The Continuum in Reactions with Light Exotic Nuclei

F.M. Nunes

Universidade Fernando Pessoa, Praça 9 de Abril, 4200 Porto, Portugal

Received on 30 October, 2002

One of the most striking features of dripline nuclei is their very low binding energy. The proximity to threshold implies that many of the standard reactions should take into account the possibility of the nucleus breaking up into the continuum and rearranging itself within the continuum, throughout the reaction process. Whereas for stable nuclei often structure could be factorized out of the reaction model, it is clear that for exotic nuclei, the specific structure features cannot be desentangled. We review a variety of different cases where the coupling to the continuum and between continuum states was shown to be crucial for the understanding of the physical phenomena. Breakup reactions of $^8$B, for various energies and on a few targets, are discussed. The effect of the continuum on the elastic cross sections as well as transfer and fusion processes is also presented.

I Introduction

Nuclei far from stability exhibit interesting features that are strongly determined by the very low energy that binds the system. As technology develops, the various laboratories around the world have provided the community with increasingly high quality reaction experiments where these exotic nuclei, constituting the beam, can be studied [1]. The primary total reaction cross section measurements [2], gave way to differential breakup cross section data, where initially only one of the fragments was measured (as for example the Notre Dame data for $^6$He [3] and the Oak Ridge data for $^{17}$F [4]), followed by complete kinematics breakup, where both fragments were detected (an example is the $^9$B GSI results [5]). Elastic scattering reactions on a variety of different targets (e.g. [6, 7, 8]) have been facing the challenge of increasing the angular range, and additionally there are also some examples of inelastic measurements, by coupling gamma detectors to the charged particle detection setup (e.g. [9]). Considerable effort in the field is now shifting into transfer processes. Here detailed spectroscopy can be learnt [10, 11] or even astrophysical information can be extracted [12]. Fusion reactions cannot go without mention as they may shed light into the synthesis of the superheavy elements (the most studied case to date has been $^{9}$He [13]).

If a decade ago theorists were aiming at unveiling the general properties of these nuclei, now that these are well understood, studies become more ambitious. As our probes are mostly reactions, one has to refine not only the structure models used to understand the properties of these exotic systems, but also to improve the reaction description. Unfortunately it has become clear that it is generally not possible to desentangle the structure from the reaction models. Thus, in order to have a reliable description of a particular experiment, one needs to take into account the specific features of these nuclei when developing the reaction model. In particular one should keep in mind that:

- these nuclei have long extended tails, and therefore finite range effects will be strong;
- many such nuclei exhibit strong few body clusterization, where recoil effects play an important role;
- given the proximity to threshold of the ground state, the continuum states may influence the reaction mechanism.

This contribution focuses on the last of these properties, the continuum, and its effect in reactions. Historically one has always divided the continuum into resonant-continuum and non-resonant pure-scattering states. Although structure models concentrate on a good description of the resonances, non-resonant continuum is often forgotten in reaction models. Yet is it present, and it plays an important role in most cases, not only contributing on its own but causing strong interferences. If the objective of the experiment is to measure the spectrum of the nucleus, one needs to determine the regime so to emphasize the effect of the resonances, and minimize the effect of the non-resonant part. Resonant structures in light dripline nuclei have been reported in the recent years [14, 15, 16]. On the other hand, for measuring the properties of the ground state, one would like to minimize the effect of all continuum states. Also in Astrophysics, the distinction between resonant and non-resonant continuum is usually made. Therein capture reactions may have a resonant contribution and a non-resonant contribution [17]. Both are often included incoherently (e.g. [18, 19]) and some cases have been shown to produce considerable interference.

In this paper we will show the importance of the non-resonant continuum states in a variety of reactions that have been measured. We will study the dependence on beam energy and binding energy in order to gain intuitive insight into the general trends of the couplings to these breakup states. We will conclude with some general remarks concerning the future of the field.
II Breakup reactions

From the early days of radioactive nuclear beams, measurements of breakup reactions have been numerous, since the cross sections are typically large. When the breakup of a stable nucleus is sufficiently peripheral (\( b > R_c + R_p \)), it is dominated by the Coulomb part. In such cases the semiclassical first order perturbation Alder and Winther theory is used [20], with possible corrections for higher order effects. However when the nucleus is spatially extended, the nuclear contribution has effects at distances much larger than what could naïvely be expected. Even if considering Coulomb processes only, it has been shown that the Coulomb interaction when appropriately folded with the extended wavefunction of the loosely bound system, produces a significant reduction of the cross section [21].

