Layered Superconductivity Models at the London Limit and Their Continuum Properties

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We write the London limit of the Lawrence Doniach free energy in terms of the local magnetic field and of the average supercurrent over the interplane distance. Starting from this formulation we study a model where the supercurrent at the buffer layers is obtained from the superconducting sheets by a Taylor expansion. The continuum limit of this model gives corrections to the anisotropic London theory due to the layered structure.

I. Introduction

Long ago superconductivity in layered compounds was studied in the intercalated transition-metal dichalcogenides [1, 2]. The discovery of the high-temperature superconductors brought a renewed interest in the study of layered superconductors. A theoretical framework to understand layered compounds for extremely type II superconductors (large Ginzburg-Landau parameter κ) is the London limit of the Lawrence-Doniach (LLD) theory [3,4,5]. The Lawrence-Doniach theory is a generalization of the Ginzburg-Landau theory that considers zero-thickness superconducting planes. Between the planes this model considers perfect insulating *buffer* layers. The London limit holds provided that the density of superconducting pairs is constant in all layers even in the presence of vortices where local variations of this density, close to the cores, are discarded. London theory was proposed long before the Ginzburg-Landau theory. Although London theory can not explain the existence of vortices in a superconductor it can treat vortices under the assumption that the vortex core is a region totally exterior to the superconductor. The London limit of the Lawrence-Doniach theory is expected to apply to a broader temperature range since the former is a free energy expansion valid close to the critical temperature whereas the latter is based on energy considerations valid at any temperature. Therefore there is superconductivity in the LLD theory only at the zero-thickness planes that interact through the (Josephson) supercurrent component, whose value is constant at each buffer layer. The Josephson current only depends on the difference between the phases of the order parameter defined at each superconducting plane.

In many situations to understand the layered superconductor is just enough to consider the standard London model that takes the superconductor anisotropic and continuous along the c-axis. This is the case of torque studies where agreement between the anisotropic London theory and experiments[6] is satisfactory as along as the applied field is not extremely aligned to the planes[7]. The vortex lines, though tilted are still three-dimensional. For an applied external field with an strong component orthogonal to the c-axis, thus along the CuO_2 planes, the ratio between the coherence length along the c axis, ξ_c, and s, the distance between two consecutive superconducting sheets, determines a crossover between two and three-dimensional regimes [8]. To see this consider a vortex line parallel to the CuO_2 plane where ξ_c is the radius of its normal core. For $\xi_c \gg s$ there are several planes inside the vortex core, the normal state is established inside the core and the vortex is surely three-dimensional. For $\xi_c < s$ the whole core fits in between two consecutive planes, coherence is small and the LLD theory must describe a stack of coupled Josephson juntions. The system displays two-dimensional behavior. Obviously this is a very restrictive criterion of dimensional crossover that applies in case the vortex line is oriented along the CuO_2 planes. A better dimensionality criterion is to check if the two-dimensional pancake vortices [9,10], which exist at each superconducting plane, are sufficiently correlated to define a three-dimensional vortex line. For an applied external field (H) along the c-axis, the diagram ($H-T$), T is the temperature, the vortices are known to be three-dimensional at low temperature and sufficiently low aplied field along the c-axis. However this is no longer the case when the external field becomes so strong, that correlation among pancake vortices in each plane surpasses the correlation among pancake vortices in In this case the flux-line lattice becomes two-dimensional[11].

Up to this point we have only considered superconductors where the buffer layers are perfectly insulating the superconducting planes. Let us consider the same dimensionality question in case the buffer layers are not perfect insulators and do show conducting properties through the proximity effect. This is not the case of the High-Tc compound $Bi_2Sr_2CaCu_2O_{8+x}$ where the assumption of perfect insulating layers between the CuO_2 is indeed a good approximation. Direct measurements of the $I-V$ characteristic curve along the direction orthogonal to the CuO_2 layers (c-axis) of $Bi_2Sr_2CaCu_2O_{8+x}$, for instance, indicate that this material behaves as a stack of superconducting CuO_2 planes separated by insulating BiO and SrO layers[12]. Recently, artificially layered superconductors [13] have been grown, like $(Pr/Y)Ba_2Cu_3O_{7-\delta}$, which do behave as a series array of Josephson junctions: the Pr insulating layers are introduced between the Y superconductor layers. However for $YBa_2Cu_3O_{6+x}$, another High-Tc superconductor, it is well-known that the buffer layers have a richer structure because of the so-called CuO chains, located between the planes. In this case the three-dimensional behavior prevails in all situations because of the proximity effect. In summary the buffer layers may display a richer behavior, e.g. metallic, and considering them as perfect insulators may not always be a good approximation. For such reasons we find worthwhile the present study, namely, the proposal of phenomenological models that provide a description of the buffer layers other than that of the *original* LLD theory.

