Resonant Tunneling of Electrons in Asymmetric Double Quantum Wells Under Crossed Electric and Magnetic Fields

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Received July 21, 1995

We have investigated the influence of an uniform electric field, applied in the growth direction, and an uniform magnetic field, perpendicular to this direction, on the resonant tunneling of electrons in a system formed by two asymmetric quantum wells separated by a thin barrier. The semiconductor heterostructure is considered in the effective mass approximation and one band model. The method we have used to calculated the electronic structure is based on the solution of the time-dependent Schrödinger equation using the split-operator technique. The tunneling dynamics in the resonance condition is studied using the time evolution of a wave-packet from which we determine the tunneling time. A comparison with recent experimental data is presented.

I. Introduction

Tunneling of carriers in semiconductor heterostructures has been extensively studied both theoretically and experimentally, because of the importance of its fundamental quantum mechanics aspect and its technological interest in applications as fast tunneling devices[1]. In the last few years, asymmetric double quantum wells structures (ADQWS's) have been widely used to investigate tunneling through a barrier, since in those systems the tunneling processes can be studied by optical means[2−5]. Due to the different size quantization of the two wells, selective excitation in one well is made possible and optical recombination, after tunneling, can be detected. Furthermore, the coupling between the wells can be modified by application of an electric field in the growth direction and/or a magnetic field either parallel or perpendicular to it. In the resonance condition the states become delocalized over the two wells. A coherent excitation of the two levels arising from the coupling, can lead to an optical observation of charge oscillations of the photocreated wave packet[2]. Heberle *et al.*[5] have shown, using time-resolved photoluminescence (PL) spectroscopy, that the decay time reaches a minimum at certain values of applied external electric and magnetic fields, and that this minimum is directly related with the resonant tunneling transfer of electrons between adjacent wells.

Following recent experimental results[5] we have investigated theoretically the influence of uniform electric field applied in the growth direction and magnetic field applied perpendicular to it on the resonant tunneling of electrons in a system formed by an $Al_xGa_{1-x}As/GaAs$ ADQWS. The wide and the thin wells have a thickness of 100Å and 50 Å respectively. The $Al_xGa_{1-x}As$ barrier is 60Å thick and has an *Al* concentration of $x = 0.35$. The resonant and nonresonant magnetotunneling times have been measured[5] using time-resolved picosecond photoluminescence spectroscopy. During the measurements electrons and holes are photoexcited and the signal of the ground state of the thin well is monitored. For the values of electric fields used only electron states are

almost in resonance while the hole states are far from it. Therefore only electronic states are involved in the tunneling process. Due to that, the dispersion relations of this structure have been calculated for different values of electric and magnetic fields using the effective mass approximation and the one band model. We thus neglect any effect related to valence band mixing and nonparabolicity. The good agreement obtained between theoretical and experimental results shows that these effects are not important to treat the resonant tunneling of electrons. The tunneling dynamics in the resonance condition is studied through the time evolution of a wave-packet using the time dependent Schrödinger equation from which we determine the tunneling time.

The Schrödinger equation of the system, for the wave function $\phi_{nk_x}(z)$, of an electron in the n^{th} subband under an electric field applied in the growth direction (*z direction*) and a perpendicular magnetic field (*y direction*) can be expressed as

$$\left[-\frac{\hbar^2}{2}\frac{d}{dz}\frac{1}{m^*(z)}\frac{d}{dz} + V_c(z) + eEz + \frac{1}{2}m^*(z)\omega_c^2(z-z_o)^2\right]\phi_{nk_x}(z) = \varepsilon_{nk_x}\phi_{nk_x}(z), \tag{1}$$

where $\varepsilon_{nk_x} = \varepsilon_n(k_x, k_y = 0)$ is the energy of the n^{th} subband, E is the applied electric field, $m^*(z)$ is the electron effective mass that varies along the different heterostructure layers and $V_c(z)$ is the profile of the conduction band due to the band gap discontinuity. The cyclotron frequency is given by $\omega_c = eB/m^*(z)c$ and $z_0 = -\hbar c k_x/eB = -l_c^2 k_x$ is the position of the cyclotron orbit center. To solve this equation we use the *split operator technique*[6,7]. This method is based on the solution of the time-dependent Schrödinger equation, which for numerical purposes is written as

