Investigations of Electron Wave Interference and Quantum Chaos in Ballistic Quantum Dots with Square Geometry

J.P. Bird[1]*, K. Ishibashi[1], R. Newbury[2], D. M. Olatana[2],
R. P Taylor[2], Y. Ochiai[3], Y. Aoyagi[1], T. Sugano[1]
1: Nanoelectronics Materials Laboratory,
Frontier Research Program, RIKEN, 2-1 Hirosawa,
Wako, Saitama 351-01, JAPAN

2: School of Physics, University of New South Wales,
Kensington, Sydney 2052, AUSTRALIA

3: Department of Materials Science, Chiba University,
1-33 Yayoi-cho, Inage-ku, Chiba 263, JAPAN

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In this report we summarise the results of low temperature magneto-resistance studies, of electron transport in ballistic quantum dots. Evidence for the ballistic nature of motion in the dots is provided by the observation of: a clear linear form, to the weak localisation peak at very low magnetic fields, and; by the temperature dependent behaviour of the aperiodic fluctuations, observed over a much wider field range, which is found to be very different to that typical of disordered systems. The square geometry employed in the devices is of particular interest in studies of quantum chaos, since while the classical motion of particles in the geometry at zero field is known to be regular, theory predicts a transition to chaotic motion in a finite magnetic field, and in this report we also discuss the experimental evidence for this transition.

I. Introduction

Recent advances in semiconductor processing technology have enabled the fabrication of quantum dot devices, in which the effects of bulk disorder are effectively eliminated. Large angle scattering of electrons in such devices occurs only at the dot boundaries, and experimental studies have demonstrated the presence of low field magneto-resistance fluctuations, due to interference between electrons ballistically confined in the dot[1,2]. A semi-classical analysis of the fluctuations suggests it should be possible to determine the electrical characteristics of such conductors, simply from a knowledge of their shape[3]. In particular, the magneto-transport characteristics of small dots are expected to take very different forms, in structures in which the classical motion is either chaotic or regular, and much recent interest has focused on the potential of such devices, as a probe of the effects of quantum chaos[3].

In this report we summarise the results of recent experimental studies, of the low temperature magneto-transport properties of ballistic quantum dots with square geometry. The square geometry is expected to be of particular interest, since while the classical motion of particles in the geometry at zero field is known to be regular, theory predicts a transition to chaotic motion in a finite magnetic field. Evidence for the ballistic nature of transport in the dots is provided by studies, of the weak localisation peak observed close to zero magnetic field. This shows a clear linear de-

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*Contact Author: JP Bird Tel: 048462-1111, Fax: 048465-8048, E-mail: bird@postman.riken.go.jp

pendence on magnetic field, consistent with the regular scattering of electrons within the dot[4,5]. Ballistic motion is also consistent with our observation of aperiodic fluctuations, over a wider range of magnetic field, with a temperature dependence very different to that typically observed in disordered systems[6]. In addition, a Fourier analysis of the fluctuations is found to be partially consistent with predictions for chaotic motion[3], and therefore provides evidence for the expected field induced transition, from regular to chaotic motion in the square geometry.

II. Sample preparation and measurement technique

Split gate quantum dots were realised in a GaAs/AlGaAs heterojunction, using standard electron-beam lithographic techniques. The wafer was patterned in to a Hall bar geometry, with a typical carrier density 4.10^{15} m^{-2}, and mobility 40 m^2/Vs. The gates consisted of a stub like design, in which a lithographically square dot was separated from the source and drain by quantum point contacts[6]. Independently tunable dots with gate dimensions 0.6-, 1.0- and 2.0-μm were defined within close separation on the same Hall bar wafer. The dots were considerably smaller than the calculated mean free path in the bulk wafer (5 μm), and electron transport within them was therefore expected to be ballistic. The samples were clamped to the mixing chamber of a dilution refrigerator, and magneto-transport measurements were made at cryostat temperatures down to 10 mK. The four probe configuration employed[6] included a series contribution due to the source and drain regions, and at low magnetic fields the resistance of this was much smaller than that of the dot. Great care was taken to ensure good thermal contact to the samples, and a source-drain excitation of less than 3 μV was employed for the current bias measurements.

III. Basic experimental results

As examples of the typical features observed at low temperatures consider Figs. 1 and 2. Fig. 1 shows a zero field peak in the magneto-resistance, with a linear lineshape at very low fields ($\leq$0.002 T). In contrast, Fig. 2 shows that over a wider magnetic field range the magneto-resistance is largely dominated by aperiodic fluctuations. The fluctuations are reminiscent of those observed in the low temperature magneto-resistance of disordered mesoscopic devices, in which they result from interference between electrons, multiply scattered by sample impurities[7]. In ballistic devices there should be no bulk disorder, however, and the fluctuations instead result from interference between electrons, trapped within the dot and forced to undergo multiple boundary scattering events[3]. In this report we focus on an analysis of both the zero field peak, and the aperiodic fluctuations, to establish two important features of electron transport in the dots: firstly to show that motion is indeed ballistic, with large angle electron scattering occurring only at the dot walls, and; secondly, to provide evidence for the expected, magnetically induced transition from regular to chaotic scattering.