One of the most interesting new aspects of the superconductivity in the high temperature superconductors is the existence of a broad region where thermal fluctuations dominate the response of the superconductor, showing transitions of first and second order. The thermal fluctuations of the flux-line lattice in these superconductors can become quite large due to the higher temperature and the elastic softness which is caused by the large London penetration length, small coherence length and specially, the pronounced anisotropy or layered structure. In conclusion all free energy processes should be always compared to the thermal energy available, $K_B\,T$, for the high temperature superconductors. The present study addresses a zero temperature question, the proposal of new models laying between the anisotropic London theory (three dimensional) and the London limit of the Lawrence Doniach model (two-dimensional). It is well-known that thermal fluctuations do play an important role for both the London theory [7] and for the LLD model [11] and surely will also play an important role for the present model, although we do not discuss this matter any further in this paper.

An interesting property of the continuous London model free energy is its twofold formulation, either in terms of the local magnetic field $\vec{h}$ and the vorticity $\vec{\nu}$, or in terms of the phase of the order parameter, ϕ, and the magnetic potential $\vec{A}$. This property can be regarded as a simple consequence to a free energy which is a sum of kinetic and magnetic field energies, where the kinetic energy density is *locally* expressed as a function of the supercurrent. Curiously this property is absent in the LLD theory, which only admits a formulation in terms of the last set of variables, ϕ and $\vec{A}$. The LLD free energy can not be expressed in terms of the *local* supercurrent and the local magnetic field, and we take this as the starting point for our theoretical considerations. We show in this paper that formulating the LLD theory in terms of the *average* value of the supercurrent, instead of its local value, provides a broader view of the LLD theory. We take here that this broader view of the LLD theory opens the way to formulate new phenomenological models. For instance, we show here that the addition of an extra assumption yields a generalized continuous London model which contains the interlayer spacing s as a paramenter. This assumption is that the supercurrent, at any point of the buffer layers, can be Taylor expanded around the layers. Taking that the layers are no longer perfect insulators yields a well-defined nearly continuum limit of the layered theory, as shown here. We recall that our goal in this paper is not the study of a particular toy model, obtained throgh the Taylor expansion assumption, but really to show that our broad interpretation of the LLD provides a route to formulate intermediate theories sitting in between the continuum London model and the standard LLD model. The zeroth-order expansion of our free energy in powers of s is the anisotropic London theory. Surprisingly, even under our simplifying assumptions, we find that our passage to the continuum is not uniquely defined and yields two nearly continumm models which are here presented.

To make our point of view clear we start reviewing the well-known continuous isotropic London theory in section II. In the next section, III, we obtain a broader formulation of the LLD theory considered so far, and regard it as a general phenomenological model for the layered superconductor. In section IV we introduce some assumptions about the supercurrent behavior at the buffer layers and obtain a new model that gives the continuous anisotropic London theory in lowest order expansion. We show that there are two possible models in this near continuum limit. Finally in section V we present our conclusions of this study.

II. London Theory

The simplest model for a superconductor is the London theory, which takes into account the magnetic and kinetic energies. The free energy is a sum of these two energies, the former expressed in terms of the local magnetic field $\vec{h}(\vec{x})$ and the latter in terms of the local superfluid velocity $\vec{V}(\vec{x})$. Call ρ the superfluid density, m and q the mass and charge of a superfluid element ($q = 2e$, $m = 2m_e$), and $\vec{J} = q\rho\vec{V}$ its local supercurrent density. The London free energy is,

$$F = \frac{1}{8\pi}\int d^3\vec{x}\,[(4\pi\lambda/c)^2\vec{J}^2 + \vec{h}^2]\,, \tag{1}$$

where $\lambda^2 = mc^2/(4\pi q^2\rho)$ is the well-known London penetration length. Obviously these two energy contributions are not independent because the superconducting fluid is charged, and so, the supercurrent also contributes to the local field. There are two possible ways to link these terms and the way this is done is important for our considerations on the LLD theory.

