Long Period Prominence Loop Oscillations Excited by Slow Magnetosonic Waves

C.A. de Azevedo*
International Centre for Theoretical Physics, Trieste, Italy
and
Division of Fusion Plasma Physics, Alfvén Laboratory,
Royal Institute of Technology, S-100 44, Stockholm, Sweden

T. Hellsten, E. Tennfors and A.S. de Assis†
Division of Fusion Plasma Physics, Alfvén Laboratory,
Royal Institute of Technology, S-100 44, Stockholm, Sweden

Received December 12, 1995

The slow magnetosonic waves can, under certain conditions, have a discrete set of regular eigenmodes similar to the discrete Alfvén wave spectrum. The latter were shown to explain the short period (2-8 min) prominence loop oscillations as global Alfvén modes. Here, we show that the discrete slow wave spectrum can explain the long period (24-84 min) oscillations of solar loop prominences. Conditions for the existence of a discrete slow wave spectrum are given. We have also estimated the $Q(Q^{-1} = I_m\omega/\omega)$ factor, due to electron and ion Cherenkov damping, for the loop cavity and that the value of $Q \sim 6$ is compatible with the observations.

I. Introduction

Oscillatory motions in solar atmosphere are common phenomenon and have been studied for decades. However, the oscillations occurring in the upper layers, in prominences, and in the corona are still not well understood ([1-5] and references therein). Contributions to the investigation of oscillations in the prominences (loop and sheet) have been given by a number of authors[1−28]. By modeling the loop prominence like a current carrying cylindrical plasma column de Azevedo et al.[29] explained the short-period prominence loop oscillation as being global (discrete) Alfvén wave. The proposal of this paper is to explain the long-period loop prominence oscillations in terms of global slow magnetosonic waves. The slow wave can be described as a sound wave propagating along a magnetic flux tube. In contrast to the Alfvén wave, the slow, wave is a longitudinal mode. It propagates along the magnetic field lines with the sound speed. The slow magnetosonic wave was first discovered by Herlofson[30] by including the pressure term to the magnetohydrodynamics (MHD) equations. The Alfvén wave and slow wave have singular behavior and do not, in general, appear as a global modes. This has been already discussed by Goedbloed[31,32] and Appert et al.[33]. In section II, we derive the conditions for the existence of global slow wave eigenmodes in a cylindrical plasma column. We find that these discrete modes form an anti-Sturmian spectrum, contrary to the Sturmian spectrum formed by the discrete Alfvén waves[29] and as far as we know, it is the first time that a practical application of the anti-Sturmian spectra is proposed. In section III, we model the solar loop prominences as a force-free cylindrical plasma and determine the frequencies of the eigenmodes, which are determined mainly by the temperature and linear dimension of the prominence.

*Permanent address: Instituto de Física, Universidade do Estado do Rio de Janeiro, 20550, Rio de Janeiro, RJ, Brazil
†Permanent address: Instituto de Matemática, Universidade Federal Fluminense, 24024, Niterói, RJ, Brazil.

II. Global slow magnetosonic waves

The MHD spectrum consists of three different waves: the Alfvén wave the slow and the fast magnetosonic waves. The slow magnegetosonic wave and the Alfvén wave propagate only along the magnetic field, giving rise to strongly anisotropic wave propagation proprieties. In an homogeneous plasma the "eigenfunctions" consist of solutions oscillating only on a magnetic flux surface. The solutions are not functions but exist only in the distribution sense. Each surface has its own eigenfrequency which depends on the equilibrium quantities. As the equilibrium quantities vary smoothly, the eigenfrequencies will form continuous sets. Regular solutions may exist outside the continuum for which the minimum or the maximum of the set can be a cluster point of a Sturmian or anti-Sturmian sequence of eigenmodes. This was discussed by Newcomb[34] in connection with the unstable Suydam modes and by Goedbloed[31] for the stable spectrum. However the existence of a discrete spectrum of Global Alfvén waves in an inhomogeneous plasma was first shown by Appert et al.[34]. These waves offer a likely explanation of the short period oscillation of the prominences suggested by de Azevedo et al.[29].

