Inelastic Light Scattering from Alkali Bose- Einstein Condensates

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We determine the spontaneous and stimulated non-resonant combination light scattering cross sections from a finite-size trapped alkali Bose-Einstein condensate (BEC) tor scattered radiation with frequencies which differ from the incident frequency by elementary vibrational-rotational-spin excitations of the BEC. We consider incident microwave frequency radiation not far from the hyperfine splitting resonance frequency, and incident visible frequency radiation cases. Bosonic stimulation (stimulation due to the population of quantum states by Bose-Einstein particles) is a telltale signature of BEC since it drastically affects the relative and absolute line intensities. The strongest feature in the scattered radiation is the elastic scattering peak; the intensities of red and blue shifted scattering peaks depend strongly on temperature even for temperatures below the critical temperature for BEC. The Stokes (red-shifted) peaks are more intense than the anti-Stokes (blue-shifted) peaks, and as $T \rightarrow O$, all blue shifted features in the scattering disappear.

The excitation spectrum of superfluid ^{4}He and superconductive materials are crucial in understanding the collective properties that determine their special character[1]. It is likely that the excitation spectrum of a Bose-Einstein condensate (BEC) of finite-size trapped alkali systems[2-4] may similarly ordain any special collective many-body characteristics that they might have. The vibrational-rotational excitations of such BEC systems can differ from those of the uncondensed system due to the interactions of the alkali atoms. Hence, these excitations can provide a means of determining the properties of the condensate. Bogoliubov[5] theoretically predicted the excitation spectrum for a homogeneous, zero-temperature weakly-interacting BEC by using linear response to a weak single-frequency probe. However, the excitation spectrum of a finite-size finite-temperature weakly-interacting alkali BEC is expected to be substantially different in character[6]. We recently suggested[7] that spontaneous and stimulated combination scattering, i.e., spontaneous and stimulated Raman-Landsberg-Mandel'shtam-Brillouin scattering, as a viable experimental method for determining the vibrational-rotational excitations of a finite-size trapped alkali BEC and worked out the theoretical formulation for analyzing such experiments. Combination scattering results in the appearance of frequencies which differ from the incident frequency by vibrational-rotational and magnetic excitations of the finite-size trapped alkali BEC. The incident frequency should be close, but not too close, to a transition in the alkali atom, so that the scattering probability is large, but not too large (perturbation theory for the radiative processes should still be valid and the probability of transitions should be negligible). For the alkali BECs of Refs. 2-4, if the incident frequency is in the GHz range (not far in frequency from the hyperfine resonance frequency), where the excitation frequencies are in the 10-1000 Hz range, the scattered microwave frequencies can be easily generated and the incident frequency can he discriminated against the scattered frequency. Unfortunately, the scattering cross section is very small in this case since it involves magnetic dipole interaction operators. If the incident frequency is in the infrared or the visible regions of the spectrum, the scattering cross section is much larger, since the transitions would involve electric dipole interaction operators, but ultra-stable lasers are needed to discriminate the incident and scattered radiation. Here we elaborate on the treatment in Ref. 7 and consider in somewhat more detail the non-spherically symmetric potential case.

Light scattering from a low density degenerate Bose gas was recently considered by Javanainen[8]. He showed that at finite-temperature, extra structure in the light scattering spectrum is expected due to the Bose-Einstein statistics of the atoms and predicted two-peaks in the light scattering, one at frequencies below the incident light frequency arising from the recoil of the gas atoms by the change in the wavevector of the photon in the scattering process, and the other above the incident light frequency arising due to the preferred scattering of an atom in the gas into the occupied ground state of the BEC. Javanainen estimated the total light scattering from a BEC and concluded that a good fraction of the photons striking the sample would be scattered, and the scattered light would be easily detectable[8]. However, the excitation spectrum of the Bose gas in a harmonic oscillator potential was not accounted for and the probabilities of combination scattering peaks in the spectrum were not calculated. Here, we consider the scattering spectrum due to excitations of the Bose gas in the harmonic potential and the external magnetic field. We estimate the fraction of the light scattered into combination scattering lines by considering scattering from an ideal BEC (no interactions) and analyze the Dicke narrowing type spectrum[9] resulting from the confinement of the atoms in the trap.

