Two-Neutron Removal Cross Section of Exotic Nuclei Using Gaussian Nucleon Densities*

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We determine the trend of the two-neutron removal cross section, σ_{-2n}, with energy for ^{11}Li+^{12}C in the high energy region. The cross section depends on the "$t\rho_A\rho_B$" optical potential. By using Gaussian nucleon densities for the nuclei, we obtain closed-form expressions for the transmission coefficient and the cross section. Several corrections such as refractive and "in-medium" Pauli-blocking are considered.

I. Introduction

In the last decade, nuclear reactions of weakly bound projectiles have been one of the central concerns in the field of direct reaction theories. The importance of the exotic nuclei is a result of the fact that they present different structural properties when compared with the stable nuclei. Typical examples are the neutron rich nuclei.

The nucleus of ^{11}Li has attracted the most attention both experimentally and theoretically. The two-neutron separation energy of ^{11}Li is only about 0.34 MeV[1)], and the two neutrons are expected to have a very low and spatially extended density distribution surrounding the ^{9}Li core, forming the so-called neutron-halo structure. The halo structure of ^{11}Li is consistent with all the experimental findings, which include the enhancement of the interaction and two-neutron removal cross sections. Such nuclear structure properties have an impact on the reactions in which those nuclei are involved. For example, take the neutron removal cross section. It presents a large 2n break-up cross section at large incident energy. The experimental determination of the neutron halo density distribution is very important to determine the trend with energy of the two-neutron removal cross section. Particularly, the radioactive beams data of energies at 100-1000 MeV/nucleon offer a transparent link between neutron halo density and two-neutron removal cross section in Glauber approach. Consequently, it is of some value to consider what might be expected for the two-neutron removal process at these energies.

Even though the structure of ^{11}Li directly calls for a three-body model of the reaction, it is highly desirable to find an approximate method to form an intuitive picture of the process. An approximate method will also be useful to extract structural information of ^{11}Li from the analysis of the experimental data.

The purpose of the present paper is to determine the trend of the two-neutron removal cross section with the energy giving a closed-form expression for it. Due to the weak correlation between ^{9}Li and the two neutrons, the approach used directly relates the picture of the ^{11}Li reaction as a superposition of the independent reactions of the $^{9}Li - target$ and the dineutron-target reactions.

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The approach is of particular importance for studying the mechanism of the fragmentation reaction and to understand the roles that are played by nuclear and Coulomb forces in the dissociation of the neutron-rich nuclei.

In this paper we will restrict ourselves to reactions caused by the nuclear force and will not discuss the reactions induced by the Coulomb force.

Our study is based on the eikonal approximation to the phase-shift plus refractive corrections, with the optical potential determined from the usual multiple-scattering series with in-medium effects taken into account.

The organization of the paper is the following: In sect. II we present the approach that is useful for analysing the trend of the two-neutron removal cross section with energy. The eikonal approximation to the phase-shift with refractive corrections is also considered. The nucleus-nucleus optical potential employed is explicitly presented. In sect. III we write closed-form expressions for the two-neutron transmission coefficient and the two-neutron removal cross section, starting from the Gaussian nucleon densities for the nuclei involved in the reaction. Sect. IV is reserved for the results from the approach used. The importance of the "in-medium" Pauli-blocking to the nucleus-nucleus optical potential at high energy region and the refractive corrections, real nuclear and Coulomb fields, to the phase-shift are studied. In sect. V conclusions are presented.

II. The nuclear two-neutron removal cross section

The total two-neutron removal process, for example, $^{11}Li \rightarrow {^9Li}$, induced by a target nucleus A accounts for all break-up channels, the inelastic and the elastic ones. In the cluster model, the exotic nucleus, ^{11}Li, is treated as an inert core, ^9Li, surrounded by two weakly-bound neutrons . The approach, that we call the "transmission method", is determined by the transmission coefficient of just the two neutrons, in which the system is considered as a product of two-binary subsystems: the core-target and the 2n-target ones. The resulting cross section presents the following form when written in terms of the impact parameter b:

$$\sigma_{-2n}(E) \;=\; 2\pi \int_0^\infty \; db\; b\; T_{-2n}(b;E)\;. \tag{1}$$

The two-neutron transmission coefficient is expressed as

$$T_{-2n}(b;E) \;=\; \mid S_{CA}(b;E)\mid^2 \;\; T_{2nA}(b;E)\;, \tag{2}$$

where $S_{CA}(b;E)$ is the elastic S-matrix term due to the subsystem consisting of the core (C) and the target nucleus (A) and the transmission coefficient T_{2nA} refers to the subsystem of the two-neutrons (2n) and the target A. For the two-neutron removal process $^{11}Li \rightarrow {^9Li}$, the core is ^9Li. The cross section is obtained by the integration over impact parameter b of the product of the two probabilities, the survival probability of the core in its ground state, $\mid S_{CA} \mid^2$, and the probability of dineutron absorption by the target, T_{2nA}.

