# Electron Mobilities in Single and Double $\delta$ -Layers

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The electron transport properties in  $\delta$ -doped semiconductors have been studied for single and double impurity layer systems. The ionized impurity scattering is considered and the screening of the electron gas is taken into account within the random-phase approximation (RPA). The subband quantum and transport mobilities are investigated as a function of the total electron density, the thickness of the doped layer, and the separation of the two impurity layers. The calculated mobilities are in good agreement with the available experimental results.

# I. Introduction

In recent years, an appreciable amount of experimental work has been done on the electron transport properties in  $\delta$ -layers<sup>[1-5]</sup>. However, rather limited number of theoretical calculations on the electron mobility in such systems are available<sup>[6-9]</sup>. The  $\delta$ -doped systems are in general characterized by a rather high electron concentration which makes them different from the other quasi-two-dimensional (Q2D) systems, such as heterojunctions and quantum wells. Typically, several subbands are occupied in a  $\delta$ -doped system and effects resulting from the occupation of several subbands are important.

In this work, we study the electron subband mobilities in heavily doped  $\delta$ -layers. The electronic structure of the  $\delta$ -layer has been obtained by solving selfconsistently the coupled Schrödinger and Poisson equations. The ionized impurity scattering is considered and the screening effects of the 2D electron gas on the scattering potential are included through the dielectric matrix within RPA. The effect of the empty subbands above the Fermi level on the electron mobility is also taken into account. The theory is applied to Si  $\delta$ -doped GaAs structures.

# II. Electronic structures

We consider a single impurity layer located in the xy-plane with thickness  $W_D$  and areal impurity concentration  $N_D$ . For the double  $\delta$ -layers, they are par-

alell in distance  $W_S$ . For typical experimental conditions we have  $N_D > 10^{12}/\text{cm}^2$  and  $W_D < 100$  Å. The background acceptors are distributed uniformly in the sample. The electronic structure of the system is determined by employing the so called self-consistent calculation within the local density approximation<sup>[8]</sup>. The subband energy  $E_n$  and wave function  $\psi_n(z)$  are obtained from the numerical self-consistent solution of the one-dimensional Poisson and Schrödinger equations. The total electron energy is given by  $E_n(\vec{k}_{\parallel}) =$  $E_n + \epsilon(\vec{k}_{\parallel})$ , where n = 1, 2, ... is the subband index,  $\epsilon(\vec{k}_{\parallel})\,=\,\hbar^2k_{\parallel}^2/2m^*$  is the electron kinetic energy, and  $k_{\parallel}$  is the electron wave vector in the *xy*-plane. The effective confinement potential  $V_{sc}(z)$  is composed as a sum of the Hartree potential  $V_H(z)$  and exchangecorrelation potential  $V_{xc}(z)$ . The Hartree potential is determined by the Poisson equation. In the calculation, we assumed that all the donors in the doping layer are ionized. The subband nonparabolicity is included through the electron density of states. The exchangecorrelation potential as a function of electron density is considered within the local density approximation.

#### III. Electron subband mobility

We consider only the ionized donor scattering because it is the most important scattering mechanism for the considered systems. The two-dimensional Fourier transform of the ionized impurities scattering potential is given by

$$v(q_{\parallel},z) = -\frac{2\pi e^2}{\epsilon_0 q_{\parallel}} \sum_{i} e^{-q_{\parallel}|z-z_i|} e^{i\vec{q}_{\parallel}\cdot\vec{R}_{\parallel i}} , \qquad (1)$$

where  $\vec{R}_i = (\vec{R}_{\parallel i}, z_i)$  is the position of the impurity, and the sum runs over all the impurities in the system which are distributed randomly in the doping layer.

In the calculation of the electron mobilities, we assume a parabolic conduction band. If ignoring the correlation between impurities, the transition matrix element due to screened scattering potential is given by  $u_{n,n'}(\vec{q}_{\parallel}) = \sum_{mm'} \epsilon_{nn',mm'}^{-1}(\vec{q}_{\parallel}) u_{m,m'}^{(0)}(\vec{q}_{\parallel})$ , where  $u_{m,m'}^{(0)}(\vec{q}_{\parallel})$  is that of the bare impurity scattering potential,  $\epsilon_{nn',mm'}^{-1}(\vec{q}_{\parallel})$  is the element of the inverse matrix of the dielectric response function, and the sum (m, m')runs over all the subbands of the system. In practical calculations, we have to limit the (m, m') sum. In most previous works, only the matrix elements of the dielectric function associated with the occupied subbands were considered. Consequently for a system of N occupied subbands the dielectric function  $\epsilon_{nn',mm'}(\vec{q}_{\parallel})$  is approximated by an  $N^2 \times N^2$  matrix<sup>[7,8]</sup>. In the present calculation, we included all the occupied subbands and up to two empty subbands above  $E_F$  in the dielectric function which is approximated by an  $(N+2)^2 \times (N+2)^2$ matrix<sup>[8]</sup>.

The electron subband quantum and transport mobilities are determined from the different scattering times connected to the average time between the scattering events. The quantum lifetime or the single particle relaxation time is the averaged elastic scattering time. On the other hand, in the transport lifetime or the momentum relaxation time, every scattering event is averaged over its projection of the outgoing wave vector on the incident direction.<sup>[1,10]</sup>





Figure 1. The subband (a) quantum mobility and (b) transport mobility as a function of the total electron density for  $W_D = 20$  Å. The experimental results are indicated by the circles (n = 1) and squares (n = 2).