The spectrum and the occupation numbers for a Bose gas in a 3D harmonic potential with Hamiltonian

$$H = \sum_{i=1}^{N} \left[\frac{\mathbf{p}_i^2}{2m} + \frac{m}{2}(\omega_x^2 x_i^2 + \omega_y^2 y_i^2 + \omega_z^2 z_i^2) \right] \qquad (1)$$

is given by $E_{j,k,l} = \hbar(\omega_x(j + 1/2) + \omega_y(k + 1/2) + \omega_z(1 + 1/2))$ where j, k and l are non-negative integers. The BE occupation numbers are given by $< n_{jkl} >= [\exp(\beta(E_{j,k,l} - \mu)) - 1)]^{-1}$ where the chemical potential μ is determined by the condition

$$\sum_{jkl=0}^{\infty} < n_{jkl} >= N \ . \qquad (2)$$

It is convenient to define the dimensionless variables $x = \beta\hbar\omega_x$, $\alpha = \mu/\hbar\omega_x$, $r_z = \omega_z/\omega_x$, $r_y = \omega_y/\omega_x$, and $\gamma = 3/2 - \alpha$, in terms of which the average occupation numbers can be written as

$$< n_{jkl} >= \frac{\sum_{n_{jkl}=0}^{\infty} n_{jkl} \exp[-x(j + r_y k + r_z l - \gamma)n_{jkl}]}{\sum_{n_{jkl}=0}^{\infty} \exp[-x(j + r_y k + r_z l - \gamma)n_{jkl}]} = \frac{1}{\exp[x(j + r_y k + r_z l + \gamma)] - 1} \ , \qquad (3)$$

Fig. 1 shows $\gamma(= 3/2 - \mu/\hbar\omega_x)$ and n_{000} vs $x^{-1} = k_B T/\hbar\omega_x$ for $N = 2000$ and $r_y = r_y = 1$ (the isotropic harmonic potential case). It is clear from this figure that $-\gamma$, which is the chemical potential minus the zero point energy in units of $\hbar\omega$, only vanishes for $T = 0$ (i.e., does not vanish for finite temperature), hence BEC does not occur for this confined finite system. Nevertheless, a sharp change in γ vs T occurs at $T_c \approx 10\hbar\omega_x/k_B$ for $N = 2000$, hence this is the "almost" critical temperature for this N. Fig. 2 compares γ vs x^{-1} for two values of the total number of Bose-Einstein particles, $N = 2000$ and 20,000. The figure clearly shows that γ decreases and T_c increases as the total number of particles increase.

The probability amplitude $c_{BA}^{(2)}(t)$ for light scattering from state A to state B can be calculated from second order perturbation theory

$$c_{BA}^{(2)}(t) = \frac{1}{(i\hbar)^2} \int_0^t dt'' \int_0^{t''} dt' \sum_I h_{BI} \exp[i(E_B - E_I - \hbar\omega)t''/\hbar] h_{IA}^\dagger \exp[i(E_I - E_A + \hbar\omega')t'/\hbar]$$

$$+ \quad h^{\dagger}_{BI} \exp[i(E_B - E_I + \hbar\omega')t''/\hbar] h_{IA} \exp[i(E_I - E_A - \hbar\omega)t'/\hbar)] \ , \tag{4}$$

where the interaction matrix elements are given by $h_{IA} =< I|e^{-i\mathbf{k}\cdot\mathbf{r}}[\mathbf{p}\cdot\mathbf{e}^{(\alpha)} + \frac{\hbar}{2}\sigma\cdot(-i\mathbf{k}\times\mathbf{e}^{(\alpha)})]|A>$. For electric dipole transitions (relevant for optical transition frequencies) the individual atomic transition matrix element reduces to $h_{IA} =< I|\mathbf{p}\cdot\mathbf{e}^{(\alpha)}|A>$ whereas for magnetic dipole transitions (trelevant for microwave transitions) $h_{IA} = -i/2(\mathbf{k}\times\mathbf{e}^{(\alpha)})\cdot < I|[\mathbf{L}+2\mathbf{s}]|A>$. In what follows we proceed as if the relevant matrix element involves $\mathbf{p}\cdot\mathbf{e}^{(\alpha)}\exp(i\mathbf{k}\cdot\mathbf{x})$, but for magnetic transitions the spin part of the matrix element should be included. Performing the time integrals and using the resulting expression for the amplitude to form the differential cross section we obtain[10]