$$\psi_n(z, t+\Delta t) = \exp\left[-\frac{iV(z)\Delta t}{2\hbar}\right]\exp\left[-\frac{ip^2\Delta t}{2m^*(z)\hbar}\right]\exp\left[-\frac{iV(z)\Delta t}{2\hbar}\right]\psi_n(z,t) + O(\Delta t^3), \tag{2}$$

where $V(z)$ is the total potential energy and p is the momentum operator. The error introduced in this expression is due to the noncommutativity of the kinetic energy and potential energy operators and since in equation (2) each operator is unitary, the norm is strictly conserved. The time evolution of an wave function $\psi_n(z,t)$ is obtained first by multiplying the initial wave function $\psi_n(z,0)$ by $\exp(-iV(z)\Delta t/2\hbar)$. To perform the second operation we use the following approximation

$$\exp\left[\frac{i\hbar\Delta t}{2}\frac{d}{dz}\frac{1}{m^*(z)}\frac{d}{dz}\right] \cong \left[1 - \frac{i\hbar\Delta t}{4}\frac{d}{dz}\frac{1}{m^*(z)}\frac{d}{dz}\right]^{-1}\left[1 + \frac{i\hbar\Delta t}{4}\frac{d}{dz}\frac{1}{m^*(z)}\frac{d}{dz}\right]\psi_n(z,t). \tag{3}$$

The operator on the right-hand side is unitary and as a consequence the norm will be preserved. The error introduced by the expansion in equation (3) is of the same order as in equation (2). In order to solve equation (3) the derivatives are performed as finite differences such that the solution is reduced to the inversion of a tridiagonal matrix. The result is then multiplied by the third operator $\exp(-iV(z)\Delta t/2\hbar)$ to obtain the wave function at time $t + \Delta t$. By using this propagation scheme in the imaginary time domain ($\tau = it$) we are able to calculate the *eigenstates* $\phi_{nk_x}(z)$ and *eigenvalues* ε_{nk_x} of the equation (1). We choose the initial wave function $\psi_n(z, 0)$ as being the *eigenstates* of the harmonic oscillator. It is important to stress that, in the imaginary time propagation the wave functions must be orthonormalized at each time step. This method is very stable, almost independent on the initial wave functions $\psi_n(z, 0)$ and the energy convergence is very fast.

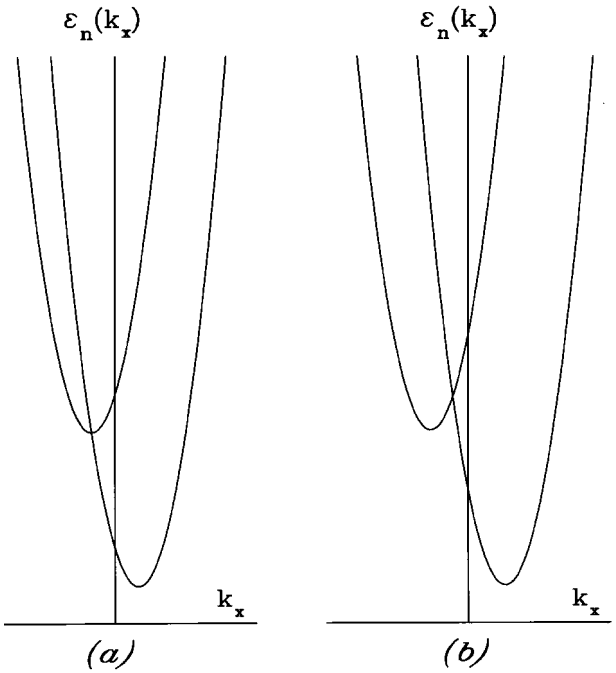

Figure 1. Schematic diagram of the dispersion relation $\varepsilon_n(k_x)$ for the first two electronic subbands of a ADQWS with a magnetic field applied in the y-direction. The intersection of the subbands allows resonant transfer of electrons from the narrow quantum well to the wide one. (*a*) resonant magnetic field; (*b*) situation where resonant transfer of electrons is not possible, since the resonant state is not occupied.