1) $\vec{F}[\vec{A},\phi]$. Assume the minimal coupling, which leads to a well defined prescrition that links the supercurrent contains to the vector potential. The minimal coupling is introduced at the level of the Ginzburg-Landau complex order parameter, $\psi = \sqrt{\rho}\exp i\phi$, where the supercurrent density is $\vec{J}(\vec{x}) = q\rho(\hbar\vec{\nabla}\phi(\vec{x}) - q/c\vec{A}(\vec{x}))/m$,. where the vector potential is $\vec{A}(\vec{x})$ and $\phi(\vec{x})$ is the phase of the condensate. Thus magnetic and kinetic energies are related in the following way,

$$F[\vec{A},\phi] = \frac{1}{8\pi}\int d^3\vec{x}\,[\frac{1}{\lambda^2}(\frac{\Phi_0}{2\pi}\vec{\nabla}\phi - \vec{A})^2 + (\vec{\nabla}\times\vec{A})^2]. \tag{2}$$

leading to the extreme equations,

$$\vec{A} + \lambda^2\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \frac{\Phi_0}{2\pi}\vec{\nabla}\phi. \tag{3}$$

2) $\vec{F}[\vec{h}]$. The current and the magnetic field are locally well defined, consequently Ampère's law, $\vec{\nabla}\times\vec{h} = 4\pi\vec{J}/c$, should work, yielding the free energy expression,

$$F[\vec{h}] = \frac{1}{8\pi}\int d^3\vec{x}\,[\lambda^2(\vec{\nabla}\times\vec{h})^2 + \vec{h}^2]\,. \tag{4}$$

and the corresponding variational equation is

$$\vec{h} - \lambda^2\vec{\nabla}^2\vec{h} = \Phi_0\vec{\nu}. \tag{5}$$

The vorticity field $\vec{\nu}(\vec{x})$ has been introduced because we think of a multiple connected space, with normal state filaments inside, like worm holes, where part of the external field penetrates. Thus $\vec{\nu}(\vec{x})$ vanishes everywhere except inside such filaments - the vortex lines - where London theory fails. London theory only applies to the superconducting space excluding the core of the vortex lines. In case the superconductor fills all the space, except the filaments, it is easy to show there are no sources or sinkholes for them: $\vec{\nabla}\cdot\vec{\nu}=0$ follows from applying the divergence of Eq.(5) and using that $\vec{\nabla}\cdot\vec{h}=0$.

We know that these two ways of seeing London theory are equivalent, and so, the phase and the vorticity are related, according to $\vec{\nu}=\vec{\nabla}\times\vec{\nabla}\phi$. Notice that the free energy $F[\vec{A},\phi]$ has one more equation than $F[\vec{h}]$, obtained from the phase ϕ, the variational equation $\vec{\nabla}\cdot\left(\frac{\Phi_0}{2\pi}\vec{\nabla}\phi-\vec{A}\right)=0$, whose content is no other than current conservation, $\vec{\nabla}\cdot\vec{J}=0$.

In summary we have just found that the continuous London theory can be formulated either in terms of the local magnetic field $\vec{h}$ or the set $(\vec{A},\phi)$. and changing from one set to the other is a straightforward transformation. We show, in the next section, that for the LLD theory the situation is quite distinct.

