Combined Effect of Impurities and Phonon Scattering in the Magnetic Transport: Diffusive Pole Approximation

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A calculation of the electron-LA phonon scattering contribution for an electron gas under the action of strong magnetic field is performed using memory function formalism. We take into account, through vertex corrections in the diffusive pole approximation, the combined effect due to the presence of impurities in the material, which renormalize the polarization function. The renormalized electron-phonon interaction is discussed and we calculate, in the low temperature regime, the dependence of the phonon channel resistivity in terms of the strength of magnetic field.

The resistance in solids appears as a result of elastic and inelastic scatterings by sources like, impurities and phonons. The Mathiessen's rule, concerning the sum of resistances of each mechanism separately, works very well at low temperature and applies equally well in the presence of an external magnetic field. In this work we investigate the combined effect of impurities on the electron-LA phonon interaction in the diffusive pole approximation for a two dimensional electron gas system under the action of a strong magnetic field. The correction to the magneto resistivity in the phonon channel, due to the presence of impurities in the material, is calculated using a memory function formalism[1]. Our results are based on the calculation of ladder diagrams. They are responsible for the appearance of a diffusive pole giving a non-zero contribution to the memory function (MF). We start with a two-dimensional electron gas (2DEG), under the action of a strong uniform magnetic field, $\vec{B} = B\hat{z}$, perpendicular to the motion plane and a vector potential $\vec{A}(\vec{r}) = (0, Bx, 0)$. The energy levels of the 2DEG are quantized into discrete Landau levels, characterized by a state $|n, k_y>$ and energy $\epsilon_n = \hbar\omega_c(n + 1/2)$, where n is the Landau level index, k_y is the wave vector in the y direction and $\omega_c = eB/m$, stands for the bare cyclotron frequency.

In our derivations the frequency-dependent MF, $M(\omega)$ is expressed in terms of the retarded force-force correlation function, $\Pi^R(\omega)$[1],

$$M(\omega) = -\frac{1}{n \cdot m\omega}[\Pi^R(\omega) - \Pi^R(0)] \qquad (1)$$

where the force-force correlation function for electron-phonon systems is written in the Matsubara finite temperature representation in terms of the phonon propagator and the density-density correlation function[2]

$$\Pi_{yy}(\tau) = -\sum_{\vec{q}} q_y^2\gamma^2(q)\langle T_\tau A_{\vec{q}}(\tau)A_{\vec{q}}(0)\rangle\langle T_r\rho(\vec{q},\tau)\rho(-\vec{q},0)\rangle \ , \tag{2}$$

where $\gamma(q)$ is the electron phonon coupling, $A_{\vec{q}} = a_{\vec{q}} + a^+_{-\vec{q}}$ is written in terns of the phonon destruction and creation operators. The density operator for a 2DEG under the action of a magnetic field is given by[3]

$$\rho(\vec{q}) \sum_{n,n',k_y} J_{nn}(-q_x,k_y,k_y+Q_y)C^+_{n,k_y+q_y}C_{n',k_y} \ , \tag{3}$$

with

$$J_{n,n'}(-q_x,k_y,k'_y) = \int d^2\vec{r}\psi^*_{n,k_y}(\vec{r})e^{i\vec{q},\vec{r}}\psi_{n',k'_y}(\vec{r}) \tag{4}$$

where

$$\psi_{n,k_y}(\vec{r}) = e^{-ik_y y}\phi_n(x+l_0^2k_y) \ , \tag{5}$$

$\phi_n(x)$ is the n-th harmonic oscillator eigenfunction, whereas $l_0^2 = \hbar/m\omega_c$ is the magnetic radius length. The retarded density-density correlation function for non-interacting electronic system at the presence of disorder is given by

$$\chi(\vec{x},\vec{y};\Omega) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\epsilon\{[n+(\epsilon+\Omega)-n+(\epsilon)]\chi^{RA}(\vec{x},\vec{y}\,';\epsilon+\Omega,\epsilon)+ \\ n+(\epsilon)\chi^{RR}(\vec{x},\vec{y},\epsilon+\Omega,\epsilon)-n+(\epsilon+\Omega)\chi^{AA}(\vec{x},\vec{y};\epsilon+\Omega,\epsilon)]\} \ , \tag{6}$$

with $n_+(\epsilon)$ denoting the usual Fermi function, whereas R and A mean retarded and advanced function, respectively. Here $\chi^{ij}(\vec{x},\vec{y};\epsilon+\Omega,\epsilon)$ satisfies a Dyson equation:

$$\chi^{ij}(\vec{x},\vec{y}\,';\epsilon+\Omega,\epsilon) = \chi_0^{ij}(\vec{x},\vec{y}\,';\epsilon+\Omega,\epsilon) + n_iu^2\int d\vec{r}\,',\chi_0^{ij}(\vec{x},\vec{r}\,';\epsilon+\Omega,\epsilon)\chi^{ij}(\vec{r}\,'\vec{y};\epsilon+\Omega,\epsilon) \tag{7}$$

n_i is the impurity density, u^2 stands for the mean-square impurity potential and χ_0^{ij} means the polarizability without vertex corrections[4]. The contributions $\chi^{ij}(\vec{x},\vec{y};\epsilon+\Omega,\epsilon)$, due to pure ladder diagrams, are responsible for a diffusive pole structure on the polarization function. After a straightforward calculation, we obtain for the Fourier transform of the impurity averaged polarization function in the limit of small $\vec{q},\Omega$[4,5],

$$\chi(\vec{q},\Omega) = -\frac{\Omega}{2\pi i}\chi_0^{RA}(\vec{q};\epsilon_F+\Omega,\epsilon_F) + N(\epsilon_F) + \mathcal{O}(\Omega,q^2) \ , \tag{8}$$

where $N(\epsilon_F)$ means the density of states at the Fermi energy, with

$$\chi_0^{RA}(\vec{q},\epsilon+\Omega,\epsilon) = \frac{1}{2\pi d_0^2}\sum_n\Big\{G_n^R(\epsilon+\Omega)G_n^A(\epsilon) - q^2l_0^2\Big[(n+1/2)G_n^R(\epsilon+\Omega)G_n^A(\epsilon)- \\ \left(\frac{n+1}{2}\right)G_{n+1}^R(\epsilon+\Omega)G_n^A(\epsilon) - \left(\frac{n+1}{2}\right)G_n^R(\epsilon+\Omega)G_{n+1}^A(\epsilon)\Big]\Big\} \ . \tag{9}$$

The summations in the above equation are evaluated by contour integrals, so that the final expression to the renormalized polarization function exhibits clearly a diffusive pole in the ladder diagram approximation:

$$\chi(\vec{q},\Omega) = N(\epsilon_F)\frac{D_B q^2}{-i\Omega + D_B q^2} , \qquad (10)$$

where the diffusion constant, D_B is magnetic field dependent, given by[5]

$$D_B = \frac{\epsilon_F \tau/m}{(1+w_c^2\tau^2)} . \qquad (11)$$

In our approach, according to Eqs. (2 and 10), we can write the Fourier transform of the force-force correlation function as

$$\Pi_{yy}(i\omega) = -\sum_{\vec{q}} q_y^2\gamma^2(q)\frac{1}{\beta}\sum_{i\Omega} D^0(i\omega - i\Omega)\chi'(\vec{q},i\Omega) . \qquad (12)$$

Here the symbol $D^0(i\omega)$ describes the bare phonon propagator and $\Omega = 2m\pi/\beta$ stands for Bose frequencies. The occurence of imaginary poles is eliminated in the Matsubara summations, Eq. (10), according to the properties of the Laplace transforrn of the spectral functions. One has

$$\chi(z) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{dw'p(w')}{w'-z}\Im\ z > 0 , \qquad (13)$$

with

$$\rho(\omega') \equiv \Im\chi(\omega') = N(\epsilon_F)\frac{D_B q^2\omega'}{\omega'^2 + (D_B q^2)^2} . \qquad (14)$$

Now, the evaluation of the Matsubara summations gives for the force-force correlation function

$$\Pi_{\nu\nu}(i\omega) = \sum_{\vec{q}} q_\nu^2\gamma^2(q)\frac{1}{\pi}\int_{-\infty}^{\infty} d\omega'\rho(\omega')\left\{\frac{[n_-(-\omega_q) - n_-(\omega')]}{(i\omega - \omega' - \omega_q)} - \text{term } (\omega_q \to -\omega_q)\right\} , \qquad (15)$$

where $\eta_-(\omega)$ means the Bose distribution function. So, perforrning an analytical continuation, we obtain the imaginary part of the retarded correlation function:

$$\Im\Pi_{\nu\nu}^R(\omega) = \sum_{\vec{q}} q_\nu^2\gamma^2(q)\left\{\rho(\omega-\omega_q)[n_-(-\omega_q) - n_-(\omega-\omega_q)] - \text{term } (\omega_q \to -\omega_q)\right\} , \qquad (16)$$