The main effect of the electric field is to vary the energy separation of the confined states of the adjacent wells, without any changes in the in-plane wave vectors. Therefore, in the resonance condition an electron can tunnel from one well to the another with any in-plane canonical momentum. On the other hand, a magnetic field applied perpendicularly to the growth direction shifts the dispersion relation of the states localized in the wide quantum well against that of the states localized in the thin one. The intersection of these subbands fulfills conservation of energy and transverse canonical momentum for resonant tunneling, and therefore, allows resonant transfer of electrons from the thin quantum well to the wide one with a well determined k_x. However, after optical excitation the electrons relax to the bottom of the subbands and resonant transfer is only possible if the applied magnetic field is such that resonant states occurs in the subband edge of the narrow well, as shown in Fig. 1. In this case, electrons can be injected resonantly at finite in-plane wave vectors in the target well. For a given value of the electric field, i.e., for a given energy separation between the energy levels of the quantum wells, we determine the magnetic field necessary to reach the resonance condition.

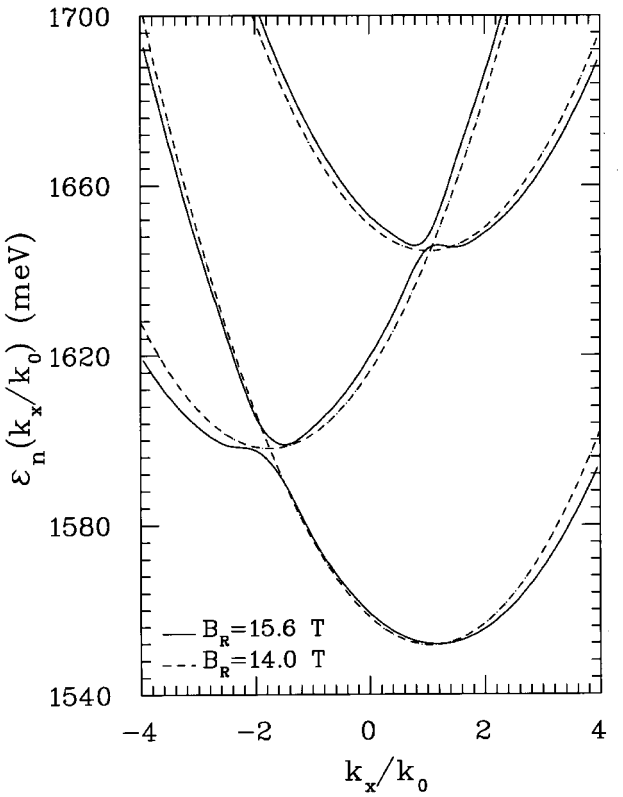

Figure 2. Dispersion relations in the absence of electric field and for a resonant magnetic field ($k_0^{-1} = a_0^*$ – effective Bohr radius). The solid line was calculated using equation (1) for a magnetic field of 15.6T and the dashed line was obtained by first order perturbation theory with a magnetic field of 14.0T.

