How to Select a Lens for Focusing of Femtosecond Pulses

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The optimal beam diameters ensuring the highest intensity in the focal point of a lens having chromatic aberration have been calculated. For large input beam diameters the pulse front distortion caused by the chromatic aberration, for small beam diameters the diffraction effects decrease the intensity in the focus. The highest intensity is obtained for intermediate beam diameters. It is shown that in case of optimal focusing the longitudinal chromatic aberration of the lens is about twice larger, than the depth of focus caused by diffraction effects. Nomograms are given for easy determination of the optimal beam diameters for different pulse shapes, pulse durations lens materials and wavelenghs.

I. Introduction

Short femtosecond laser pulses are formed just by a few optical cycles[1−3] therefore optical distortions (spherical aberration, coma, astigmatism, chromatic aberration e.t.c.) of a lens can considerable change the temporal shape of the pulse in the focus.

The effect of chromatic aberration is especially important since it is known to introduce large optical distortions of the pulse front[4−9]. The pulse front distortion introduced by chromatic aberration is proportional to the square of the beam diameter. Thus large beam diameters result in low intensity in the focal plane. On the other hand small beam diameters result in large diffraction spot size, i.e. in low intensity of the focused beam. It is therefore expected, that the highest intensity is obtained at intermediate beam sizes. In this paper we derive expressions to be used for selection of the input beam diameter resulting the highest peak power. Throughout the paper we suppose that all lens distortions are negligible except of chromatic aberration and the energy carried by beams having different diameters and pulse shapes are the same for all beam diameters and pulse shapes. Pulse broadening caused by the second and higher order dispersion terms of the refractive index was also neglected, since it was shown to be much smaller than the delay caused by pulse front distortion[4].

II. Intensity in the focal point

Let us consider a thin lens illuminated by a beam carrying a femtosecond pulse having duration of τ (FWHM in intensity) and central wavelength λ_0. Furthermore assume that the illumination of the lens is spatially homogeneous. It was shown[8] that if the input electric field has the form of

$$E(t) = E_0 s(t) e^{i\omega_0 t} \tag{1}$$

than the electric field in the focal point can be calculated as

$$E_f(t) = -\frac{iE_0 f_0}{f'(\omega_0)} \int_t^{t+T} s(\mu) d\mu \; e^{i(\omega_0 t - \Phi_0} \quad , \tag{2}$$

where $f_0 = f(\lambda_0)$, $f'(\omega_0) = df/d\omega$ at central frequency given by $\omega_0 = 2\pi c/\lambda_0$ (c is the velocity of the light in vacuum and $f(\lambda)$ is the focal length of the lens). T in Eq. (2) is the pulse broadening due to chromatic aberration. It is given by

$$T = -\frac{a^2}{2cf_0^2}\omega_0 f'(\omega_0) = \frac{a^2}{2cf_0(n_0-1)}\omega_0 n'(\omega_0) \ , \tag{3}$$

where $2a$ is the input beam diameter, $n_0 = n(\omega_0)$ and $n'(\omega_0)$ is $dn/d\omega$ at ω_0 (n is the refractive index of the lens material). The intensity in the focal point is given by $I_f(t) = |E_f(t)|^2$. If $s(t)$ is even and continuous function the intensity reaches its maximum in the moment of $t = -T/2$ [8]. This peak intensity depends on the input beam diameter. It is given by

$$I_{\max}(a) = \left[\frac{2 \cdot f_0}{a \cdot f'(\omega_0)} F_s\left(\frac{T}{2}\right)\right]^2 , \tag{4}$$

where $E_0 = 1/a$ condition is assumed to ensure the same energies for different beam diameters[8] and $F_s(t)$ is the integral function of $s(t)$ given by

