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Non-Mínímal Interaetíon of Gravíty wíth Other Physícal Fíelds: An Overview

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Abstract We present a review of some modern developments concerning the interaction of gravity with other physical fields. It is argued that a suitable context for an account of their dynamical interplay is that of the non-minirnal (e.g. conformal) coupling of these fields to gravity. Some interesting features of non-minimal coupling, such as the connection with Weyl-integrable spacetime (wIST) structure, the generation of eterna1 Universes, the appearance of a cosmlogical constant and the possible induction of repulsive gravity via spontaneous symmetry breaking (SSB) rnechanisms, are discussed. In particular, we examine a simple case of strong-interacting scalar particles (such as the well-known elastic reaction *TK* → ***M),*** in a curved background, thereby obtaining the curious result that the actual, observed value of the strong coupling constant and the minirnurn allowable value, in order to preclude antigravity, are related by Eddington's number 10 ~ ~.

1. INTRODUCTION

Though Einstein's old program towards the unification of the fundamental forces of Nature has experienced, during the last years, a remarkable impulse, due mainly to the successes achieved by gauge theories, nevertheless *gravity* still remains peculiarly apart from the other forces. This situation, it seems, will persist for some time, at least as long as the metric aspects of spacetime are attached to gravitation only.

This ***state*** of ***the art*** justifies alternative approaches that have been attempted by many physicists in the course of the examination of reciproca1 effects in the interaction of gravity with the other forces, searching to understand how strong, weak and electromagnetic interactions are affected by gravity, that is, when they are described in a curved spacetime and, conversely, what the effects of these forces are upon the characteristics of spacetime.

Gravitational effects on electromagnetism have been studied exhaustively. It has been obtained, for example, that the influence of gravity can be thought of as being equivalent to the presence of a material medium described by dielectric constants ε, μ that dependonly upon the geometry $[\varepsilon = \varepsilon(g_{\mu\nu})$, $\mu = \mu(g_{\mu\nu})]$, thus constituting a microscopic responde to the curved background[1]. Along the other direction of inquiry, many interesting spacetime configurations, either localized (geons)[2] or global (cosmological solutions)[3], can be generated by means of photons' energy. Moreover, according to the type of development of the interaction, an electromagnetic field may even generate non--singular expanding universe models, surmounting in this way the strong restrictions that the *singularity theorems* have established[4]. The origin of this feature is to be found in the forn ofthephoton-gravity interaction picture, which allows the violation of one of the theorems' basic inequalities, $(R_{\mu\nu} - \frac{1}{2} Rg_{\mu\nu})V^{\mu}V^{\nu} < 0$ (for arbitrary observers endowed with four-velocity V^{μ}). A concrete, solvable example of a situation in which this inequality does *not* hold has been obtained in ref. 5, and examined in further detail in ref. 6, where a non-singular, spatially homogeneous and isotropic cosmic solution is exhibited, representing an eternal Universe, without beginning or end.

Mutual influences and feedback are to be expected when one attempts to analyse physical processes involving aspects of distinct realms of physics. A remarkable example of this inter-association can he found, in recent years, in the approaching between the physics of elementary particles and cosmology, a connection that has benefited both. Some scientists, apparently warier, have called attention upon the fact that there is, in practice, a large disparity in the approximation quoted above: while we have plenty of observations at the microscopic level, our experimental knowledge about the Universe at large is scarce. Hence, they argue, we should restrict ourselves to inferences only from elementary particle physics into cosmology, and not the other way round. Other scientists, however, disregarding this reasoning as inconclusive, have considered the other direction of inference and derived conclusions about microphysical properties of matter from special characteristics of our Universe. Indeed, even dealing with a rather simplified global model of the Universe, which still poses some as yet unsolved questions, cosmologists were able to produce some valuable quantitative relations, relevant for the description of the microworld domain. To quote just one simple but very interesting example, it has been possible, due to the examination of primordial element abundances, to set up an upper bound for admissible massless neutrino types[7].

In this same trend, it has been suggested that elementary interaction processes may depend on some characteristics of the Universe. Consider, for instance, the phenomenon of disintegration, described by weak interaction theory. Following evidence put forth by various authors, it was finally established by Lee, Yang and Wu that weak processes can be associated to structures violating specular symmetry (parity non-conservation)[8]. The interaction Lagrangian of a weak process can be written, for example, as

$$L_{int} = g_F \bar{\psi}_{(e)} \gamma^{\mu}(1 + \gamma_5)\psi_{(\nu)} W_{\mu} , \qquad (1)$$

where the vector boson W_{μ} is the mediator between the electron and neutrino weak currents. How could the effects of, say, the expansion of the Universe, be reflected in a direct way on this interaction picture?

One could follow the approach of Dirac, Jordan and others, replacing the Fermi constant g_F by a *coupling* scalar field that should then exhibit a dependence on the Universe radius, or better, on cosmic time. Due to the negative results concerning a similar phenomenon in electromagnetism, this attempt can be practically discarded in view of the electroweak unification we possess today. Nevertheless, if one considers the γ_5 weak part of the interaction, a new, distinct possibility arises, which has no electromagnetic counterpart. Since one is dealing here with two types of current, both axial and vector, there could exist a cosmic dependence of the rate of parity violation; this rate might be not maximal, as in the above example, but rather described by the current

$$j_{\mu} = \bar{\psi}_{(e)} \gamma^{\mu}(1 + \varepsilon(t)\gamma_5)\psi_{(\nu)} \qquad (2)$$

where the function $\varepsilon(t)$ that measures the violation rate depends on cosmic time only, due to the observed homogeneity of the Universe. In this case, neutrinos and antineutrinos are produced in mixtures of both left and right polarization states, the ratio of the mixtures depending on their (cosmological) instant of creation. Note, observer, that since we now live in a world in which $\varepsilon=1$, if neutrinos are indeed massless then right-(left-) handed neutrinos (antineutrinos) would be completely invisible to any detection apparatus (save one employing gravitational interaction). Thus, the only observable consequence of the model given by eq. (2) with respect to the detection of neutrinos (or antineutrinos) on the Earth would be the effective decreasing of the Universal Fermi constant. In an early paper[9], it has been even hinted that, in the same vein, CP violation could also be a (cosmic) time-dependent phenomenon, that should possibly vanish as the Universe expands. Recently[10], this question was reexamined in connection with the problem of primordial element abundances, and a suggestion was made on how a value of $\varepsilon(t)$ different from unity could be made to appear and be useful in the removal of some disagreements between the observed rate of (primordial) Helium formation and the standard cosmological model prediction.