As shown in an earlier work[2], the above quantities can be related to the imaginary part of the phase shift $\delta(b;E)$ as follows:

$$\mid S(b;E)\mid^2 \;=\; e^{4\; Im\; \delta(b;E)}\;, \tag{3}$$

where the S-matrix and the transmission coefficient, T(b;E), obey the flux conservation expression,

$$\mid S(b;E)\mid^2 \;+ T(b;E) \;=\; 1\;. \tag{4}$$

At relative high energies, the eikonal approximation to the phase shift is reliable,

$$Im\; \delta_{eikonal}(b;E) = -\frac{1}{\hbar v}\int_0^\infty \; dz\; Im\; V_{opt}(r;E)\;. \tag{5}$$

However, since we are also interested in studying refractive effects in this approach, it is more appropriate to write the phase shift as[9)]

$$Im\ \delta_{eikonal}(b;E) = -\frac{1}{\hbar v}\int_{r_0}^{\infty} dr\ \frac{ImV_{opt}(r;E)}{\sqrt{1\ -\ \frac{b^2}{r^2}\ -\ \frac{Re\ V_{opt}(r;E)}{E}}}\ . \qquad (6)$$

The usual expression for the eikonal phase shift, Eq. (5), can be obtained from the above equation, by taking $Re\ V_{opt}(r;E)$ to be zero. It is equivalent to considering a straight line approximation to the trajectory in classical language. The general equation, given in the expression above, fully incorporates the refractive effects in the eikonal expression of the phase shift. The turning point r_0 presents a simple closed expression if the real part of the optical potential only includes only the Coulomb interaction[3], that is,

$$kr_0\ =\ \eta\ +\sqrt{\eta^2+(kb)^2}\ , \qquad (7)$$

η being the Sommerfeld parameter. However, since the real part of the nuclear optical potential will be included, the turning point in general has no simple expression.

For the calculation, it is necessary to adopt a model for the nuclear part of the optical potential. A simple and a valid expression for our study is the impulse-approximation for composite projectile nuclei, of Kerman, McManaus and Thaler[4],

$$V_{opt}(\vec{r};E) = < t_{NN}(E) > \int d\vec{r'}\rho_A(\vec{r'})\rho_B(\vec{r}-\vec{r'})\ , \quad (8)$$

where $\rho_i(r)$ is the single-particle density of nucleus i and $< t_{NN}(E) >$ is the free nucleon-nucleon amplitude in the forward direction θ=0°. The latter can be written as

$$< t_{NN}(E) >= -\frac{1}{2}\hbar v < \sigma_{NN}(E) > (< \alpha_{NN} > +i)\ , \quad (9)$$

where the brackets represent an isospin average over the projectile and target nucleons. The parameter $< \alpha_{NN} >$ expresses the ratio of the real to the imaginary parts of the optical potential. Commonly, it is given by an adjusted number[5].

III. Closed-form analytical expressions

Simple expressions can be obtained for the two-neutron transmission coefficient and the two-neutron removal cross section when a Gaussian form is used for the nucleon density distribution:

$$\rho_A(r) = \rho_A(0)e^{-(r/a_A)^2}\ , \qquad (10)$$

with

$$\rho_A(0) = \frac{A}{a_A^3\pi^{\frac{3}{2}}}\ , \qquad (11)$$

where A is the mass number of the nucleus. The parameter a_A can be considered as free or related to the root mean square radius, R_{rms}, by

$$a_A = \frac{R_{rms}}{\sqrt{1.5}}\ , \qquad (12)$$

where experimental R_{rms} values may be employed. When a_A is determined by the R_{rms}, the approximation is parameter free.