III. The London Limit of The Lawrence-Doniach Theory

In the previous section we have defined the continuous London theory free energy as a sum of kinetic and magnetic field energies of the superconductor (Eq.(1)), naturally expressed in terms of the local supercurrent and magnetic field. There is no similar formulation of the LLD theory. The LLD free energy functional is obtained from the Lawrence-Doniach theory by taking the order parameter as $\Psi(\vec{r},n)=\sqrt{\rho}\exp\left[i\phi(\vec{r},n)\right]$, where ρ is constant. The superconducting planes are located at $z=ns$, n an integer, and $\vec{r}$ is a two-dimensional vector along the layers.

$$F=\frac{1}{8\pi}\int d^3\vec{x}\;\vec{h}(\vec{x})^2+s\sum_n\int d^2\vec{r}\;f_n(\vec{r}),$$

$$f_n(\vec{r})=\frac{\hbar^2\rho}{2m_a}\Big(\frac{\partial\phi_n}{\partial\vec{r}}-\frac{2\pi}{\Phi_0}\vec{A}_n\Big)^2+\frac{\hbar^2\rho}{m_c s^2}\left[1-\cos(\phi_{n+1}-\phi_n-\chi_{n+1,n})\right]\;, \tag{6}$$

where $\chi_{n+1,n}(\vec{r})=\frac{2\pi}{\Phi_0}\int_{ns}^{(n+1)s}A_c(\vec{r},z')\;dz'$ and m_c and m_a are the mass parameters, along and orthogonal to the c-axis respectively. Hence the LLD free energy is naturally expressed in terms of the vector potential $\vec{A}(\vec{x})$ and the phase of the order parameter on the layers, $\phi(\vec{r},ns)$, $\vec{x}=(\vec{r},z)$.

Minimizing this free energy with respect to the vector potential yields Ampère's law form where we obtain the supercurrent components, along the plane and along the c-axis.

$$\vec{J}_\perp(\vec{r},z)=s\sum_n(q\rho\hbar/m_a)(\partial\phi_n/\partial\vec{r}-2\pi\vec{A}_n/\Phi_0)\delta(z-ns) \tag{7}$$

$$J_c(\vec{r},z)=\sum_n(q\rho\hbar/m_c s)\sin(\phi_{n+1}-\phi_n-\chi_{n+1,n})S_{n,n+1}(z)\;, \tag{8}$$

$S_{n,n+1}(z)$ is a function defined to be one in the interval $ns\le z<(n+1)s$ and zero elsewhere. The Lawrence-Doniach [3,4,5] supercurrent component perpendicular to the c-axis, $\vec{J}_\perp(\vec{r},z)$, diverges over the superconducting planes because they have no thickness. This component vanishes elsewhere because the regions between the buffer layers are perfect insulators in the LLD theory. The (Josephson) component along the c-axis, $J_c(\vec{r},z)$, is constant in the region between any two layers since it only depends on the difference between phases at layers n and $n+1$.

The counterpart of Eq.(1) for the LLD theory follows by expressing the LLD kinetic part in term of *average* values of the supercurrent components:

$$(q\rho\hbar/m_a)(\partial\phi_n/\partial\vec{r}-2\pi\vec{A}_n/\Phi_0)=\langle\vec{J}_\perp(\vec{r},n)\rangle_{n+1/2,n-1/2}, \tag{9}$$

and

$$(q\rho\hbar/m_c s)\sin(\phi_{n+1}-\phi_n-\chi_{n+1,n})=\langle J_c(\vec{r},n)\rangle_{n+1,n}, \tag{10}$$

where the average values are defined over s, the distance between two superconducting sheets,

$$\langle \vec{J}_\perp(\vec{r},n)\rangle_{n+1/2,n-1/2} \equiv (1/s)\int_{(n-1/2)s}^{(n+1/2)s} dz' \vec{J}_\perp(\vec{r},z') \ , \tag{11}$$

and

$$\langle J_c(\vec{r},n)\rangle_{n+1,n} \equiv (1/s)\int_{ns}^{(n+1)s} dz' J_c(\vec{r},z') \ . \tag{12}$$

In terms of these average values we propose the following free energy,

$$F = \frac{1}{8\pi}\int d^3\vec{x}\ \vec{h}(\vec{x})^2 + s\sum_n \int d^2\vec{r} f_n(\vec{r}) \tag{13}$$

$$f_n(\vec{r}) = (4\pi\lambda_a/c)^2[\langle \vec{J}_\perp(\vec{r},n)\rangle_{(n+1/2,n-1/2)}]^2 +$$

$$+ \quad 2(4\pi\lambda_c/c)^2 J_m^2 \big(1 - \sqrt{1 - \frac{[\langle \vec{J}_c(\vec{r},n)\rangle_{(n+1,n)}]^2}{J_m^2}}\big) \ . \tag{14}$$

as the LLD counterpart of Eq.(1).