Along these same lines of investigation, what could be said about strong processes? The situation here is more involved since a complete field theory for strong interactions is yet lacking. However, one can choose certain simple processes and, from a direct generalization to curved spacetime, proceed to investigate the effects one would possibly find. Thebest candidate for such analysis seems to be given by elastic interactions among scalar particles, analogous to the $\pi k \to \pi k$ reaction. We shall see later on a simple model, allowing a complete analytical treatment of such elastic process in a curved background, which imposes restrictions upon the admissible values that the strong coupling constant could possess in our Universe. It turns out that a very curious numerical relation, involving old Eddington's number, appears along this quest.

In fact, the examination of the values presented by certain constants of Nature has given origin to a vast series of speculations. An approach that has gathered lots of attention, in recent times, is provided by the so-called anthropic principle[11], which intends to correlate the actual, effective values of several constants of physicswith some properties of the world, mainly those which imply the necessity of man's presence, as an observer. This conception has had its origin in the very peculiar observation that certain adimensional ratios, built up with usual constants of physics, all seem to be related to Eddington's number 10^{39}. If Nature, it is argued, had not contrived such amazing calibration of the physical constants, and provided for certain stability conditions, man could not have come into existence — so the world *must* be what it is, since man exists. Unfortunately, neither this program, now any other, so far, has actually been able todevisea complete rationale for these peculiar values. We shall see in the following that in the present model the restriction implied by the presence of gravitation upon the strong coupling constant g_s is such that the ratio between the minimum value allowable in principle, and the actual value observed, results surprisingly enough to be given by

$$\frac{\text{Minimum possible value of } g_s}{\text{Actual value of } g_s} \sim 10^{-39} \quad .$$

For an extremely curious *coincidence*, Eddington's numbers appears in this domain as well, suggesting that we may have approached a new Pythagoric mystery.

2. DYNAMICAL ORIGIN OF WEYL — INTEGRABLE SPACETIMES

Recently[12], it has been shown how a Weyl-integrable spacetime (WIST) structure arises naturally when one treats the variational problem of the interaction of (non-minimally coupled) matter and gravity by means of Palatini's method[13], which consists of taking both the variations $\delta g_{\mu\nu}$ of the metric and $\delta\Gamma^{\alpha}_{\mu\nu}$ of the affine connections as independent, in the variational procedure. The argument is simple and direct: consider, for instance, a coupling term representing the interaction of a scalar field Φ and gravity, such as

$$L_1 = \sqrt{-g}\ \Phi^2 R \quad . \qquad (3)$$

By varying in Palatini's fashion, one obtains

$$\delta \int \sqrt{-g}\ \Phi^2 R = \int \sqrt{-g}\ \Phi^2\ (R_{\mu\nu} - \frac{1}{2}\ R_{\mu\nu})\delta g^{\mu\nu}$$

$$+ \int [(\sqrt{-g}\ \Phi^2\ g^{\mu\nu})_{;\alpha} - \frac{1}{2}\ (\sqrt{-g}\ \Phi^2\ g^{\lambda\mu})_{;\lambda}\delta^{\nu}_{\alpha}$$

$$- \frac{1}{2}\ (\sqrt{-g}\ \Phi^2\ g^{\nu\lambda})_{;\lambda}\delta^{\mu}_{\alpha}]\ \delta\Gamma^{\ \alpha}_{\mu\nu}\ . \qquad (4)$$

Since the variationsare presumed to be independent, a straightforward calculation provides Einstein's equations for the vacuum and also the condition

$$g_{\mu\nu;\alpha} = - \frac{\Phi^2_{,\alpha}}{\Phi^2}\ g_{\mu\nu}\ , \qquad (5)$$

which indeed characterizes a conformally-Riemannian or Weyl- integrable spacetirne structure[14]. It is worthy at this point to observe that, though in effect a WIST can be conformally related to a Riemannian spacetime (RST), the actual importance of condition eq. (5), in the context of any physically reasonable theory on this subject, depends fundamentally upon the other terms present in the Lagrangian describingthe interaction; more precisely, it depends upon the behaviour of these other terms under conformal transformations. Consider, for example, what happens in the theory expressed by

$$L_2 = \sqrt{-g}\ \left[\frac{1}{k}\ R - \frac{1}{6}\ R\phi^2 + \phi_{,\mu}\phi_{,\nu}g^{\mu\nu} + V(\phi) + 2\Lambda\right] \qquad (6)$$

Palatini's variation of this Lagrangian gives the following equations

$$g_{\mu\nu;\alpha} = \left[-\log(\frac{1}{k} - \frac{\phi^2}{6})\right]_{,\alpha} g_{\mu\nu} \qquad (7a)$$

$$\Box\phi + \frac{1}{6}\ R\phi - \frac{\delta V}{\delta\phi} = 0 \qquad (7b)$$

$$\left(\frac{1}{k} - \frac{1}{6}\ \phi^2\right)G_{\mu\nu} = - \phi_{,\mu}\phi_{,\nu} + \frac{1}{2}\ g_{\mu\nu}(\phi_{,\lambda}\phi^{,\lambda} + V + 2\Lambda)\ . \qquad (7c)$$

The first relafion is just WIST characterization, again; in turn, the equation for the scalar field results to be non-linear, due to the presence of the scalar of curvature R, which in a WIST depends on ϕ and its derivatives; and lastly, we observe the renormalization of the gravitational constant k. In virtue of Einstein's term $\frac{1}{k}$ R in the Lagrangian, this system of equations is *not* conformally invariant, and so the above theory describes a Weylian structure that cannot be reduced by means of a conformal transformation to a Riemannian one. In this case, one would expect lengths to vary differently under parallel transport from one point to another, according to the chosen path (Einstein's criticism against Weyl's original attempt to unify gravity and electromagnetism[15] rested precisely on this apparent difficulty). On a closed loop, however, any length variation induced by the transportation would be compensated by gravitational interaction on the way back and so the total length deviation cancels out. Thus, although a WIST always displays a local length variation $\Delta\ell = \ell\phi_{,\alpha}\Delta x^{\alpha}$, on a closed circuit one still achieves a total conservation, $\oint d\ell = 0$, just as in the Riemannian case.