$$F_s(t) = \int_0^t s(\mu) d\mu \ . \tag{5}$$

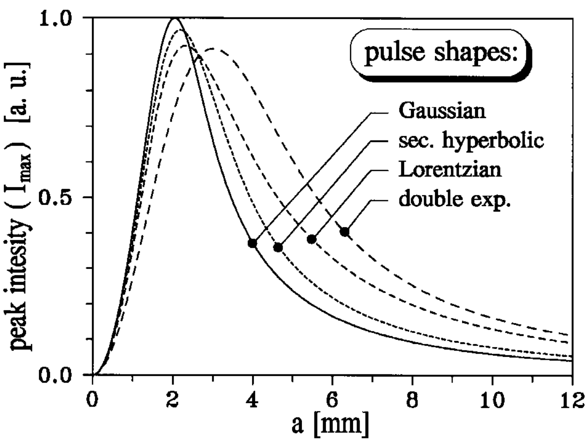

Figure 1. Maximal intensity depending on the input beam radius in the focus of a lens having chromatic aberration. The energy of the beams are supposed to be the same for different input diameters and pulse shapes.

Table 1 shows four examples for the envelopes and its integral functions for different pulse shapes. Inserting one of these integral functions and Eq. (3) into Eq. (4) one can calculate the explicit dependence of the maximal intensity. Fig. 1 shows these functions in the case of BK7 lens having f_0 = 30 mm at λ_0 = 620 nm (ω_0 = 3.038 PHz[1]), $f'(\omega_0)$ = −0.429 mm/PHz ($df/d\lambda|_{\lambda_0}$ = 2.1 mm/μm) and τ =6 fs.

III. Condition of optimal focusing

Fig. 1 also shows that the function $I_{\max}(a)$ has a maximum as it was expected. The input beam diameter ($2a_{\text{opt}}$) ensuring the highest intensity is determined by[8]

[1] $1PHz = 10^{15}hz$.

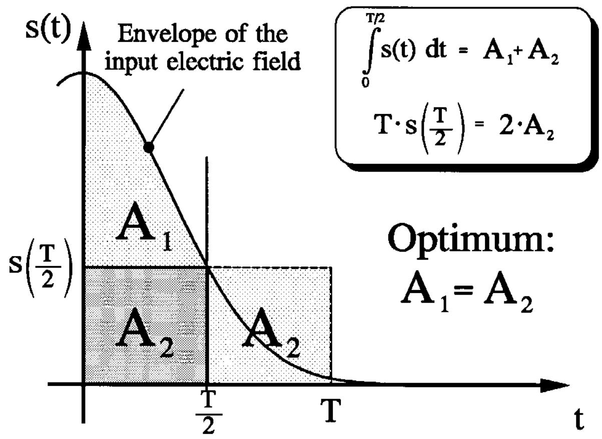

Figure 2. Drawing a vertical line at a certain point of the t-axis (denoted by $T/2$) one can obtain two areas A_1 and A_2. The left and the right side of Eq. (6) is represented by $A_1 + A_2$ and $A_2 + A_2$, respectively. The $A_1 = A_2$ condition determines the value of T_{opt} i.e. the pulse broadening in the focus for the optimal case. Then the optimal beam radius is given by Eq. (7). The condition $A_1 = A_2$ means that T_{opt} is somewhat larger than the input pulse duration.

$$\int_0^{T/2} s(\mu) d\mu = Ts(T/2) \ . \tag{6}$$

Solving Eq. (6) one can obtain the value of T_{opt} which is the pulse broadening in the focus related to a_{opt}. Then the optimal beam radius is given by (see Eq. (3))

$$a_{opt} = f_0 \sqrt{\frac{2cT_{opc}}{-\omega_0 f'(\omega_0)}} = \sqrt{\frac{(n_0 - 1)2cf_0}{-\omega_0 n'(\omega_0)} T_{opt}} \ . \tag{7}$$

