52] and a mixture of Na and K at São Carlos [53]. The former is especially interesting for EDM experiments, due to the low  $Z$  of Na and the large ratio  $R$  for Cs ( $R_{Cs} = 114$ ).

Before we conclude, we briefly address the potential problem of loss of coherence due to collisions. Although collisional shifts do not change sign under reversal of

E-fields, their presence can threaten the experimental resolution. This problem was thoroughly investigated in ref. [47], where it was found that the collisional dephasing rate should be a minor trouble: it is virtually negligible for atomic fountains, could limit densities to values below  $10^{12} \text{ cm}^{-3}$  for dipole traps with interrogation times longer than 1 s (frequently, much lower densities are found in such traps).

## V. Conclusions and perspectives

The proposed studies discussed above involve sophisticated experimental techniques that span knowledge from widely different areas in physics: from optics and atomic physics to nuclear and high-energy physics. Realistically, a quick outlook indicates that it will still take a few more years for these experiments to yield reliable results. However, the prospects of finding important evidences for physics beyond the SM or even to once more confirm the almost unshakable predictions of this model are very stimulating.

Beta-decay experiments have already gained a strong impulse after the laser trapping of interesting potassium isotopes at TRIUMF and  $^{21}\text{Na}$  at Berkeley. On the APNC front, the prospects for much more accurate results from a new generation of beam experiments using stable cesium at Colorado [29] push the theoretical physics community to improve the calculations relating  $E_{APNC}$  to  $Q_W$  (eq. 26) and, at the same time, motivate the experimentalists to begin tests with unstable cesium and eventually francium at the ISOLDE facility at Cern. Finally, EDM experiments using laser cooled atoms still need to have their debut, but the prospects are good after the development of different kinds of dipole traps and the demonstration of precise spectroscopy performed with atomic fountains.

In conclusion, it will not be surprising if the next decade witnesses a wealth of new results from weak interaction studies using ultra-cold atoms.

This work was supported by the NSF, the David and Lucile Packard Foundation, CNPq and Fapesp.

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