Optical Spectra of a Cantor Superlattice

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We investigate the optical (reflectance and transmission) spectra of a quasi-periodic structure, which obeys a Cantor sequence. The Cantor quasi-periodic layered systems are created when two different layers of different materials A and B are arranged according to the following recursion relations: $C_1 = AB_1A$; $C_2 = C_1B_2C_1; ...C_n = C_{n-1}B_nC_{n-1}$ where B_n is the same medium as layer B, but with different thickness. The simplest experimental set up for the measurement of these spectra is to place the Cantor type superlattice in the form of a slab surrounded by a transparent medium. Using a transfer matrix approach, to simplify the algebra which is otherwise much involved, it is easy to find the reflectance and transmission theoretical expressions. Then, we present numerical calculations of these spectra which show, among other interesting properties, a rich structure of self-similarity, with a series of scaling point in the reflection (and transmission) pattern.

I. Introduction

There is currently a great interest to investigate the so-called quasi-periodic structures, where two (or more) incommensurate periods are superposed, so that they can be defined as intermediate systems between a periodic crystal and the random amorphous solids[1]. Experimental evidences toward understanding these new class of quasi-crystal were given by Schechtman et al[2], and Levine and Steinhardt[3]. Furthermore, it has been recognized that quasi-periodic systems also could lead to localized states (for a review see[4−5]).

Localization due to the electronic properties of a tight-binding one-dimensional Schrödinger equation were studied by several groups. For instance, for the quasi-periodic structure based on the Fibonacci sequence, it was shown that all the electronic states were critical, with the wave functions only weakly localized, with a rich structure pattern including scaling[6−10].

On the other hand, it has been shown[11−13] that propagation of light in quasi-periodic layered materials could provide an excellent way to probe experimentally these localized states. The reason for that is because localization phenomenon is essentially due to the wave nature of the electronic states, and thus could be found in any wave phenomena. Raman scattering experiments[14,15] have also been used to study the localized states in these systems.

It is the aim of this work to present the reflectance and transmission spectra of multiple layers in a quasi-periodic arrangement forming a Cantor set. We choose this kind of quasi- periodic system since its fractal structure has recently been used to study fractons. Indeed, by setting up an artificial Cantor heterostructure, Craciun et al[16] found the direct evidence of the localized and self-similar character of the fractons, measuring the frequency spectra and amplitudes of the vibration modes.

The plan of this work is as follows: in section II, we present the method of calculation employed here, which is based on the transfer matrix approach. Section III is devoted to the presentation of the numerical results, together with the discussion of their main features.

II. Transfer matrix method

We consider a quasi-periodic layered system, which are created when two different layers of different materials A and B are arranged according to a Cantor set, that is:

$$C_1 = AB_1A;\ C_2 = C_1B_2C_1; ...C_n = C_{n-1}B_nC_{n-1} \tag{1}$$

where B_n is the same medium as layer B but with different thickness $d_{Bn} = 3^{n-1} d_{B1}$, and n means the nth generation of the Cantor sequence. The layers A and B are characterized by dielectric functions $\epsilon_A(\omega)$ and $\epsilon_B(\omega)$, respectively, with thickness d_A and d_{Bj} ($j = 1$ to n). The fractal dimension of this structure is given, according to its definition, by ln $2/\ln 3$[17].

Consider that light of frequency ω and polarization p (TM waves) is incident at an arbitrary angle θ with respect to the normal direction of the layers (see Fig. 1). The reflectance and the transmission coefficients are simply given by:

$$R = \left|\frac{M_{21}}{M_{11}}\right|^2, T = \left|\frac{1}{M_{11}}\right|^2 \tag{2}$$

where M_{ij} are the elements of the optical transfer matrix M, which links the coefficients of the electromagnetic fields in the region $z < 0$ to the coefficients of the electromagnetic fields in the region $z > L, L$ meaning the size of the Cantor superlattice.

In order to determine the transfer matrix for the nth generation of the Cantor superlattice, let us use the method of induction. For the 1st generation of the Cantor sequence, the transfer matrix M_1 is given by:

$$M_1 = T_{CA} T_A T_{AB_1} T_{B_1} T_{B_1 A} T_A T_{AC} \tag{3}$$

If we define

$$M_1' = T_A T_{AB_1} T_{B_1} T_{B_1 A} T_A \tag{4}$$

then

$$M_1 = T_{CA} M_1' T_{AC} \ . \tag{5}$$

For the 2nd generation, we have:

$$M_2 = T_{CA} T_A T_{AB_1} T_{B_1} T_{B_1 A} T_A T_{AB_2} T_{B_2} T_{B_2 A} T_A T_{AB_1} T_{B_1} T_{B_1 A} T_A T_{AC} \tag{6}$$