$$\begin{aligned} \frac{d\sigma_{BA}(E=\hbar\omega')}{d^2\Omega} &= r_e^2/m_e^2\frac{\omega'}{\omega}\Big|\sum_I \Bigg[\frac{< B|\mathbf{p}\cdot\mathbf{e}^{(\alpha')}\exp(-i\mathbf{k}'\cdot\mathbf{x})|I>< I|\mathbf{p}\cdot\mathbf{e}^{(\alpha)}\exp(i\mathbf{k}\cdot\mathbf{x})|A>}{E_I - E_A - \hbar\omega} \\ &+ \frac{< B|\mathbf{p}\cdot\mathbf{e}^{(\alpha)}\exp(i\mathbf{k}'\cdot\mathbf{x})|I>< I|\mathbf{p}\cdot\mathbf{e}^{(\alpha')}\exp(-i\mathbf{k}\cdot\mathbf{x})|A>}{E_I - E_A + \hbar\omega}\Bigg]\Big|^2 \ . \end{aligned} \tag{5}$$

where r_e the classical electron radius $r_e = e^2/(m_e c^2)$. State $|A>$ involves an internal electronic and an external harmonic oscillator state for all the atoms and a state vector for the photon field degrees of freedom, and is therefore given by a product state of form $\prod_{i=0}^{N}|\eta_i F_i M_i>|j_i k_i l_i>\prod_{k\alpha}|n_{k\alpha}>$ where η_i represents additional internal state quantum numbers which must he properly symmetrized. For large detuning Eq. 5 reduces for electric dipole transitions to

$$\frac{d\sigma_{BA}(E=\hbar\omega')}{d^2\Omega} = \frac{m_e^2 r_e^2\omega^3\omega'}{\Delta^2}|\mathbf{e}^{(\alpha)}\cdot < \tilde{B}|\mathbf{x}\exp < \tilde{B}|\mathbf{x}\exp[-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}]\mathbf{x}|\tilde{A}> \cdot\mathbf{e}^{(\alpha)}|^2 \ . \tag{6}$$

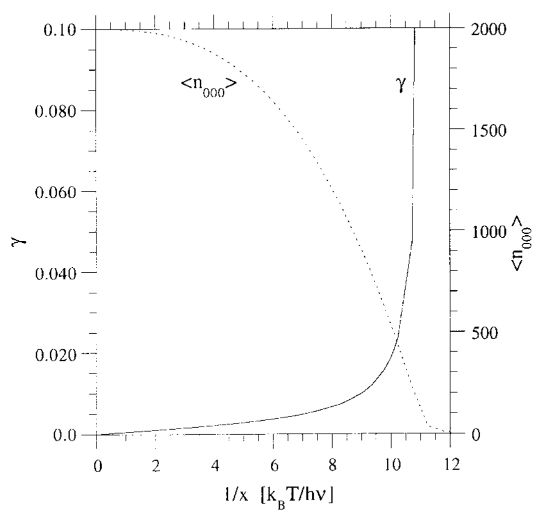

Figure 1. $\gamma(=3/2-\mu/\hbar\omega_p)$ and n_{000} vs $x^{-1} = k_BT/\hbar\omega_p$ (i.e., chemical potential and ground state population vs temperature) for $N = 2000$.