Note also an additional property of system (7): the condition for Riemannization is given by ϕ = constant; this solution, however, will be compatible with the remaining relations of the system only for some special types of potential $V(\phi)$ and hence, in general, it will depend considerably on V. Furthermore, the solution ϕ = const implies that Einstein's equations reduce to the form $G_{\mu\nu} = \Lambda_{eff} g_{\mu\nu}$, thus inducing a transformation of the bare cosmological constant Λ (or even generating one. Together with the renormalization of constant k (in the presence of matter), this feature permits a gravitational behaviour far richer and more complex than in usual General Relativity.

3. COSMIC REPULSION

Highly daring and speculative ideas are commonplace in physical investigations today, arousing fewer metaphysical quandaries than in earlier times, and so nowadays it is quite admissible to discuss, say, antigravitational aspects conveyed by theories under current scrutiny (for example, it has been shown[16] that Supergravity endowed' with N = 2,3,...,8 fermionic generators leads to antigravity). Nevertheless, it seems that the great majority of physicists still supports tradition and takes only purely attractive gravitational phenomena into consideration.

It is surely convenient, however, to examine the countless possibilities that Nature would have at hand in order to turn antigravity into a true phenomenon of the actual world, since we will be learning, at the same time, why certain conditions are forbidden, or why certain properties may exist (see section 6). Still more interesting would be those schemes in which no common law of physics were violated and situations could be elicited where common matter (such as photon, neutrinos, etc) accounted for the repulsive effects. One such scheme has been presented recently[17], and it might be worthy, for completeness, to depict here a brief outline of the main ideas concerned.

Let us consider the interaction of a scalar field with gravitation according to the theory described by the non-minimal Lagrangian

$$L = \sqrt{-g}\,\left[\phi^*_{,\mu}\phi_{,\nu}g^{\mu\nu} - \frac{1}{6}R\phi^*\phi + V(\phi^*\phi) + \frac{1}{k}R + 2\Lambda\right] \tag{8}$$

The equation of motion, if we assume a *priori* that spacetime is Riemannian, are

$$\Box\phi + \frac{1}{6}R\phi - \frac{\delta V}{\delta\phi^*} = 0\ , \tag{9a}$$

$$\left(\frac{1}{k} - \frac{1}{6}\phi^2\right)G_{\mu\nu} = -\frac{1}{2}\left(\phi^*_{,\mu}\phi_{,\nu}+\phi^*_{,\nu}\phi_{,\mu}\right) + \frac{1}{2}g_{\mu\nu}\left[\phi^*_{,\sigma}\phi^{,\sigma}+V+2\Lambda\right]$$

$$-\frac{1}{6}\left[\phi^*\Box\phi + \phi\Box\phi^* + 2\phi^*_{,\lambda}\phi^{,\lambda}\right]g_{\mu\nu}$$

$$+\frac{1}{6}\left(\phi^*_{,\mu\|\nu}\phi + \phi^*\phi_{,\mu\|\nu} + \phi^*_{,\mu}\phi_{,\nu} + \phi^*_{,\nu}\phi_{,\mu}\right)\ , \tag{9b}$$

in which the double bar means covariant differentiation in RST sense. If we limit ourselves to a quartic potential such as

$$V(\phi^*\phi) = -\mu^2\phi^*\phi + \sigma(\phi^*\phi)^2\ , \tag{10}$$

then the trace of Einstein's equations reduces to the form

$$\frac{1}{k} R = \mu^2\phi^*\phi - 4\Lambda \ . \tag{11}$$

Consequently, eq. (9a) for the scalar field turns into

$$\Box \phi + (\mu^2 - \frac{2}{3} k\Lambda)\phi - [2\sigma - \frac{1}{6} k\mu^2]\phi^2\phi = 0 \tag{12}$$

(where $\phi^2 = \phi^*\phi$), which means that the gravitational effects upon the field amount just to the renormalization of the mass

$$\mu^2 \rightarrow \mu^2_{eff} = \mu^2 - \frac{2}{3} k\Lambda \quad , \tag{13}$$

and self-interaction constant,

$$\sigma \rightarrow \sigma_{eff} = \sigma - \frac{1}{12} k\mu^2 \quad . \tag{14}$$

The next step should be the search for a fundamental solution $\phi = \phi_0$ = constant, which would also be a minimum of the energy of field ϕ. Here a small difficulty presents itself, concerning the appropriate definition of the energy of a field coupled non-minimally with gravitation. This obstruction is easily surmounted if we adhere to the conventional approach, that is, if we take the energy expression from

$$\delta \int \sqrt{-g}\, L_\phi = \int \sqrt{-g}\, T_{\mu\nu}\, \delta g^{\mu\nu} \ . \tag{15}$$

In this case, we obtain

$$T_{\mu\nu} = t_{\mu\nu} - \frac{1}{6}\phi^2 G_{\mu\nu} + \frac{1}{6}(\Box\phi^2 g_{\mu\nu} - \phi^2{}_{,\mu||\nu}) \ . \tag{16}$$

where $t_{\mu\nu}$ is the energy-momentum tensor of minimal coupling,

$$t_{\mu\nu} = \frac{1}{2}(\phi^*_{,\mu}\phi_{,\nu} + \phi^*_{,\nu}\phi_{,\nu}) - \frac{1}{2} g_{\mu\nu}[\phi^*_{,\sigma}\phi^{,\sigma} + V] . \tag{17}$$