In the spirit of the Gaussian form for the densities, we adopt a similar model for the optical potential:

$$V_{opt}(r;E) = - < t_{NN}(E) > \pi^{\frac{3}{2}} \rho_A(0)\rho_B(0) \frac{a_A^3 a_B^3}{a_{AB}^3} \; e^{-(\frac{r}{a_{AB}})^2} \;, \tag{13}$$

where

$$a_{AB} = \sqrt{a_A^2 + a_B^2} \; . \tag{14}$$

The expression is well justified for the energy range of the present study[6−7]. Note that Eq. (13) determines the central part of the optical potential, with both the imaginary and real parts are taken into account. The advantage of using a convenient optical potential in order to describe the subsystems $C-A$ and $2n-A$, which is necessary to determining $T_{-2n}(b;E)$, is the possibility of having a simple expression for the two-neutron transmission coefficient,

$$T_{-2n}(b;E) = \; e^{-\tau_{CA}(E) e^{-\frac{b^2}{a_{CA}^2}}} \; [\, 1 - e^{-\tau_{2nA}(E) \; e^{-\frac{b^2}{a_{2nA}^2}}} \,], \tag{15}$$

where the dynamical quantities $\tau_{CA}(E)$ and $\tau_{2nA}(E)$ are determined by:

$$\tau_{AB}(E) =< \sigma_{NN}^{(tot)}(E) > \frac{\pi^2 a_A^3 a_B^3}{(a_A^2 + a_B^2)} \rho_A(0)\rho_B(0) \; . \tag{16}$$

The simple closed-form expression of Eq. (15) is a result of two ingredients: i)-The eikonal phase-shift expression, given in Eq. (5), with no refractive corrections and ii)- The well known "$t\rho_A\rho_B$", where the nucleus density ρ_i has a Gaussian form. The parameter $\tau_{AB}(E)$ contains information about the adopted nuclear structure of nuclei involved.

The above expression for $T_{-2n}(b;E)$ allows us to write down the two-neutron removal cross section as an infinity series

$$\sigma_{-2n}(E) = 2\pi \sum_{n=1}^{\infty} \frac{[\tau_{2nA}(E)]^n \; \gamma(\mu(n), \tau_{CA}(E))}{n! \; 2 \; a_{CA}^2 \; [\tau_{CA}(E)]^{\mu(n)}}, \tag{17}$$

with

$$\mu(n) = \frac{n a_{CA}^2}{a_{2nA}^2}, \tag{18}$$

where the incomplete gamma function is defined[8] by:

$$\gamma(\alpha, x) = \int_0^x e^{-t} t^{\alpha-1} dt \;, \quad [Re \; \alpha > 0]. \tag{19}$$

The importance of the expression for $\sigma_{-2n}(E)$ is that it might provide a pratical way of determining the cross section, if it is a highly convergent series. If so, a few terms might be sufficient to reasonably account for the $\sigma_{-2n}(E)$. In this case, we would have a fast way of determining this cross section.

IV. Results

The KMT formalism was developed to give a practical means of obtaining the optical potential for the nucleon-nucleus system. At sufficiently high energies, calculation using this lowest order optical potential provides a reasonable qualitative description of the total reaction cross section data[9]. The formalism assumes valid the use of the free two-body t-matrix, which is known as the "impulse approximation" (IA). If the IA is to be valid, effects of the nuclear medium must be negligible. Among these effects, the Pauli-blocking "in-medium" correction is the most important one for the nucleus-nucleus system, in the range of energy of the present study[10]. Its effect is to reduce the magnitude of the optical potential turning the imaginary part of the optical potential less absorptive. The correction is taken into account by considering that each nucleus is described by a Fermi sphere of momentum K_F in which each nucleon has a momentum k. In the collision process, the scattering of nucleons is restricted by the Pauli-principle in order to avoid the occupations of states already occupied. The model, developed for nuclear matter, makes use of the "local density approximation" in order to consider finite nuclei. This is done through the well known relation between the Fermi momenta of nuclei and the their nucleon densities by $K_{Fi} = (\frac{3}{2}\pi^2\rho_i)^{1/3}$. We will use the following convenient expression[11]

$$\sigma_{NN}^{eff}(E) = P(E, K_{F1}, K_{F2})\sigma_{NN}^{free}(E), \tag{20}$$

where the reduction factor P above is parametrized in terms of the variables $\xi = \frac{k}{K_{F>}}$ and $\eta = \frac{K_{F<}}{K_{F>}}$, where k is the relative momentum between the two nucleons in consideration and $K_{F>}(K_{F<})$ is the larger(smaller) of K_{F1} and K_{F2}. See Ref. 11 for details. We use $K_F(^9Li)$=0.89 fm^{-1} and $K_F(^{12}C)$=0.86 fm^{-1}, which correspond to Fermi momenta calculated at the surface of the respective nuclei.