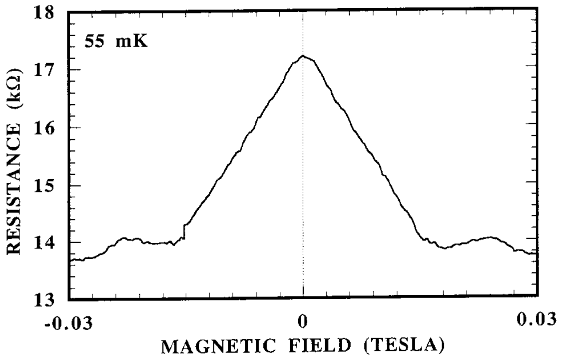

Figure 1. Zero field magneto-resistance peak observed in a 0.6-μm quantum dot at low temperatures.

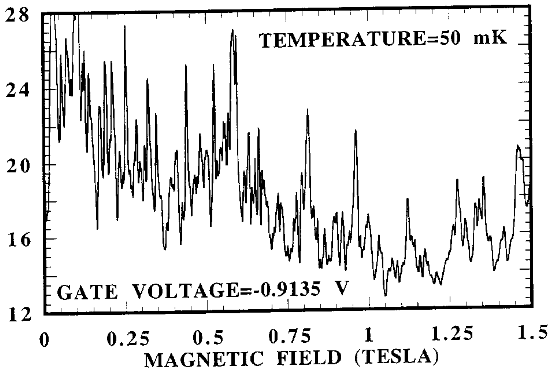

Figure 2. Over a wider range of magnetic field than shown in Fig. 1, reproducible aperiodic fluctuations are observed in the magneto-resistance of a 1-μm dot. Note that this is not the same dot as measured in Fig. 1.

IV. Evidence for ballistic motion in the dots

Evidence for ballistic motion of electrons in the dots is provided by the lineshape of the zero field magnetoresistance peak. The zero field peak is understood to result from weak localisation of electrons ballistically scattered within the dot, and its lineshape has been predicted to depend strongly upon the nature of scattering in the dot[4,5]. In particular, the peak is expected to show a linear dependence on magnetic field when the scattering is regular, whereas a Lorentzian form is expected for chaotic scattering[4]. Motion in the square geometry is expected to be regular at extremely low magnetic fields, in which case the ballistic weak localisation peak should show a linear dependence on magnetic field. Since this is indeed what we observe experimentally (Fig. 1), we conclude that large angle electron scattering occurs only at the walls of the dots. Further evidence for ballistic motion is provided by the temperature dependence of the aperiodic fluctuations. In particular, increasing temperature the fluctuations are found to weaken, until they are no longer resolved at temperatures in excess of a few degrees Kelvin (Fig. 3). In order to quantify the temperature dependent characteristics of the fluctuations, we first define their correlation function[3,7]:

$$F(\Delta B) =< [g(B)- < g(B) >] \cdot [g(B+\Delta B)- < g(B) >] > , \tag{1}$$

where $g(B)$ is the conductance in units of e^2/h at magnetic field B, and the angled brackets indicate an average over a suitably large field range. The correlation field B_c is then defined from the half-width $F(B_c) = F(0)/2$, while the root mean square fluctuation $\delta g = \sqrt{[F(0)]}$. Applying these considerations to our data we find that δg decreases *exponentially* with increasing temperature, while B_c remains *unaltered*[2]. Such scaling is very different to that observed for conductance fluctuations in disordered systems, in which δg typically only decays as a weak *power law* of temperature[8], while B_c shows a simultaneous *increase*. Such a difference between disordered and ballistic systems was previously theoretically anticipated, however, and is associated with the absence of large scale disorder in ballistic dots[3]. In particular, the large number of impurities in disordered systems ensure that, even if the inelastic scattering length of the electrons becomes much shorter than the sample dimensions, there will still be regions of the system which exhibit coherent interference. In ballistic systems, however, once the inelastic length becomes shorter than the dot dimensions, it should not be possible to consider coherently interfering sub-segments, since the scattering of electrons occurs only at the device boundaries. The fluctuations are therefore expected to be much more sensitive to temperature in ballistic systems.