Solution $\phi = \phi_0$ = constant is given, according to eq.(12), by either

$$\phi_0 = 0 \tag{18a}$$

or

$$\phi_0^2 = 2\left[\frac{2k\Lambda - 3\mu^2}{k\mu^2 - 12\sigma}\right] \quad . \tag{18b}$$

The energy E is provided by the formula

$$E = T^0{}_0 = \frac{3\mu^2\phi^2 - 3\sigma\phi_0{}^4 - k\Lambda\phi_0{}^2}{6 - k\phi_0{}^2} \tag{19}$$

which has extrema, for the non-trivial case, given by the solutions of an algebraic eq'uation of fourth degree,

$$k\sigma\phi_0{}^4 - 12\sigma\phi_0{}^2 + 6\mu^2 - 2\Lambda = 0 \ . \tag{20}$$

A straightforward calculation allows one to see that eqs. (18b) and (20) are not always compatible; this will be the case only if among themass μ, the self-interaction constant σ and the bare cosmological constant Λ the following relation holds

$$\mu^4 = 8\sigma\Lambda \quad . \tag{21}$$

This is certainly a strong and peculiar condition, but nevertheless a plausible one[18]. Curiously, in this case the ground-state fundamental solution eq. (18b) coincides with its corresponding value in Minkowski space,

$$\phi_0{}^2 = \frac{\mu^2}{2\sigma} \quad . \tag{22}$$

Since gauge symmetry is broken, we call this an induced symmetry breaking (ISB) mechanism.

Some final observations are worthy of note. Consider the graph of the energy $E(\phi_0)$ with respect to ϕ_0 (fig.1).

Observe that points $\phi_0 = 0$ and $\phi_\pm = \pm 2\sqrt{\Lambda}/\mu$ are all minima, provided that $(2k\Lambda - 3\mu^2) > 0$, but the non-trivial solutions represent states that are more stable against arbitrary perturbations; at these extremum points, furtherrnore, the equation of motion for geometryturns

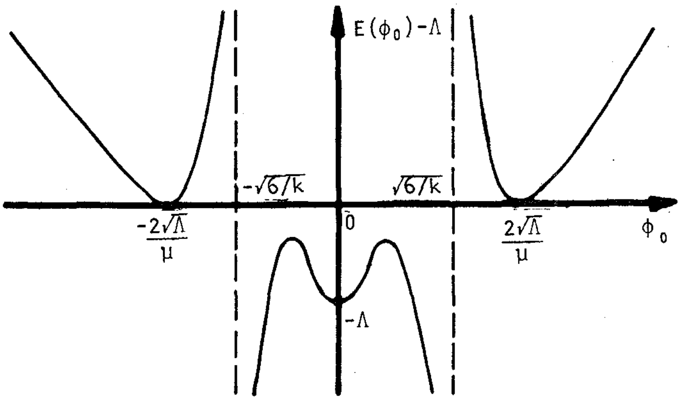

Fig.1 - Plot of $E(\phi_0) - \Lambda$ versus ϕ_0, when $2k\Lambda - 3\mu^2 > 0$.

into the free-field form $G_{\mu\nu} = 0$. Hence, when the field ϕ is at ISB ground states $\phi_\pm$, the system behaves as matter-free in what concerns gravitation. What would happen, on the other hand, if some other matter were present and the field ϕ had reached either of states $\phi_\pm$? We already know that constant solutions $\phi = \phi_0$ lead to the renormalization of the gravitational constant k; thus, a rapid calculation will give

$$G_{\mu\nu} = - k_{ren} \cdot T_{\mu\nu}(\text{matter}) = - \left[\frac{3k\mu^2}{3\mu^2-2k\Lambda}\right] \cdot T_{\mu\nu} \ . \tag{23}$$

When the minimum condition $(2k\Lambda-3\mu^2) > 0$ holds, solutions $\phi_\pm$ are stable and

$$k_{ren} = \frac{3k\mu^2}{(3\mu^2-2k\Lambda)} > 0 \quad , \tag{24}$$

as it should. However, is the mass μ is sufficiently small ($\mu < 10^{-33}$ eV), so that $(3\mu^2-2k\Lambda) < 0$, then the system is at an unstable (maximum) condition in which *antigravity* (i.e., $k_{ren} < 0$) can be generated.

The free-field form $G_{\mu\nu} = 0$ of Einstein's equations is due to the relation imposed upon μ, σ and Λ (eq. (21)). Let us now discard this condition. The energy picture in this case, in the absence of other matter constituents, is displayed on fig. 2.

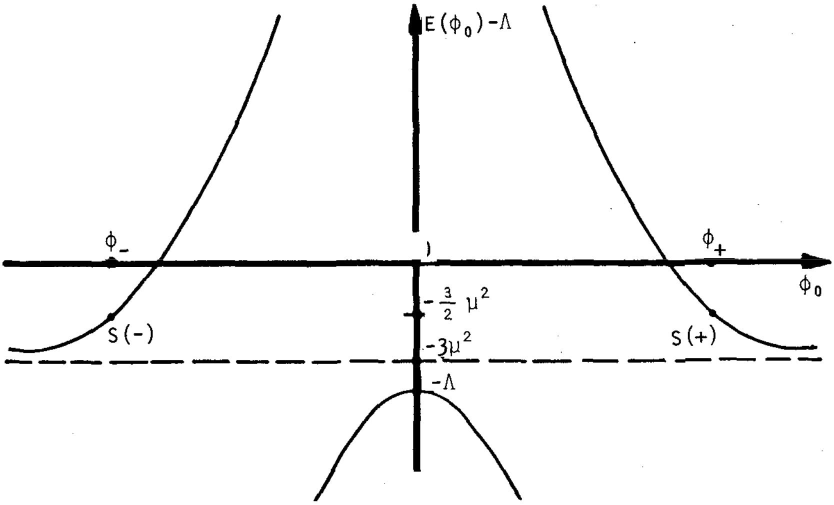

Fig.2 - Plot of $E(\phi_0)$ - A versus ϕ_0 for the a = 0 case.