Fig. 1 shows the effect of the Pauli-blocking "in-medium" correction on the two-neutron removal cross section of the system $^{11}Li + ^{12}C$ over a large energy range. We use $a_{2n} = 4.42 fm^{15)}$. The parameters $a_{^9Li}$=1.952 fm and $a_{^{12}C}$=2.120 fm are from Ref. 12 and they are the ones that fit the nucleon density distribution calculated by Bertsch and coworkers[13]. The first remark to make is that the Pauli-blocking is important mainly at lower energies. At sufficiently high energies the correction due to the blocking is negligible[9], although in Fig. 1 its effect persists up to the highest energies shown. This is related to the fact that the Pauli-blocking was estimated at the surface of the nuclei involved. At these energies the use of a free two-body t-matrix is a valid approximation. Therefore, the impulse approximation optical potential, given in Eq. (8) and Eq. (9), is valid at sufficiently high bombarding energies.

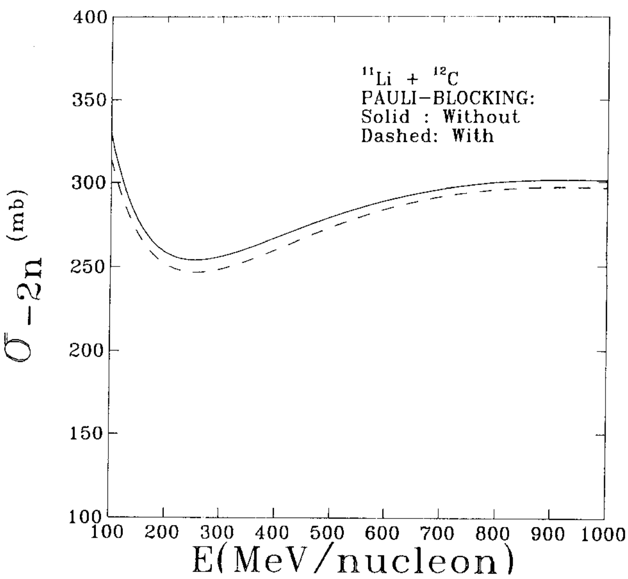

Figure 1. The effect of Pauli-blocking on the two-neutron removal cross section as a function of bombarding energy in the system ^{11}Li+^{12}C. The dashed (solid) curve include (omit) it, respectively. See the text for details.

The two-neutron removal cross section based on the two-neutron transmission coefficient method, given in Eq. (1), is a peripheral process. This is illustrated in Fig. 2. The two-neutron transmission coefficient, T_{-2n}, is a product of the S-matrix of the binary system $C - A$, $| S_{CA} |^2$, and the transmission coefficient of the $2n - A$ system, T_{2nA}. It displays the geometry of the reaction very clearly. As Fig. 2 indicates, one of the terms excludes small values of the impact parameter b, while the other excludes large values of b. The product $| S_{CA} |^2 T_{2nA}$ clearly peaks at the surface of the target nucleus. This characteristic can result in a sensitivity of the cross section with respect to the density of the two outermost neutrons since in the exotic nuclei, the weakly bound neutrons are far away from the core. The solid line in Fig. 2, which represents T_{-2n}, has a maximum at the surface of the system. If we sum the radius of ^{11}Li, R=3.14 fm with ^{12}C, R=2.45 fm we have $R_{^{11}Li} + R_{^{12}C}$=5.59 fm. Clearly, it indicates that the density of the outermost neutron will play a significant role in the cross section.

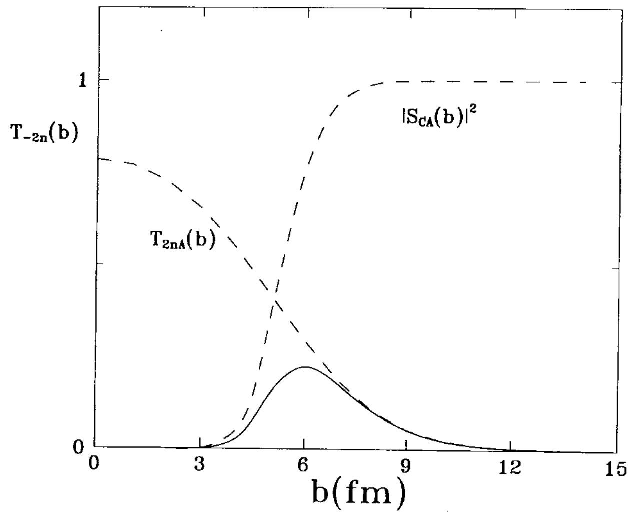

Figure 2. Schematic representation of the two-neutron transmission coefficient, indicating its surface-peaked origin.