One of the most successful ways of including the continuum in reaction models consists on the Continuum Discretized Coupled Channels method (CDCC) [22]. This method is non-perturbative as it includes all multistep paths within the continuum of the projectile for its breakup into core + fragment. The continuum states of the projectile are integrated over an energy bin, and subsequently the Schrödinger coupled channel scattering equation for the relative motion between projectile and target is solved. The coupling potentials consist on the sum of the interactions between the projectile fragment and the target, as well as the projectile core and the target, folded by either the bound state wavefunction of the projectile or the continuum bins. Here both resonant and non-resonant continuum are automatically included, depending only on the definition of the Hamiltonian of the projectile used to generate the scattering wave functions. As the bins are square integrable averages of scattering states over energy intervals, continuum-continuum couplings can be included in a straightforward way. Therefore, one can distinguish the breakup couplings that connect the bound state and a specific continuum bin, from the continuum-continuum couplings. If the later are left out of the calculation, the breakup process can still involve multiple steps, yet the various continuum-continuum couplings were shown to have a decisive influence [23].

The CDCC formulation is fully quantum mechanical, dealing with nuclear and Coulomb contributions on the same footing. This method has been compared with other semiclassical formulations that also include continuum couplings [24, 25]. It was verified that these so-called dynamical time-dependent approaches agree with the CDCC results under the same conditions.

Also reassuring is the good agreement with data under the most difficult theoretical conditions, where all typical approximations cannot be made: large angles low energy breakup data. The Notre Dame experiment for the breakup of \(^8\)B on a \(^{58}\)Ni target at \(E_{lab} = 25.8\) MeV, detected only the outgoing \(^7\)Be [26]. Note that \(^8\)B is a very loosely bound system \(S_p = 0.137\) MeV, and it is the lightest and more convincing proton halo candidate, suggested by its very large quadrupole moment [27]. In order to make a comparison with the Notre Dame data [26], the three body kinematics was correctly derived and the full three body observables where constructed from the CDCC calculations [28]. Direct comparison with the data shows excellent agreement for both the angular distributions and the energy distributions. In that work [28] the importance of the continuum couplings is clearly demonstrated. In particular there is a very strong suppression of the nuclear peak at large angles due to the continuum-continuum couplings. Besides, these couplings are responsible for the symmetry of the energy distributions, which otherwise would be asymmetric (see the 1-step DWBA calculations in [28]).

Whenever performing a CDCC calculation, convergence aspects need to be seriously considered. The discretization of the continuum truncates the space by considering a limited number of partial waves and a maximum relative energy for the motion of the core-fragment within the projectile. There is also a truncation in the multipole expansion of the operator. Usually only dipole and quadrupole terms are considered but there are circumstances where higher terms need to be included [28]. Essentially, depending on the observable desired, one finds different ranges of sensitivity and needs to take care in checking that the set of parameters spans the necessary subspace. For this reason it is often the case that calculations become cumbersome and some times even unfeasible. We will come back to this point at a later stage.

In order to get insight into the effect of the continuum couplings in the reaction mechanism, we have performed a set of calculations that illustrate the dependence with beam energy, binding energy and other properties. In Figs. 1, 2 and 3, we present the cross sections for the breakup of \(^8\)B on \(^{58}\)Ni, for three beam energies: the first well below the Coulomb barrier, where only Coulomb effects are relevant, the second on the Coulomb barrier, where nuclear effects become important, and the third above the Coulomb barrier, where both nuclear and Coulomb effects are important. These angular distributions are given in the centre of mass of \(^8\)B*. For these calculations we have used the same optical potentials as in [28], as well as the same \(^7\)Be-p interaction, binding \(^8\)B and generating its continuum structure. The curves shown in the figures include the full CDCC calculation, the CDCC calculation without including couplings within the continuum, and the 1-step calculation. In all cases continuum couplings play a crucial role and generally reduce the cross section. This reduction is more accentuated for the nuclear part than the Coulomb part.
Figure 1. Angular distribution and L-distributions for the breakup of $^8$B on $^{58}$Ni, well below the barrier ($E_{lab} = 15$ MeV): full CDCC calculations (solid line), truncated CDCC calculation where no continuum-continuum couplings are included (dashed line) and a 1-step calculation (dot-dashed line).

Figure 2. Same as before with $E_{lab} = 23$ MeV (on barrier).

Figure 3. Same as before with $E_{lab} = 30$ MeV (on barrier).
On the righthand side of Figs. 1, 2 and 3 are the L-distributions for the same three cases mentioned above. As the beam energy increases, the peak of the distribution is shifted to larger values of L implying that the reaction is more peripheral.