For any value of A, there is no extremum condition compatible with the non-trivial constant solutions $\phi_{\pm} = \pm(\sqrt{6\mu^2-4\Lambda})/\mu$; their corresponding energy states, which we labelled $S(\pm)$, are highly unstable and decay asymptotically to the De Sitter solution. The equation of motion for the geometry, in turn, results to be

$$G_{\mu\nu} = \Lambda_{eff}\, g_{\mu\nu} \quad , \tag{25}$$

where the effective cosmological constant is $A_{eff} = +\frac{3}{2}\mu^2$.

This analysis shows that the presence of a self-interaction constant σ, in this theory, helps to increase the stability of symmetry breaking solutions.

4. WEYL-RIEMANN REDUCTION AND THE COSMOLOGICAL CONSTANT

In order to examine some consequences of spacetime Weylization, let us consider the same non-minimal coupling between a scalar field ϕ and gravity as in the previous section, but here in the context of Palatini's variation. Non-minimal Lagrangian eq.(8), in this case, allows dynamical equations similar to eqs. (7):

$$\Box\phi + \frac{1}{6}R\phi - \frac{\partial V}{\partial\phi^*} = 0 \tag{26}$$

$$\left(\frac{1}{k} - \frac{\phi^2}{6}\right) G_{\mu\nu} = - \phi^*_{,(\mu}\phi_{,\nu)} + \frac{1}{2} g_{\mu\nu}\left[\phi^*_{,\lambda}\phi^{,\lambda} + V(\phi^2) + 2\Lambda\right] \tag{27}$$

$$g_{\mu\nu;\alpha} = g_{\mu\nu}\,\omega_\alpha \quad , \tag{28}$$

where $\phi^2 = \phi^*\phi$ and $\omega_\alpha = \left[-\log\left(\frac{1}{k} - \frac{\phi^2}{6}\right)\right]_{,\alpha}$ (WIST characteriration).

For a quartic potential $V(\phi^2) = -m^2\phi^2 + \sigma(\phi^2)^2$, contraction of Einstein's equations, eq. (27), and the use of eq. (26) give

$$\frac{1}{k} R = m^2\phi^2 - 4\Lambda - \frac{1}{2} \Box \phi^2 \quad , \tag{29}$$

so that the equation for the scalar field ϕ turns out to be

$$\Box\phi + \left[-\frac{k}{12}\Box\phi^2 + \left(m^2 - \frac{2}{3}k\Lambda\right)\right]\phi + \left(\frac{km^2}{6} - 2\sigma\right)\phi^2\phi = 0 \tag{30}$$

We also known from WIST theory[19] that

$$G_{\mu\nu}(\text{WIST}) = G_{\mu\nu}(\text{RST}) - \omega_{\mu\|\nu} - \frac{1}{2}\omega_\mu\omega_\nu + \left(\omega^\alpha{}_{\|\alpha} - \frac{1}{4}\omega^2\right) g_{\mu\nu} \; , \tag{31}$$

where $\omega^2 = w_\mu \omega^\mu$, and the double bar refers to RST covariant differentiation, employing Christoffel symbols only.

Now we try to elaborate an homogeneous and isotropic universe, choosing a Friedrnann-Robertson-Walker line element such as

$$ds^2 = dt^2 - S^2(t)\left[d\chi^2 + \Sigma^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)\right] \; . \tag{32}$$

For the sake of simplicity, we take $\phi = \phi^*$ from now on and adopt the ansatz $\phi = \phi(t)$, so that

$$\omega_\alpha = a(t)_{,\alpha} = \dot{a}\delta^0{}_\alpha \; , \tag{33}$$

where $a(t) = \left[-\log\left(\frac{1}{k} - \frac{\phi^2(t)}{6}\right)\right]$; conversely, $\phi^2(t) = \frac{6}{k}\left(1 - ke^{-a(t)}\right)$.

Eq. (30) for the scalar field then reads

$$\frac{k}{2}\,\frac{\left(\ddot{a} + \frac{3}{2}\dot{a}\dot{b} - \dot{a}^2\right)}{(e^a - k)} - \frac{k}{4}\,\frac{e^a\,\dot{a}^2}{(e^a-k)^2} + \frac{e^a}{k}\left(\gamma + \frac{6\beta}{k}\right) - \frac{6\beta}{k} = 0 \tag{34}$$

(with $b(t) = \log S^2(t)$, $\beta = (\frac{km}{6} - 2\sigma)$, $\gamma = (m^2 - \frac{2}{3} k\Lambda))$, while components (0-0) and (1-1) of Einstein's equations eq. (27) give, respectively,

$$(6-b)' - 4\varepsilon e^{-b} = \frac{2}{3}(Q-P) \qquad (35)$$

$$(\ddot{a}-\ddot{b}) - \frac{1}{2}(\dot{a}-\dot{b})^2 + \frac{1}{4}(\dot{a}^2-\dot{b}^2) + \varepsilon e^{-b} = \frac{1}{2}(Q+P) \quad , \qquad (36)$$

in which $Q[a,\dot{a}] = \frac{3}{2}\frac{k\dot{a}^2}{(e^{a}-k)}$, $P[a] = e^{a}[V+2\Lambda]$. and where, in view of the required equality of the spatial components of the mixed tensor $G^{\mu}{}_{\nu}$, we have taken $\frac{1}{\Sigma}\frac{d^2\Sigma}{d\chi^2} = \varepsilon = \text{const.} = (0, \pm 1)$.

In the case of Euclidean section, $\varepsilon = 0$, and we obtain

$$\dot{b} = \dot{a} \pm F \qquad (37)$$

$$\dot{F} + \frac{\dot{a}}{2} F = \pm Q \quad , \qquad (38)$$

in which $F[a,\dot{a}] = (\frac{2}{3}(Q-P))^{1/2}$.