Fig. 3 shows the two-neutron distribution using the Gaussian form given in Eq. (10). The parameter a_{2n}=3.30 fm is adjusted in order to better reproduce the elastic scattering of protons off ^{11}Li at 62 MeV[14], while a_{2n}=4.42 fm is determined by the r.m.s. radii[15] of ^9Li and ^{11}Li. We can see how the tail of the distribution is sensitive to the a_{2n} parameter. That difference in the densities of the nucleus will affect strongly the σ_{-2n} due to the peripheral nature of the reaction. In fact, in Fig. 4, we observe this. At an incident energy of 800 MeV/nucleon, we note that the experimental value of $\sigma_{-2n}(^{11}Li +^{12} C) = 220 \pm 10 mb$[16], falls between the results with a_{2n}=3.30 fm and a_{2n}=4.42 fm. Therefore, we can say that the two-neutron removal cross section determined by the "transmission method", given in Eq. (1), is sensitive to the variation of a_{2n}, the two-neutron halo parameter.

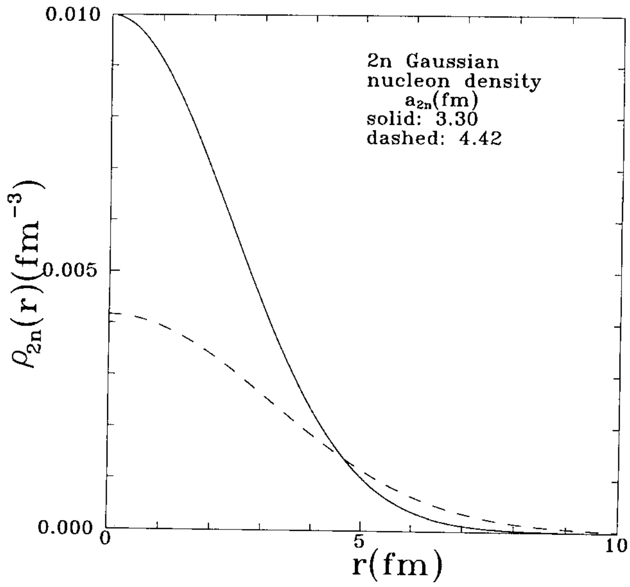

Figure 3. The two-neutron Gaussian nucleon density distribution as a function of the parameter a_{2n}. See the text for details.

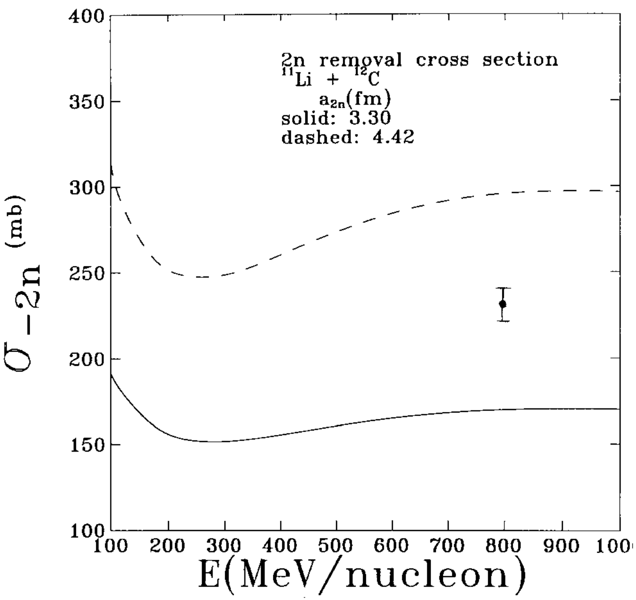

Figure 4. The trend with energy of the two-neutron removal cross section for ^{11}Li+^{12}C as a function of the parameter a_{2n}. See the text for details.

The eikonal approximation to the phase shift implies the assumption of straight line propagation in classical language. When the Coulomb and the nuclear fields are considered, they produce a deviation of the trajectory from a straight line. The consequence is a change in the absorption. While the Coulomb potential is of a repulsive nature, favoring the scattering process, the inclusion of the (real) nuclear potential, will increase the absorption, since the projectile will be attracted by the target nucleus. Therefore, they act in opposite directions. The net contribution will depend on the dynamic of the reaction.