In order to evaluate the relative importance of couplings in percentage, we plot in Fig. 4 the difference between the full CDCC calculation and the DWBA calculation, on the lefthand side, and the difference between the full CDCC calculation and CDCC calculation that does not include continuum-continuum couplings. The difference is divided by the average and multiplied by 100. By comparing the two, the preponderant effect can be attributed to the continuum-continuum couplings. Fig. 4 indicates that the larger the beam energy, the more significant the effect becomes.

Figure 4. Relative importance of the continuum coupling for the breakup of \(^8\text{B}\) on \(^{58}\text{Ni}\): the percentage difference between the full CDCC calculation and the 1-step results (left) and the percentage difference between the full CDCC calculations and the truncated CDCC calculation where no continuum-continuum couplings are included. The solid, dashed and dot-dashed lines correspond to the three energies \(E_{\text{lab}} = 15, 23, 30\) MeV, respectively.

Another experiment to measure the breakup of \(^8\text{B}\) was performed in MSU, at higher energy \([29]\), on heavier targets: \(^{108}\text{Ag}\) and \(^{208}\text{Pb}\) at 44 MeV/A and 82 MeV/A. Measurements focused at forward angles, where clearly the larger Coulomb field dominates. A CDCC calculation of the three body observables necessary for the analysis of this data was performed in \([30]\). Here we compare the angular distributions in the centre of mass of \(^8\text{B}^*\) for the various cases (see Figs. 5 and 6). Again we verify that the effect of continuum couplings is considerable even if not as pronounced as in the lower energy cases. Mostly it is the couplings between two continuum states that cause the reduction of the cross section. The effect is less important for the 82 MeV/A case.

Figure 5. Angular distribution for the breakup of \(^8\text{B}\) on \(^{108}\text{Ag}\), at both \(E_{\text{lab}} = 44, 82\) MeV per nucleon: full CDCC calculations (solid line), truncated CDCC calculation where no continuum-continuum couplings are included (dashed line) and a 1-step calculation (dot-dashed line).
Figure 6. Same as before but using $^{208}$Pb as a target instead.

Figure 7. Angular distribution and L-distributions for the breakup of neutron $^8$B on $^{58}$Ni, well below the barrier ($E_{lab} = 15$ MeV): full CDCC calculations (solid line), truncated CDCC calculation where no continuum-continuum couplings are included (dashed line) and a 1-step calculation (dot-dashed line).

Figure 8. Same as before with $E_{lab} = 23$ MeV.
One of the obvious questions that would arise when trying to gain intuition into the effect of these continuum couplings, is whether this is a specific feature of proton rich systems or whether it would also be found near the neutron dripline. We have replaced the proton in the \( ^8B \) nucleus by a neutron, and composed a system \(^7Be+n\) which is \(^7Be\) bound by a nuclear interaction with the same geometry as that used for the previous \(^8B\) calculations, adjusting only the depth to reproduce the same 0.137 MeV binding energy. In Figs. 7, 8 and 9, we show the results for the breakup of this fictitious nucleus on \(^{58}Ni\) for the same three beam energies. As the fragment is not charged, here the Coulomb barrier does not play such an important role, so identical shapes for the angular distributions are found for all three beam energies. The Coulomb peak, now only a result of the core contribution to the coupling potential, is not significantly affected by the continuum couplings. Yet at larger angles there is again a strong continuum effect. The corresponding L-distributions are presented on the righthand side of the Figs. 7, 8 and 9.

The other obvious question is whether these features are strongly dependent on the binding energy of the projectile. We have performed a set of calculations where the binding energy of the \(^8B\) projectile is increased to \(S_p = 0.5\) MeV and reduced further to \(S_p = 0.06\) MeV, without changing the optical potentials. Results for the angular distribution of the breakup on \(^{58}Ni\) at \(E_{lab} = 30\) MeV are shown in Fig. 10. On the lefthand side are the DWBA calculations for the three considered binding energies, and on the righthand side are the corresponding CDCC calculations. The systematic reduction of the Coulomb peak and a strong suppression of the nuclear peak is present for all three binding energies. In the corresponding L-distributions shown in Fig. 11 one can clearly see the shift into higher partial waves as one decreases the binding energy, reflecting the fact that most of the interaction happens at larger distances. Furthermore we find that the L-distributions for the full CDCC calculations have longer tails than those for the 1-step results, suggesting that continuum couplings shift the breakup to a more peripheral region.
Figure 11. Dependence on the binding energy of the L-distribution for the breakup of $^8$B on $^{58}$Ni ($E_{\text{lab}} = 30$ MeV): 1-step calculations on the lefthand side and full CDCC calculation on the righthand side. Dashed, solid and dot-dashed correspond to $S_p = 0.06$ MeV, $S_p = 0.137$ MeV and $S_p = 0.50$ MeV, respectively.