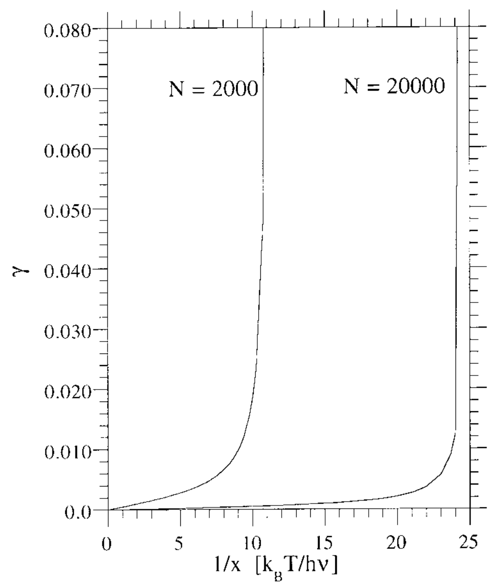

Figure 2. γ vs $x^{-1} = k_BT/\hbar\omega_p$ for $N = 2000$ and 20,000. $T_c \approx 10\hbar\omega_x/k_B(\approx 24\hbar\omega_x/k_b)$ for $N = 2000(20.0000)$.

where state $|\tilde{A}>$ indicates the part of the state $|A>$ without the photon degrees of freedom included, $\mathbf{x}$ is the electronic position coordinate, $\mathbf{r}$ is the position coordinate for the atom (in the dipole approximation, the electronic part of the position coordinate in the exponent of the exponential can be neglected, but the position of the atom can not be neglected) and Δ is the detuning. The sum over atoms, is implied but not indicated in the operators appearing in Eq. 6.

To evaluate the amplitude on the right hand side of Eq. 6, it is convenient to employ a second quantization formulation. Let us denote the quantum state for a single bosonic atom by the quantum state label $\xi = \eta FMjkl$, and the number of bosons in state ξ, by $n\xi$. The Fock space state vectors $|n\xi_1\ n\xi_2\ n\xi_3 ... >$ can be constructed, and the operator in the matrix element of Eq. 6 can be written in Fock space as $S_{\xi\xi}$, $b_\xi^\dagger$, b_ξ. Here $S_{\xi\xi}$ is the transition amplitude to be evaluated below and $b_\xi^\dagger$, and h_ξ are the Fock space bosonic raising and lowering operators for states ξ' and ξ, respectively. The amplitude in Eq. 6 is given by the product of $S_{\xi\xi}$, and the matrix elements of $b_\xi^\dagger b_\xi$, in Fock space[11]. The only nonvanishing matrix elements of $b_\xi^\dagger, b_\xi$ in Fock space are

$$< n-1_\xi n+1_{\xi'} | b_{\xi'}^\dagger b_\xi | n_\xi n_{\xi'} >= \sqrt{(n+1)_{\eta' F' M' j' k' l'} (n)_{\eta FMjkl}} \ . \tag{7}$$

The matrix element $S_{\xi'\xi}$ can be evaluated as follows:

$$S_{\eta' FM' j' k' l', \eta FMjkl} = \mathbf{e}^{(\alpha')} \cdot < \eta FM' | \mathbf{xx} | \eta FM > \cdot \mathbf{e}^{(\alpha)} < j'k'l' | \exp[i(\mathbf{k}-\mathbf{k}')\cdot \mathbf{r}] | jkl > \ , \tag{8}$$

where the first term on the right hand side is the electronic matrix element, and the last term is the single particle harmonic oscillator matrix element.

For the symmetric potential case, $\omega_x = \omega_y = \omega_z$ the matrix elements can be analytically evaluated by choosing a coordinate system in which the momentum transfer $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ lies along the Z axis (not the same axis of quantization defining the azimuthal quantum number M):

$$< j'k'l' | \exp[i(\mathbf{k}-\mathbf{k}')\cdot \mathbf{r}] | jkl >=< j'k'l' | \exp(iqz) | jkl >= \delta_{j'j} \delta_{k'k} < l' | \exp(iqz) | l > \ , \tag{9}$$

$$< l' | \exp(iqz) | l >= \frac{1}{\sqrt{\pi 2^l 2^{l'} l! l'!}} \int_{-\infty}^{\infty} dz \exp(-z^2 + iqLz) H_{l'}(z) H_l(z), \tag{10}$$