With relation to the dependence on the impact parameter b, we can say that the refractive effects are more pronounced the smaller b becomes. As fragmentation is a peripheral process, and due to the energy region of the study, the refractive effects make a negligible correction. We note that the S-matrix, $| S_{CA} |^2$, and transmission coefficient, T_{2nA}, are little modified by the Coulomb and the nuclear fields. Therefore, the refractive effects, contained in Eq. (6), do not alter significantly the physics included by using the eikonal expression of Eq. (5), where refractive effects are neglected. The nuclear field was considered by taking $< \alpha_{NN} >= 1$.

The fact that the two-neutron removal transmission coefficient, given in Eq. (2), is determined by the values in the surface region, makes a series representation of T_{2nA} reliable, in which the terms are determined by expanding the exponential function in brackets of Eq. (15). The result for σ_{-2n} is the expression given in Eq. (17). In the table below we compare values of $\sigma_{-2n}(E)$ from the exact calculation, given in Eq. (1), with those of the series expansion, given in Eq. (17), with a_{2n}=4.42 fm. We note that the first term of the series is already sufficient to give a good representation of the cross section. The agreement becomes better when more terms of the series are considered. We also note that the agreement with the exact results becomes better as the energy increases.

Table I: Two-neutron Removal Cross Section for the system $^{11}Li + ^{12}C$ with a_{2n}=4.42 fm

	$\sigma_{-2n}(E)(mb)$		
E(MeV/nucleon)	*EXACT*	*serie(N$^{\underline{o}}$ of terms)*	
		1 term	*2 terms*
100	314.10	355.72	307.88
200	251.36	277.19	248.15
300	248.56	273.78	245.43
400	260.09	287.85	256.53
500	273.32	304.16	269.44
600	284.17	317.28	279.93
700	291.53	326.28	286.18
800	295.58	333.06	291.30
900	296.97	336.39	296.05
1000	296.52	333.14	292.25

In the Table I, the series with just the first term falls short of the exact values by no more than 15%. Therefore, the two-neutron removal cross section can be fairly described by a closed-form expression, given by just the first term of the series, in Eq. (17). The series with two terms gives an agreement of better than 2% with the exact values.

V. Conclusions

We have analysed the trend of the two-neutron removal cross section, as a function of bombarding energies for the system $^{11}Li + ^{12}C$, using the "transmission method", where the cross section depends directly on the flux which feeds the two-neutron removal channel. The method used is able to give a value in agreement with experimental data at 800MeV/nucleon.

The transmission method presented a high sensitivity to the variation of the spatial distribution of the neutron-halo. A small variation in the halo density makes a big variation in the σ_{-2n}. Therefore, it is important to know the neutron halo distribution in exotic nuclei, in order to estimate reliably the trend of σ_{-2n} with energy.

The Pauli-blocking "in-medium" correction to the optical potential was shown to be important in reducing the absorption and giving smaller values for the cross sections. While the effect is strong at lower energies, we can neglect it at sufficiently high energies.

The refractive corrections to the usual straight line approximation to the trajectory were considered. This correction to the eikonal phase shift represented a small effect, in the high energy range analysed here. So, the nuclear and Coulomb fields were shown to have a minimal effect on the cross section.

Using a Gaussian form for the nucleon density of the nuclei involved in the reaction, we presented closed-form expression for the two-neutron transmission coefficient and for the two-neutron removal cross section. The two-neutron removal cross section was written in a series involving the incomplete gamma function. We showed that it is rapidly convergent series and that just the first term is sufficient to give a good representation of σ_{-2n}. All the parameters are fixed by the experimental values, but the ratio of real to imaginary part of the optical potential. Therefore, contrary to the common analysis with the optical potential model, in which the energy dependent parameters are adjusted for each bombarding energy, in order to explain the experimental data, our trend for the σ_{-2n} as a function of the bombarding energy is obtained by once adjusted the parameters they are kept fixed for the whole range of energy.

It is worth noting that the closed-form expression for the cross section, given in Eq. (17), and the transmission coefficient, given in Eq. (15), are general expressions valid whenever Gaussian nucleon densities are used.

In our expressions we have included only the nuclear forces. Therefore, neither the Coulomb dissociation process nor its interference with the nuclear process were studied.

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