The slow magnetosonic wave is essentially a sound wave propagating along the magnetic field. This wave has also been named the cusp wave, because the wave front produced by a point source has a cusp. In a cylindrical plasma column with circular cross section the equation for the slow wave can, in the limit of low β, i.e. the square of the ratio between sound speed and Alfvén speed, be obtained by taking the low frequency limit of the Hain-Lüst equation[35] yielding:

$$\frac{d}{dr}\left[\frac{\rho\omega_A^2}{rk_0^2}\frac{(\omega^2-\omega_s^2)}{(\omega^2-\omega_c^2)}\frac{d}{dr}(r\xi)\right]-\rho\omega_A^2\xi=0 \qquad (1)$$

where

$$\omega^2=\frac{(k_zB_z+mB_\theta/r)^2}{\mu_0\rho}=\frac{k_\parallel^2B^2}{\mu_0\rho}$$

$$\omega_s^2=\frac{\beta}{1+\beta}\omega_A^2$$

$$\omega_c^2=\omega_s^2\left\{1+\frac{\omega_A^2\beta}{k_0^2v_A^2(1+\beta)^2}\right\}=\omega_s^2\left\{1+\frac{\beta k_\parallel^2}{k_0^2(1+\beta)^2}\right\}$$

ω_A and ω_s are the frequencies defining the Alfvén and slow wave continua, respectively, ω_c is the mathematical separator between the slow and Alfvén spectra, and ξ is the radial displacement. The sets of ξ_A, ω_s and ω_c are functions of $r(\omega_A=\omega_A(r)$, $\omega_s=\omega_s(r)$, and $\omega_s=\omega_c(r))$ and these functions depend on the equilibrium profile. The other quantities are defined as follows:

$$k_0^2=k_z^2+m^2/r^2;\quad \beta=v_s^2/v_A^2;\quad v_s^2=\gamma p/p;\quad v_A^2=B^2/\mu_0\rho$$

where ρ is the density, p is the pressure, γ is the specific heat ratio, $B = (0, B_0, B_z)$ is the equilibrium magnetic field, v_A is the Alfvén wave velocity, and v_s is the sound wave velocity, m and k_z are the azimuthal and axial wave numbers, respectively. To visualize the behavior of the solutions of Equation (1) we make the WKB ansatz $\xi = \xi \exp(ik_r r)$, representing the lowest order of WKB expansion. Introducing this ansatz into Equation (1) we obtain the dispersion relation for the slow wave

$$k_r^2 = -k_0^2 \frac{(\omega^2 - \omega_c^2)}{(\omega^2 - \omega_s^2)} \qquad (2)$$

As can be seen from the above expression the slow wave has resonances at $\omega = \pm\omega_s$ and cut-offs at $\omega = \pm\omega_c$. Since $\omega_c^2 > \omega_s^2$, the wave has an oscillatory behavior for $\omega_c^2 > \omega^2 > \omega_s^2$ and an evanescent behavior for $\omega^2 > \omega_c^2$ or $\omega^2 < \omega_s^2$, This means for this case (low β) we have only the possibility of the anti-Sturmian discrete slow waves. Depending on the equilibrium profiles there may exist regular solutions of Equation (1), for $\omega^2 > \omega_s^2$ everywhere in the plasma. If $\omega^2 = \omega_s^2$ somewhere in the plasma the solution becomes singular at this or these points. The eigenvalues corresponding to singular solutions are said to belong to the continuum. These eigenvalues also form a continuous set. The maximum and the minimum values of the continuum (ω_M^2 and ω_m^2, respectively) either correspond to local maximum or minimum or to boundary values.

From Equation (1) we can see that as ω^2 approximates ω_M^2 from above $|k_r|$ increases. Thus, as we approach, WM we expect the solution to oscillate more rapidly with respect to r. To investigate whether regular eigenmodes exist or not we have to solve the Equation (1) instead of the dispersion relation, since the WKB expansion is not valid close to singular points. To do so we make the expansion of Equation (1) around the maximum value, in powers of x, where $x = r - r_M$, and $r_m \neq 0$ is the radius where ω takes its maximum value:

$$\omega_s^2 = \omega_M^2 + 2\omega_M(x\omega_s' + x^2\omega"_s/2 + ...) , \qquad (3)$$

The maximum value can either be a local maximum or an end point depending on the profiles. In the case the maximum of the continuum is an end point we include only the lowest correction yielding $\omega_s^2 = \omega_M + 2\omega_M x\omega_s'$. Substituting this expression into ω_c^2 we have $\omega_c^2 = \omega_M^2 + 2\omega_M x\omega_s^2 + \delta$, where $\delta = \omega_M \beta k_\parallel^2 / k_0^2 (1+\beta)^2$. Assuming the other variables to be constant we obtain

$$\frac{d}{dx}\left[\frac{x}{x + \frac{\delta}{2\omega_M\omega_s'}}\frac{d}{dx}\xi\right] = k_0^2\xi \qquad (4)$$