where $L = (\hbar / m\omega_p)^{1/2}$. The integral in Eq. (10) can be evaluated in terms of the Hermite polynomial power series expansion

$$H_l(z) = \sum_{v=0}^{[\ell/2]} a_v z^{\ell - 2v} \quad , \quad a_v = \frac{(-1)^v l! 2^{l-2v}}{v!(l-2v)!} \ , \tag{11}$$

and the expression,

$$\int_{-\infty}^{\infty} dz \exp(-z^2 + iqLz) = \sqrt{\pi} \exp\left(\frac{-q^2 L^2}{4} \right),$$

to obtain

$$< l' | \exp(iqz) | l >= \frac{1}{\sqrt{\pi 2^l 2^{l'} l! l'!}} \sum_{v=0}^{[\ell/2]} \sum_{v'=0}^{[\ell'/2]} a_v a_{v'} (-i)^{\ell + l' - 2(v+v')} \partial_x^{l+l'-2(v+v')} \left[\exp\left(\frac{-x^2}{4} \right) \right]_{x=qL} \ . \tag{12}$$

If $qL << l$, i.e., if the wavelength of the light is much larger than the trap size L, Eq. (12) reduces to

$$< l' | \exp(iqz) | l > \approx \delta_{l',l} + iqL \left[\sqrt{(1+1)/2} \delta_{l',l+1} + \sqrt{1/2} \delta_{l',l-1} \right] \ . \tag{13}$$

Substituting Eqs. 7, 8 and 9 into 6, we obtain the expression for the differential cross section,

$$\frac{d\sigma_{BA}(\omega')=\omega-\omega_p(l-l')-\mu_0 H(M-M'))}{d^2\Omega}=\frac{m_e^2 r_e^2\omega^3\omega'}{\Delta^2}(n+1)_{\eta F'M'j'k'l'}(n)_{\eta FMjkl}$$
$$|\mathbf{e}^{(\alpha')}(\hat{\mathbf{k}'})\cdot<\eta FM'|\mathbf{xx}|\eta FM>\cdot\mathbf{e}^{(\alpha)}(\hat{\mathbf{k}})\delta_{j'j}\delta_{k'k}<l'|\exp(iqz)|l>|^2\,. \tag{14}$$

Here H is the magnetic field and μ_0 the magnetic moment.

For an asymmetric harmonic oscillator with potential,

$$V(\mathbf{r})=\frac{m}{2}(\omega_x^2x_i^2+\omega_y^2y_i^2+\omega_z^2z_i^2),$$

matrix elements in Eq. (9) must be evaluated with the $\mathbf{q}$-vector properly oriented relative to the vector $\mathbf{L}=(\hbar/m)^{1/2}(\omega_x^{1/2},\omega_y^{1/2},\omega_z^{1/2})$. The calculation of the matrix element in this case is more complicated. The spectrum will have peaks at $\omega'=\omega-\omega_x(j-j')+\omega_y(k-k')-\omega_z(l-l')-\mu_0H\Delta M$.

To obtain the total differential cross section, we must average the cross section $\sigma_{BA}(\omega')$ over the initial set of states A and sum over all states B that can yield photons of frequency ω'. Using the grand canonical probability for independent Bose-Einstein particles[12], and the expression for the average occupation number given in Eq. (3) we obtain for the symmetric potential case,

$$\frac{d\sigma_(\omega'=\omega-\omega_p(l-l')-\mu_0H(M-M'))}{d^2\Omega}=\frac{m_e^2r_e^2\omega^3\omega'}{\Delta^2}|\mathbf{e}^{(\alpha')}(\hat{\mathbf{k}'})\cdot<\eta FM'|\mathbf{xx}|\eta FM>\cdot\mathbf{e}^{(\alpha)}(\hat{\mathbf{k}})|^2$$
$$\sum_{jkll'}<(n+1)_{\eta FM'jkl'}><n_{\eta FMjkl}>|<l'|\exp(iqz)|l>|^2\,. \tag{15}$$