Fig. 2 shows the geometrical interpretation of Eq. (6). The left side of the equation equals to the area of $A_1 + A_2$. The right side of equation equals to the area $A_2 + A_2$. So the condition of $A_1 = A_2$ gives the value of T_{opt} which is the root of Eq. (6). This simple geometrical condition is useful for the estimation of T_{opt} and the optimal beam radius as well. The condition $A_1 = A_2$ means that T_{opt} is in the order of τ, but usually a somewhat larger than τ. Table 2 shows the parameter K_s for different pulse shapes defined as

$$K_s = T_{opt}/\tau \ . \tag{8}$$

Combining Eq. (8) and (7) one can obtain the optimal beam radius as the function of $f_0 cr$:

$$a_{opt} = \sqrt{\frac{2K_s(n_0 - 1)}{\omega_0 n'(\omega_0)} f_0 cr} \ . \tag{9}$$

Fig. 3 shows this function for different wavelengths (a: 620 nm, 780 nm; b: 249 nm, 620 nm, 780 nm), pulse shapes (secant hyperbolic and Gaussian) and lens materials (a: BK7; b: fused silica).

Below we also would like to give a condition for the optimal focusing which is connected with clear physical quantities. As it was mentioned above, the intensity of the pulse in the focus is effected by the chromatic aberration and the diffraction. We define the depth of focus caused by the diffraction as the FWHM of the intensity distribution along the optical axis of a monochromatic beam with wavelength λ_0 and diameter $2a$ [10]. It is given by

$$\Delta z = K_d \frac{2\lambda_0}{\pi}\left(\frac{f_0}{a}\right)^2 , \qquad (10)$$

where $K_d = 2.7831$. The longitudinal chromatic aberration is defined as

$$L_{ch} = f'(\omega_0)\Delta\omega = f'(\omega_0)\frac{K_{ch}}{\tau} , \qquad (11)$$

where $\Delta\omega$ is the FWHM of the power spectrum of the pulse and $K_{ch} = \Delta\omega \cdot \tau$ is depending on the pulse shape. Combining Eq. (7) and Eq. (8) one can obtain that

$$a_{opt}^2 = f_0^2 \frac{K_s 2c\tau}{\omega_0 |f'(\omega_0)|} . \qquad (12)$$

From Eq. (10)-(12) it follows that

$$\frac{\Delta z}{L_{ch}} = \left(\frac{a_{opt}}{a}\right)^2 \frac{2 \cdot K_d}{K_{ch} \cdot K_s} . \qquad (13)$$

Table 2 shows for different pulse shapes that $2K_d/(K_{ch}K_s)$ is between 1 and 3. It means that in the optimal case, that is $a = a_{opt}$, the depth of focus and the longitudinal chromatic aberration are approximately equal (see Eq. (13)).