The constant J_m is the maximum Josephson supercurrent, $(4\pi J_m/c = \Phi_0/(2\pi\lambda_c^2 s))$, a natural consequence of the layered structure where the minimum wavelength along the c-axis must be the interlayer separation s ($J_m = q\rho v_m$, $v_m = (\hbar/s)/m_c$); Notice that the Josephson contribution is expressed in terms of a square root because whereas the Josephson energy is proportional to *cosine*, the average Josephson current is proportional to the *sine* of the phase difference. The choice of the positive square root assures that the continuous London theory is obtained in the limit $s \to 0$.

Our claim in this paper is that Eq.(14) is a broader view of the LLD theory than the original one, Eq.(6), because it can allow a description of the buffer layers, other than just purely insulating. In the next section we propose a new toy model starting from Eq.(14), to show that this is indeed possible.

IV. Beyond the London-Lawrence-Doniach Model

The picture of strictly two-dimensional superconducting sheets surrounded by an absolute vacuum enviroment, as in the original Lawrence-Doniach model, is no longer a requirement for the Eq.(14) model. The *average* values of the supercurrent over the interplane distance s, is the route for the proposal of new three-dimensional models. We show this by adding a working assumption on the behavior of the supercurrent in the buffer layers, namely, they can be smoothly obtained from their values at the superconducting sheets. In the buffer regions, $ns \le z < (n+1)s$, the supercurrent can be determined from its value on the superconducting layer ns by a Taylor expansion:

$$\vec{J}(\vec{r},z) = \sum_{q=0}^{\infty} \frac{(z-ns)^q}{q!}\ \frac{\partial^q \vec{J}(\vec{r},z)}{\partial z^q}|_{z=ns} \ , \tag{15}$$

In this way the mean supercurrents over s just become power series around the layers,

$$\langle \vec{J}_\perp(\vec{r},ns)\rangle_{n+1/2,n-1/2} = d_z(s) J_\perp(\vec{r},z)|_{z=ns} \ , \qquad d_z(s) = \frac{\sinh\left(\frac{s}{2}\frac{\partial}{\partial z}\right)}{\frac{s}{2}\frac{\partial}{\partial z}} \ , \tag{16}$$

$$\langle J_c(\vec{r},n)\rangle_{n+1,n} = d'_z(s) J_c(\vec{r},z)|_{z=ns} \ , \qquad d'_z(s) = \frac{\exp\left(s\frac{\partial}{\partial z}\right) - 1}{s\frac{\partial}{\partial z}} \ . \tag{17}$$

Two major features of the layered structure remain, the upper limit J_m, and the differential operators $d'_z(s)$ and $d_z(s)$, this last representing the energy cost for a siginificant supercurrent change on a scale defined by s. At the expense of introducing these differential operators, the new model recovers, the local dependence on the supercurrent components at the superconducting planes.

$$F = \frac{1}{8\pi}\int d^3\vec{x}\; \vec{h}(\vec{x})^2 + s\sum_n \int d^2\vec{r}\; f_n(\vec{r})$$
$$f_n(\vec{r}) = (4\pi\lambda_a/c)^2[d_z(s)\vec{J}_\perp(\vec{r},z)|_{z=ns}]^2 +$$
$$+ \quad 2(4\pi\lambda_c/c)^2 J_m^2\big(1-\sqrt{1-[d'_z(s)\vec{J}_c(\vec{r},z)|_{z=ns}/J_m]^2}\big)\;. \qquad (18)$$

In summary starting from our broad intepretation of the LLD theory, given by Eq.(14), we have reached Eq.(18), which contains the Taylor expansion assumption. Notice that the free energies of Eq.(14) and Eq.(18) have not been completely determined yet, it remains to connect the local supercurrent at the planes to $\vec{h}$ or $(\vec{A},\phi)$, similarly to our introductory treatment of the continuous London theory. Because we are ultimately interested in the continuum limit, this question is treated next, after the passage to this limit.