In the Fig. 2 we show the dispersion relation, $\varepsilon_n(k_x)$, in the resonance condition obtained from the

solution of the equation (1) in the absence of electric field. Due to coupling between the wells we can observe an anticrossing of the resonant states, which produces a gap $\Delta\varepsilon$ in the dispersion relation (solid line). The dashed line was obtain treating the magnetic field as a perturbation of the first order. In this case the energy is given by

$$\varepsilon_n = \varepsilon_n^0 + \frac{1}{2m^*}\left(\hbar k_x + eB\langle z\rangle_n\right)^2 + \frac{e^2B^2}{2m^*}\left(\langle z^2\rangle_n - \langle z\rangle_n^2\right). \tag{4}$$

where ε_n^0 is the n^{th} subband energy of the unperturbed problem. The second term represents the original parabolic k_x dispersion shifted by an amount $eB\langle z\rangle_n$, where $\langle z\rangle_n$ is the expectation value of z and the last one is the diamagnetic shift. The condition for resonant transfer of the first two levels ($n = 1$ and $n = 2$) is therefore

$$\Delta E = \frac{e^2B_R^2}{2m^*}\left[(\Delta z)^2 + (\Delta z_1)^2 - (\Delta z_2)^2\right] \tag{5}$$

where $\Delta E = \varepsilon_2^0 - \varepsilon_1^0$, which is determined by the applied electric field, and B_R is the magnetic field necessary to reach the resonant condition with $(\Delta z)^2 = (\langle z\rangle_2 - \langle z\rangle_1)^2$ and $(\Delta z_n)^2 = \langle z^2\rangle_n - \langle z\rangle_n^2$. Since we are using first order perturbation theory, we do not expect any interaction between the levels, i.e., we cannot observe any anticrossing between them. This results in a lower value for the resonant magnetic field as can be noted in the Fig. 2.

Applying an electric field in the growth direction we can vary the subband energy separation, ΔE, between the energy levels localized in the different quantum wells. Decreasing ΔE shifts the resonance to lower magnetic fields, as shown in Fig. 3. The solid line was calculated with equation (1) where, for a given value of electric field we determine the magnetic field necessary to reach the resonance condition. The experimental results were obtained from the time resolved PL spectroscopy[5]. In this reference the authors claimed that the systematic difference observed between the experimental data and that obtained from the first order perturbation theory originates from the excitonic interaction. We show here that this difference is in fact due to the coupling between the levels, which is not take into account by the perturbation theory. As can be observed in Fig. 3, our results are in excellent agreement with the experiment.

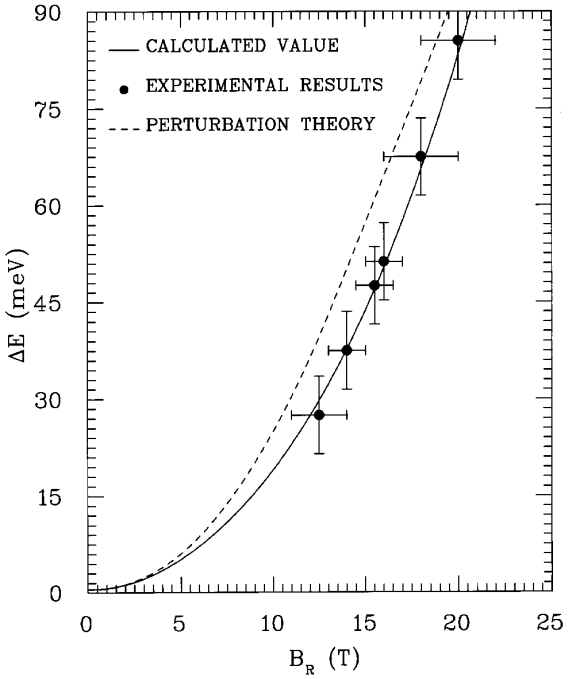

Figure 3. Subband energy separation (which is given by the applied electric field), ΔE, for several applied magnetic fields in resonance condition. The experimental results were obtained by time resolved luminescence spectroscopy[3]. The dashed line was calculated using first order perturbation theory (equation (4)) and the solid line was obtained solving equation (1).