The same analysis was performed when taking a neutron fragment in $^{10}$B. In Fig. 12 we show the results for the angular differential cross section when the neutron is bound by $S_n = 0.06$ MeV and $S_n = 0.5$ MeV. We show the full CDCC calculations versus the truncated CDCC calculations (where no continuum-continuum couplings are included) and the DWBA calculations. Although the Coulomb part is not strongly affected, there is a strong suppression of the nuclear peak in both examples, caused fundamentally by the continuum-continuum couplings. From both Fig. 10 and Fig. 12, we conclude that the effect of the continuum couplings does not change significantly when approaching threshold. Nevertheless, it would still be interesting to compare these illustrative examples with other well bound systems ($S_p > 3$ MeV), although then an appropriate set of optical potentials would have to be found and the adequate structure of the projectile would have to be included.

Figure 12. Dependence on the binding energy of the angular distribution for the breakup of $^8$B on $^{58}$Ni ($E_{\text{lab}} = 30$ MeV): calculations for $S_p = 0.06$ MeV on the lefthand side and for $S_p = 0.5$ MeV on the righthand side. Solid, dashed and dot-dashed correspond to the full CDCC, the truncated CDCC without continuum-continuum couplings and the 1-step calculations, respectively.

III Other reactions

A. Elastic Scattering

Elastic scattering is one of the most important tools to determine the size of the nucleus and the optical potentials that should be used in reactions (e.g. [32]). Not surprisingly there is an overwhelming quantity of elastic scattering publications for stable nuclei. Unfortunately such has not been the case for dripline nuclei. Although some elastic studies have been performed, often optical potentials for these exotic systems have to be extrapolated from the parameters deduced from stable nuclei. Recently there was a systematic experimental study performed for nuclei neighbouring $^8$B [33] with the specific aim of reducing the uncertainties in
the optical potentials. As in most cases, the analysis of those data was based on a 1-step optical model approach, although an elaborate microscopic JLM description was used to generate the optical potential. By including the possibility of breakup channels in the reaction mechanism (using CDCC), it was shown that ground state to continuum couplings become very important at larger angles [34]. In that work, the elastic scattering of $^8$B on $^{13}$C at $E_{lab} = 78$ MeV was studied. It is found that, taking into account the adequate $p-^{13}$C and $^7$Be-$^{13}$C optical potentials, 2-step processes enhance the cross section beyond 40 degrees when comparing with an optical model calculation using the extrapolated $^8$B optical potential from [33]. The major effect comes from the fact that the ground state to ground state couplings are not the same in the two calculations (CDCC and OM), but there is also an explicit contribution from the breakup channels. The data corresponding to this particular elastic scattering process has now been measured and will be published soon [31].

**B. Transfer reactions**

After decades of using transfer reactions for spectroscopy in the valley of stability, it is unnecessary to argue for transfer reactions on unstable nuclei. The recent systematic program in MSU measuring knock-out reactions on a range of nuclei from $A=6$ to 40 [10] is a proof that transfer reactions are a necessary path for a more detailed understanding of the structure of dripline nuclei [35]. In fact, a specific detection system has been developed at GANIL for measuring inverse kinematic transfers and has already produced results for $^{11}$Be [11].

Another motivation for developing the transfer reaction tool for nuclei on the dripline is Astrophysical. Several studies have been performed in order to establish the so-called ANC (Asymptotic Normalization Coefficient) method [36, 37]. This method is an alternative to the traditional direct capture reaction measurements relevant in Astrophysics. The ANC method becomes useful when the direct capture cross sections are impossibly low or in situations where the capturing nucleus is too unstable to work as a target. First one should realise that the zero energy capture rate of $X$ into $Y$ depends only on the ANC for the overlap of the wavefunction $< Y | X >$. Consequently, by measuring a transfer reaction $A(X,Y)B$, and making sure the reaction is direct and completely peripheral, in other words, well described within DWBA, one can extract the product of the ANCs of the two overlaps $< A|B >$ and $< Y|X >$. As long as $< A|B >$ can be determined in an independent way, $< Y|X >$ can be extracted by scaling the DWBA calculation to the measured transfer cross section. The great advantage of this method is that transfer cross sections (measured at any beam energy) are much larger than the direct capture cross sections needed typically at very low energy.