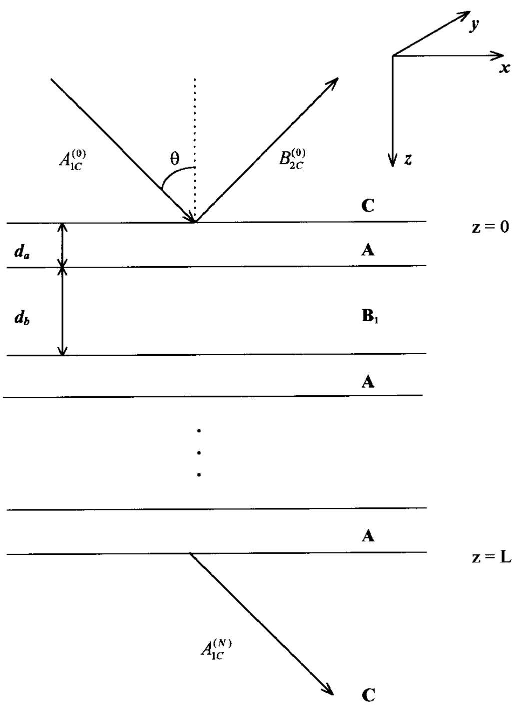


Figure 1. Quasi-periodic superlattice of Cantor type surrounded by a transparent medium C.

i.e.:

$$M_2 = T_{CA} M_1' T_{AB_2 A} M_1' T_{AC} \tag{7}$$

where

$$T_{AB_2 A} = T_{AB_2} T_{B_2} T_{B_2 A} \tag{8}$$

For the 3rd generation we have:

$$M_3 = T_{CA} M_2' T_{AB_3 A} M_2 T_{AC} \tag{9}$$

with

$$M_2' = M_1' T_{AB_2 A} M_1' \tag{10}$$

Finally, it is easy to induce that for the nth generation we have:

$$M_n = T_{CA} M_{n-1}' T_{AB_n A} M_{n-1}' T_{AC} \tag{11}$$

with

$$M_{n-1}' = M_{n-2}' T_{AB_{n-1} A} M_{n-2}' \quad (n \geq 3) \tag{12}$$

Also, (α, β equal to A, B_n or C):

$$T_{\alpha\beta} = \frac{1}{2}(\epsilon_{\beta\alpha})^{1/2} \begin{bmatrix} 1 + k_{\beta\alpha}\epsilon_{\alpha\beta} & 1 - k_{\beta\alpha}\epsilon_{\alpha\beta} \\ 1 - k_{\beta\alpha}\epsilon_{\alpha\beta} & 1 + k_{\beta\alpha}\epsilon_{\alpha\beta} \end{bmatrix} \tag{13}$$

$$k_{\alpha\beta} = k_{z\alpha}/k_{z\beta} \tag{14}$$

$$k_{z\alpha} = [\epsilon_\alpha(\omega/c)^2 - k_x^2]^{1/2},$$

with

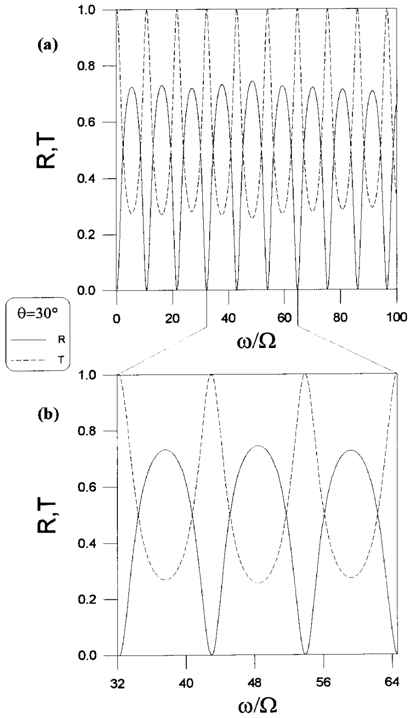
$$k_x = \epsilon_c^{1/2}(\omega/c)\sin\theta \tag{15}$$

$$\epsilon_{\alpha\beta} = \epsilon_\alpha/\epsilon_\beta \tag{16}$$

and ($\gamma = A,\ or\ B_n$)

$$T_\gamma = \frac{1}{2}\begin{bmatrix} \bar{f}_\gamma & 0 \\ 0 & f_\gamma \end{bmatrix} \tag{17}$$

$$f_\gamma = \exp(ik_{z\gamma}d_\gamma),\ \ \bar{f}_\gamma = 1/f_\gamma \tag{18}$$



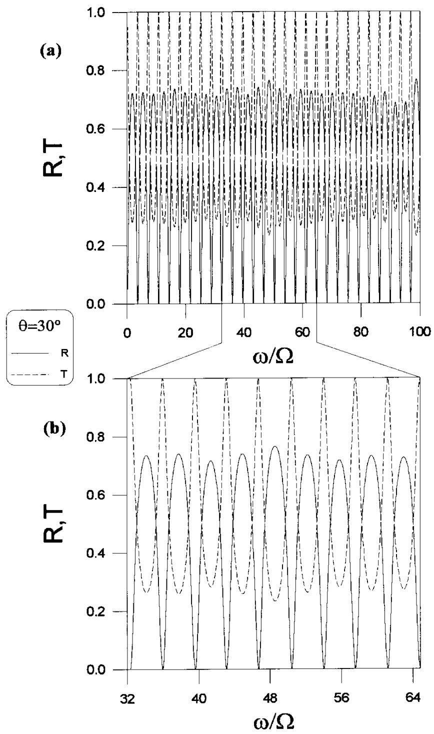
Figs. 2 (a) and (b)