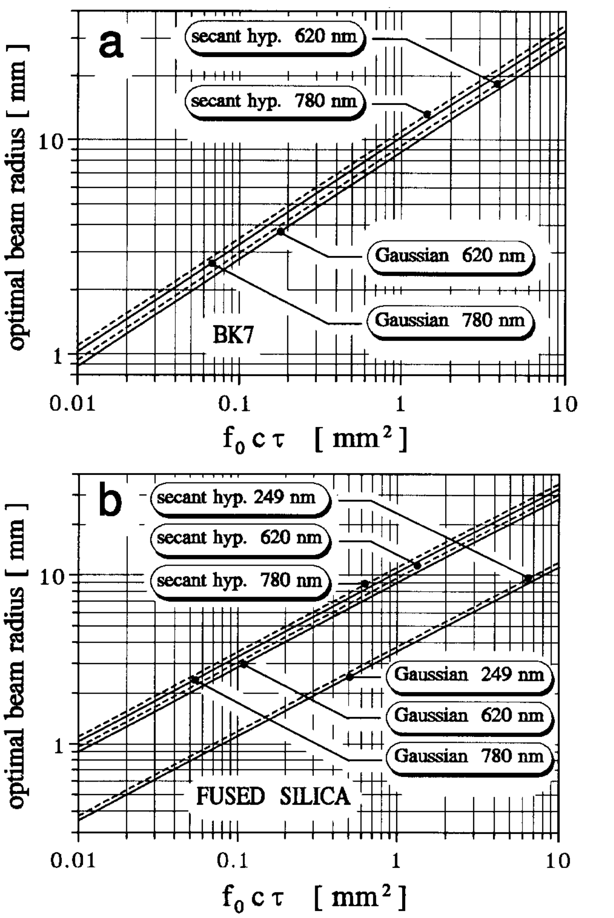

Figure 3. Optimal beam radius depending on $f_0 c\tau$ for different lens materials, pulse shapes and central wavelengths, Here f_0 is the focal length of the lens at the central frequency, c is the speed of light in vacuum and τ is the input pulse duration.

It is important to notice that optimal focusing depends on the beam diameter and not on lens diameter, supposing that the lens aperture does not clip the beam. However the use of unnecessary large lenses should be avoided, since for large central thicknesses of the lens pulse broadening due to the second derivative of the refractive can increase the pulse duration.

IV. Conclusions

In this paper equations and nomograms were calculated for easy determination of the input beam diameter, ensuring the highest peak power of the focused femtosecond pulse. A thumb rule was also derived, which states that the highest intensity can be obtained with a beam having such diameter $2a_{opt}$ that the depth of focus caused by diffraction (see Eq. (10)) was about twice larger, than the longitudinal chromatic aberration of the lens (see Eq. (11)). The exact value of a_{opt}

pulse shape	$s(t)$	$A^2(q)$	$q(\tau)$	$F_s(t) = \int_0^t s(\mu)\,d\mu$
Gaussian	$A(q) \cdot e^{-(qt)^2}$	$q \cdot \sqrt{2/\pi}$	$\sqrt{2 \cdot \ln 2}\,/\tau$	$\frac{A(q) \cdot \sqrt{\pi}}{2q} \cdot \mathrm{erf}(qt)$
sec. hyperbolic	$A(q) \cdot \mathrm{sech}(qt)$	$q/2$	$2 \cdot \mathrm{acosh}\,\sqrt{2}\,/\tau$	$\frac{2\,A(q)}{q} \cdot \arctan(\tanh \frac{qt}{2})$
Lorentzian	$\frac{A(q)}{1+(qt)^2}$	$2q\,/\pi$	$2 \cdot \sqrt{\sqrt{2}-1}\,/\tau$	$\frac{A(q)}{q} \cdot \arctan(qt)$
double exp.	$A(q) \cdot e^{-q\lvert t \rvert}$	q	$\ln 2\,/\tau$	$\frac{A(q)}{q} \cdot [\,1 - e^{-qt}\,]$

Table 1: The envelope of the electric field and its integral function for different pulse shapes. Here τ is the duration of the pulse. The function $A(q)$ ensures that the total energies represented by all spectral components are same for different pulse shapes.

pulse shape	K_{ch}	K_s	$\frac{2 \cdot K_d}{K_{ch} \cdot K_s}$
Gaussian	$4 \ln 2 =$ 2.7726	1.6816	1.1939
sec. hyperbolic	$2\,\left(2\ \mathrm{acosh}\,\sqrt{2}\right)^2/\pi =$ 1.9782	1.9181	1.4670
Lorentzian	$2\,\sqrt{\sqrt{2}-1}\,\ln 2 =$ 0.8922	2.1625	2.8850
double exp.	$2\,\sqrt{\sqrt{2}-1}\,\ln 2 =$ 0.8922	3.6253	1.7209
square	$2 \cdot K_d =$ 5.5662	1	1

Table 2: The values of the parameters used in the equations for different pulse shapes.

is determined by Eq. (6) which gives the pulse duration (T_{opt}) for the optimal case. Then a_{opt} is given by Eq. (7).

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