In this way, we can derive a well-behaved frequency-dependent imaginary part of the MF, in the static limit, as

$$\Im M(0) = -\frac{1}{n_e m}\frac{d}{d\omega}[\Im\Pi^R(\omega)]_{\omega=0} , \qquad (17)$$

The cross effect on the correction of the dc phonon resistivity ($\delta\rho_{dc}$), for 2DEG, can be evaluated in the memory function formalism, as[4]

$$\delta\rho_{dc} = -\frac{2N(\epsilon_F)D_B\beta}{dn_e^2e^2}\sum_{\vec{q}}^{2D}\gamma^2(q)\frac{q^4\omega_q}{(\omega_q^2 + D_B^2q^4)}\frac{e^{\beta\omega_q}}{(e^{\beta\omega_q}-1)^2} , \qquad (18)$$

where $\beta = (k_BT)^{-1}$ and $\omega_q = c_s q$, with c_s, denoting the velocity of the sound in the material. Finally, the negative correction effect on the phonon channel resistivity, obtained in ours calculations, can be written as

$$\delta\rho_{dc}^{eff} = \left|\frac{\delta\rho_{dc}}{-N(E_F)\zeta_d^2 D_B/4\pi N_i m_i \eta_e^2 e^2 c_s^8 \beta^5 \hbar^4}\right| = \int_0^\infty \frac{x^5e^x dx}{(e^x-1)^2(1+Ax^2)} , \qquad (19)$$

where we have assumed usual values for a two-dimensional system, e.g., Si-MOSFET[6], and we have taken $x = \hbar\beta c_s q$ and $A = D_B^2/\hbar^2\beta^2 c_s^4$.

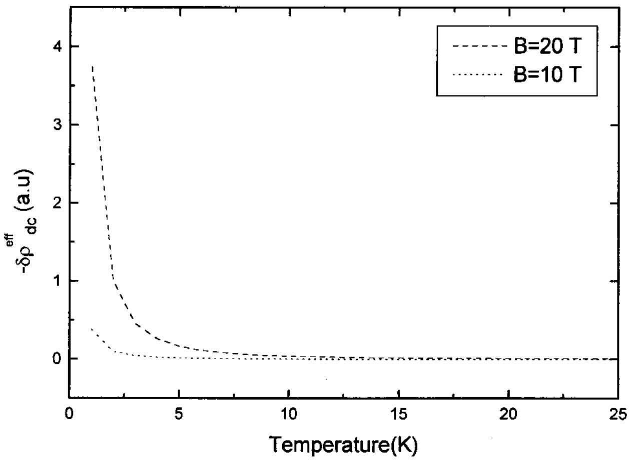

Figure 1. The behavior of $\delta\rho_{dc}^{eff}$ in the low temperature regime for some magnetic field intensities.

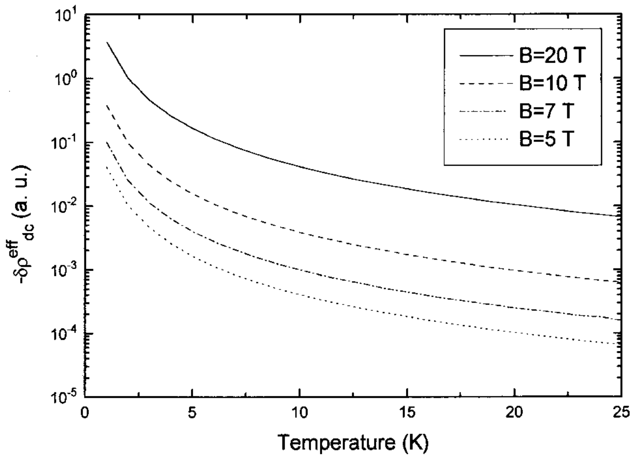

Figure 2. The log-plot for $\delta\rho_{dc}^{eff}$, in arbitrary units, as a function of the temperature for B=5,7,10,20 Tesla.,

Figs. 1 and 2 show the behavior of this correction effect on the resistivity as function of the temperature, at low temperature regime, for several magnetic field strengths. As one knows, by applying a magnetic field the flux through the path creates differences in the electron phases which are much bigger than 2π and after averaging on impurity configuration gives rise to a decrease on the resistivity[7]. In the present case, the negative correction term on the resistivity is very muchlike the negative magnetoresistance: before returning to the starting point in the self-crossing path the electron has its phase changed randomly through the scattering by the LA phonons. As in the case of randomizing the phase through the magnetic flux, this phonon scattering mechanism is expected to decrease the resistivity, as obtained in the above calculations. Furthermore, the calculations presented throughout this paper reveals that the correction $\delta\rho_{dc}^{eff}$ is significant only to high magnetic field regime.

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