Here l' is restricted in the sum on the right hand side so that $l-l'$ is fixed. In the experiments of Refs. 2-4, only one FM state is populated, so $<n_{\eta FMjkl}>=<n_{jkl}>$ of Eq. 3 for the trapped hyperfine state ηFM and is zero otherwise, and $<(n+1)_{\eta FM'jkl'}>=<(n+1)_{jkl'}>$ for this state and is unity otherwise. Bosonic stimulation due to the factor $<(n+1)_{\eta FM'jkl'}><n_{\eta FMjkl}>$ is a telltale signature of BEC since it drastically affects the relative and absolute line intensities. The optically stimulated cross section is given by Eq. (15) times a factor equal to the number of photons in the field at frequency ω'.

For small qL, and in the limit of weakly interacting particles, the frequencies of the emitted radiation are given by $\omega'=\omega-\mu_0H(\Delta M)$ and $\omega'=\omega\pm\omega_p-\mu_0H(\Delta M)$. The cross sections for $\omega'=\omega\pm\omega_p$ and $\omega'=\omega\pm\omega_p-\mu_0H(\Delta M)$ with $\Delta M\neq0$ in Eq. (15) are proportional to the quantities

$$\begin{aligned}F(\omega'=\omega-\omega_p,T) &= \sum_{jkl}(l+2)/2<(n+1)_{jkl+l}><n_{jkl}>\\ &= \sum_{jkl}\left[\frac{1}{\exp[x(j+k+l+1+\gamma)]-1}+1\right]\frac{(l+1)/2)}{\exp[x(j+k+l+\gamma)]-1}\,,\end{aligned} \tag{16}$$

$$F(\omega' = \omega - \omega_p, T) = \sum_{jkl} l/2 < (n+l)_{jkl-l} >< n_{jkl} >$$
$$= \sum_{jkl} \left[\frac{1}{\exp[x(j+k+l-1+\gamma)]-1} + 1\right] \frac{l/2}{\exp[x(j+k+l+\gamma)]-1} , \quad (17)$$

$$F(\omega' = \omega - \omega_p - \mu_0 H\Delta M, T) = \sum_{jkl} (l+2)/2 < n_{jkl} = \sum_{jkl} \frac{(l+1)/2}{\exp[x(j+k+l+\gamma)]-1} , \quad (18)$$

$$F(\omega' = \omega - \omega_p - \mu_0 H\Delta M, T) = \sum_{jkl} l/2 < n_{jkl} = \sum_{jkl} \frac{l/2}{\exp[x(j+k+l+\gamma)]-1} . \quad (19)$$