For the homogeneous equation for F, that is, when $Q=0$, we get

$$F = \text{const. } e^{-a/2} \qquad (39)$$

But Q = 0 implies a = const., so that F = H = const.; then,

$$S^2 = e^{b} = \text{const. } e^{\pm Ht} \quad . \qquad (40)$$

Eq. (34) for the scalar field settles the value of the constant H, in terms of the field's basic quantities:

$$H = \left(\frac{\beta-\gamma}{k\beta}\right)^{1/2} \quad . \qquad (41)$$

Since a = constant amounts to $\omega_\alpha = 0$, we achieve for the homogeneous case a De Sitter solution in a Riemannian spacetime structure. As we have seen in the last section, a spontaneous symmetry breaking mechanism can generate a cosmological constant, thus inducing De Sitter-type universes. In the present theory, however, it appears that the existence of a cosmological constant can be associated to the Riernannian nature of spacetime. Hence, the dynarnical interplay between gravity and scalar fields, in this context, allows an interesting interpretation: the presence of a cosmological constant in our Universe as a consequence of a reduction from a WIST structure to a Riemannian one.

5. AN ETERNAL UNIVERSE

As we have comrnented at the Introduction, it is possible to generate non-singular cosmological models through the non-rninimal coupling of a vector field and gravitation. We shall now exhibit a concrete exarnple of a solution displaying this property[5,6].

Consider, for an a *priori* RST structure, the theory given by

$$L = \sqrt{-g}\left[\frac{1}{k} R - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \beta R A_\mu A^\mu\right] , \qquad (42)$$

in which, as usual, $f_{\mu\nu} = A_{[\mu,\nu]}$. The equations of motion are

$$f^{\mu\nu}{}_{||\nu} = - \beta R A^\mu , \qquad (43)$$

$$\left(\frac{1}{k} + \beta A^2\right) G_{\mu\nu} = -E_{\mu\nu} - \beta R A_\mu A_\nu + \beta(\Box A^2 g_{\mu\nu} - A^2{}_{,\mu;\nu}), \qquad (44)$$

with

$$A^2 = A_\mu A^\mu \quad \text{and} \quad E_{\mu\nu} = f_{\mu\alpha} f^\alpha{}_\nu + \frac{1}{4} f_{\alpha\beta} f^{\alpha\beta} g_{\mu\nu}$$

It is then possible to show that, choosing a Friedrnan-Robertson-Walker line elernent such as eq. (31) and the ansatz $A^2 = A^2(t)$, a solution of this system is given by

$$\begin{cases} A^2(t) = \dfrac{1}{k}\left(1 + \dfrac{1}{\sqrt{t^2+P^2}}\right) , & (45) \\[2ex] S(t) = \sqrt{t^2 + P^2} , & (46) \end{cases}$$

(Novello-Salim solution) in which P is a constant (in fact, the minimum of the Universal radius) that measures the intensity of the vector field at $t=0$; when $P=0$, one obtains Minkowski spacetime in Milne coordinates. In the $P\neq 0$ case, one may indeed built models that present no singularitity, and thus can be infinitely old.

Such models can be better visualized by means of a dynamical system analysis of the generic equations of motion (eq.(44)) for a FRW metric, which are

$$3\,\frac{\ddot{S}}{S} = -\,\frac{\ddot{\Omega}}{\Omega} \tag{47}$$

$$\frac{\ddot{S}}{S} + 2\,\frac{\dot{S}^2}{S^2} + \frac{2\Sigma}{S^2} = -\,\frac{\dot{S}}{S}\,\frac{\dot{\Omega}}{\Omega} \tag{48}$$

where we have put $\Omega = (\frac{1}{k} + \beta A_\mu A^\mu)$; calling $x = 3\dot{S}/S$, $y = \dot{\Omega}/\Omega$, this set reduces to an autonomous planar system:

$$\begin{cases} \dot{x} = -\,\frac{1}{3}\,x^2 + xy\,, & (49a)\\ \dot{y} = -\,y^2 - xy\,. & (49b)\end{cases}$$

The characteristic aspects of this dynamical system are summarized in the following phase portrait, projected on the Poincaré sphere (fig. 3):

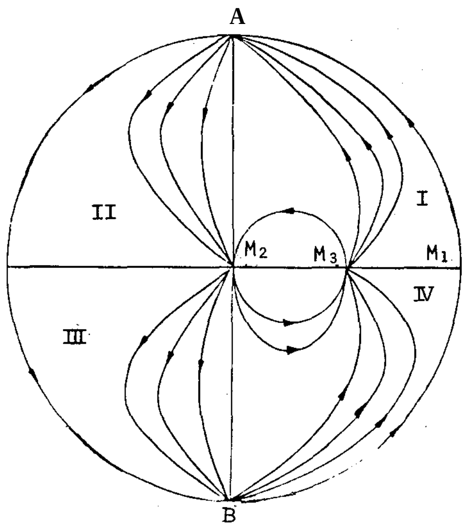

Fig.3 - Graph of the non-minimally coupled photon- gravity system eq. (49). Novello-Salim solutions belong to quadrants II and III.

Some considerations about this dynamical picture: Novello-Salim solutions belong to quadrants II and III (the remaining solutions are of interest in their own right and were examined exaustively in C. Romero's' work Some *Remarks Concerning and Eternal Universe*, to be published). Each solution is Minkowskian when it begins at point $A(t=-\infty)$; its point of maximum contraction is reached at $M_2(t=0)$, and it terminates at point $B(t=+\infty)$, Minkowskian again. Hence, we may say that in this model the Universe initiates from Minkowskian Nothing, at the infinite past, contracts up to the minimum radius P, at $(t=0)$ — and so presents no singularity — and then expands indefinitely until it wears out in Minkowskian Nothin again, at the infinite future.

We observe further that an intriguing solution of this very non-minimal scenario, but using Palatini's method instead, has been recently obtained[20], in which a non-minimally coupled vector field (which can be regarded as the physical cause of the evolution of the metric properties of the Universe) results to be an indeterminate function of time, related to the degree of Weylization of the spacetime structure, and which is not settled down by the dynamics. Such property, that can be conceived as a 'driven-from-without' feature, endows this model with a curious *marionette-like* character.