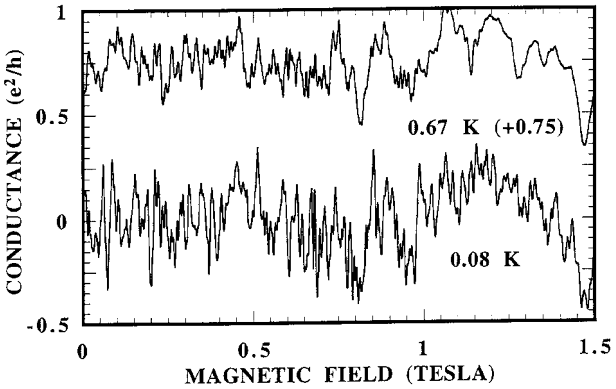

Figure 3. Temperature dependent decay of the conductance fluctuations, obtained in the 1-μm dot of Fig. 2 (the higher temperature trace has been shifted upwards by 0.75.e^2/h for clarity.) The traces were obtained by subtracting a smoothed polynomial fit from the raw data, with an average resistance of roughly 16 kΩ.

V. Evidence for a magnetically induced transition to chaos

The transport properties of ballistic conductors are expected to be strongly influenced by geometrical considerations. In particular, for geometries in which the classical motion is chaotic, the Fourier spectrum of the fluctuations should take the form[3]:

$$S(f) = S(0) \cdot [1 + 2\pi\alpha\phi_0 f] \cdot \exp[-2\pi\alpha\phi_0 f] \ , \tag{2}$$

where $1/\alpha$ is essentially the average coherent area enclosed by electrons, ϕ_0 is the single flux quantum, and the magnetic frequency f corresponds to an effective electron path area $A = \phi_0/f$. Close to zero field we have already shown above that motion in the square geometry we study is regular. In a finite magnetic field, however, recent calculations predict a transition to chaotic behaviour and it is therefore of interest to compare the spectral characteristics of the aperiodic fluctuations with the predictions of Eqn. 2. In Fig. 4 we show the smoothed Fourier spectra of the fluctuations, observed in measurements on 1- and 2-μm dots (each spectrum represents an average of eight individual spectra, evaluated over half-overlapping ranges of magnetic field[1].) The solid curves are least squares fits to the form of Eq. 2. Interestingly, these only give good agreement at low frequencies, corresponding to trajectory areas much smaller that of the dot, while for larger path areas significantly enhanced content is observed. In particular, the Fourier amplitude is found to saturate at high frequencies, at values significantly in excess of the noise floor. Since the form of Eqn. 2 is derived by assuming an exponential distribution of trajectory areas within the dot[3], deviation from this form at high frequencies suggests a non exponential distribution of large area paths in the dot. Further theoretical studies are required to determine if this is in turn related to some interesting intrinsic effect, such as the formation of stable orbits within the dot[5].

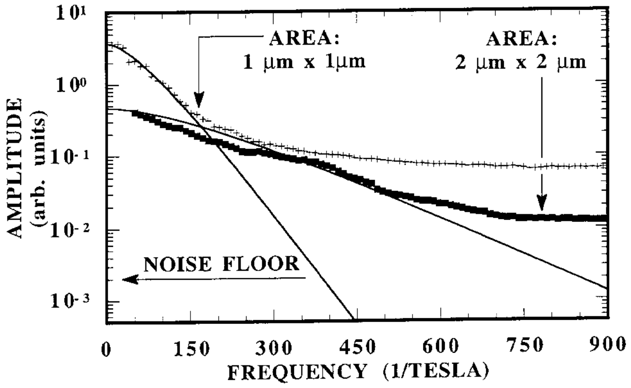

Figure 4. Smoothed Fourier spectra of the fluctuations observed in 1-μm (crosses) and 2-μm (squares) dots at 0.01 K. The spectra were determined over the range from 0-0.4 Tesla, while the solid lines are single parameter, S(0), least-squares fits to the form of Eq. 2. Arrows indicate the frequencies at which the trajectory areas ($A = f.\phi_0$) are calculated to be the same as that of the dots.

VI. Conclusions

In conclusion, we have used low temperature magneto-resistance studies to obtain important information on the nature of electron scattering in ballistic quantum dots, realised using a split gate technique. Evidence for the ballistic nature of motion in the dots is provided by the observation of: a clear linear form, to the weak localisation peak at very low magnetic fields, and; by the temperature dependent behaviour of the aperiodic fluctuations, observed over a much wider field range, which is found to be very different to that typical of disordered systems. Since the linear form of the weak localisation peak is consistent with regular scattering in the dots at zero field, it also implies that the square shape of the gates is reasonably well preserved in the two dimensional electron gas layer itself. Evidence for the expected transition from regular to chaotic motion at weak magnetic fields was only partially conclusive. In particular, the Fourier spectra of the fluctuations were only found to be consistent with predictions for chaotic dots, over a relatively small dynamic range, and at higher frequencies an undetermined size effect gave rise to enhanced power content. This in turn suggests a non-exponential distribution of electron paths, with areas comparable to or larger than that of the dot.

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