At the limit where all relevant lengths are much larger than s a passage to the continum limit is justified, $s\sum_n \int d^2\vec{r} \rightarrow \int d^3\vec{x}$.

$$F = \frac{1}{8\pi}\int d^3\vec{x}\;\{\vec{h}^2 + (4\pi\lambda_a/c)^2(d_z(s)\vec{J}_\perp)^2 +$$
$$+ \quad 2(4\pi\lambda_c/c)^2 J_m^2[1-\sqrt{1-(d'_z(s)\vec{J}_c/J_m)^2}]\}\;. \qquad (19)$$

Obviously all terms introduced by the full expansions of the differential operators, $d'_z(s)$ and $d_z(s)$, are not necessary, At any moment a Taylor expansion in s and the maximum Josephson current, J_m, can be applied, producing corrections to the anisotropic continuous London theory. Let us look at the lowest order corrections to the continuous anisotropic London theory. The differential operator along the c-axis gives linear corrections in s, $d'_z(s) = 1 + (s\partial/\partial z)/2 + (s\partial/\partial z)^2/6 + O(s^3)$, and along the planes, corrections appear in quadratic order, $d_z(s) = 1 + (s\partial/\partial z)^2/6 + O(s^4)$. Up to lowest order, which corresponds to terms proportionaL to s^2 the theory is already non-linear because of the J_m contributions, since $1/J_m^2$ is proportional to s^2. The contribution of such non-linear term to the torque has been igonored in past[14]

$$F = \frac{1}{8\pi}\int d^3\vec{x}\;\{\vec{h}^2 + (4\pi\lambda_a/c)^2\vec{J}_\perp^2 + (4\pi\lambda_c/c)^2 J_c^2 +$$
$$+\frac{s^2}{8\pi}[-\frac{1}{3}(4\pi\lambda_a/c)^2\big(\frac{\partial\vec{J}_\perp}{\partial z}\big)^2 + (4\pi\lambda_c/c)^2\big(\frac{\partial J_c}{\partial z}\big)^2 + \frac{(4\pi\lambda_c/c)^2}{4(q\rho\hbar/m_c)^2}J_c^4]\} \qquad (20)$$

Such nonlinear contribution is expected to contribute in many situations and can be treated approximately by a mean field theory: $J_c^4 \approx \langle J_c^2\rangle\, J_c^2$, where the average is taken in all space. This shows that the effect of this non-linear term is the enhancement of the London penetration length along the c-axis. Applications of the present study to the magnetic torque will be seen elsewhere.

To proceed any further and obtain the variational equations for this continuum model, we have to relate the supercurrent to the variational variables $\vec{h}$ or $(\vec{A},\phi)$, similarly to our section II discussion of the continuous London theory. Varying the free energy under the small displacements, $\delta\vec{J}_\perp(\vec{x})$ and $\delta\vec{J}_c(\vec{x})$, gives that,

$$\delta F = \frac{1}{4\pi}\int d^3\vec{x}\;\{\delta\vec{h}\cdot\vec{h} + (4\pi\lambda_a/c)^2\delta\vec{J}_\perp\cdot d_z^2(s)\vec{J}_\perp +$$
$$+ \quad (4\pi\lambda_c/c)^2\delta J_c d'_z(-s)[\frac{d'_z(s)\vec{J}_c}{\sqrt{1-(d'_z(s)\vec{J}_c/J_m)^2}}]\}. \qquad (21)$$

At this point we introduce the two possible choices:

1) $\vec{J}[\vec{A},\phi]$ - We assume the familiar relations $\vec{J}_\perp = q\rho(\hbar\vec{\partial}\phi - q/c\vec{A}_\perp)/m_a$, and $J_c = q\rho(\hbar\partial_c\phi - q/cA_c)/m_c$ to be valid at the planes. Ampère's law becomes

$$(\vec{\nabla}\times\vec{h})_\perp = 4\pi d_z^2(s)\vec{J}_\perp/c\;(\vec{\nabla}\times\vec{h})_c = \frac{4\pi}{c}d'_z(-s)\{\frac{d'_z(s)J_c}{\sqrt{1-(d'_z(s)J_c/J_m)^2}}\}, \qquad (22)$$