III. Numerical results and conclusions

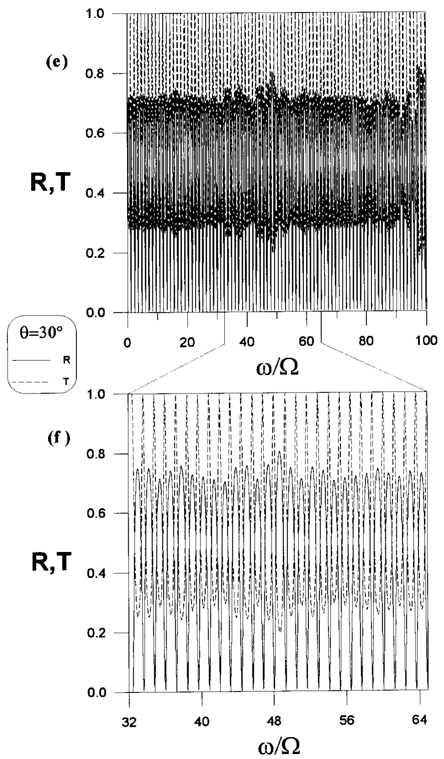
Now, we present the optical spectra (reflectance and transmission) of a quasi-periodic superlattice which obeys a Cantor sequence. We consider that the building blocks A and B (organized according to a Cantor sequence) are GaAs and SiO_2, respectively. The frequency dependent dielectric function of GaAs, apropriated for the interaction of the electromagnetic radiation with plasmons, is given by

$$\epsilon_A(\omega) = \epsilon_{\infty A}\left(1 - \frac{\omega_{PA}^2}{\omega(\omega + i\Gamma_A)}\right) \tag{19}$$

where $\omega_{PA}(\Gamma_A)$ is the plasma's frequency (damping) in medium A. For SiO_2 (medium B), we consider the frequency independent dielectric function $\epsilon_B = 12.26$. Medium C, which surrounds the superlattice, is considered to be vacuum. The other physical parameters are: $\epsilon_{\infty A} = 12.9$, $\omega_{PA} = 4.04$ THz, $m^* = 6.4 \times 10^{-32}$ kg, $d_A = d_{B1} = 40$nm and $\Gamma_A = 0$. The number of layers for the first seven generations are, respectively, 3, 7, 15, 31, 63, 127 and 255. Let us consider also that the light is incident at an angle $\theta = 30°$ (see Fig. 1).



Figs. 2 (c) and (d)



Figs. 2 (e) and (f)

Figure 2. Reflection and Transmission spectra for the physical parameters given in the main text, and for the following generations of the Cantor sequence: (a) 3rd genaration; (b) 3rd genaration in a reduced frequency interval; (c) 4th generation; (d) 4th generation in a reduced frequency interval; (e) 5th generation; (f) 5th generation in a reduced frequency interval.

Figs. 2(a) to 2(f) show the reflectance (full lines) and the transmission (dotted lines) spectra versus the reduced frequency ω/Ω, where

$$\Omega = \left[\frac{n_A e^2}{m^* \epsilon_0 \epsilon_{\infty A} d_A}\right]^{1/2} . \qquad (20)$$

Here, n_A, the carrier concentration in GaAs, is equal to $6 \times 10^{15} m^{-2}$.

From these figures, we can infer the following properties:

a) They have a rich, well defined, self-similarity pattern. This pattern can be better understood if we compare Fig. 2(a) (3rd generation of the Cantor sequence) with Fig. 2(d) (4th generation of the Cantor sequence in a reduced scale), as well as Fig. 2(c) (4th generation of the Cantor sequence) with Fig. 2(f) (5th generation of the Cantor sequence in a reduced scale);

b) They show a series of scaling points ω^*, physically characterized by a dip (peak) of the reflectance (transmission) spectrum These points are given by: $\omega^*/\Omega = 95.23\ p/3^{n-1}$, where $p = 0, 1, 3^{n-1}$, n being the nth Cantor generation. Of course, these scale points form theirself another Cantor set;

c) They are repeated at each range of frequency $(q-1) \leq \omega^*/\Omega \leq 95.23\ q, q = 1, 2, ...$;

d) The number of peaks and dips in the spectra increases as the number of generation of the Cantor sequence increases. For instance, the number of dips (peaks) of the reflectance (transmission) spectrum obeys the relation $S_n = 3^{n-1}$ (within the range of frequency defined by $0 \leq \omega^*/\Omega \leq 95.23$). Here S_n is the number of peaks (or dips) at the nth Cantor generation.

Summarizing, we have presented a fractal pattern for the reflectance and transmission spectra of a quasi-periodic superlattice of a Cantor type. The self-similarity behavior of the spectra is a rich one, with a series of interesting scaling point at the frequency. As it is shown elsewhere[18], the quahtative aspects of these spectra are in agreement with the plasmon- polaritons spectra for the same type of superlattice.

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