If a nucleon is transferred from nucleus $Y$ into nucleus $A$ where either $Y$ or $A$ is very loosely bound, one can expect that multi-step paths through the continuum may contribute to the transfer cross section. Breakup channels generated within the CDCC method can be easily included in the transfer formalism (the CDCC-BA method) although calculations easily become very large [34]. Such calculations were recently performed for the reaction $^{14}$N($^7$Be,$^8$B)$^{13}$C at $E_{lab} = 84$ MeV. Data for this reaction has been used to extract the $^7$Be proton capture rate at zero energy [38]. The calculations show that at these intermediate energies, continuum couplings do not play a role.

We have performed additional calculations at much lower energy, where one could guess the effect to be more significant. In Fig. 13 we show the differential cross section for transfer calculations of $^8$B($^{60}$Ni,$^{60}$Cu)$^7$Be at $E_{lab} = 30$ MeV using the same optical potentials as in [28]. The angles are the centre of mass angles for the outgoing particle. We have included the continuum of $^8$B in the transfer process. Comparing the DWBA results with the full CDCC-BA results we find that continuum effects are not significant. If a truncated calculation is performed without continuum-continuum couplings, there is a decrease in the overall distribution and a slight shift to lower angles. If additionally one also removes couplings from the ground state into the continuum, then there is an increase of the cross section and the result is very close to the original full CDCC result (disagreement up to 5%). This result is valid both above and below the Coulomb barrier. However the effect of the continuum couplings in transfer reactions can increase up to 20% if the beam energy is on the Coulomb barrier.

**Figure 13.** One proton transfer cross section for $^8$B on $^{58}$Ni: the full CDCC calculation (solid), the truncated CDCC calculation without continuum-continuum couplings (dashed) and the 1-step calculation (dot-dashed).

The dependence on binding energy is shown in Fig. 14. Black curves correspond to the full CDCC-BA calculations, whilst the grey curves are the DWBA results. For all three binding energies there is a subtle decrease of the cross section, due to continuum couplings. We conclude that continuum effects do not change significantly with binding energy on the dripline.
in transfer processes. When including the continuum couplings in a reaction model, one should definitely take care of non-resonant continuum as well as the resonant states. Couplings between two continuum states may be equally (or even more) important as couplings between the ground state and the continuum.

All the results here presented use the well established CDCC method to discretize the continuum. However, given the computational demand of the traditional CDCC calculations, research into new methods is being developed. One of the most promising alternative methods for discretizing the continuum uses transformed harmonic oscillators (THO). Benchmark calculations comparing the CDCC and the THO methods for the elastic scattering and breakup of deuterons on $^{208}$Pb are very encouraging [44]. We expect that, in the future, the optimization of the continuum discretization will make it feasible to tackle reactions involving three body systems, such as $^{11}$Li, by including the three body continuum properly.

Another point that should be high in the agenda concerns the structure information that can be incorporated in the reaction model, in particular the continuum structure. In the past, spectroscopic factors were the easy was of including structure information. How accurate are shell model calculations for determining continuum spectroscopic factors? Considerable effort is being developed by Shell Model theorists to tackle these and related issues [45, 46]. A further improvement would also take into account microscopic information on the radial dependence, i.e. one-body overlap functions. Although, in recent years, some advances have been made in calculations for reactions with nuclei on the dripline, we are aware that there are still obstacles to overcome, before the above mentioned improvements can be implemented.

Conclusion remarks

Continuum couplings are crucial to understand some reaction processes involving light exotic nuclei. It is in the breakup process that these coupling effects are better seen, but we have shown that there is an influence on elastic scattering and fusion reactions. From the examples so far studied, couplings to the continuum seem to be less important

C. Fusion reactions

The interest on fusion reactions with light exotic nuclei was initiated in the early nineties. At that time the simple theoretical models producing either suppression [39] or enhancement [40] of fusion due to the proximity to threshold and the coupling to breakup channels, could not be compared with experiment. Meanwhile several experiments have been performed using beams of $^6$He, $^{11}$Be, $^{17}$F but, regarding a general behaviour of the process, results are still unconclusive [41]. In fact, the experimental issue on whether measurements concern complete fusion only or also incomplete fusion (fusion of the core and fragments after breaking up infight, whilst approaching the target) has caused enormous debate.

Independently on the experimental points that need to be sorted, it is now possible to perform accurate calculations for the fusion process including the couplings to the continuum [42, 43]. Both cited studies present fusion calculations for $^{11}$Be on $^{208}$Pb, the main difference being that in [43] continuum-continuum couplings are included, whereas [42] contains only ground state to continuum couplings. The results show that couplings from the ground state to the continuum enhance subbarrier fusion and reduce slightly the fusion above the barrier. A rather surprising result is produced when the full calculation is performed, including all couplings: continuum-continuum couplings reduce the overall cross section by an order of magnitude [43]. More work is need for a better understanding of this striking result.

References