For small x we can neglect x in the denominator yielding

$$\frac{d}{dx}\left[x\frac{d}{dx}\xi\right] = +\frac{\delta k_0^2}{2\omega_M\omega_s'}\xi \qquad (5)$$

The most singular term is obtained by assuming that the solution can be written as $X = \Sigma a_n x^n + \Sigma b_n x^n \ln x$ near the maximum value. After substituting this ansatz into Equation (5) one finds that the most singular term behaves like $\ln|x|$. Since this is not an oscillating function for small x, ω_M will hence not be a cluster point of a set of discrete eigenmodes. Although $\ln|x|$ is a square integrable function, the displacement parallel to the magnetic field has a single pole, which is not square integrable. Hence, the solution exists only in distribution sense.

If instead we have a local maximum with a finite second derivative at $r = r_M$, $r_M \neq 0$, we include only the first non-vanishing correction yielding $\omega_s^2 = \omega_M^2 + \omega_M x^2 \omega_s'$, where $x = r - r_M$. We then have $\omega_c^2 = \omega_M^2 + \omega_M\omega"_s x^2 + \delta$ which yields

$$\frac{d}{dx}\left[\frac{x^2}{x^2 + \frac{\delta}{\omega_M\omega"_s}}\frac{d}{dx}\xi\right] = k_0^2\xi \qquad (6)$$

For small x we neglect x^2 in the denominator

$$\frac{d}{dx}\left[x^2\frac{d}{dx}\xi\right] = \frac{\delta k_0^2}{2\omega\omega"_s}\xi \qquad (7)$$

Assuming ξ can be written as

$$\xi = \sum_{N=n}^{\infty} a_n x^n$$

and substituting this expression into Equation (7) yields

$$N^2 - N - \frac{\delta k_0^2}{\omega_M \omega"_s} = 0 \tag{8}$$

when x^N is the most singular term. Complex rapidly oscillating solutions near the singular point is obtained if N is a complex number and has a negative real part. This is satisfied if

$$-\frac{\delta k_0^2}{\omega_M \omega"_s} < \frac{1}{4}$$

which is the same as

$$0 < -\frac{\omega"_s}{\omega_M} < \frac{4k_\parallel^2 \beta}{(1+\beta)^2} \tag{9}$$

If the above condition is satisfied ω_M will be a cluster point of an anti-Sturmian spectrum of discrete slow waves. Resonances on the magnetic axis has to be treated separately. Assuming

$$\omega_s^2 = \omega_M^2 + \omega_M \omega"_s r^2 \tag{10}$$

we then have

$$\omega_c^2 = [\omega_M^2 + \omega_M \omega"_s r^2]\left[1 + \frac{\beta k_\parallel^2}{k_0^2(1+\beta)^2}\right] \tag{11}$$

Inserting ω_s^2 and ω_c^2 into Equation (1) yields

$$\frac{d}{dr}\left[\frac{r}{m^2 + \frac{\omega_M \beta k_\parallel^2}{\omega_s"(1+\beta)^2} + \left[k_z^2 + \frac{\beta k_\parallel^2}{(1+\beta)^2}\right] r^2}\frac{d}{dr}(r\xi)\right] = \xi \tag{12}$$

Assuming that ξ can be written as

$$\xi = \sum_{n=N}^{\infty} a_n r^n$$

we obtain

$$(N+1)^2 = m^2 + \frac{\omega_M \beta k_\parallel^2}{\omega"_s(1+\beta)^2} \tag{13}$$

where N is complex if

$$0 < -\frac{\omega"_s}{\omega_M} < \frac{k_\parallel^2 \beta}{m^2(1+\beta)^2} \tag{14}$$

The first inequality implies that ω_s^2 has a parabolic maximum on axis and the second ω_c^2 has a minimum. For $m \neq 0$, this condition is stronger than that in Eq (10) and is satisfied only in a narrow regime close to where the second order term vanishes and the maximum moves off-axis. For $m = 0$, the second condition is automatically satisfied.