The corresponding Euler-Lagrange equation for ϕ gives no content other than current conservation, whose expression involves the operators $d'_z(s)$ and $d_z(s)$. Multiplying these equations by $d_z^2(s)$ and $d'_z(-s)$, respectively, and solving for $d'_z(s)J_c/J_m$ first, yields that

$$(\vec{\nabla}\times\vec{h})_\perp = 4\pi d_z^2(s)\vec{J}_\perp/c \qquad (23)$$

$$\frac{4\pi}{c}J_c = (d'_z(s))^{-1}\{\frac{d'_z(s)^{-1}(\vec{\nabla}\times\vec{h})_c}{\sqrt{1+(d'_z(s))^{-1}(\vec{\nabla}\times\vec{h})_c)^2/4\pi J_c/c}}\}. \qquad (24)$$

Thus one obtains that

$$\vec{A}+\tilde{\Lambda}^2(\vec{h},s)\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \frac{\Phi_0}{2\pi}\vec{\nabla}\phi \qquad (25)$$

$$\tilde{\Lambda}^2(\vec{h},s) =$$

$$= \begin{pmatrix} \lambda_a^2 d_z^{-2}(s) & 0 & 0 \\ 0 & \lambda_a^2 d_z^{-2}(s) & 0 \\ 0 & 0 & \lambda_c^2 d'_z(s)^{-1}\frac{d'_z(-s)^{-1}}{\sqrt{1-(d'_z(-s)^{-1}\vec{\nabla}\times\vec{h})_c^2/(4\pi J_m/c)^2}} \end{pmatrix}$$

2) $\vec{J}[\vec{h}]$- Imposing Ampère's law one obtains that

$$\vec{h}+\vec{\nabla}\times[\Lambda^2(\vec{h},s)\cdot\vec{\nabla}\times\vec{h}] = \Phi_0\vec{\nu}, \qquad (26)$$

$$\Lambda^2(\vec{h},s) = \begin{pmatrix} \lambda_a^2 d_z^2(s) & 0 & 0 \\ 0 & \lambda_a^2 d_z^2(s) & 0 \\ 0 & 0 & \lambda_c^2 d_z(-s)\frac{d_z(s)}{\sqrt{1-(\vec{\nabla}\times d_z(s)\vec{h})_c^2/(4\pi J_m/c)^2}} \end{pmatrix}$$

Applying the curl to Eq.(26) leads to an equation for the local magnetic field, like Eq.(26). However they are not the same, the latter has $\Lambda(\vec{h},s)$ whereas the former has $\tilde{\Lambda}(\vec{h},s)$. Thus the vorticity $\vec{\nu}$ is not simply related to the condensate's phase ϕ as it is for the continuous London theory.

V. Conclusion

The LLD theory is naturally expressed as a function of $(\vec{A},\phi)$, the local magnetic potential and the phase of the order parameter. Both three-dimensional London and LLD free energies are sums of magnetic field and of kinetic energies. The kinetic energy of the three-dimensional London theory depends locally on the supercurrent. The LLD kinetic energy can be expressed at best as a function of the *average* value of the supercurrent over s, as shown here. We take this formulation of the LLD free energy as the starting point for the proposal of new models. In particular we consider here a very simple choice of *local* supercurrent behavior inside the buffer layers that leads to a model distinct from the original LLD theory. We assume that at the buffer layers the supercurrent is obtained by a Taylor expansion around the superconducting sheets. Dependence on the local supercurrent is retrieved at the expense of introducing some infinite order diffential operators along the c-axis. In this nearly continuum limit the model turns into the anisotropic London (AL) theory with additional s^2 dependent interactions. We find that even in the lowest order corrections, proportional to s^2, there must be non-linear corrections to the London theory, previously ignored by other authors [14]. To completely determine the model it is necessary to express the supercurrent at the layers as a function of $\vec{h}$ (Ampère's law) and $\vec{v}$, or of $(\vec{A}, \phi)$ (minimal coupling). Within the Taylor expansion picture previously discussed, we consider these two possible models in this nearly continuum limit. We show that their corresponding free energies, $F[\vec{h}]$ and $F[\vec{A}, \phi]$ yield distinct properties. A more detailed study of the properties of the vortex lattice[15], including applications to the torque[7], and its elastic properites[16] will be seen elsewhere.

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