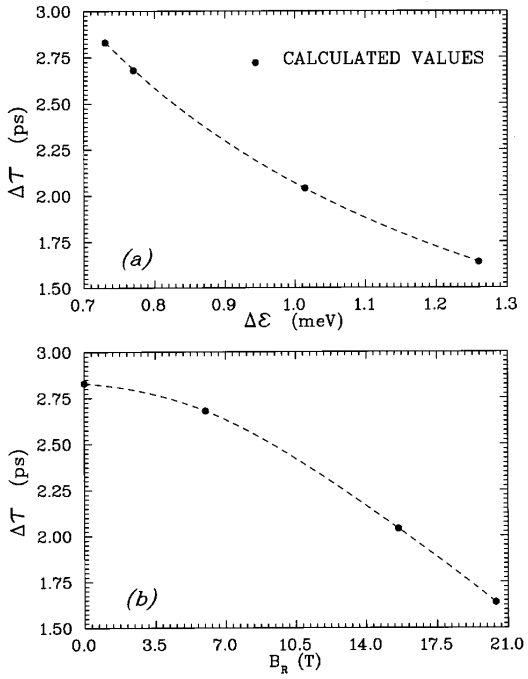

Figure 4. Resonant tunneling time as a function of the anticrossing energy gap (a) and as function of the resonant magnetic field (b). The dashed lines are a guide to the eye.

In the Fig. 4 we present the dependence of the tunneling time, $\Delta\tau$, on the applied magnetic field and on the anticrossing energy gap, $\Delta\varepsilon$. This results were obtained using the temporal evolution (equation (2)) of the states which satisfy the resonance condition. Fig. 4a shows that the dependence of the tunneling time on $\Delta\varepsilon$ follows the well known expression $\Delta\tau = \pi\hbar/\Delta\varepsilon$ obtained for a general two levels system. As can be noted from Fig. 4b the tunneling time decreases as the magnetic field, necessary to reach the resonance condition, increases. This is due to the coupling between the states which becomes stronger as the magnetic field increases, since the splitting between the levels also increases. The values obtained for the tunneling time are in good agreement with the experimental data measured using time resolved degenerated four-wave-mixing and pump-probe spectroscopy [2] and time resolved coherent detection of the submillimeter-wave radiation [8].

In conclusion, we have presented theoretical results on the resonant tunneling of electrons and the coherent wave packet oscillations in the asymmetric double quantum well structure under crossed electric and magnetic fields. The consistency between our results and the reported experiments shows that the one band model and the effective mass approximation gives us a good description of the electronic states. However, the approximation of independent bands does not provide good results to the hole states [9,10], since the inclusion of valence band mixing is crucial to understand the hole tunneling transfer.

References

1. For a review see F. Capasso, ed., Physics of Quantum Electron Devices, Springer Series in Electronics and Photonics, Vol. 28 (Springer, Berlin, 1990).
2. K. Leo, J. Shah, E. O. Göbel, T. C. Damen, S. Schmitt-Rink, W. Schäfer, K. Köhler, Phys. Rev. Lett. **66**(2), 201-204 (1991).
3. J. Kuhl, R. Strobel, R. Eccleston, K. Köhler, Semicond. Sci. Technol. **7**, B418–B420 (1992).
4. A. P. Heberle, W. W. Rühle, M. G. W. Alexander, K. Köhler, Semicond. Sci. Technol. **7**, B421–B423 (1992).
5. A. P. Heberle, M. Oestreich, S. Haacke, W. W. Rühle, J. C. Maan and K. Köler, Phys. Rev. Lett. **72**(10), 1522-1525 (1994).
6. R. Kosloff, J. Phys. Chem. **92**(8), 2087 (1988).
7. M. H. Degani, Appl. Phys. Lett. **59**(1), 57 (1991).
8. H. G. Roskos, M. C. Nuss, J. Shah, K. Leo, D. A. B. Miller, A. M. Fox, S. Schmitt-Rink, K. Köhler, Phys. Rev. Lett. **68**(14), 2216-2219 (1992).
9. R. Ferreira and G. Bastard, Europhys. Lett. **10**(3), 279-284 (1989).
10. Ph. Roussignol, A. Vinattieri, L. Carraresi, M. Colocci, A. Fasolino, Phys. Rev. B **44**(16), 8873-8879 (1991).