For higher order off-axis zeros of $(\omega_M - \omega_s)$ we make the approximation $\omega_s^2 \simeq \omega_M^2 + \kappa x^q$. Equation (1) then takes the form

$$\frac{d}{dx}\left[\frac{\kappa x^q}{\kappa\left(1+\frac{\beta}{1+\beta}\right)^2 x^q + \delta}\frac{d}{dx}\xi\right] = k_0^2\xi \tag{15}$$

For $q > 2$ the most singular term is of the form $\exp(-C/x^p)$. We then expand ξ as

$$\xi = \sum_{n=N}^{\infty} a_n x^n \exp(-C/x^p) \tag{16}$$

Sserting Eq. (10) into Eq. (15) we obtain two conditions for the most singular terms to cancel

$$p = \frac{q-2}{2} \tag{17}$$

and

$$C = \sqrt{\frac{\delta k_0^2}{\kappa p^2}} = \frac{k_\parallel \omega_M}{(1+\beta)p}\sqrt{\frac{\beta}{\kappa}} \tag{18}$$

If $\kappa < 0$ the solution is rapidly oscillating around $\omega = \omega_M$. For higher order on-axis zeros of $(\omega_M - \omega_s)$ we use again $\omega_s^2 \approx \omega_M^2 + \kappa r^q$, and Eq. (1) takes the form

$$\frac{d}{dr}\left\{\frac{\kappa r^{q-1}}{\omega_M^2\frac{\beta k_\parallel^2}{(1+\beta)^2}+m^2\kappa r^{q-2}+\kappa\left[k_z^2+\frac{\beta k_\parallel^2}{(1+\beta)^2}\right]r^q}\frac{d}{dr}r\xi\right\}=\xi \tag{19}$$

For $q > 2$ the most singular term is again of the form $\exp(-C/x^p)$. The conditions for the cancelation of the most singular terms are again given by equations (17)-(18) and the condition $\kappa < 0$.

Thus, when the maximum value of the slow continuum represents a higher order zero of $(\omega_M - \omega_s)$, there always exists a discrete set of eigenmodes. For $m = 0$, also a second order on-axis maximum leads to discrete eigenmodes. Eigenmodes with $m \neq 0$ exist only for special temperature and magnetic field profiles. If the maximum is a end point with a non-zero derivative no regular solutions of an end point with a mon-zero derivative no regular solutions of the slow wave equation exist.

III. Model of the solar prominences

The model the solar loop prominence as a cylindrical plasma. Typical parameters are length $L = 3.7\times10^7 m$, radius $a = 1.5\times10^6 m$, longitudinal magnetic field $B = 67G$, longitudinal current $I = 6x10^{10}A$, density $10^{17}m^{-3}$ and temperature 10^4K [1]-[5]. Due to the large plasma current, small minor radius and low pressure, the magnetic field configurations has to be nearly force-free to last long enough for observing oscillations of several hours. A force-free equilibrium is given by

$$\nabla\times\vec{B}=\mu\vec{B} \tag{20}$$

Assuming μ to be constant and the solution to be cylindrical symmetric, the magnetic field is given by

$$B_\theta = B_0J_1(\mu r);\quad B_z = B_0J_0(\mu r) \tag{21}$$

where J_0 and J_1 are the Bessel functions, B_0 is the longitudinal magnetic on the axis and $\mu = \mu_0 I/(\pi a^2) < B_z >$ and $< B_z >$ is the averaged longitudinal magnetic field in the absence of plasma currents. To calculate the parallel wave number we expand the Bessel functions near the magnetic axis it yields

$$k_\parallel^2 b^2 = b_0^2\left[\frac{m\mu}{2}+k_z-\frac{k_z\mu^2r^2}{4}\right]^2 \tag{22}$$

or

$$k_\parallel^2 B^2 = b_0^2\left[\frac{m\mu}{2}+k_z\right]^2\left[1-\frac{k_z\mu^2r^2}{4\left(\frac{m\mu}{2}+k_z\right)}\right]^2 \tag{23}$$

We further assume the temperature profile to be of the form. $T = T_0(1-r^2/a^2)^\alpha$. The slow wave continuum then becomes

$$\omega_s^2=\frac{2\gamma\kappa T_0\left[1-\frac{r^2}{a^2}\right]^\alpha\left[\frac{m\mu}{2}+k_z\right]^2\left[1-\frac{k_z\mu^2r^2}{4\left(\frac{m\mu}{2}+k_z\right)}\right]^2}{m_i\left\{\frac{\mu^2r^2}{4}+\left[1-\frac{\mu^2r^2}{4}\right]^2\right\}} \tag{24}$$