Fig. 3 plots $F(\omega' = \omega - \omega_p, T)$, $F(\omega' = \omega + \omega_p, T)$, $F(\omega' = \omega - \omega_p - \mu_0 H\Delta M, T)$ and $F(\omega' = \omega - \omega_p - \mu_0 H\Delta M, T)$ for $\Delta M \neq 0$) as a function of x for $N = 2000$ and small qL. At $T = 0$, $F(\omega' = \omega + \omega_p, 0) = 0$, $F(\omega' = \omega - \omega_p, 0) = 1000$, $F(\omega' = \omega + \omega_p - \mu_0 H\Delta M, 0) = 0$, and $F(\omega' = \omega - \omega_p - \mu_0 H\Delta M, 0) = 1000$, since all atoms are in the lowest harmonic oscillator state, transitions to lower energy states are impossible, and $< n_{000} >= 2000$, $< (n+l)_{jkl} >= 1$ for jkl not all zero. All Fs increase with increasing T. The blue shifted Fs are smaller than the red shifted intensities, and the intensities of the lines corresponding to $\Delta M \neq 0$ are smaller than those with $\Delta M = 0$. When qL is not small, additional scattered frequencies at $\omega' = \omega \pm (\Delta l)\omega_p - \mu_0 H\Delta M$ with $\Delta l \neq l$ will become significant, as determined from Eq. (15). The width of the scattered lines is related to the natural linewidth of the atoms and is determined by a number of factors[8,13]. Hence, the spectrum of the scattered radiation is of the following general form: the strongest peak appears at $\omega' = \omega$,, peaks at $\omega' = \omega \pm (\Delta l)\omega_p$ are less intense with the $\omega' = \omega - (\Delta l)\omega_p$ peak more intense than at $\omega' = \omega + (\Delta l)\omega_p$. For $qL << 1$, only the $\Delta l = 1$ peaks contribute. Satellite peaks due to ΔM transitions are displaced in frequency from the $\Delta M = 0$ peaks by $-\mu_0 H(\Delta M)$. The intensities of these peaks at low T do not increase with T as quickly as the $\Delta M = 0$ peaks but as T increases they become important. The peaks at $\omega - (\Delta l)\omega_p - \mu_0 H\Delta M$ are more intense than for $\omega + (\Delta l)\omega_p - \mu_0 H\Delta M$.. The polarization of the scattered radiation depends strongly on ΔM and is determined by the factor $|\mathbf{e}^{(\alpha')}(\hat{\mathbf{k}'})\cdot < \eta FM'|\mathbf{xx}|\eta FM > \cdot \mathbf{e}^{(\alpha)}(\hat{\mathbf{k}})|^2$ appearing in Eq. (15). As $T \to 0$, all blue shifted features in the scattering disappear. The temperature dependence of the relative and absolute intensities of the peaks is a strong indication of BEC, and in particular the temperature dependence of the ratio of the intensities of the peaks is a clear signature of BEC. For an asymmetric harmonic oscillator potential, the spectrum has peaks at $\omega' = \omega - \omega_x(j-j') + \omega_y(kk') - \omega_z(l-l') - \mu_0 H\Delta M$, and for small $\mathbf{q}\cdot\mathbf{L}$, where $\mathbf{L} = (\hbar/m)^{1/2}(\omega_x^{1/2}(\omega_x^{1/2}, \omega_y^{1/2}, \omega_z^{1/2})$, the strong peaks are at $\omega' = \omega - \mu_0 H\Delta M$, $\omega' = \omega \pm \omega_x - \mu_0 H\Delta M$, $\omega' = \omega \pm \omega_y - mu_0 H\Delta M$, and $\omega' = \omega \pm \omega_z - \mu_0 H\Delta M$, (with $\Delta M = 0, 1$ and 2 if the trapped state has $M = F$).

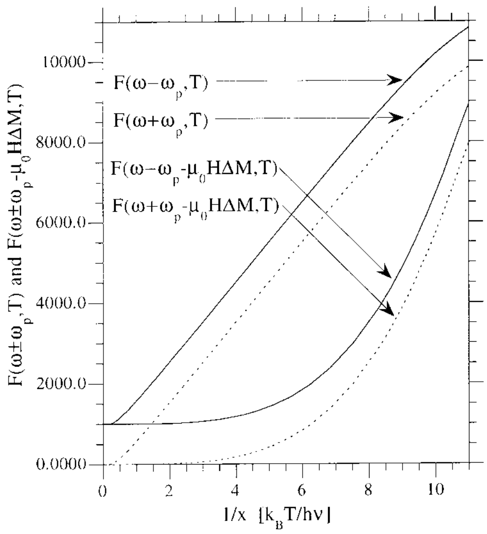

Figure 3. $F(\omega' = \omega - \omega_p, T)$, $F(\omega' = \omega + \omega_p, T)$, $F(\omega' = \omega - \omega_p - \mu_0 H\Delta M, T)$, and $F(\omega' = \omega + \omega_p - \mu_0 H\Delta M, T)$, (for $\Delta M \neq 0$) vs $x^{-1} = k_B T/\hbar\omega_p$ for $N = 2000$ and $qL \ll 1$.

The comparison with the $\omega_p \to 0$ (no confining potential) limit considered by Javanainen is considerably different than what is considered here; his limit should correspond to our $qL >> 1$ limit. Note that even in this limit, the intensities of the blue detuned features of the spectrum vanish and the red detuned features decrease as $T \to 0$.

It remains to determine via light scattering studies the extent to which interaction of the atoms modify the elementary excitations of the BEC.

Acknowledgments

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