6. STRONG INTERACTION IN A CURVED SPACETIME

We have attempted to survey, in preceding sections, some modern approaches towards the problem of the reciprocal influences between weak and electromagnetic processes and gravitation. Let us now pose the question: what could one say about *strong* interaction processes in a curved spacetime? This is, doubtless, a much more complex domain, since a complete field theory for the strong interaction is still lacking. However, notwithstanding the present theoretical limitations, we shall be concerned in what follows with a strong process that does possess a Lagrangian representation, viz., the interaction among π and k mesonic scalar fields, described by

$$L_{\pi k} = g_s(\phi^*\phi)_\pi(\psi^*\psi)_k \qquad (50)$$

This must be understood as a phenomenological Lagrangian in which g_s stands for a strong coupling constant (observe that L is conformally invariant). How can we investigate gravitational effects upon this interaction? Two points must be clear from the outset: first, the following generalization from a flat space-time context to a curved one is by no means unique; second, the order of magnitude of the perturbation suffered by this interaction, due to the curved background, might be so small that one could, in principle, disregard *any* gravitational effects as physically irrelevant. Though this account seems to be true at first sight, we will see later on that in fact it is *not* correct in general.

A suitable extension of the flat-spacetime (ϕ,ψ) interaction outlined above to a curved spacetime $(\phi,\psi,g_{\mu\nu})$ configuration may be given by the *quasi-conformal* theory

$$L = \sqrt{-g}\ \Big[\phi^*_{,(\mu}\phi_{,\nu)}g^{\mu\nu} + V(\phi^2) + \psi^*_{,(\mu}\psi_{,\nu)}g^{\mu\nu} + W(\psi^2) + \xi\phi^2\psi^2 - \frac{1}{6}R(\phi^2+\psi^2) + \frac{1}{k}R\Big] \quad , \tag{51}$$

in which, instead of using the actual, experimental value g_s of the strong constant, we have chosen a free parameter ξ in order to find out whether the coupling with gravitation imposes any constraints upon the spectrum of admissible values of ξ. Observe that *both* scalar fields are non-minimally coupled to gravity.

If spacetime is *a priori* supposed to be Riemannian, variation of Lagrangian eq. (51) yields

$$\Box\phi + \frac{1}{6}R\phi - \xi\psi^2\phi - \frac{\partial V}{\partial\phi^*} = 0 \ , \tag{52}$$

$$\Box\psi + \frac{1}{6}R\psi - \xi\phi^2\psi - \frac{\partial W}{\partial\psi^*} = 0 \ , \tag{53}$$

$$\left(\frac{1}{k} - \frac{1}{6}(\phi^2+\psi^2)\right)G_{\mu\nu} = -T_{\mu\nu}\boxed{\phi} - T_{\mu\nu}\boxed{\psi} - \frac{1}{2}\xi\phi^2\psi^2 g_{\mu\nu} + \Lambda g_{\mu\nu} \quad , \tag{54}$$

where $T_{\mu\nu}[\phi]$ has the form

$$T_{\mu\nu}[\phi] = \phi^*_{,(\mu}\phi_{,\nu)} - \frac{1}{2}[\phi^*_{,(\lambda}\phi_{,\varepsilon)}g^{\lambda\varepsilon} + V(\phi^2)]g_{\mu\nu} + \frac{1}{6}\Box\phi^2 g_{\mu\nu} - \frac{1}{6}\phi^2_{,\mu;\nu} \tag{55}$$

and analogously for $T_{\mu\nu}[\psi]$; once again, we take potentials V and W given by quartic expressions,

$$V(\phi^2) = - m^2\phi^2 + \sigma\phi^4 \quad , \tag{56a}$$

$$W(\psi^2) = - M^2\psi^2 + \eta\psi^4 \quad . \tag{56b}$$

Contraction of Einstein's equations eq. (54) and the use of eqs.(52) and (53) give

$$\frac{1}{k} R = m^2\phi^2 + M^2\psi^2 - 4A \ . \tag{57}$$

Remark that self-interaction constants σ, η do not appear explicitly in the trace expression; hence, from now on we will specialize to the case of *null* self-interaction constants ($\sigma = \eta = 0$).

If now we ask whether there exists a solution of this system for non-trivial constant values $\phi = \phi_0$, $\psi = \psi_0$, the answer is affirmative. Indeed, from eqs. (52) and (53) we obtain

$$\phi_0{}^2 = \frac{(\frac{1}{6} R + M^2)}{\xi} \quad , \tag{58a}$$

$$\psi_0{}^2 = \frac{(\frac{1}{6} R + m^2)}{\xi} \tag{58b}$$

Use of eq. (57) for the trace then yields

$$\phi_0{}^2 = \frac{kM^2(m^2-M^2)-2\xi(2k\Lambda-3M^2)}{\xi[6\xi-k(M^2+m^2)]} \quad , \tag{59}$$

$$\psi_0{}^2 = \frac{km^2(M^2-m^2)-2\xi(2k\Lambda-3m^2)}{\xi[6\xi-k(M^2+m^2)]} \tag{60}$$

Thus, when the system $(\phi,\psi,g_{\mu\nu})$ is found in state $(\phi = \phi_0,\ \psi = \psi_0,\ g_{\mu\nu})$, the equation for geometry reduces to

$$\left[\frac{1}{k} - \frac{1}{6}\,(\phi_0{}^2+\psi_0{}^2)\right]G_{\mu\nu} = \frac{1}{2}\left[-m^2\phi_0{}^2-M^2\psi_0{}^2 + \xi\phi_0{}^2\psi_0{}^2 + 2\Lambda\right]g_{\mu\nu}\ , \qquad (61)$$

so that we can write, in general, that

$$G_{\mu\nu} = \Lambda_{eff}\, g_{\mu\nu} - k_{ren}\, T_{\mu\nu}(\text{matter})\ . \qquad (62)$$

[Since $(\Lambda/\mu_\pi^2) \sim 10^{-80}$, the presence of the bare cosmological constant Λ is of no importance for our later results and so we drop it from now on.]