In Fig. 1, the electron subband (a) quantum mobility and (b) transport mobility for the single  $\delta$ -layer of Si doped GaAs structures of  $W_D = 20$  Å are plotted as a function of the total electron density. The background acceptor concentration is taken as  $n_A = 10^{14}$  $cm^{-3}$ . It is seen that the quantum mobility increases with increasing subband index and decreases with increasing total electron density (or donor concentration). The n = 3 and 4 subbands begin to be populated at  $N_e = 1.62$  and  $4.76 \times 10^{12} / \text{cm}^2$ , respectively. At the onset of occupation of a new subband, the theoretical subband mobility exhibits an abrupt jump due to the intersubband interaction. For the transport mobility, however,  $\mu_2^t > \mu_3^t$  when only three subbands are populated. This is mainly due to the fact that the wave function  $\psi_2(z)$  is antisymmetric and has a node at z = 0. For a narrow doped layer, electrons in this subband have a smaller overlap with the impurities than those in the third subband and, consequently, the scattering is weaker. After the onset of occupation of the n = 4 subband,  $\mu_2^t$  becomes smaller than  $\mu_3^t$ . This is because  $\psi_2(z)$  and  $\psi_4(z)$  have the same parity, and the intersubband scattering between them is strong. The experimental results of subband transport and quantum mobilities [1-3,5] for the lowest two subbands are indicated by circles and squares. Experimentally, the quantum mobility is obtained by the SdH measurements<sup>[1,2]</sup> and the transport mobility is obtained by the so called mobility spectrum technique or by the Hall measurements combining with the subband electron density from the SdH measurements. Our calculation shows a quite good agreement with experimental results of the quantum mobility. For the transport mobility, quantitatively,  $\mu_1^t$  has a better agreement with experimental measurements than  $\mu_2^t$  which is about a factor of 2 larger than observed experimentally for  $N_e > 3.0 \times 10^{12}/\text{cm}^2$ .



Figure 2. The dependence of quantum mobility on the width of the layer for Si  $\delta$ -doped GaAs with  $N_D = 8 \times 10^{12} / \text{cm}^2$ .



Figure 3. The subband quantum mobility as a function of the separation of the two  $\delta$ -layers for  $N_D = 2.5 \times 10^{12}/\text{cm}^2$ and  $W_D = 10$  Å in Si  $\delta$ -doped GaAs.  $n_A = 10^{15}$  cm/<sup>3</sup>. The solid, dashed, dotted and dotted-dashed curves present the results of the n = 1, 2, 3, and 4 subbands, respectively. The experimental results are indicated by the circles, squares, triangles, and diamonds, which correspond to n = 1, 2, 3, and 4, respectively (See Ref. [5]).

In Fig. 2, the quantum mobility is given as a function of the width of doped layer  $W_D$  for Si  $\delta$ -doped GaAs of  $N_D = 8 \times 10^{12} \text{ cm}^{-2}$ . We find that, by increasing  $W_D$ ,  $\mu_1^q$  increases slowly and  $\mu_2^q$  tends to  $\mu_1^q$  at  $W_D > 100 \text{ Å}$ .  $\mu_3^q$  is about a factor 2 larger than  $\mu_2^q$ , and  $\mu_4^q$  decreases rapidly with increasing  $W_D$ . Fig. 3 shows the quantum mobility as a function of  $W_S$  in a double  $\delta$ -layer system of  $N_D = 2.5 \times 10^{12}/\text{cm}^2$ and  $W_D = 10$  Å for each layer, and  $n_A = 10^{15}/\text{cm}^3$ . We see that  $\mu_1^q$  increases slightly with increasing  $W_S$ until  $W_S = 130$  Å and, then, turns to be a decreasing function.  $\mu_2^q$  decreases monotonously as a function of  $W_S$ . At  $W_S \rightarrow 0$ ,  $\mu_2^q$  is about five times larger than  $\mu_1^q$ . But when  $W_S > 82$  Å,  $\mu_2^q$  becomes smaller than  $\mu_1^q$  and they are very close to each other. The mobility  $\mu_3^q$  is about a factor of 3 larger than  $\mu_1^q$  and  $\mu_4^q$  is close to  $\mu_3^q$  and it increases slowly with increasing  $W_S$ . The

to  $\mu_3^q$  and it increases slowly with increasing  $W_S$ . The experimental results of the quantum mobility<sup>[5]</sup> are indicated by the different symbols. Our calculation is in quite good agreement with the experimental results<sup>[5]</sup> for the four subbands.

# IV. Conclusions

In summary, the electron transport properties in  $\delta$ doped semiconductor systems have been studied for single and double  $\delta$ -layers. Our calculation shows that the electrons in the lowest subband have a low mobility which is not much influenced by the doping concentration and the thickness of the doped layer. The mobilities of higher subbands are much larger than those of the lowest one at small  $W_D$  for single  $\delta$ -layer and at small  $W_S$  for double  $\delta$ -layers. They are strongly dependent on the thickness of the doping layer and on the distance between the double layers. We demonstrated that intersubband scattering is also important for the mobilities of the higher subbands. Our calculated quantum mobilities, both for single and double  $\delta$ -layers, are in quite good agreement with experimental results from  $SdH^{[1,5]}$ . The transport mobility of the second subband has the same behavior as observed experimentally but, quantitatively, it is larger than experimental results.

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