For $m = 0$ this ω_s^2 has a parabolic maximum at modes exits according to section II. These modes cluster at the maximum value of the continuous spectrum. For $k_z = n\pi/L$, the frequency at the cluster point is given by

$$\omega_s^2 = \frac{2\gamma\kappa T_0(n\pi/L)^2}{m_i} \quad (25)$$

The oscillation frequency is this determined essentially by the temperature and the axial wavelength. With the previously given data and $n = 1$ we arrive at $\omega_s = 1.5 \times 10^{-3}s^{-1}$, corresponding to an oscillation period of 70min. For $m \neq 0$, the condition for a parabolic maximum at $r = 0$ is

$$\alpha > \left[\frac{\mu a}{2}\right]^2 \left\{\frac{1 - \frac{2k_z}{m\mu}}{1 + \frac{2k_z}{m\mu}}\right\} \quad (26)$$

and the maximum value given by [36]:

$$\omega_s^2 = \frac{2\gamma\kappa T_0\left(\frac{m\mu}{2} \pm \frac{n\pi}{L}\right)^2}{m_i} \quad (27)$$

The data give $\mu = 1.6 \times 10^{-6}m^{-1}$, which for $m + 1$, and $n = 1$ yields $\omega_s = 1.6 \times 10^{-2}s^{-1}$ and $1.2 \times 10^{-2}s^{-1}$, respectively corresponding to oscillation periods of 6.5 and 8.7 minutes. Condition (27) requires $\alpha > 1.1$ for $m = 1$ and $\alpha > 1.9$ for $m = -1$. For slightly higher exponents α, also ω_c has a maximum, violating the second part of condition (15). Slightly lower α results in a parabolic minimum on-axis and a parabolic maximum off-axis. Just at the limit a higher order on-axis maximum appears on. It is questionable if the $m \neq 0$ oscillations can be observed more than sporadically since they require rather special profiles.

In order to determine whether or not the slow mode can oscillate in the prominence structure, we have to estimate its Cherenkov damping (Landau and TTMP) due to electrons and ions. The damping term is calculated by Shafranov[37]

$$\frac{I_m\omega}{\omega} = Q^{-1} \approx \frac{\sqrt{\pi}}{4}\left\{\sqrt{\frac{2m_e}{m_i}} + \left[\frac{2T_e}{T_i}\right]^{3/2} e^{-2T_e/T_i}\right\} \quad (28)$$

where the first term is the electron contribution and the second is due to the ions. For the prominences we can consider $T_e \approx T_i$, to obtain

$$\frac{I_m\omega}{\omega} = \sqrt{\frac{\pi}{2}}e^{-2} \simeq 0.17 \; . \quad (29)$$

Thus, $Q = 6$, which means that oscillation and heating happen in this structure due to the anti-Sturmian slow mode.

IV. Conclusions

By modeling the solar loop prominences as being nearly force-free cylindrically symmetric equilibria we have analyzed their long period oscillations. We found that these can be explained as being global slow magnetosonic waves analogous to the short period oscillations explained as global Alfvén waves. The ratio in oscillation frequency between the two waves are roughly proportional to the square root of the β-value being of the order of a tenth. Further evidences may be obtained by studing the polarization of the wave, in good agreement with observed loop oscillations periods[1-5]. An alternative explanation of the short period oscillations could by sporadic appearance of global slow waves with $m = \pm 1$, when the profile requirements are met.

When investigating the slow wave spectrum, we found conditions for the existence of global waves. If the frequency maximum of the $m = 0$ continuum is located on the magnetic axis (or if the slow wave resonance frequency profile for general m values has a sufficient flat minimum), a set of discrete global modes cluster at the maximum frequency. These discrete modes form an anti-Sturmian spectrum for the radial eigenvalue problem, contrary to the Sturmian spectrum formed by the discrete Alfvén wave[29]. For the profiles considered here, the $m = 0$ maximum value is located on the magnetic axis and corresponds to the oscillations periods $\approx$ 70min., and the Q factor, due to the electron and ion Cherenkov damping, $Q \approx 6$, which agrees with the observations[1-5].

When the resonance frequency, ω_s, maximum is located on-axis, the condition for discrete modes is that the corresponding cut-off-frequency, ω_c has a minimum.

Further evidence may be obtained by studying the polarization of the oscillations. The frequency is determined mainly by temperature and linear dimension of the prominence. Observations of temperature would, thus, be essential for the judgment of our proposed explanation of the long period oscillations.

Acknowledgements

It is a pleasure to thank Dr. P. Carlqvist for stimulating discussions and suggestions. We are grateful to IAEA - the International Atomic Energy Agency, who financed the visit of one authors in Sweden. This work was financially supported by the Swedish Natural Science Research Council and by the European Communities under an association contract between Euraton and Sweden.

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