Two significant features deserve comment: the gravitational constant k is renormalized,

$$k_{ren} = \frac{\xi\left[6\xi-k(M^2+m^2)\right]}{6(\xi-\xi_{(+)})(\xi-\xi_{(-)})}\,k\ , \qquad (63)$$

in which

$$\xi_{(\pm)} = \frac{k}{6}\,(M \pm m)^2\ , \qquad (64)$$

and an effective cosmological constant is generated,

$$\Lambda_{eff} = \frac{-\,3m^2M^2}{6\xi-k(M^2+m^2)}\,k\ . \qquad (65)$$

Figures 4 and 5 display the behaviour of both k_{ren} and Λ_{eff} as functions of the strong parameter 5.

Observe that the signs of both k_{ren} and Λ_{eff} change according to the range of values of 5. Indeed, we can see from fig. 6 below that there are ranges of ξ in which k_{ren} is negative, and so repulsive gravity is generated.

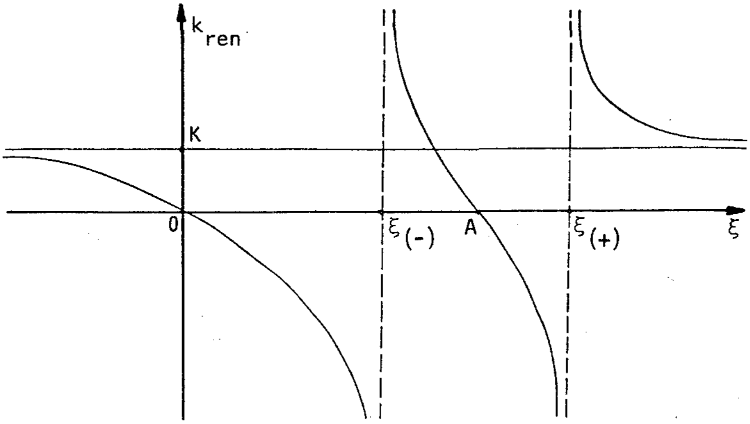

Fig.4 - Graph of the renormalized gravitational constant k_{ren} (eq.(63)) as a function of the strong parameter ξ, in the state $(\phi_0,\psi_0,g_{\mu\nu})$. Point A corresponds to $\xi=k(M^2+m^2)/6$.

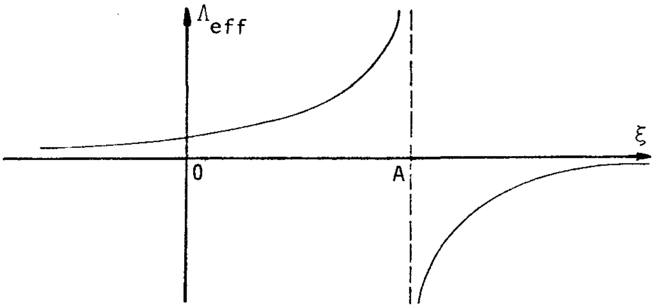

Fig.5 - Graph of the effective cosmological constant Λ_{eff} (eq.(65)) as a function of the strong parameter ξ, in the state $(\phi_0,\psi_0,g_{\mu\nu})$. Point A corresponds to $\xi = k(M^2+m^2)/6$.

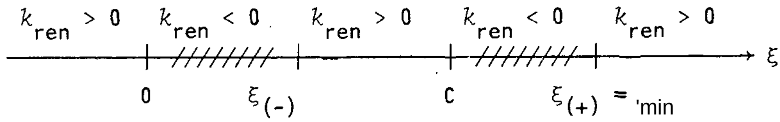

Fig.6 - Analysis of the sign behaviour of k_{ren} uith respect to the parameter ξ. Dashed sections correspond to negative values of k_{ren} and hence to antigravity. Point C corresponds to $\xi = k(M^2+m^2)/6$.

Thus, if we require gravity to be strictly attractive, we are led to impose a lower bound for parameter ξ, in order to ensure that k_{ren} remains always positive[21]. It is certainly a remarkable fact that this simple, basic property of gravity implies a restriction upon the strength of the strong interaction.

What is the ratio between the minimum value allowed for parameter ξ, $\xi_{min} = \xi_{(+)} = \frac{k}{6}(M+m)^2$, and the actual, observed value $g_s^2 \sim 15\hbar c$? Taking into account that $\mu_\pi \sim \mu_k$, we obtain that

$$\frac{g_s}{\xi_{min}} \sim 10^{39}.$$

Though one would surely expect that Nature had selected a very large value for the actual strong constant, very far from the minimum allowed, it is indeed surprising that such ratio reproduced precisely old Eddington's number!

As we have said at the Introduction, the relation of Eddington's number with large, adimensional ratios of physical constants has troubled physicists since long ago. Though this new *coincidence*, arising from an as yet unexpected physical domain, adds no further understanding to this puzzle, it certainly increases the mystery surrounding this apparent connection between the microscopic and the macroscopic worlds.

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Resumo

Apresentamos um panorama de alguns desenvolvimentos recentes no estudo da interação da gravitação com outros campos físicos, adotando o acoplamento não-mínimo entres estes campos e a gravitação como o cenário apropriado para a descrição dos fenômenos envolvidos. Discutimos algumas características de interêsse exibidas por acoplamentos não-mínimos, tais como a relação com espaços-tempo de Weyl integráveis, a geração de universos eternos, a aparição de uma constante cosmológica como resultado de uma transição entre estruturas geométricas de Weyl e Riemann, e a possível indução de gravidade repulsiva através de mecanismos de quebra expontánea de simetria. Em particular, examinamos um caso simples de interação forte entre partículas escalares (como no caso da bem conhecida reação elástica $\pi K \rightarrow \pi K$), acopladas não-minimalmente a um espaço curvo, que fornece um novo e curioso contexto para a aparição do número de Eddington (10^{-39}).